

On the notion of internal energy

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The notion of internal energy at a level appropriate for students of the Introductory Physics Course (IPC) is examined. The methodology proposed offers a general frame for analysing practically any physical system or process in a system of particles, from thermal phenomena and phase transitions to chemical and nuclear reactions, including processes involving elementary particles. The role of interactions among particles in a system is emphasized. The notion of statistical equilibrium of a system is discussed in this context. Finally, two *structure factors* are introduced to show the relation between the kinetic, interaction and particle energies of a system in maintaining stable structures.

In the Introductory Physics Course (IPC) students usually get a piecemeal or fragmentary view of energy. They first learn about the kinetic and potential energy of a particle. Later on, as the course progresses, they learn how the energy notion is used in a variety of physical situations. An amazing fact of physics is that, although the kinetic and potential energies of a single particle are basically a consequence of its equation of motion, the notion of energy has been extended to many complex situations, involving systems composed of many units or particles and for which the concept of *internal energy* is essential.

Until the time when these processes were well understood in terms of interactions between particles and their structures, and the role of the internal energy of a system was clarified, people invented several empirical forms of energy, such as chemical and thermal, to explain them, without relating these energies directly to interactions between particles. When the notion of internal

energy is well understood, students can see that these empirical energies are related to the particle energies previously introduced, making it clear that they are not truly 'new forms' of energy.

The internal energy of a system of particles

One important idea that should be stressed to students is that most physical systems we deal with or observe in nature, such as gases, liquids, solids and plasmas (as well as molecules, atoms, nuclei and hadrons), are composed of interacting 'particles', where by 'particles' we mean the recognizable units composing the system at any time, each of which may have an 'internal structure'. That is, the particles themselves may be systems and are not strictly geometrical points (an erroneous idea that often appears in the mechanics part of the IPC). Also it is important that students recognize that some 'particles' may be stable, but others may change in time as a result of processes occurring in the system (e.g. chemical and nuclear reactions) or related to the internal structure of the particles themselves (e.g. decay).

To understand the dynamics of a system of particles, it is necessary to introduce the notion of *internal kinetic energy* of the system, defined as the sum of the kinetic energies $E_k(i)$ of the particles, measured relative to the CM of the system[†]. Since *internal* forces can do work on the particles as a consequence of their relative motion, one may associate an 'interaction' energy with these. Thus the *internal interaction energy* of the system, associated with the

[†] Otherwise, the internal kinetic energy of a system would depend on the frame of reference used to measure the velocities of the particles; therefore, the internal kinetic energy would not be a property of the system, independent of its motion relative to the observer [1].

internal forces, is defined as the sum of the particles' 'interaction' energies, which we may assume can be expressed as interactions between pairs of particles, $E_{\text{int}}(ij)$ [†]. Also, since the particles in a system may have an internal structure that can be modified as a result of inelastic collisions and other interactions among the particles or by external agents, one must take into account the energy associated with that structure. This energy is called the *intrinsic particle energy*, $E_{\text{part}}(i)$. Therefore, the *internal energy* U of a system is defined as the sum of the three energies we have just considered, kinetic, interaction and intrinsic particle energies; that is,

$$U = \sum_i E_k(i) + \sum_{ij} E_{\text{int}}(ij) + \sum_i E_{\text{part}}(i) \quad (1a)$$

or

$$U = E(\text{kinetic}) + E(\text{interaction}) + E(\text{particle}) \quad (1b)$$

or more simply

$$U = E(K) + E(I) + E(P). \quad (1c)$$

The first term is the total internal kinetic energy, the second the total interaction energy and the third the total intrinsic particle energy of the system. Equation (1) contains all the elements needed to know and analyse what may happen in or to any physical system and this is why we like to call it an **equation for everything**^{††}.

Students should be made aware that, because the interaction energy $E_{\text{int}}(ij)$ corresponds to a coupling between particles, individual particle energies are not well defined and therefore the internal energy of a system is *not* the sum of individual particle

energies. Only the total internal energy of the system is well defined.

What is meant by 'particle' in a physical system?

It is most important that students clearly understand what is meant by a 'particle' in a physical system and the meaning and relevance of the third term in equation (1). The best way to help students grasp the meaning of the third term, $E_{\text{part}}(i)$, is by providing some concrete illustrations. Students easily see, for example, that for a system whose 'particles' are atoms, $E_{\text{part}}(i) = E(\text{electrons})$, which is the energy associated with the electronic motion, since very seldom do nuclei participate in atomic processes. For a system whose 'particles' are molecules, $E_{\text{part}}(i) = E(\text{rotation}) + E(\text{vibration}) + E(\text{electrons})$, where the first term corresponds to the rotation of the molecule as a whole, the second to the relative vibrational motion of the nuclei, and the third is associated with the electronic motion in the molecule. In the case of a system composed of nuclei, $E_{\text{part}}(i) = E(\text{collective}) + E(\text{nucleons})$, where the first term corresponds to the collective rotational and vibrational motions in deformed nuclei and the second to the individual motions of the nucleons. For a system of hadrons, one could write $E_{\text{part}}(i) = E(\text{quarks})$, which is the energy associated with the dynamics of the quarks composing the hadrons, although their dynamics is not yet well known.

The internal structure of particles may sometimes be ignored, as we do for planetary motion relative to the Sun or with atoms and molecules in gases at low temperatures and densities. In other cases, one may consolidate $E_{\text{part}}(i)$ into a mass energy term, mc^2 .

[†] In certain cases the interaction energy can be expressed as a potential energy depending on the relative coordinates of the particles, in which case we can talk of an *internal potential energy*. When that is not possible, other descriptions of the interaction might be necessary. This is particularly true at high energies, when the interaction is interpreted as an exchange of bosons, an idea students can visualize without having to mention the quantum theory of fields, and accept that the notion of potential energy is not applicable in this case.

^{††} Our term 'equation for everything' should not be confused with the more fashionable and colourful term 'theory of everything' that has become popular in quantum theory to refer to an ambitious theory for comprehensively describing the fundamental particles and their interactions. Our term refers to a general way of looking at the physical systems we find in nature in a way that clearly identifies the three energy components that determine their properties and behaviour. Also, the reader may recognize that equation (1) is a somewhat disguised form of the hamiltonian of the system, which plays a crucial role in the more advanced treatment of many particle systems. The second term is the interaction hamiltonian, fundamental to the description of most physical processes. In fact, one task of theoretical physicists is to identify a possible interaction hamiltonian for any system or process, under the assumption that once it is known, we can explain everything we need to know about a system. In turn, experimental physicists perform experiments to discover if an assumed interaction hamiltonian works or to provide clues for theoretical physicists to invent an interaction hamiltonian.

This is particularly useful when dealing with nuclear processes and with some processes involving fundamental particles. And, of course, some particles are assumed to be structureless, such as the leptons and quarks, and thus have only mass energy. Students must recognize that how one treats the 'particles' of a system depends on the relative value of the three terms in equation (1) and the type of process involved. For example, in polyatomic gases the rotational excitation energy is of the order of 0.01 eV, the vibrational excitation energy is about 0.5 eV and the electron excitation energy is of the order of 1 eV, while the average kinetic energy of the molecules is of the order of $10^{-4}T$ eV, where T is the temperature of the system expressed in kelvin. Therefore, depending on the temperature of the gas, one may have to consider different internal properties of the molecules; for example, below 100 K most molecules can be treated as structureless. In the case of a monatomic gas such as He, whose electronic excitation energy is of the order of 20 eV, the internal structure of the atoms may be ignored at temperatures below 10^5 K[†]. Examples such as these help students recognize that 'models' are flexible and must be adapted to each specific situation.

A system may be composed of several kinds of 'particles' P, P^I, P^{II}, \dots each kind with a different internal structure, as in a volume of air which is a mixture of O_2, N_2, H_2, \dots molecules, a mixture of several chemical compounds, or a mixture of nuclei in a star. For such cases the third term in equation (1) must be replaced by a sum of terms, one for each kind of particle; that is, $E(P) + E(P^I) + E(P^{II}) + \dots$. In some processes in these systems, particles may exchange intrinsic energies or even change their own nature, under the influence of the kinetic and interaction terms as well as external factors. This is what happens, for example, in an inelastic collision and in chemical and nuclear reactions. Particle accelerators are designed to optimize the transfer of kinetic energy to interaction and intrinsic particle energies, often creating new particles, so that the number and kind of particles do not always remain constant in such systems.

There may also be more than one kind of interaction, as may happen in a star where gravitational,

[†] Atoms of course, may be excited at lower temperatures using other mechanisms such as an electric discharge.

electromagnetic and nuclear interactions compete under certain circumstances. In most instances only one interaction is dominant and the others can be ignored. Then the dominant interaction determines the physics of the system. This is the case, for example, of molecules in a gas, for which only electric forces have to be considered, or in processes involving hadrons, for which the strong interaction is dominant, or for planets in a solar system, where the gravitational interaction is primary. A further point is that, in general, the dominant interactions are the ones that characterize the traditional macroscopic branches of physics. In this way students acquire a dynamical picture of physical systems, in opposition to the static and uninteresting view provided in the mechanics part of the IPC, where the internal structure of particles is ignored and systems do not change.

What we consider 'particles' in a system may themselves be systems composed of other 'particles', and so on. In this case the particle energy $E(P)$, when the 'particle' is part of a system, becomes an internal energy when the particle is treated as a new system. For example, atoms may be considered as the particles in a gas, but atoms are systems composed of electrons and a nucleus, and this is why we have a theory of atomic structure. In turn, nuclei can be considered as systems composed of nucleons, whose study is the subject of nuclear physics, and so on. This chain can be expressed as a ladder as follows[†]:

$$\begin{aligned}
 U &= E(K) + E(I) + E(P) \\
 &\quad \downarrow \\
 U^I &= E^I(K^I) + E^I(I^I) + E^I(P^I) \quad (2) \\
 &\quad \downarrow \\
 U^{II} &= E^{II}(K^{II}) + E^{II}(I^{II}) + E^{II}(P^{II}) \\
 &\quad \downarrow \\
 &\dots\dots
 \end{aligned}$$

Physicists spend a lot of effort trying to probe the inner structure of the 'particles' composing the systems they deal with. It is a goal of physicists to reach the last rung in the ladder of equation (2) and identify the ultimate components of matter, hoping

[†] See Chapter 20 of [1] for examples of high energy processes, where particles must be treated as systems as they appear at successive stages.

they will be structureless. In fact one may even tell students that physics has progressed by going further down the ladder, while trying to understand the physics of each rung, which in itself means doing a lot of good physics.

The conservation of energy

An important problem, at the very core of physics, is to relate the changes in the internal energy of a system, which generally consist in changes of the three terms in equation (1), to energy *exchanges* with other systems, such as the environment. For example, increasing the temperature (heating) of a system means increasing the kinetic energy of the particles (the first term in equation (1)), while, depending on how it is done, there may also result a change in the interaction term (expansion or compression) or in the particle term (excitation or dissociation). Modifying the configuration of the system (change of shape or volume), which results in a change in the interaction term, may also affect the other two terms (change in the temperature of the system and/or electronic excitations). Changing the third term (absorption of radiation, i.e. photons; ionization; dissociation) may also affect the first and second terms of equation (1), such as what happens in an oven where microwave or infrared radiation is the external source of energy. Of course, one can change all three terms at the same time by combining different external actions.

Designate the change in the internal energy of a system by ΔU and the energy exchanged with another system by E_{exch} . Then the conservation of energy requires that $\Delta U = E_{\text{exch}}$, a relation students have no difficulty accepting[†]. Next, one can explain that it is convenient to split E_{exch} into components, according to the procedures for exchanging energy. Thus we write (see ch 16 of [1] and Note 1 of [2])

$$\Delta U = Q + W + R \quad (3)$$

where Q is the heat absorbed by the system, W the work done on the system and R the radiation

[†] A critical point that puzzles many students is *why* energy is conserved at all. The main problem is the difficulty in showing IPC students how to relate energy conservation to the invariance of the laws of motion relative to a change in the origin of the time coordinate and, at best, can only be mentioned in passing.

(photons) absorbed by the particles of the system. The quantities Q , W and R are positive or negative, depending on whether the system gains or loses energy. Equation (3) is the usual expression for the first law of thermodynamics, except for the radiation term R , which is unfortunately often omitted in spite of its importance. (Actually, conservation of energy is so important in all of physics that equation (3) should be called the first law of physics.)

Using equation (1), one may write equation (3) as

$$\Delta E(K) + \Delta E(I) + \Delta E(P) = Q + W + R \quad (4)$$

which expresses a balance between the changes in the three components of the internal energy of a system and the energy exchanges with the surroundings. The advantage of equation (4) is that it explicitly shows how the conservation of energy is related to the three terms in equation (1), which is not apparent in equation (3). Of course, there is not a one-to-one correlation between the terms on the two sides of equation (4). However, Q usually is associated with the change in the kinetic energy of the particles (primarily through collisions with particles outside the system), W is associated with a change in the value of the interaction term (through a change of shape or volume of the system, in the orientation or configuration of the particles, etc) and R is associated with a change in the particle energy term (by absorption or emission of photons) but radiation may also affect the kinetic and interaction energies as well (as, for example, in a microwave oven).

When the 'system' is just one particle so that there is no interaction energy $E(I)$ and the notion of heat Q does not apply, equation (4) reduces to

$$\Delta E(K) + \Delta E(P) = W + R \quad (5)$$

which is the logical generalization of the standard work-kinetic energy theorem, $\Delta E(K) = W$, for the dynamics of a single particle. It is most unfortunate that in the traditional treatment of the dynamics of a single particle this generalization is not considered, in spite of its importance. For example, in the special case of the absorption and/or emission of radiation by a nucleus, atom or molecule, no work is done and equation (5) becomes

$$\Delta E(K) + \Delta E(P) = R \quad (6)$$

where $\Delta E(K)$ refers to the recoil energy $\frac{1}{2}mv_{\text{recoil}}^2$ of the nucleus, atom or molecule, $\Delta E(P)$ corresponds to the energy difference $E_f - E_i$ of the levels between which the transition occurs, while $R = \pm hf$ is the energy of the photon absorbed or emitted and f is the frequency of the radiation. Written in the conventional form, we have $\frac{1}{2}mv_{\text{recoil}}^2 + (E_f - E_i) = \pm hf$. This is a refinement of the Bohr equation $\Delta E = \pm hf$, where the recoil energy is ignored. In the operation of a laser, where the 'system' is the atoms of the material, one may ignore $\Delta E(I)$, and Q and W do not apply. Then we again write $\Delta E(K) + \Delta E(P) = R$, where $E(K)$ and $E(P)$ now refer to the atoms or molecules of the material.

Photoelectric emission by metals is a further application of equation (4). The system is composed of conduction electrons and ions in the emitter, but the radiation affects only the electrons. Then the particle energy $E(P)$ may be ignored, since we are dealing with electrons and Q and W are again not applicable, so that equation (4) becomes $\Delta E(K) + \Delta E(I) = R$. Written in the conventional form, this relation is $\frac{1}{2}mv^2 + e\phi = hf$, where ϕ is the work function of the emitter and $\frac{1}{2}mv^2$ is the kinetic energy of the electrons.

For an *isolated* system, that is, a system not subject to any external action, the three terms on the right-hand side of equation (3) are zero, so that $\Delta U = 0$ and the internal energy of the system remains constant, $U = \text{constant}$. Then equation (4) becomes

$$\Delta E(K) + \Delta E(I) + \Delta E(P) = 0 \quad (7)$$

which expresses energy conservation in an isolated system. This equation is more informative than the traditional expression $\Delta U = 0$ and clearly displays the restrictions on the possible processes that may occur in an isolated system, since one energy component cannot change without appropriate changes in the others. When energy conservation is expressed by equation (4) or (7), it acquires a new dimension in students' eyes by allowing them to visualize, in a more general way, what happens in all processes involving physical systems.

Illustrations of how to use the equation for everything

It is important to emphasize to students that whatever happens or may happen to a system is the

result of the interplay among the three energy components that appear in equation (1), as well as the actions of external agents that may also change those components as expressed by equation (4) or, if the system is isolated, by the internal energy exchanges as expressed by equation (7). One may point out to students that the real and interesting physics is contained in the interaction energy $E(I)$ and the intrinsic particle energy $E(P)$ terms. Most research in physics concentrates on these two terms, which are essential to explain those processes that may occur or what structures may appear in a physical system. For example, chemistry, nuclear physics and high energy physics deal with processes mainly involving the second and third terms in equation (1), although the first term is important in many processes and reactions.

Without interactions, no structures can appear in a system, and when particles have no internal structure the processes that may occur are usually not very interesting. Also structures and processes in a system provide hints about interactions and the internal structure of the particles. A good deal of what experimental physicists do is to induce processes in the laboratory and analyse existing structures so that theoretical physicists can test their models or guess the kind of interactions and the internal properties of the particles of the system, although students do not always realize this kind of relation. However, what structures exist or what processes can occur in a system depends critically on the relative values of the three terms appearing in equation (1), and this allows many diverse physical situations to be explained. In some instances the intrinsic particle energy can be ignored and the particles can be considered in a first approximation as structureless. In other cases, the interaction energy is negligible in a first approximation. In other situations only the first term has to be considered, as happens with 'ideal' gases. Of course, in most cases, as with 'real' gases and plasmas, one may have to consider the three terms of equation (1).

The most common, and elementary, application of equations (1) and (4) is in the analysis of the processes usually discussed in the thermodynamics part of the IPC. Students easily see that by supplying energy as heat Q to a liquid or gas at constant volume, $E(K)$ increases with no changes in

$E(I)$ or $E(P)$. However, large increases in $E(K)$ may result in changes in $E(P)$, with consequent possible changes in $E(I)$, as happens in inelastic collisions, phase transitions or chemical reactions. In the case of some solids, when the increase in $E(K)$ is very large, the change in $E(P)$ may also be so large that radiation is emitted (negative R), as in the filament of an electric bulb or a 'red hot' iron bar. Supplying energy as compression work (positive W) changes $E(I)$ and $E(K)$; but when the compression work is very large, as in the magnetic compression of a plasma in a fusion reactor, $E(P)$ may also change. Isothermal compression work (W positive) requires extraction of energy as heat (negative Q), with or without changes in $E(K)$, depending on the system, but with changes in $E(I)$ and perhaps also in $E(P)$. In most of the processes traditionally considered in classical thermodynamics one may ignore the energy $E(P)$ because thermodynamics deals only with relatively low energy processes during which $E(P)$ stays constant.

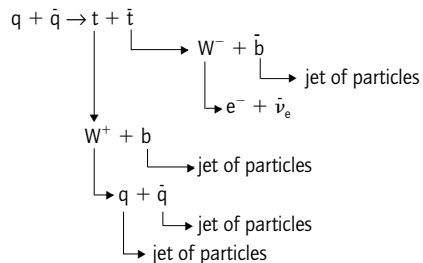
A simple and instructive example is the case of a metallic rod initially at a uniform temperature. If one end of the rod is placed in a hot furnace, the vibrational kinetic energy of the atoms increases appreciably ($\Delta E(K) > 0$) as energy, Q , is absorbed at the heated end. Two effects then occur. As a result of *inelastic collisions* among the vibrating atoms at the hot end, the electrons in those atoms are excited (i.e., the intrinsic particle energy at the hot end increases, $\Delta E(P) > 0$) and the rod begins to glow (emit radiation R). Also, as a result of *elastic collisions*, the kinetic energy at the hot end is gradually transferred along the rod toward the cooler end (thermal conduction). A further increase in the kinetic energy of the atoms at the hot end may result in the modification of their regular arrangement in the rod because their kinetic energy overcomes the interaction between them (melting and annealing, $\Delta E(I) > 0$).

An example of an isolated system is the elastic and inelastic collisions of two particles. The system is initially composed of two particles and is isolated because the only forces are those due to the interaction between the colliding particles, so that equation (7) applies. During the collision the three terms in equation (7) may change. However, when equation (7) is applied to the initial and final states of the system, the interaction energy is negligible and can therefore be ignored (because, in both

states, the particles are very distant); that is, $\Delta E(I) = 0$. In elastic collisions the particles suffer no change in their internal structure; i.e., $\Delta E(P) = 0$. Therefore, equation (7) reduces to $\Delta E(K) = 0$ and the initial and final states have the same total kinetic energy. In an inelastic collision, there is a change in the total kinetic energy of the particles and the internal structure also changes so that equation (7) becomes $\Delta E(K) + \Delta E(P) = 0$, which is conventionally written as $\Delta E(K) = Q$, with $\Delta E(P) = -Q$. For the case where the number of particles in the final state is different from the initial state, the analysis is the same. A similar analysis can be extended to chemical and nuclear reactions, where the initial and final 'particles' are different.

A collision experiment that interests students and shows the interplay among the three terms in equation (7) in a more elaborate way for an isolated system is the system composed of a pulse of particles in an incoming beam and the nuclei in the target of a particle accelerator, or that composed of the two colliding pulses or bunches of particles in a collider. The processes that occur, which generally result in the production of new particles, depend on the kinetic energy in the CM of the colliding particles, on their interaction at collision and on the intrinsic particle energy of the projectiles and the target. The system can be considered as isolated because no external forces are applied, only the interactions among the particles involved. The discovery of the top quark[†] is a beautiful illustration

[†]See [3, 4]. In this experiment the initial system is composed of a proton and an antiproton; that is $p(uud) + \bar{p}(\bar{u}\bar{u}\bar{d})$, but at the high collision energy of the Tevatron, we may consider the initial system as composed of three quarks and three antiquarks and the final composition, after going through steps of the ladder portrayed by equation (2), is composed of leptons and several jets of other particles, some containing bottom (or b) quarks. The top quarks, t and \bar{t} , are not observed directly, but are inferred from the analysis of the jets of particles and leptons, which constitute the final system. The processes assumed to occur are as follows:



of the applicability of equation (7). Thus we see that equations (4) and (7) provide deeper insight into the analysis of collisions and reactions than that usually presented in the standard IPC.

Statistical equilibrium

An important notion in statistical physics and thermodynamics is that of *statistical equilibrium*. Usually students are told that in a system in statistical (or thermal) equilibrium its macroscopic parameters (temperature, pressure, density, etc) are well defined and remain constant. However, it is more informative to say that, in a system in statistical equilibrium, no *net* internal energy exchanges occur and therefore the three component energies of equation (1) remain statistically constant (over time), experiencing very small fluctuations; i.e., the time averages $\langle E(K) \rangle_{av}$, $\langle E(I) \rangle_{av}$ and $\langle E(P) \rangle_{av}$ are constant. This is equivalent to having the average of each of the three terms in equation (7) be individually zero; that is,

$$\langle \Delta E(K) \rangle_{av} = 0, \langle \Delta E(I) \rangle_{av} = 0, \langle \Delta E(P) \rangle_{av} = 0 \quad (8)$$

where each of the terms in equation (8) refers to the time average of the fluctuations of each energy term in equation (1). This helps students understand that statistical equilibrium is dynamic. Any structures that may have appeared in the system remain essentially unchanged and nothing relevant happens in the system until it is disturbed from the equilibrium state by an external agent[†]. Electromagnetic radiation (i.e. photons) trapped inside a cavity and the atoms in the walls of the cavity is one of the most important examples of systems in equilibrium.

One illustration of the interplay between the three terms of equation (1) in achieving equilibrium is the density 'anomaly' of water. As is well known, solid water (ice) can be in several phases and liquid water reaches its maximum density at 4 °C. In this case the 'particles' are water molecules and their equilibrium arrangement is determined by some

[†]Examples include a substance in one or several phases coexisting in equilibrium (a solid in several phases, a solid in equilibrium with its liquid, a liquid in equilibrium with its vapour, a substance at a triple point, etc), in which case there may be a different value of $E(I)$ and $E(P)$ for each phase.

combination of their kinetic energy, their interaction energy with neighbours (hydrogen bonds) and the bending of the molecules, which corresponds to $E(P)$ [5].

Initially a system may be in a non-equilibrium state, or may be thrown out of equilibrium by some external action, such as subjecting it to intense electromagnetic radiation (photon flux) or by suddenly changing some macroscopic parameter, as in the free expansion of a gas (Joule's experiment). Then several processes may occur in the system simultaneously. Students should be made to recognize, as a well accepted assumption in physics, that an isolated system not in equilibrium will eventually evolve, as a result of an internal adjustment of the three terms in equation (1), to an equilibrium state. In the process, the internal energy must remain constant (i.e. $\Delta U = 0$) and equation (7) applies as the system evolves toward equilibrium. At equilibrium the three energy terms of equation (1) reach their 'optimum' values compatible with the system's composition and internal energy and equation (8) applies. This is the actual meaning of the second law of thermodynamics. In the terminology of statistical mechanics, an isolated system evolves toward the most probable, or optimum, configuration, which is also the state of maximum entropy, at which point stable structures may appear in the system. By no means does the system evolve toward a more disordered state, as is often incorrectly stated[†]. One fundamental task of statistical mechanics is to obtain the equilibrium configuration of a system. This is a rather difficult problem for the case of a system of strongly interacting particles but it is relatively easy for systems where the single particle energy approximation is valid (that is, when $\sum_{ij} E_{int}(ij)$ can be replaced by $\sum_i E_{int, av}(i)$, so that $U = \sum_i E_i$ where $E_i = E_k(i) + E_{int, av}(i) + E_{part}(i)$ because one can calculate the distribution of particles among

[†]With regard to this point, students should be made aware that there may be several, very similar, 'optimum' *microscopic* arrangements or configurations, all compatible with equation (8), where the system is in equilibrium with the same *macroscopic* parameters. The system may fluctuate from one microscopic configuration to another, but that does not imply that the system is disordered. For a more elaborate discussion see [2].

available energy states using probabilistic considerations.

As we all know, the most interesting physics occurs in systems that are not in equilibrium; that is, systems where the three components of equation (1) or (4) are changing continuously. Stars are a good example of such systems. Their evolution responds to the interplay between the three terms in equation (1), with the second corresponding to gravitational, electromagnetic and nuclear interactions; which interaction is dominant depends on the level of evolution of the star (see section 41.9 on p 1091 of [1]; also refer to Note 11.4 on p 278 of [1] for the role of gravitation). The evolution of the universe is perhaps the most dramatic example of the interplay between the three terms of equation (1). As the universe cooled down from a very hot beginning (decrease in the kinetic energy term in equation (1)) as a result of its expansion, new structures began to appear (hadrons, nuclei, atoms, molecules, gases, liquids, solids, and eventually living systems) depending on the dominant interaction, the kinetic energy term at each phase of the expansion and local conditions.

Students should realize that, despite the fact that nothing happens in a system in equilibrium unless acted upon, they are nonetheless interesting. In fact, most of classical thermodynamics deals with systems in equilibrium or slightly disturbed from equilibrium (reversible processes) so that the macroscopic parameters remain well defined.

The structure factor

The relation between kinetic and interaction energies in the formation of structures in a system can be expressed conveniently by the *interaction structure factor* F , defined as the ratio of the average interaction energy between pairs of particles and the average kinetic energy of the particles of the system; that is,

$$F = \frac{\langle \Delta E_{\text{int}}(ij) \rangle_{\text{av}}}{\langle E_k(i) \rangle_{\text{av}}} \quad (9)$$

Systems for which $F = 0$ (i.e., the interaction energy between particles is zero) are called 'ideal' and show no structure. No physical system is truly ideal, but some systems, like dilute gases and low density plasmas, can be treated as ideal in a first approximation. As F increases (i.e., as the interaction energy becomes larger relative to the average kinetic

energy), systems begin to show some degree of correlation among the motion of their particles, as in liquids, dense plasmas and some massive nuclei. When F becomes very large, the motion of the particles is strongly correlated and stable structures may appear. This is the case of electrons in atoms, nucleons in nuclei, atoms or molecules in solids and quarks in hadrons.

One may also introduce a *particle structure factor*, F' , defined by

$$F' = \frac{\langle \Delta E_{\text{part}}(i) \rangle_{\text{min}}}{\langle E_k(i) \rangle_{\text{av}}} \quad (10)$$

where $\langle \Delta E_{\text{part}}(i) \rangle_{\text{min}}$ is the minimum energy required to modify the internal structure of the particles of the system (excitation, dissociation, ionization, etc). This factor serves to indicate to what extent the internal structure of the particles is stable and can therefore sometimes be ignored. The larger F' is relative to one, the more stable are the particles; i.e. the kinetic energy is not sufficient to alter the particles. When F' is small compared to one, the particles are easily excited or may be highly unstable.

Both structure factors, F and F' , are temperature dependent, since changing the temperature of the system is equivalent to changing the kinetic energy of the particles. In general F and F' decrease as the temperature increases and some structures disappear or some changes occur among the particles.

Conclusion

We hope this brief and elementary discussion of the internal energy of systems of particles serves to point out the importance of equation (1) and the role of interactions and internal particle energies in physical processes. We believe the methodology presented helps students obtain an adequate holistic view of energy so they are able to analyse and explain a multitude of physical situations and processes using equations (4) and (7). In other words, an understanding of equation (1) should provide students with a unified conceptual view about how the physical world functions at all levels. As mentioned previously, the detailed physics in each case is incorporated into the interaction and particle terms of equation (1). This applies whether one refers to gravitation, electromagnetic or nuclear processes, to the standard model of fundamental

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particles, or, at a more elementary level, to thermal and chemical processes. The specific treatment may vary according to the particular situation or process and whether one uses classical or quantum methods, but the overall conceptual frame remains the same. Recognizing the universal value of equation (1), the equation for everything, and fully understanding how to use it, is, we believe, an important methodology that contributes toward students acquiring a conceptual unification of physics. Unfortunately, the structure of most IPCs is not conducive to this goal and we recognize that, given the current ambitious scope of the IPC, it may not be possible to provide such a conceptual unification unless profound adjustments are made in the content and organization of the course.

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