Heavy Quarkonium and QCD Nonrelativistic Effective Field Theories

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(Workshop: Charmonium Spectroscopy
Genova, 7-8 June 2001)

Quarkonium is a key issue for many experiments

- Search of gluonic degrees of freedom: hybrids, glueballs (BNL, BES, CEBAF, ...)
- Determination of CKM matrix elements (B factories)
- Physics at next linear collider (TESLA,CLIC)
- Physics of quarkonium production (Tevatron)
- Quark-gluon plasma (RHIC, ...)

More on the theoretical side it is a very interesting system to probe

- Confinement and to answer questions as How do theories with a nontrivial vacuum structure work? Relation to strings and SUSY.
- the Low/high energy parameters consistency.
- the Mixing of different energy scales in the bound state
  $\Rightarrow$ effective theories

A) Short History

Phenomenological potential models

Non-potential effects

Wilson loops and potentials

B) A hierarchy of scales, a hierarchy of EFT

The existence of a hierarchy of energy scales in quarkonium systems allows the construction of EFT with less and less degrees of freedom. These lead to a field theory derived quantum mechanical description of those systems. We call pNRQCD the corresponding EFT.

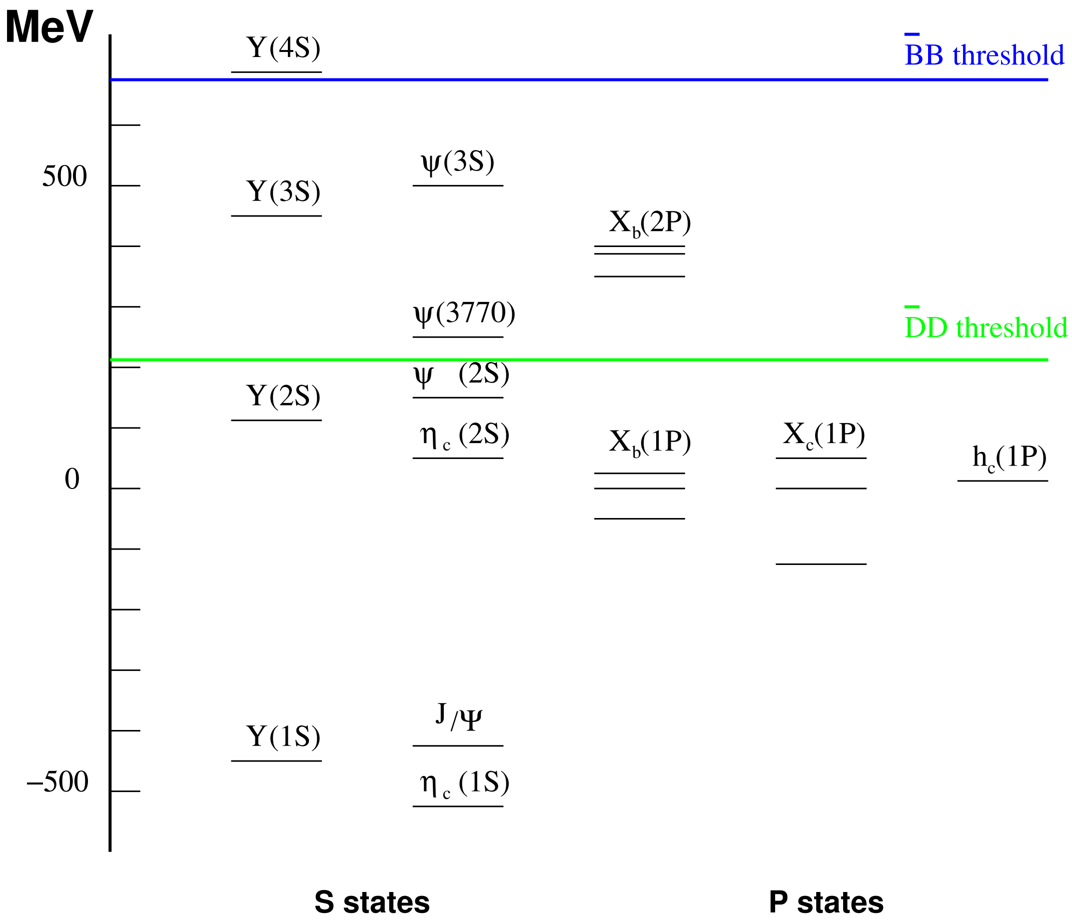
C) Short-range quarkonium ($t\bar{t}$, lowest levels of $b\bar{b}$, $c\bar{c}$, hybrids, ...)

Results of pNRQCD calculated at order $1/m^2$ and at the next-to-leading order in the multipole expansion.

D) Long-range quarkonium (excited $c\bar{c}$ and $b\bar{b}$ states)

Results of pNRQCD calculated at order $1/m^2$ and at the leading order in the ultrasoft expansion.

§ *Quarkonium: the masses*



(Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$)

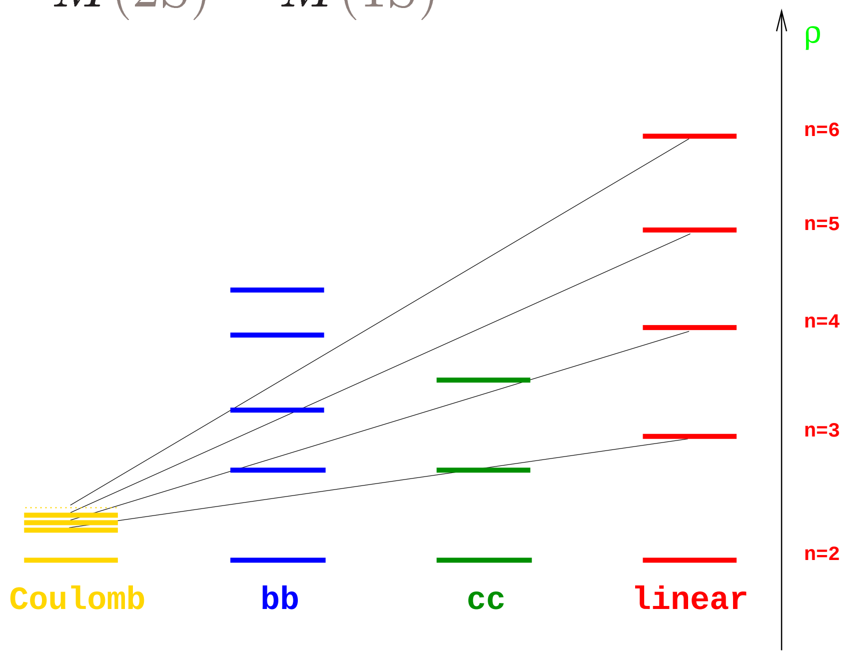
$m_b \simeq 5$ GeV, $m_c \simeq 1.6$ GeV $\Rightarrow \Delta E \ll m$

- The mass scale is perturbative.
- The system is non-relativistic:

$r \sim 1/mv,\ E \sim mv^2 \qquad v \ll 1$

Quarkonium phenomenology

$$\rho(nS) \equiv \frac{M(nS) - M(1S)}{M(2S) - M(1S)}$$ gives the level spacing.



This suggested a whole zoo of QCD potentials all essentially of the type

$$V = -\frac{\kappa}{r} + \sigma r + C \quad \textit{(Cornell potential)}$$

The parameters are fitted to the data:

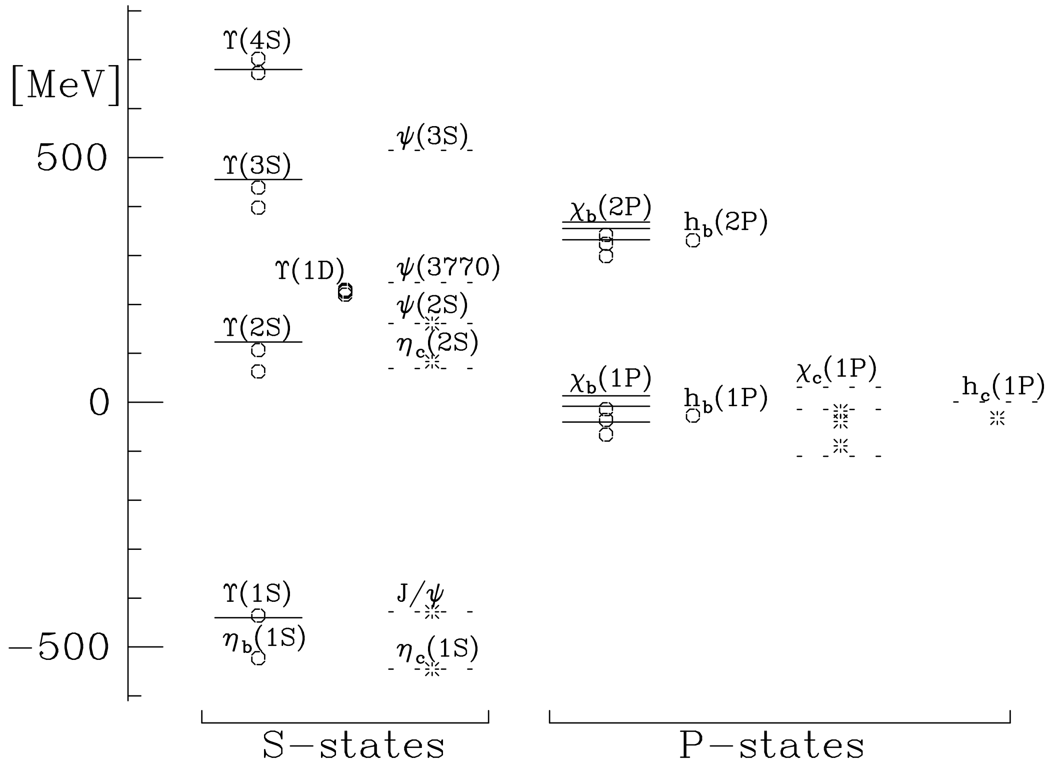
$\sigma \simeq \Delta E / \Delta r \simeq 1\mathrm{GeV/fm}$ *(string tension)*,

$\kappa \simeq 4\alpha_s/3$ *at some scale.*

The success of the quark models signals that

- quarkonium can be described by a potential;
- perturbative QCD cannot explain it.

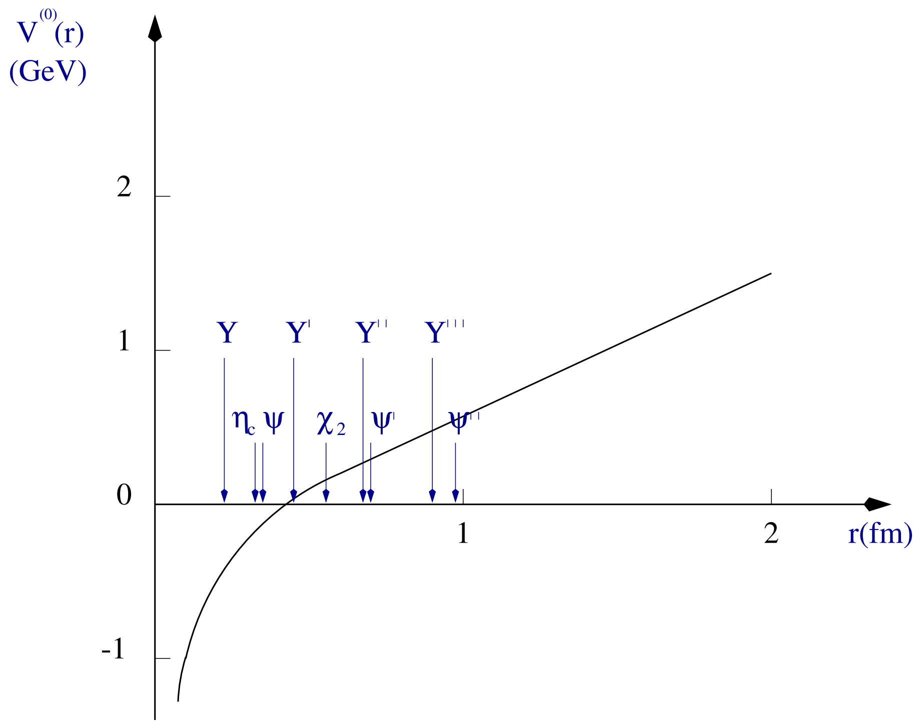
E.g. Eichten and Quigg 94



But in spite of their success the limitations of potential models are clear:

- The theoretical understanding is poor.
- There is no room for systematic improvement.
- There is no clear relation between the parameters of the potential models and the fundamental parameters of QCD.

§ *Quarkonium: the radius*



$r_{q\bar{q}}$ vs Cornell potential Godfrey Isgur 85

For ground state quarkonia $r \ll 1/\Lambda_{\mathrm{QCD}}$

- The scale $1/r$ is perturbative.

Most quarkonia have $r \sim 1/\Lambda_{\mathrm{QCD}}$

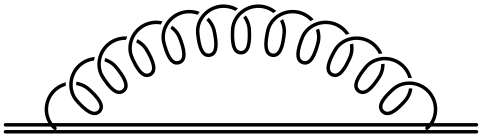
- The binding is nonperturbative.

Condensates and Quarkonium

The leading contribution coming of the gluon local condensate is to a Coulombic n, l state is (Voloshin 79 and Leutwyler 81)

$$\boxed{\delta E_{nl} = m\frac{\epsilon_{nl} n^6 \pi \langle \alpha_s F^2(0)\rangle}{(mC_F\alpha_s)^4}}$$

- It is of non potential type.
  Like



$\sim \alpha_s^5 \ln \alpha_s$

- It grows like $\sim n^6$ and is out of control already at $n > 1$.

! Warning. Local condensates arise from expanding around short distances:

$$\langle F(x)F(0)\rangle \sim \langle F(0)F(0)\rangle + O(x^2)$$

Is x always small ?

§ *Potentials from QCD*

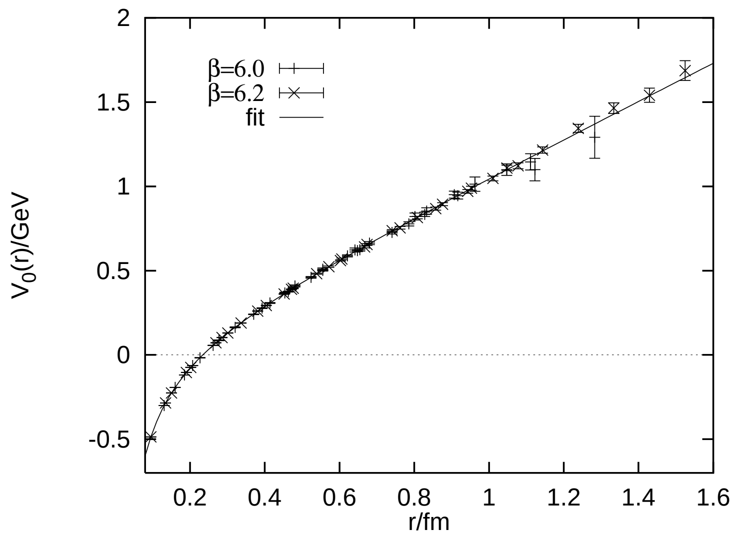
* Wilson 74

The Wilson loop as order parameter of confinement:

$$\boxed{\langle W(\Gamma)\rangle = \int \mathcal{D}A\, e^{-S_{\rm YM}}\, \mathrm{P}\, \mathrm{Tr}\, \exp\{ig \oint_\Gamma dz^\mu A_\mu\}}$$

* Creutz et al. 82

First lattice measure (Creutz ratio)



Bali et al. 97

* Susskind 77 Fischler 77 Brown Weisberger 79

The Wilson loop as energy of static sources:

$$\langle \mathrm{vac}|\chi^\dagger(x_2)\phi(x_2,x_1)\psi(x_1)\psi^\dagger(y_1)\phi(y_1,y_2)\chi(y_2)|\mathrm{vac}\rangle$$

$$= \delta(\mathbf{x}_1 - \mathbf{y}_1)\,\delta(\mathbf{x}_2 - \mathbf{y}_2)\left(\frac{1+\gamma_0}{2}\right)^{(1)}\left(\frac{1-\gamma_0}{2}\right)^{(2)}$$

$$\times e^{-i(m_1+m_2)T}\langle W(\boxed{\,_{r}\quad ^{T}\quad\,})\rangle$$

$\rightarrow$ *(Feynman-Kac)* $V_0 = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(\boxed{\,_{r}\quad ^{T}\quad\,})\rangle$

V_0 is the static energy of the sources

$\stackrel{?}{=}$ static potential

From the Foldy–Wouthuysen reduction of the QCD Lagrangian one may express

* Eichten Feinberg 81 Gromes 84

the spin dependent potentials up to $1/m^2$,

* Brambilla, Prosperi et al.. 88 90

the momentum dependent potentials up to $1/m^2$,

in terms of Wilson loops and operator insertions.

* Campostrini 85 ...

Several lattice simulations.

Various Problems:

1) Mismatch with perturbation theory.
In particular what about $\ln(m)$?

2) Potential vs non-potential.

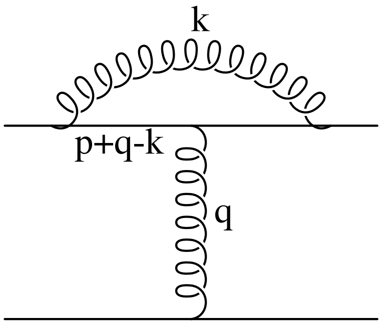
3) What about $1/m$ and $1/m^2$ momentum and spin independent terms?

There are several assumptions out of control, mainly connected with the different scales of the problem: a) is α_s small or big, b) can condensates be locally expanded, ... and different effects

- Different physical scales in the bound state
- Perturbative (Hard scale)/ Nonperturbative (Low scale) effects
- Potential and Nonpotential effects
- Local and Nonlocal condensates

Effective Field Theories give a systematic solution

- Technical problem of disentangling the scales which get mixed up in the bound state.



$$q^0 \sim m\,v^2$$
$$\vec{q} \sim m\,v$$
$$k^0 \sim \vec{k}$$

- More fundamental problem of disentangling perturbative from nonperturbative effects.

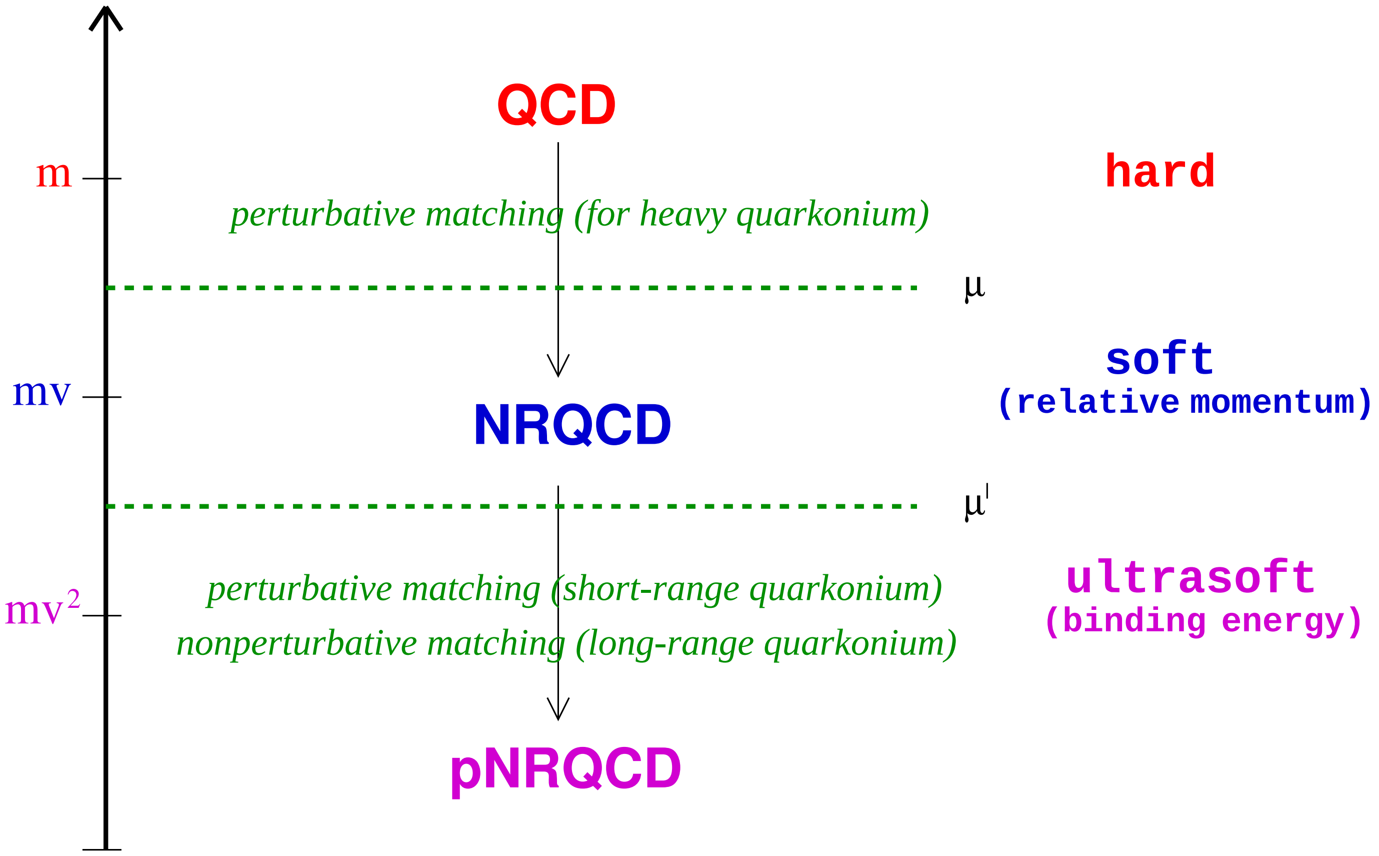
Also in a lattice calculation it is difficult to simulate quantities that involves two very different scales $Q \gg q$, since it should hold

$$L^{-1} \ll q \ll Q \ll a^{-1}$$

$a =$ lattice spacing, $L =$ lattice size

The idea is to substitute scale by scale the initial more complicate theory with a simpler but equivalent effective theory.

As a non relativistic bound state, quarkonium is characterized by three energy scales, hierarchically ordered by the quark velocity.



In QCD another scale is relevant:

$$\Lambda_{\rm QCD} \sim \text{scale of nonpert. physics}$$

A way to disentangle rigorously the scales is to substitute QCD scale by scale with simpler but equivalent effective theories.

Note that in an actual system other scales can be present. E.g. Γ_t in $t\bar{t}$ scattering.

Perturbatively $v_{pert} \sim 4/3\, \alpha_s/n$.

A (rough) classification of the bound states which can be treated perturbatively up to a scale mv^n can be made assuming as a criterion $\dfrac{mv^n_{pert}}{\Lambda_{\rm QCD}} \gg 1$.

μ	bound states
m	$\psi(2S)$, $\chi_{c0}(1P)$, ...
	$\Upsilon(2S)$, $\chi_{b0}(1P)$, ...
	long-range hybrids, ...
mv	$\eta_c(1S)$, $J/\psi(1S)$, ...
	short-range hybrids, ...
mv^2	$t\bar{t}$, $\Upsilon(1S)$, $B_c(?)$, ...
$< mv^2$	QED bound states

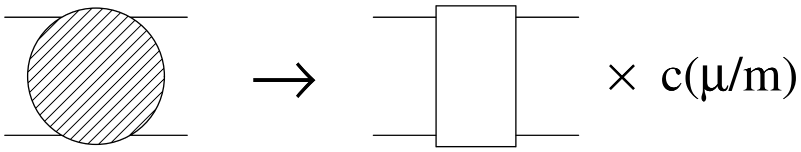
E.g.

	m	mv	mv^2
$b\bar{b}(1S)$	4.7 GeV	1.5 GeV	0.5 GeV
$c\bar{c}(1S)$	1.3 GeV	0.7 GeV	0.4 GeV

But only a consistency check can clearly state the applicability of the perturbative calculation.

§ *NRQCD (up to order $1/m^2$)*

The hard scale m is integrated out by comparing on shell amplitudes in QCD and NRQCD.



- The matching scale is $m > \mu > mv$.
- Since $\mu \gg \Lambda_{\rm QCD}$ the matching is perturbative.

$$L = \psi^\dagger \Bigg(iD_0 + c_2 \frac{\mathbf{D}^2}{2m} + c_4 \frac{\mathbf{D}^4}{8m^3} + c_F g \frac{\mathbf{S}\cdot\mathbf{B}}{m} + c_D g \frac{\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D}}{8m^2} + i c_S g \frac{\mathbf{S}\cdot(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D})}{4m^2} \Bigg) \psi$$

$$+ \text{ antiquark terms} \qquad + \text{ light quarks}$$

$$-\frac{b_1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \frac{b_2}{m^2} F^a_{\mu\nu} D^2 F^{a\,\mu\nu} + \frac{b_3}{m^2}\, g\, f_{abc} F^a_{\mu\nu} F^b_{\mu\alpha} F^c_{\nu\alpha}$$

$$+\frac{d_1}{m^2} \psi^\dagger \psi \chi^\dagger \chi + \frac{d_2}{m^2} \psi^\dagger \mathbf{S} \psi \chi^\dagger \mathbf{S} \chi$$
$$+\frac{d_3}{m^2} \psi^\dagger T^a \psi \chi^\dagger T^a \chi + \frac{d_4}{m^2} \psi^\dagger T^a \mathbf{S} \psi \chi^\dagger T^a \mathbf{S} \chi$$

Caswell and Lepage 86, Manohar 97, ...

§ *NRQCD matching coefficients*

If $\mu \gg \Lambda_{\rm QCD}$ then the matching is perturbative.

- b_1 fixes the new β-function ($N_f \to N_f - 2$)
  $b_2, \ldots d_1, \ldots$ are $\sim \alpha_{\rm s}$ and known at 1-loop
  Manohar 97, Pineda and Soto 98

- Reparameterization invariance fixes
  $c_2 = c_4 = 1$ $\qquad\qquad c_S = 2c_F - 1$

- Amoros et al. 97

$$c_F = \left(\frac{\alpha_{\rm s}(m)}{\alpha_{\rm s}(\mu)}\right)^{\gamma_0/2\beta_0} \left[1 + \frac{\alpha_{\rm s}(m)}{2\pi}(C_A + C_F) + \frac{\alpha_{\rm s}(m) - \alpha_{\rm s}(\mu)}{4\pi} \frac{\gamma_1\beta_0 - \gamma_0\beta_1}{2\beta_0^2}\right]$$

  γ_0, γ_1 anomalous dimensions.

- Bauer and Manohar 97

$$c_D = \left(\frac{20}{13} + \frac{32}{13}\frac{C_F}{C_A}\right)\left[1 - \left(\frac{\alpha_{\rm s}(m)}{\alpha_{\rm s}(\mu)}\right)^{13C_A/6\beta_0}\right] + \left(\frac{\alpha_{\rm s}(m)}{\alpha_{\rm s}(\mu)}\right)^{2C_A/\beta_0}$$

§ *pNRQCD for $mv \gg \Lambda_{QCD}$: degrees of freedom*

a) The matching.

We assume $mv \gg \mu' \gg mv^2,\ \Lambda_{QCD}$.

- The matching is perturbative.
- At the scale μ' we have quarks and gluons.

b) Power counting.

$\mathbf{R} \equiv$ centre-of-mass and $\mathbf{r} =$ relative coordinate of the bound state. Since $\mathbf{r} \sim 1/mv$, the Lagrangian will be an expansion in

$$\frac{p}{m}, \quad \frac{1}{rm}, \quad O(\Lambda_{QCD}, mv^2) \times \underset{\uparrow\ \text{multipole expansion}}{r} \quad \ll 1.$$

The non-analytic behaviour in r is encoded in the potential-like matching coefficients.

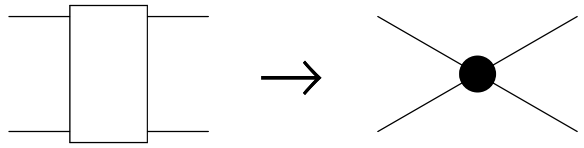
c) Degrees of freedom.

The effective degrees of freedom are

- Q-$\bar{Q}$ states, with energy $O(\Lambda_{QCD}, mv^2)$ and momentum $\lesssim mv$.
  $\Rightarrow$ i) singlet $\mathrm{S}(\mathbf{R}, \mathbf{r}, t)$ ii) octet $\mathrm{O}(\mathbf{R}, \mathbf{r}, t)$.
- Gluons with energy and momentum of $O(\Lambda_{QCD}, mv^2) \Rightarrow A_\mu(\mathbf{R}, t)$.

§ *pNRQCD for $mv \gg \Lambda_{\rm QCD}$: the Lagrangian*

The soft scale mv is integrated out by comparing off shell amplitudes.



$\times$ V(μ'/q)

NRQCD pNRQCD

At the next-to-leading order (multipole expansion)

$$\boxed{\begin{aligned} L^{(1)}{}_{\rm pNRQCD} = {\rm Tr}\Bigg\{ {\rm S}^\dagger \left(i\partial_0 - \frac{{\bf p}^2}{m} - \sum_n \frac{V_s^{(n)}}{m^n} \right) {\rm S} \\ + {\rm O}^\dagger \left(iD_0 - \frac{{\bf p}^2}{m} - \sum_n \frac{V_o^{(n)}}{m^n} \right) {\rm O} \Bigg\} \\ + gV_A {\rm Tr}\left\{ {\rm O}^\dagger {\bf r}\cdot{\bf E}\,{\rm S} + {\rm S}^\dagger {\bf r}\cdot{\bf E}\,{\rm O} \right\} \\ + g\frac{V_B}{2} {\rm Tr}\left\{ {\rm O}^\dagger {\bf r}\cdot{\bf E}\,{\rm O} + {\rm O}^\dagger {\rm O}{\bf r}\cdot{\bf E} \right\} - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\,a} \end{aligned}}$$

- The matching potentials $V(\mu', {\bf r}, {\bf p}, {\bf S_1}, {\bf S_2})$ are evaluated at a scale $mv \gg \mu' \gg mv^2$, $\Lambda_{\rm QCD}$.
- All the gauge fields are evaluated in R and t.
  $\Rightarrow$ power counting: $r \sim \dfrac{1}{mv}$ and $t, R \sim \dfrac{1}{mv^2}$.

Pineda Soto 97, Brambilla Pineda Soto Vairo 99

§ *pNRQCD for $mv \gg \Lambda_{\rm QCD}$: Feynman rules*

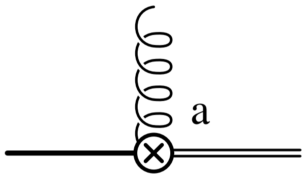
At order $(1/m)^0$ the pNRQCD Feynman rules are

$$= \theta(T) e^{-iV_s T}$$

`singlet propagator`

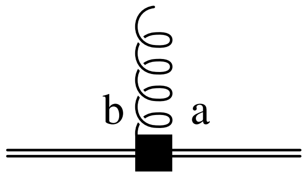
$$= \theta(T) \left(e^{-iV_o T} e^{-ig \int_{-T/2}^{T/2} dt \, A_0^{\rm adj}} \right)_{ab}$$

`octet propagator`



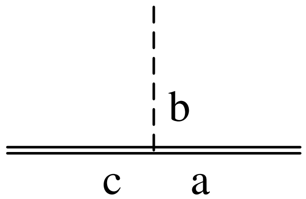
$$= igV_A \sqrt{\frac{T_F}{N_c}} \mathbf{r} \cdot \mathbf{E}^a$$

`singlet-octet vertex`



$$= ig \frac{V_B}{2} d^{abc} \mathbf{r} \cdot \mathbf{E}^c$$

`octet-octet vertex`



$$= g f^{abc}$$

`Coulomb octet-octet vertex`

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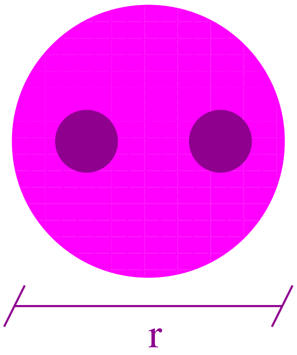


$mv \gg \Lambda_{\rm QCD}$

The matching to pNRQCD can be done perturbatively, the system is not always Coulombic.

1) $mv^2 \gg \Lambda_{\rm QCD}$

The potential is a Coulomb potential.
Nonperturbative effects are of nonpotential type and are carried by local condensates.

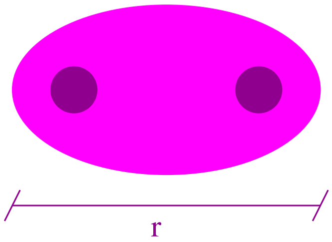
2) $mv^2 \simeq \Lambda_{\rm QCD}$

The potential is a Coulomb potential.
Nonperturbative effects are of nonpotential type and are carried by non local condensates.

3) $\Lambda_{\rm QCD} \gg mv^2$

The potential is affected by short-range nonperturbative effects.

We call these *Coulombic or Quasi-Coulombic Systems.*



$mv \lesssim \Lambda_{\mathrm{QCD}}$

The potential is non-perturbative and written in terms of Wilson loops.

The hard part of the dynamics is factorized.

pNRQCD for short-range quarkonium: applications

- Top quark production.
  At the next Linear Collider the top mass is expected to be measurable with an error of $\sim$ 100 MeV from top-antitop production near threshold. A recent theoretical estimate gives (Beneke et al. 99, Hoang Teubner 99)
  $$m_{t,PS}(20\,\mathrm{GeV}) = 165.0 + \underset{\text{1-loop}}{6.7} + \underset{\text{2-loop}}{1.2} + \underset{\text{3-loop estimate}}{0.3}\ \mathrm{GeV}$$

- Bottom and charm mass.
  Obtained from the perturbative plus condensate determination of the ground state.

- Perturbative calculation of the levels.
  For the lowest level of B_c and $c\bar{c}$ and up to $3S$ excited $b\bar{b}$ states.

- Heavy quarkonium scattering.
  Van Der Waals interaction.

- Heavy Hybrids potentials in the short range.
  pNRQCD gives model independent predictions on the gluelumps pattern.

§ *The status of quarkonium in perturbative QCD*

In the situation $\Lambda_{\rm QCD} \lesssim m\alpha_{\rm s}^2$

$$\boxed{\begin{aligned} E_X = {} & m_{1,\rm pole} + m_{2,\rm pole} \\ & - \frac{8}{9n^2}\,\alpha_{\rm s}(\mu)^2 m_r \sum_{k=0}^{2}\left(\frac{\alpha_{\rm s}(\mu)}{\pi}\right)^k P_k(L_{nl}) \\ & + \delta E_X(\alpha_{\rm s}^5 \log) + O(\alpha_{\rm s}^5) \end{aligned}}$$

where

$$m_r = m_{1,\rm pole} m_{2,\rm pole}/(m_{1,\rm pole} + m_{2,\rm pole})$$

$$L_{nl} = \log[\,3n\mu/(8\,\alpha_{\rm s}(\mu)\,m_r)\,] + \sum_{k=1}^{n+l}\frac{1}{k} + \frac{5}{6}$$

$$P_0 = 1, \qquad P_1 = \beta_0\,L_{nl} - 4,$$

$$P_2 = \frac{3}{4}\beta_0^2\,{L_{nl}}^2 + \left(-\frac{\beta_0^2}{2} + \frac{\beta_1}{4} + -6\beta_0\right) L_{nl} + f_X$$

known function of the masses

Titard Yndurain 94 Pineda Yndurain 98

Brambilla Vairo 00

§ *Quarkonium spectrum at* $m\alpha_s^5 \ln \alpha_s$

In the situation $\Lambda_{\rm QCD} \ll m\alpha_s^2$

$$\boxed{\begin{aligned}\delta E_X^{\rm pert} = -m\frac{\alpha_s^5 \log \alpha_s}{\pi}\frac{4}{9n^2}\left\{\frac{9}{2} - \frac{32}{9n^2} + \frac{400}{9n(2l+1)}\right.\\ \left. + \frac{136\,\delta_{l0}}{9n} - \frac{112\,\delta_{l0}\delta_{s1}}{9n} - \frac{8(1-\delta_{l0})\delta_{s1}}{3l(2l+1)(l+1)n}C_{j,l}\right\}\end{aligned}}$$

where

$$C_{j,l} = \begin{cases} -\dfrac{(l+1)(4\,l-1)}{2\,l-1} & , j = l-1 \\ -1 & , j = l \\ \dfrac{l(4\,l+5)}{2\,l+3} & , j = l+1 \end{cases}$$

$*$ $\delta E_X^{\rm n.pert}$ is given by local condensates (à la Voloshin–Leutwyler).

Brambilla Pineda Soto Vairo 99

But the perturbative expansion (in the pole mass)

1) shows bad convergence:

$$E_{\Upsilon(1S)} = 2 \times 4.96 - 0.10 - 0.16 - 0.22 \text{ GeV}$$

$$2 \times m_{b,\text{pole}} \qquad \alpha_s^2 \qquad \alpha_s^3 \qquad \alpha_s^4$$

at $\mu = \overline{m}_b = 4.22$ GeV $\qquad (\alpha_s(\overline{m}_b) = 0.22)$

and $n_l = 4$

* δE_X^{pert} is 30 MeV at this scale

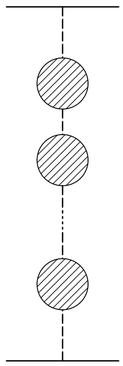
but as large as 100 MeV for more natural scales

(α_s^5 terms needed).

2) and the level splittings are not reproduced.

§ *Renormalons and perturbative series*

V_s is affected by renormalon ambiguities.



$$V_s \underset{\text{large}\,\beta_0}{\simeq} -\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q}\mathbf{r}}}{\mathbf{q}^2} 4\pi C_F\, \alpha_s(q)^{1\,\mathrm{loop}}$$

$$\underset{1/r \gg \mu \gg \Lambda}{=} -4\pi C_F \alpha_s(\mu) \sum_{n=0}^{\infty} \int^{\mu} \frac{d^3q}{(2\pi)^3} \frac{\{1 + i\mathbf{q}\mathbf{r} - (\mathbf{q}\mathbf{r})^2/2 + \ldots\}}{\mathbf{q}^2}$$

$$\times \left\{ \frac{-\beta_0 \alpha_s(\mu)}{4\pi} \ln\left(\frac{q^2}{\mu^2}\right) \right\}^n$$

$$\boxed{\begin{aligned} &V_s(\mathit{renormalon}) = C_0 + C_2 r^2 + \ldots \\ &C_0 = -2\frac{C_F \alpha_s(\mu)}{\pi} \mu \sum_{n=0}^{\infty} n! \left(\frac{\beta_0 \alpha_s(\mu)}{2\pi}\right)^n \equiv \sum_{n=0}^{\infty} V_{\beta_0}^{(n)} \\ &\qquad \Rightarrow \delta C_0 \sim \Lambda_{\mathrm{QCD}} \\ &C_2 = \frac{1}{9}\frac{C_F \alpha_s(\mu)}{\pi} \mu^3 \sum_{n=0}^{\infty} n! \left(\frac{\beta_0 \alpha_s(\mu)}{6\pi}\right)^n \\ &\qquad \Rightarrow \delta C_2 \sim \Lambda_{\mathrm{QCD}}^3 \end{aligned}}$$

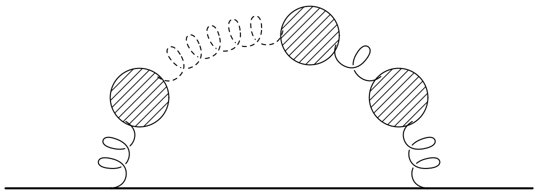
Bigi et al. 94

§ $\mathcal{O}(\Lambda_{\rm QCD})$ *renormalon cancellation*

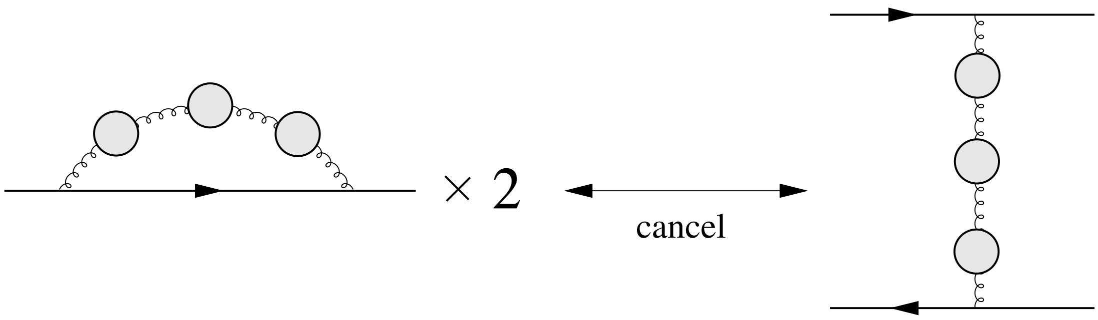
The pole mass extracted from the quarkonium spectrum has an ambiguity of order $\mathcal{O}(\Lambda_{\rm QCD})$, *which comes from the static potential renormalon. The pole mass –* $\overline{\rm MS}$ *mass relation is also affected by a* $\mathcal{O}(\Lambda_{\rm QCD})$ *renormalon of opposite sign. The renormalon cancel in a physical observable.*

$$m_{\rm pole} \underset{\text{large}\,\beta_0}{\simeq} m_{\overline{\rm MS}}(\mu) + \frac{1}{2}\int^{\mu} \frac{d^3q}{(2\pi)^3}\frac{1}{\mathbf{q}^2} 4\pi C_F\, \alpha_s(q)^{1\,\text{loop}}$$

$$= m_{\overline{\rm MS}}(\mu) - \frac{C_0}{2}$$



The $O(\Lambda_{\rm QCD})$ ambiguity (of IR origin) cancels against the $O(\Lambda_{\rm QCD})$ ambiguity (also of IR origin) in the pole mass.



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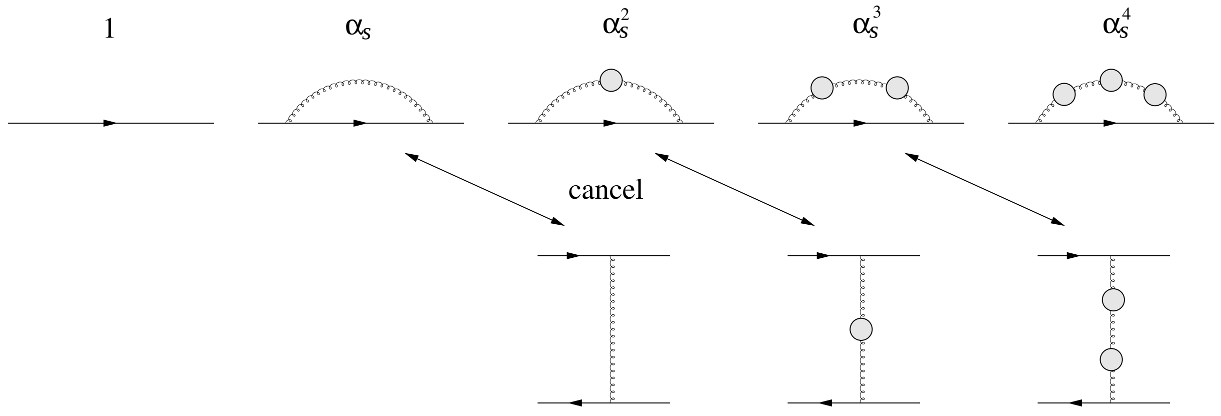
§ $\mathcal{O}(\Lambda_{\rm QCD})$ *renormalon cancellation in the quarkonium spectrum*

When the pole mass and the binding energy are given as series in α_s, renormalon cancellation takes place between terms whose orders in α_s is shifted by one , since an additional power of α_s in the binding energy is provided by the inverse of the Bohr radius :

$$\langle r^{-1} \rangle \sim \alpha_s m$$



This is often realized by counting the expansion of the pole mass as $\varepsilon^n \alpha_s^n$ and the expansion of the binding energy as $\varepsilon^n \alpha_s^{n+1}$ $(\varepsilon = 1)$ and combining same orders in ε $(\varepsilon$ *expansion*$)$.

Hoang et al. 99

§ *The heavy quarkonium masses without massive loop effects*

In this case one may chose to express the pole mass in terms of an infrared safe mass like the $\overline{\rm MS}$ mass ($\overline{m} \equiv m_{\overline{\rm MS}}(m_{\overline{\rm MS}})$):

$$m_{\rm pole} = \overline{m}\left\{1 + \frac{4}{3}\frac{\alpha_{\rm s}(\overline{m})}{\pi} + \left(\frac{\alpha_{\rm s}(\overline{m})}{\pi}\right)^2 d_1 + \left(\frac{\alpha_{\rm s}(\overline{m})}{\pi}\right)^3 d_2\right\}$$

Gray et al. 90 Melnikov Ritbergen 00

The strategy is then the following

1) fix $\overline{m}_b$ and $\overline{m}_c$ on the physical values of $E^{\rm exp}_{\Upsilon(1S)}$ and $E^{\rm exp}_{J/\psi}$ respectively.

2) fix the μ dependence through

$$A) \quad \frac{d}{d\mu}E_X(\mu, \alpha_{\rm s}(\mu), \overline{m}_i)\Big|_{\mu=\mu_X} = 0$$

$$B) \quad \frac{d}{d\mu}|E_X^{(3)}(\mu, \alpha_{\rm s}(\mu), \overline{m}_i)|\Big|_{\mu=\mu_X} = 0$$

and varying around this scale.

This fixes all the dependences and makes the approach predictive

$\rightarrow$ *the goal is to look for deviation from the experimental data in order to estimate size and nature of the non-perturbative corrections.*

Central values at the minimal sensitivity scale and $\alpha_s^{(5)}(M_Z) = 0.1181 \pm 0.0020$.

$$\overline{m}_b = 4.203^{-19}_{+18}\ \mathrm{GeV}, \qquad \overline{m}_c = 1.243^{-16}_{+15}\ \mathrm{GeV}$$

To be compared with the sum-rule results

$$\overline{m}_b = 4.20 \pm 0.05\ \mathrm{GeV} \quad \text{(Hoang 00)}$$

$$\overline{m}_c = 1.2\ \mathrm{GeV} \quad \text{(Eidemüller Jamin 00)}$$

State X	E_X^{exp}	E_X	ΔE_X	$E_X^{(1)}$	$E_X^{(2)}$	$E_X^{(3)}$	μ_X
J/ψ	3.097	3.097	0	0.362	0.205	0.043	1.07
η_c	2.980	3.056	-0.076	0.333	0.195	0.042	1.23
$\Upsilon(1^3S_1)$	9.460	9.460	0	0.837	0.204	0.013	2.49
$\Upsilon(1^3P_0)$	9.860	9.905	-0.045	1.38	0.115	0.003	1.18
$\Upsilon(1^3P_1)$	9.893	9.904	-0.011	1.40	0.098	0.002	1.15
$\Upsilon(1^3P_2)$	9.913	9.916	-0.003	1.42	0.086	0.003	1.13
$\Upsilon(2^3S_1)$	10.023	9.966	$+0.057$	1.46	0.093	0.009	1.09
$\Upsilon(2^3P_0)$	10.232	10.268	-0.036	2.37	-0.66	0.15	0.693
$\Upsilon(2^3P_1)$	10.255	$10.316^\sharp$	$-0.061^\sharp$	3.97	-3.56	1.50	$0.552^\sharp$
$\Upsilon(2^3P_2)$	10.268	$10.457^\sharp$	$-0.189^\sharp$	4.55	-5.03	2.53	$0.537^\sharp$
$\Upsilon(3^3S_1)$	10.355	10.327	$+0.028$	2.34	-0.583	0.163	0.698
$\Upsilon(4^3S_1)$	10.580	$11.760^\sharp$	$-1.180^\sharp$	5.45	-6.47	4.38	$0.527^\sharp$
B_c	6.4 ± 0.4	6.324	0.08 ± 0.4	0.668	0.187	0.022	1.64

$E_X^{(n)}$ is the order ε^n contribution.

$(*)$ Higher charmonium levels are unreliable.

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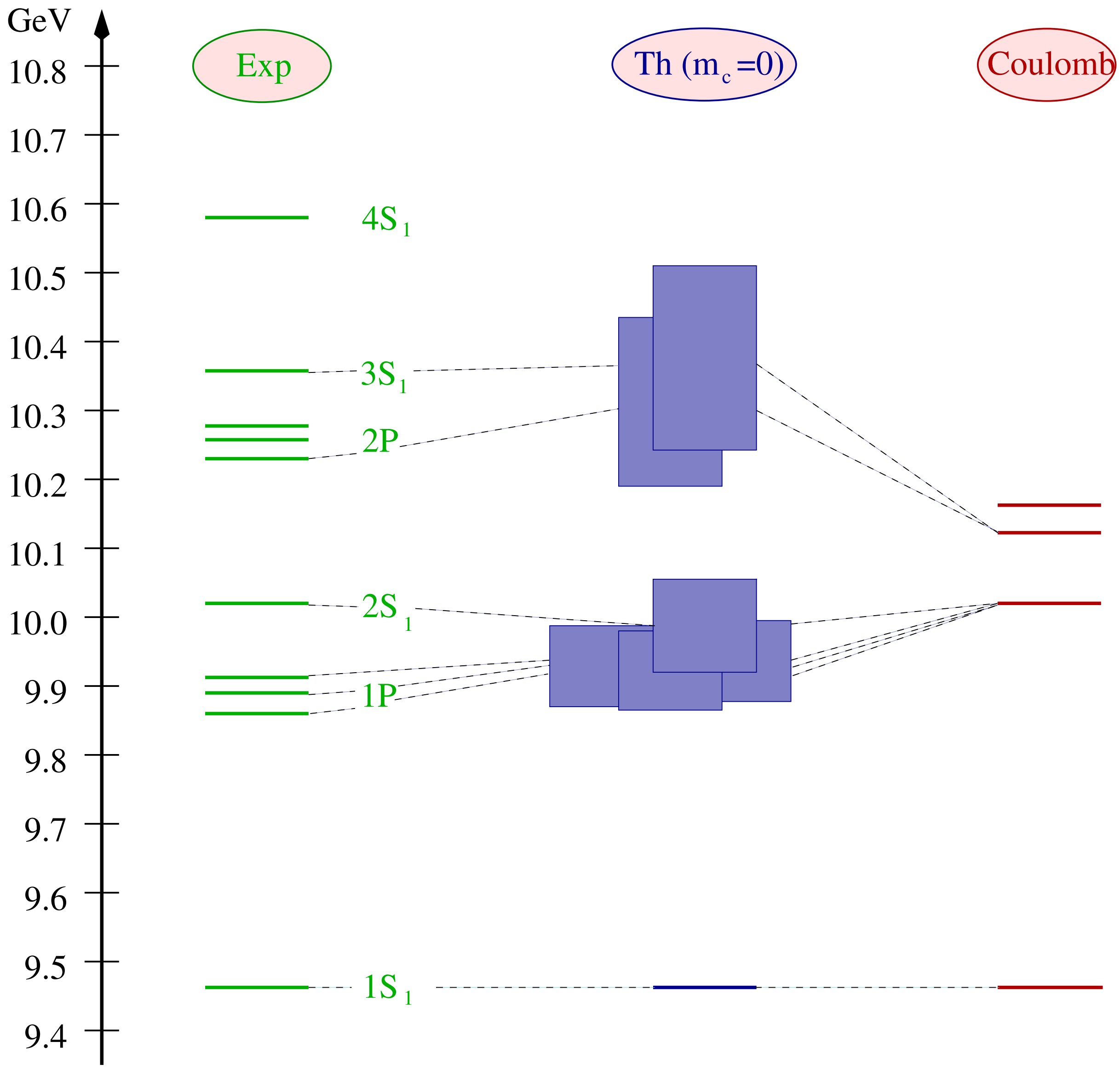
§ *The bottomonium spectrum ($m_c = 0$)*



The box refer to $\alpha_s^{(5)}(M_Z) = 0.1181 \pm 0.0020$.

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We list the main uncertainties

	Estimates of higher-order corrections						
	(i)	(ii)	(iii)	(iv)	(v)	$\pm$max	$\mu = 2\mu_X$
$\delta\overline{m}_b$	-2	$+1$	0	±7	$+5$	±7	$+16$
$\delta\overline{m}_c$	-5	$+4$	$+3$	±21	$+20$	±21	$+37$
$\Upsilon(1^3P_0)$	$+15$	-14	0	±3		±15	-53
$\Upsilon(1^3P_1)$	$+22$	-8	$+7$	±2		±22	-48
$\Upsilon(1^3P_2)$	$+16$	-16	0	±3		±16	-54
$\Upsilon(2^3S_1)$	$+18$	-19	-1	±9		±19	-73
$\Upsilon(2^3P_0)$	$+33$	-57	-21	±150		±150	-120
$\Upsilon(2^3P_1)$	-4	$+1637^\sharp$	$-66^\sharp$	$\pm1500^\sharp$		$\pm1637^\sharp$	-97
$\Upsilon(2^3P_2)$	-136	$+2136^\sharp$	$-206^\sharp$	$\pm2530^\sharp$		$\pm2530^\sharp$	-229
$\Upsilon(3^3S_1)$	$+37$	-63	-36	±163		±163	-161
$\Upsilon(4^3S_1)$	$-639^\sharp$	$+2936^\sharp$	$-1425^\sharp$	$\pm4380^\sharp$		$\pm4380^\sharp$	-1361
B_c	-4	0	-1	±22		±22	$+1$

We may compare with the $\mathcal{O}(\Lambda^3_{\rm QCD})$ renormalon contribution ($\Lambda_{\rm QCD} = 300 - 500$ MeV)

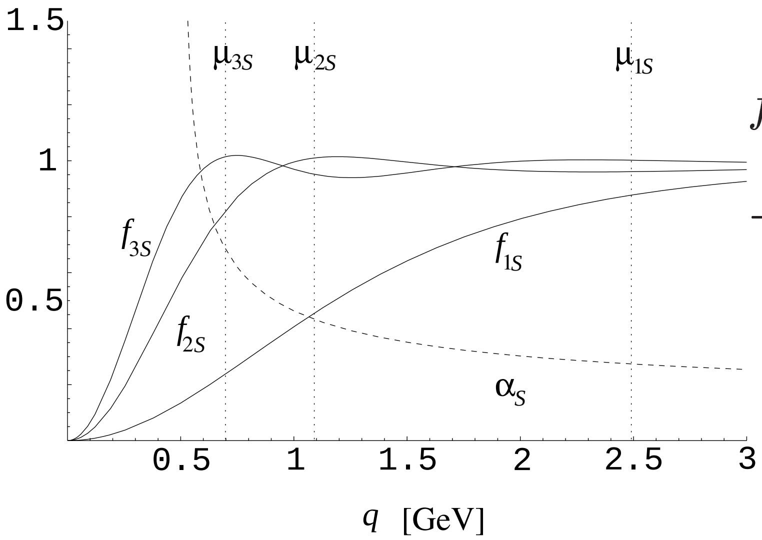
$$\begin{aligned}
(\Lambda^3_{\rm QCD}/\mu_X^2) &\simeq \pm(5-20)\ \text{MeV} \quad \text{for } n=1 \quad b\bar{b} \text{ states} \\
&\simeq \pm(20-110)\ \text{MeV} \quad \text{for } n=2 \quad b\bar{b} \text{ states} \\
&\simeq \pm(50-250)\ \text{MeV} \quad \text{for } n=3 \quad b\bar{b} \text{ states} \\
&\simeq \pm(20-110)\ \text{MeV} \quad \text{for } n=1 \quad c\bar{c} \text{ states}
\end{aligned}$$

In the presented theoretical predictions for the bottomonium spectrum the level spacings between consecutive n's are almost constant.

$$2m_{\rm pole} \simeq 2\overline{m} + \int_{|\vec{q}|<\overline{m}} \frac{d^3q}{(2\pi)^3}\,|V_s(q)| = 2\overline{m} + \frac{2C_F}{\pi}\int_0^{\overline{m}} dq\,\alpha_{\rm s}(q)$$

$$E_{{\rm bin},X} \simeq \langle X\,|\,\frac{\mathbf{p}^2}{m_{\rm pole}} + V_s\,|\,X\,\rangle$$

$$E_X \simeq 2\overline{m} + \frac{2C_F}{\pi}\int_0^{\infty} dq\,\alpha_{\rm s}(q)\,f_X(q).$$



$$f_X(q) = \theta(\overline{m}-q) - \int_0^{\infty} dr\,r^2|R_X(r)|^2\,\frac{\sin(qr)}{qr}$$

The major contribution to the levels comes from the region $1/a_X \lesssim q \lesssim \overline{m}$ of the self-energy. Level spacing between consecutive n's increases with n because the self-energy contributions grow rapidly (as $\alpha_{\rm s}(\mu_X)$) as the physical size, $\mu_X \sim 1/a_X$, of the bound-state increases.

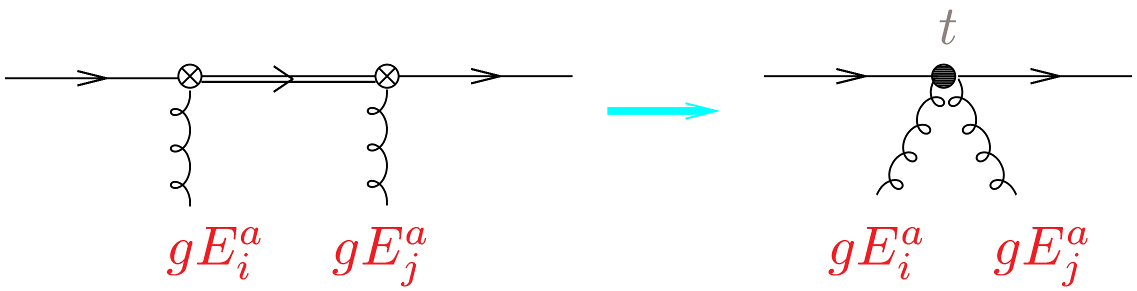
§ *Heavy quarkonium scattering*

We consider heavy quarkonium (Φ) scattering in the situation

$$\epsilon^{\rm bind} \gg \epsilon_\Phi$$

binding energy — hadron energy

Integrating out the binding energy $\epsilon^{\rm bind}$ we get



$$= a^{ij} e^{-iT\epsilon^{\rm bind}} \int_{-\infty}^{\infty} dt\, gE_j^a(t) gE_i^a(t)$$

The matching coefficient a^{ij} comes from the matching to pNRQCD:

$$a^{ij} = -V_A^2 \frac{T_F}{N_c} \langle \Phi | r_j \frac{-i}{H_o - \epsilon^{\rm bind}} r_i | \Phi \rangle$$

where H_o is the pNRQCD octet Hamiltonian.

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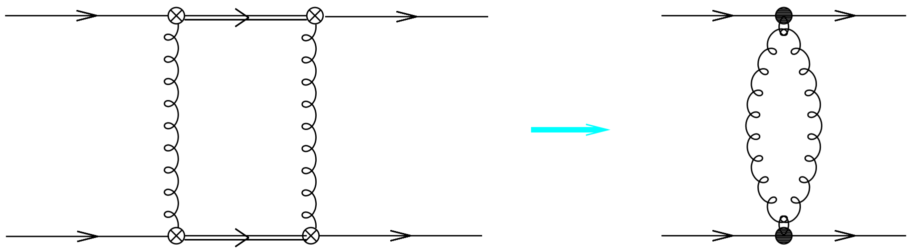
§ J/ψ-J/ψ static potential



$$= e^{-iV(R)T}$$

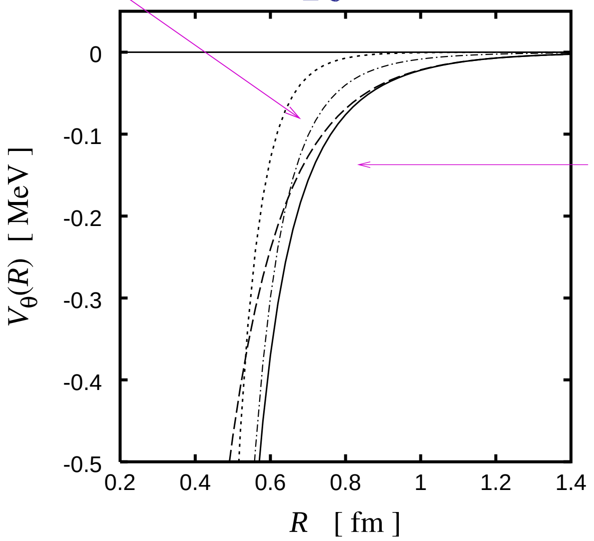
The van der Waals potential V is given by

$$V(R) = i a^{ij} a^{lm} g^4 \int_{-\infty}^{\infty} dt \, \langle E_j^a E_i^a (t, R) E_l^b E_m^b (0,0) \rangle$$

$$\simeq (a^{ii})^2 \left(\frac{8\pi^2}{\beta_0} \right)^2 \int d\sigma^2 \rho(\sigma^2) \frac{e^{-\sigma R}}{4\pi R}$$

where ρ is the spectral density.

$$V^{\mathrm{pert}} \propto -\frac{\alpha_s^2}{R^7}$$



$$\rho = \begin{cases} \rho^{\pi\pi} & 2m_\pi < \sigma < 2\mathrm{GeV} \\ \rho^{\mathrm{pert}} & 4\mathrm{GeV} < \sigma \end{cases}$$

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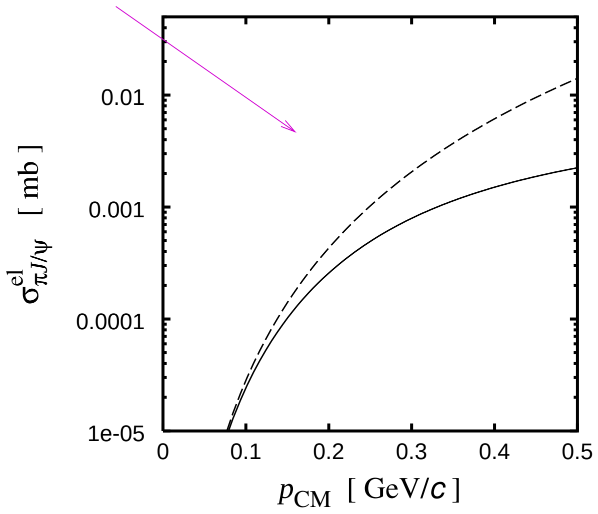
where $\rho^{\pi\pi} \sim \langle \pi(p)\pi(p')|g^2 E^{a\,2}|0\rangle^2 \sim (p - p')^4$ is the $\pi\pi$ spectral density. But in principle there are other contributions as well (glueballs ...).

§ $J/\psi + \pi \to J/\psi + \pi$

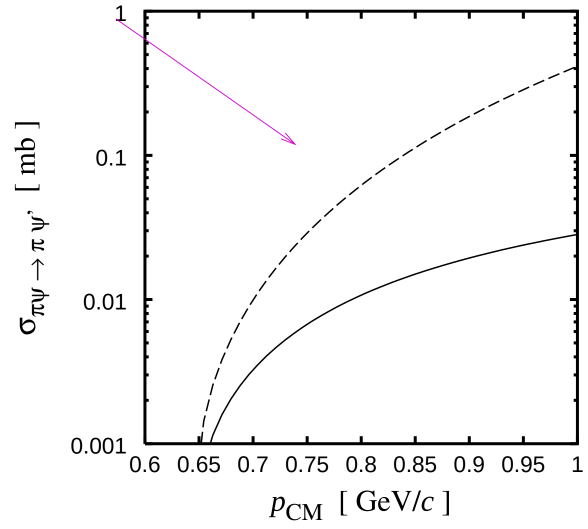
$F = 1$



where $\langle \pi(p) | g^2 E^{a\,2} | \pi(p') \rangle \sim (p - p')^2 F((p - p')^2)$ and F is the pion scalar form factor.

§ $J/\psi + \pi \to \psi' + \pi$

$F = 1$



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§ *Long-range quarkonium:* $1/r \sim \Lambda_{\rm QCD}$

In the situation:

$$\frac{1}{r} \sim \Lambda_{\rm QCD}$$

the scale $1/r$ can be integrated out only via a nonperturbative matching.

Two equivalent approaches:

- $1/m$ expansion of $\langle 0|\chi^\dagger(x_2)\phi(x_2,x_1)\psi(x_1)\psi^\dagger(y_1)\phi(y_1,y_2)\chi(y_2)|0\rangle$ in terms of Wilson loops.

  Wilson 74, lattice, ...

  Eichten et al. 81, Gromes 84, Brambilla et al. 90

- Quantum-mechanical perturbation theory on the QCD Hamiltonian.
  Some advantages:
  a) fixes in a natural way some ambiguities;
  b) gluonic excitations.

  lattice, ...

  Szczepaniak et al. 96, Brambilla et al. 00

§ *pNRQCD for long-range quarkonium*

- The singlet S has an energy of order mv^2. This is the only dynamical degree of freedom of pNRQCD, if all other states are assumed to have an energy mv and are integrated out.

$$\boxed{L^{(0)}{}_{\text{pNRQCD}} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - \sum_n \frac{V_s^{(n)}}{m^n} \right) \mathrm{S} \right\}}$$

- We perform the matching order by order in $1/m$ and at the leading order in the US degrees of freedom.

- The nonperturbative matching conditions from NRQCD to pNRQCD are:

$$|\underline{0}\rangle = S^\dagger|\text{vac}\rangle \qquad\qquad E_0 = \frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}.$$

- The potentials $V_s^{(n)}$ are unknown:
  a) to be determined from the lattice;
  b) to be determined from QCD vacuum models.

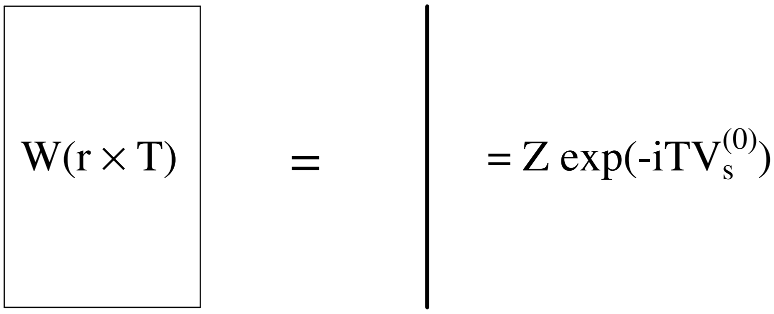
§ *The quarkonium potential*

The quantum-mechanical expressions are valid for any gluonic excitation between static quarks.
For the heavy quarkonium potential,

$$E_0 \equiv V_s = {V_s}^{(0)} + \frac{{V_s}^{(1)}}{m} + \frac{{V_s}^{(2)}}{m^2} + \ldots,$$

it is easy to translate these expressions in terms of Wilson loops, calculable on the lattice.

Since for $T \to \infty$



one gets

$$\boxed{{V_s}^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r \times T)\rangle}$$

• Note that in the perturbative case this formula has ultrasoft corrections.

§ *The quarkonium 1/m potential* ${V_s}^{(1)}$

$$ {V_s}^{(1)} = -{}^{(0)}\langle 0|\mathbf{D}^2|0\rangle^{(0)} = \underbrace{-\boldsymbol{\nabla}^2}_{\text{kinetic energy}} + \sum_{k\neq 0} \left| \frac{{}^{(0)}\langle k|g\mathbf{E}|0\rangle^{(0)}}{E_0^{(0)} - E_k^{(0)}} \right|^2 $$

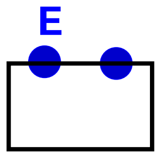
Since

$$ \langle\!\langle \mathbf{E}(t)\cdot\mathbf{E}(0)\rangle\!\rangle_\square = \sum_k |{}^{(0)}\langle 0|g\mathbf{E}|k\rangle^{(0)}|^2 e^{-iE_0^{(0)}T - i(E_k^{(0)} - E_0^{(0)})t} $$

one gets

$$ {V_s}^{(1)} = \lim_{T\to\infty} \left(-\frac{g^2}{2} \int_0^T dt\, t \langle\!\langle \mathbf{E}(t)\cdot\mathbf{E}(0)\rangle\!\rangle_\square^c \right) $$

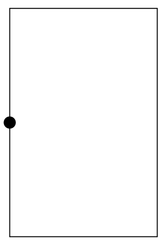
$$ \equiv \lim_{T\to\infty} \left(-\frac{1}{2} \int_0^T dt\, t \langle \quad \rangle \right) $$



$$ \langle\!\langle F \rangle\!\rangle_\square^c \equiv \langle F W(r\times\infty)\rangle / \langle W(r\times\infty)\rangle - \text{ disc. piece} $$

Same result from (for $T\to\infty$)

$$ \int dt \ \frac{iD^2}{2m} $$



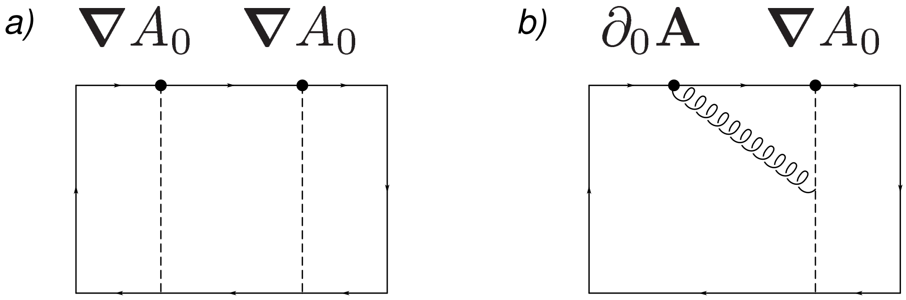
$$ = \quad = Z^{1/2} \exp(-iT(V_s^{(0)} + p^2/m + V_s^{(1)}/m))\, Z^{1/2} $$

Brambilla Pineda Soto Vairo 00

- If $\Lambda_{\rm QCD} \sim mv$ then in principle ${V_s}^{(1)} \sim mv^2$ and therefore ${V_s}^{(1)}$ is as much important as the static potential.

- There is no lattice analysis available yet.

- Perturbatively the situation is different:



$$\frac{V^{(1)}_{\rm pert}}{m} = -\frac{C_F C_A}{m}\frac{\alpha_{\rm s}^2}{4\,r^2} \sim m\alpha_{\rm s}^4 \ll V^{(0)}_{\rm pert} \sim m\alpha_{\rm s}^2$$

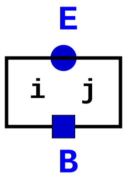
- ${V_s}^{(1)}$ vanishes in quenched QED and in exact Gaussian models:

$$\langle W(r\times T)\rangle = e^{-\frac{g^2}{2}\int ds\, ds' \int dt\, dt'\, r_i r_j f_{ij}(t-t',(s-s')\mathbf{r})}$$

therefore

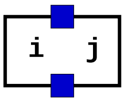
$$\langle\langle \mathbf{E}(t)\cdot\mathbf{E}(0)\rangle\rangle^c_\Box = f_{ii}(t,\mathbf{0}).$$

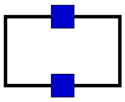
§ *The* $1/m^2$ *spin-dependent potential* $V_{\rm SD}^{(2)}$

$V_{\rm SD}^{(2)} = \lim\limits_{T\to\infty} \Bigg\{ \dfrac{1}{r} \Bigg(c_F \epsilon^{kij} \dfrac{2r^k}{r} i \displaystyle\int_0^T dt\, t \, \Big\langle$ 

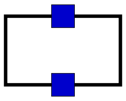
$\Big\rangle$

$\quad - \dfrac{1}{2} \dfrac{d}{dr} \dfrac{i}{T} \log \Big\langle$  $\Big\rangle \Bigg) (\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}$

$- {c_F}^2 \hat{r}_i \hat{r}_j i \displaystyle\int_0^T dt \Bigg(\Big\langle$ 

$\Big\rangle - \dfrac{\delta_{ij}}{3} \Big\langle$  $\Big\rangle \Bigg)$

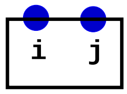
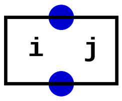
$\quad \times \Big(\mathbf{S}_1 \cdot \mathbf{S_2} - 3 (\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}}) \Big)$

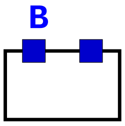
$+ \Bigg(\dfrac{2}{3} {c_F}^2 i \displaystyle\int_0^T dt \, \Big\langle$  $\Big\rangle$

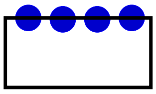
$\quad -4(d_2 + C_F d_4) \delta^{(3)}(\mathbf{r}) \Bigg) \mathbf{S}_1 \cdot \mathbf{S_2} \Bigg\}$

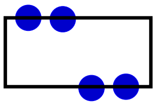
Eichten Feinberg 81, Gromes 84, Chen et al. 95
Brambilla Vairo 99

§ *The $1/m^2$ spin-independent potential $V_{\rm SI}^{(2)}$*

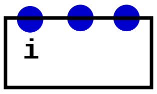
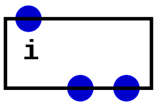
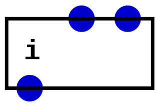
$V_{\rm SI}^{(2)} = \lim_{T\to\infty} \Big\{ p^i \Big(i \int_0^T dt\, t^2 \,\langle$  $\rangle + \langle$  $\rangle \Big) p^j$

$- \frac{c_F^2}{2} i \int_0^T dt \,\langle$  $\rangle + (d_1 + C_F d_3 + \pi C_F \alpha_{\rm s} c_D)\delta^{(3)}(\mathbf{r})$

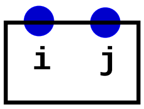
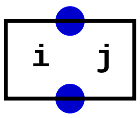
$- i \int_0^T dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 (t_2 - t_3)^2 \Big(\langle$  $\rangle$

$+ \langle$  $\rangle \Big)$

$+ \int_0^T dt_1 \int_0^{t_1} dt_2 (t_1 - t_2)^2 (\nabla^i - i(t_1 - t_2)(\nabla^i V_0))$

$\times \Big(\langle$  $\rangle + \frac{1}{2} \langle$  $\rangle + \frac{1}{2} \langle$  $\rangle \Big)$

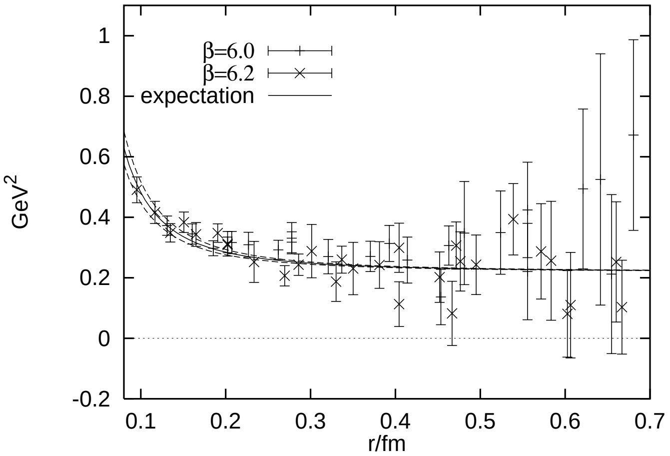
$+ \frac{1}{2} \int_0^T dt\, t^3 \,(\nabla^i - i \frac{t}{3} (\nabla^i V_0))(\nabla^j V_0)$

$\times \Big(\langle$  $\rangle + \langle$  $\rangle \Big)$

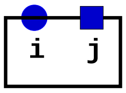
$- 2 b_3 f_{abc} \int d^3\mathbf{x}\, g \langle\!\langle G^a_{\mu\nu}(\mathbf{x}) G^b_{\mu\alpha}(\mathbf{x}) G^c_{\nu\alpha}(\mathbf{x}) \rangle\!\rangle^c_\Box \Big\}$

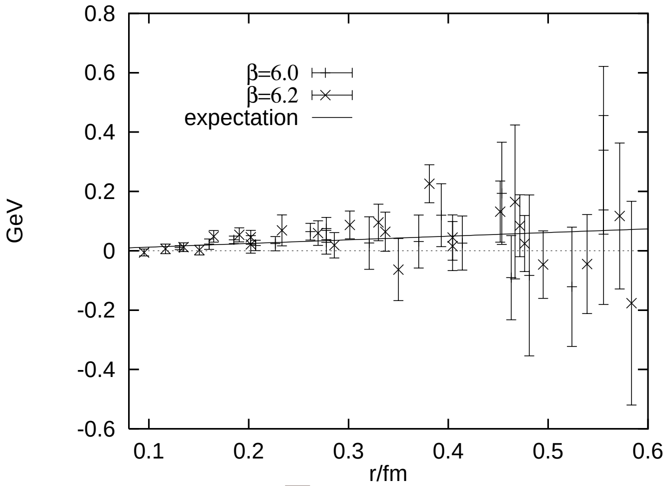
Tafelmayer 86, Brambilla et al. 88 90, Pineda Vairo 00

A (incomplete) lattice determination is available

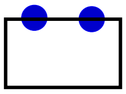


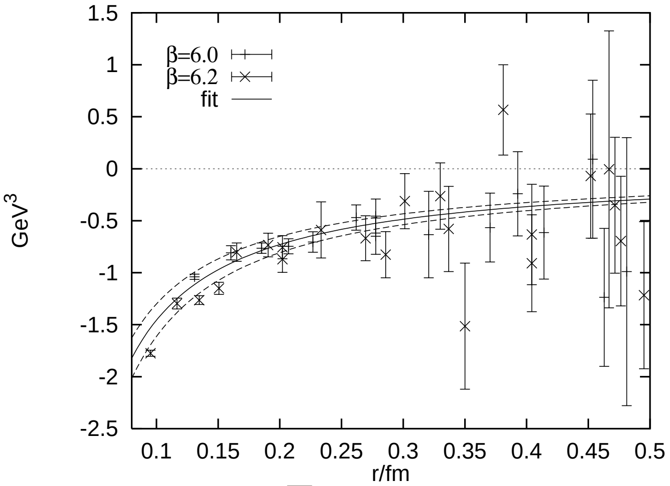
$$-2\epsilon^{kij}\,\hat{r}^k \int_0^T dt\, t \,\langle \quad \rangle$$



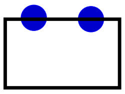


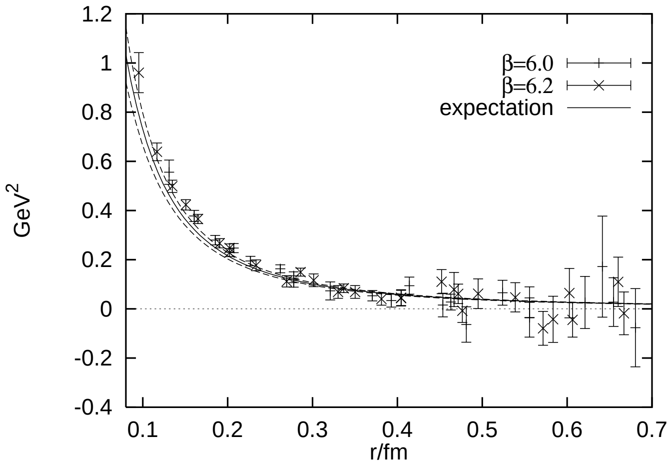
$$-\frac{1}{6}\int_0^T dt\, t^2 \,\langle \quad \rangle$$



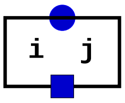


$$2\int_0^T dt\,\langle \quad \rangle$$





$$\epsilon^{kij}\,\hat{r}^k \int_0^T dt\, t \,\langle \quad \rangle$$



Bali et al. 97
Bali 00 ?

§ *Remarks*

- So far we have neglected ultrasoft degrees of freedom (i.e. scales lower than mv).
  a) if $\Lambda_{\rm QCD} \sim mv$: light fermions.
  b) if $\Lambda_{\rm QCD} \sim mv^2$: light fermions
  $+$ ultrasoft gluons.

- We have assumed $E_n \sim \Lambda_{\rm QCD}$ for $n \neq 0$. If some gluonic excitations are ultrasoft the mixing with quarkonium states may become relevant.

- Up to ultrasoft corrections our expression are exact in perturbation theory.

- All the quarkonium potentials (with the exception of one) can be expressed in terms of the Wilson loop and field insertions on it.

pNRQCD for long-range quarkonium: applications

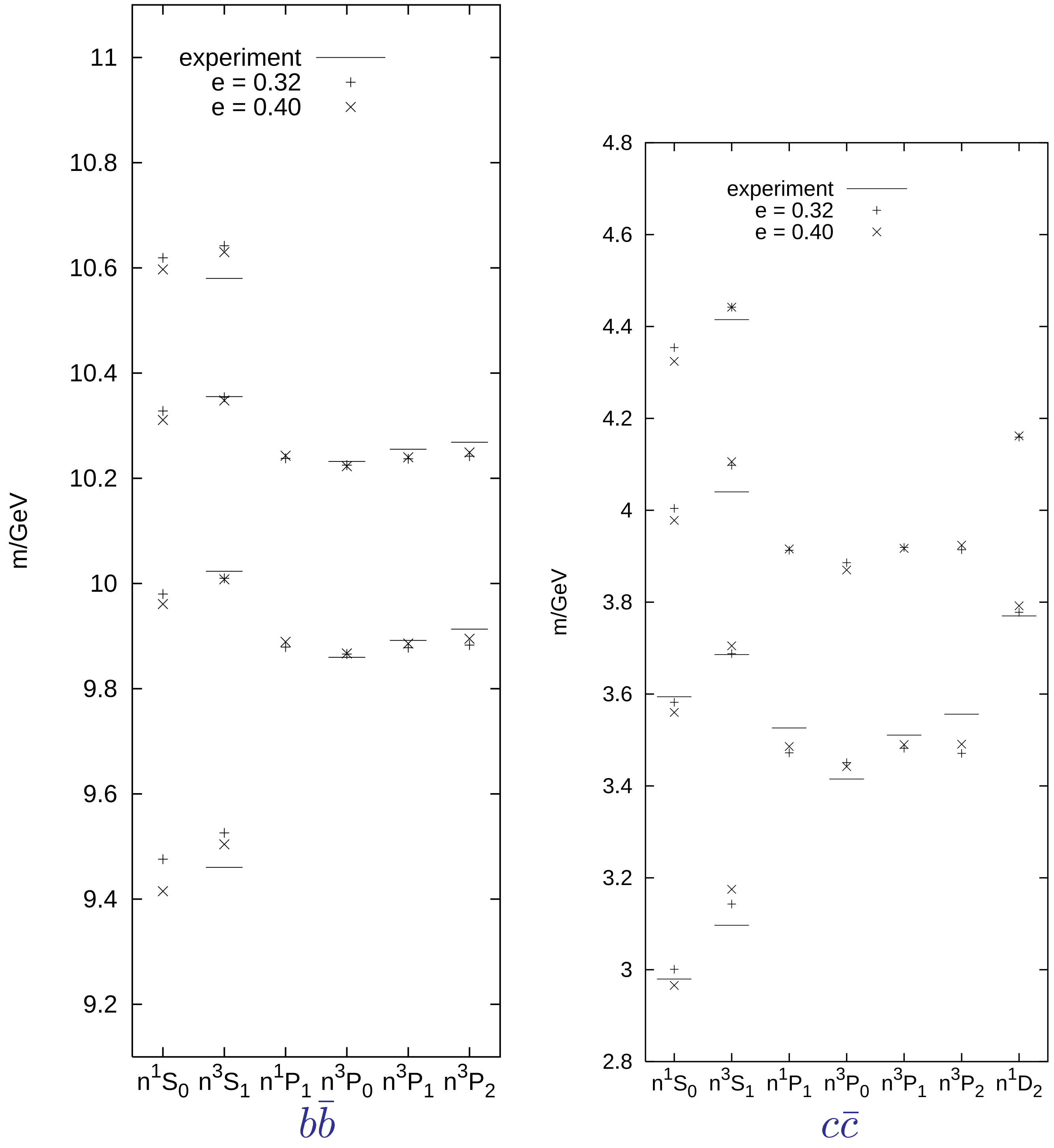
- Spectrum of charmonium.
  Hard dynamics factorized in the matching coefficients, Soft dynamics contained in the Wilson loops.

- Information on the confinement mechanism.
  Lattice evalution of the Wilson loops operators versus QCD vacuum model evaluation of the Wilson loops versus the data.

- Decays.
  Use the symmetries of pNRQCD to constrain the decays, e.g. $\chi_c \to \text{light had}, \gamma\gamma$, ...

- Heavy quarkonium production.
  Apply the power counting of pNRQCD to get predictions in quarkonium production.
  Can give a solution to the still open problems? (Prompt production of J/ψ in NRQCD ...)

Spectrum from the Lattice



Bali et al. 97

Quarkonium levels from a fit of the lattice potential.

§ *The "standard" power counting of NRQCD*

Lepage et al. 92 suggests the following counting

1) $p \sim mv$, kinetic energy $\sim mv^2$

2) $\int d^3\mathbf{x}\, \psi^\dagger \psi \simeq 1 \Rightarrow |\psi|^2 \sim \frac{1}{(\Delta x)^3} \sim m^3 v^3$

3) $\frac{\mathbf{D}^2}{2m} \sim mv^2 \Rightarrow \mathbf{D} \sim mv$

4) $\left(iD_t + \frac{\mathbf{D}^2}{2m} \right) \psi = 0$

$$(\partial_t^2 - \boldsymbol{\nabla}^2)\, g\mathbf{A} = \frac{g^2}{m} \psi^\dagger \boldsymbol{\nabla} \psi + gA_0 \boldsymbol{\nabla} gA_0$$

$\Rightarrow$ i) $D_t \sim \mathbf{D}^2/2m \sim mv^2$

ii) in Coulomb gauge $A_0 \sim mv^2$ (virial theorem)

$$g\mathbf{A} \sim mv^2 g^2 \sim mv^3 \quad (g^2 \sim v)$$

iii) $g\mathbf{E} \sim m^2 v^3$, $g\mathbf{B} \sim m^2 v^4$

- *The power counting is not unique.*
- *The power counting uses perturbation theory arguments.*
- *The power counting uses argumets rigorously suited only for bound states in a non-relativistic quantum-mechanical context.*

§ *The power counting in pNRQCD*

* *The power counting of pNRQCD is simpler since the scale mv has been integrated out.*
* *The problem of the power counting of NRQCD becomes the problem of the power counting of the different potentials in pNRQCD.*

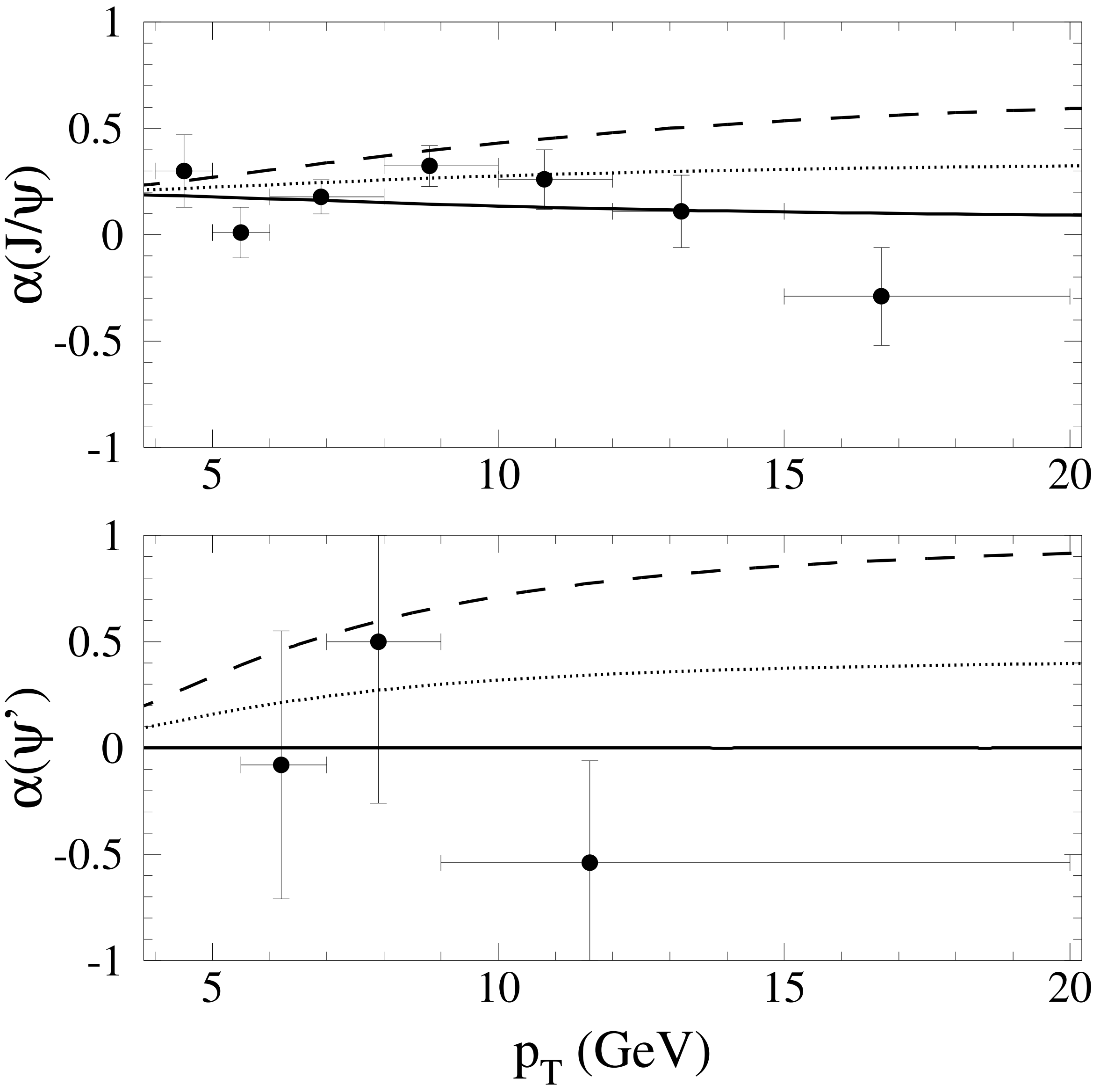
The power counting is organized in powers of mv and α_s(hard).

A "natural" counting is
$V^{(n)} \sim (mv)^{n+1}$ + constraints:
i) virial theorem $\Rightarrow V^{(0)} \sim mv^2$ $\quad (V^{(0)}_{\rm pert} \sim \alpha_s mv)$
ii) Lorentz invariance
iii) lattice measurements

E.g.
A) $V^{(1)} \sim mv^2$
B) $V^{(2)}_{\rm SD} \sim V^{(2)}_{\rm SI} \sim m^2v^3$

but terms containing $V^{(0)}$ in $V^{(2)}_{\rm SI}$ are suppressed by an extra v

but terms from the gluonic and 4-fermion sector in $V^{(2)}_{\rm SD}$ and $V^{(2)}_{\rm SI}$ are suppressed by extra α_s(hard).



Predicted J/ψ and ψ' polarizations at Tevatron with different power countings.

Fleming et al. 00

Conclusions

- An effective theory of QCD which describes quarkonium has been constructed systematically and within a controlled expansion.
- This theory disentangles the different scales of the bound state and has a definite power counting.
- All known perturbative and non-perturbative regimes (potential, non-potential) are dynamically present in the theory.
- Quarkonium properties are related to the fundamental parameters of QCD.