

Potential models of Charmonium^a

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Outline

1. Introduction
2. Level order
3. Tests of flavour independence
4. Short-range correlations
5. Spin forces

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Introduction

See other talks for justifying potential models as a limit of the exact QCD dynamics.

Born–Oppenheimer type of approximation: many other degrees of freedom (light-quark pairs, gluons) integrated out. The heavy Q and \bar{Q} evolve in a effective potential, sum of their direct interaction and the energy of the gluon field.

In this limit: Fictitious world with simplified quark dynamics: static potential with **flavour independence** and non-relativistic kinematics.

Many regularity patterns of the spectrum reproduced from the properties of Schrödinger equation.

Dramatic progress on the Schrödinger equation with confining and flavour-independent forces.

Applications to other fields, in particular **Atomic physics**. Here, also, the **same** potential is experienced by different masses (e^- , \bar{p} , μ^- , ..).

Already some results for **relativistic** equations, or spin-dependent forces.

Charmonium spectrum-1

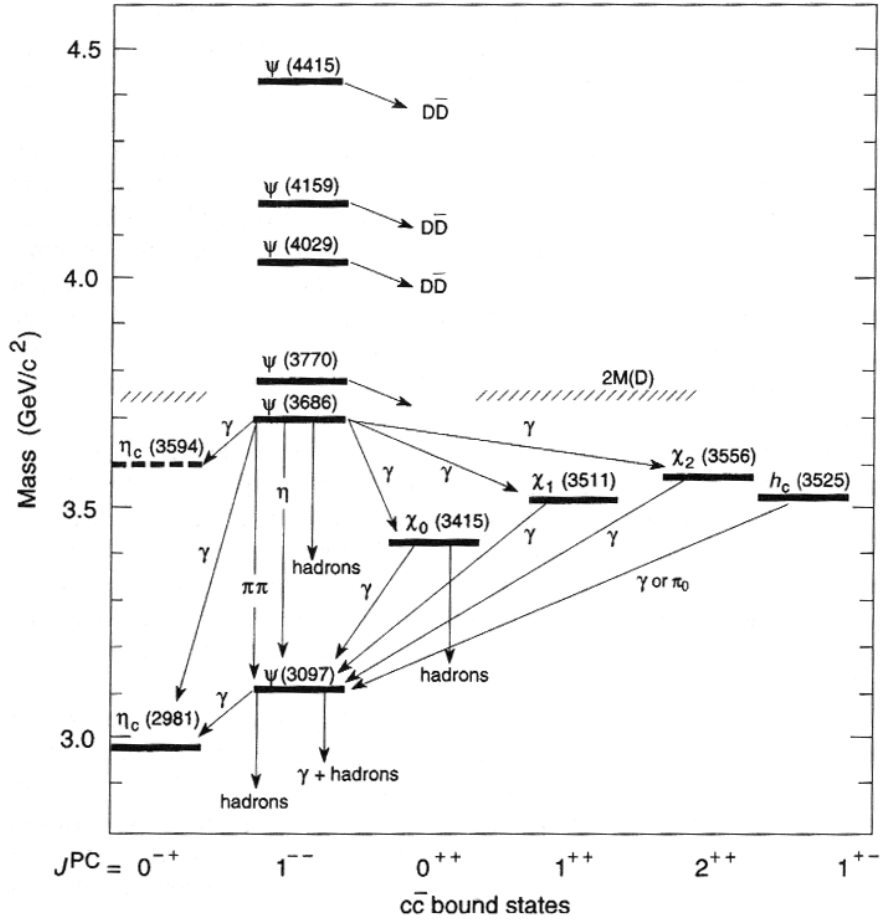


Figure 1: Experimental states of Charmonium (Figure borrowed from the book by Martin and Grosse).

Charmonium spectrum-2

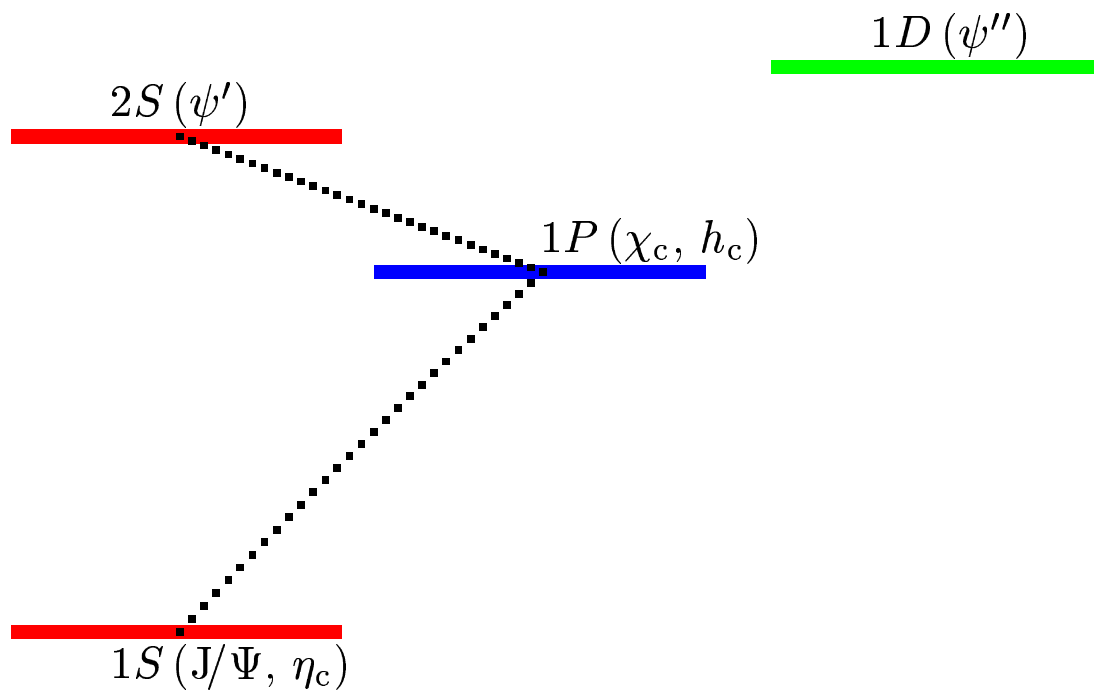


Figure 2: Spin-averaged Charmonium spectrum, for the first levels: 1S and 2S (red), 1P (blue) and 1D (green).

Level order 1

$E(1P) < E(2S)$ observed in quarkonium.

Notation (n, ℓ) : the radial wave function has $n - 1$ nodes, the principal number of atomic physics is $n + \ell$.

Coulomb theorem $E(n + 1, \ell) < E(n, \ell + 1)$ if $\Delta V > 0$, the reverse if $\Delta V < 0$.

Note: Gauss theorem: ΔV indicates whether the charge $Q(r)$ seen at distance r increases (asymptotic freedom), decreases, or remains constant (gravitation or electrostatics).

Klein–Gordon equation. Theorem extended on one side.

Also $E(1D) > E(2S)$ observed.

H.O. theorem

H.O. degeneracy $E(n + 1, \ell) = E(n, \ell + 2)$ broken according to the sign of $d^2V/d(r^2)^2$.

Level order 2

Phenomenological potentials

These results explain why potentials with different functional dependence reproduce the main features of the spectrum:

- χ states below Ψ'
- $\ell = 2$ states (ψ'' , etc.) above $2S$ (ψ')

Among these potentials: the famous Coulomb + linear

$$V(r) = -\frac{a}{r} + br + c.$$

or more sophisticated : A.F. $a \rightarrow a(r)$, screening of confinement, etc.

or simpler

$$V(r) = A r^\alpha + B, \quad (1)$$

with $V = a \ln r + b$, for $\alpha \rightarrow 0$.

Comparison Cb./H.O./ $Q\bar{Q}$

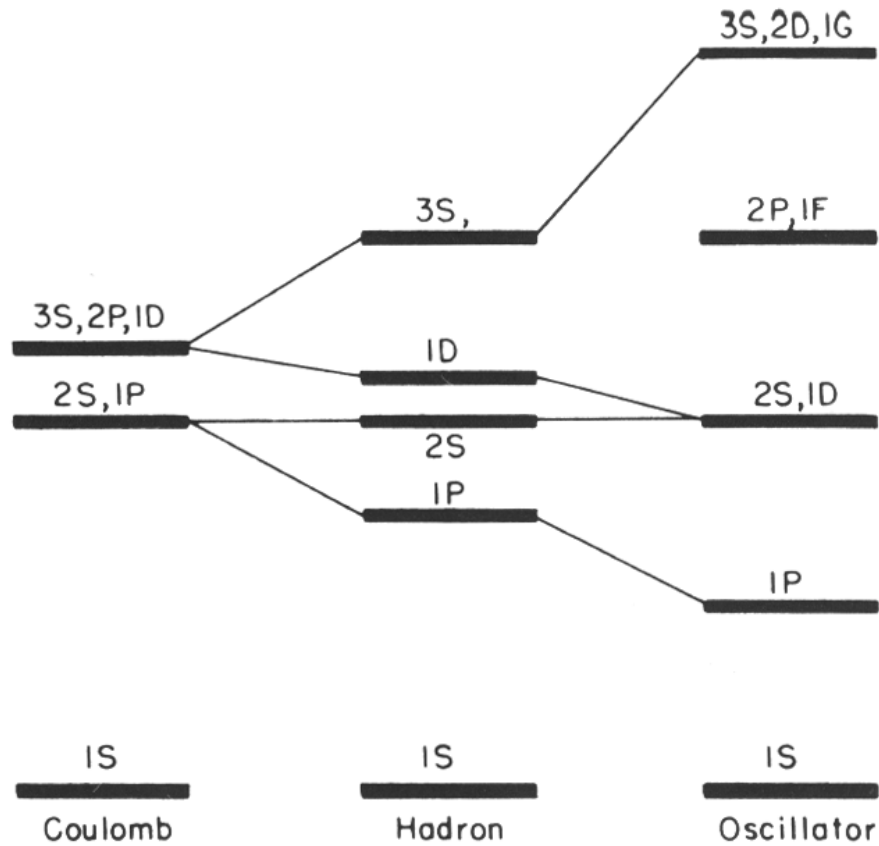


Figure 3: Comparison of levels of Charmonium vs. Coulomb and harmonic oscillator (borrowed from a paper by the Cornell group).

Method of inverse problem-1

In ordinary M.Q., two canonical problems:

- ★ # 1. Reconstruct the radial potential from the bound states and the phase-shift $\delta_\ell(E)$ from $E = 0$ to $E \rightarrow \infty$.
- ★ # 2. Reconstruct the central potential from the phase-shifts $\delta_\ell(E)$ at given E from $\ell = 0$ to ∞ .

Several attempts: Thacker, Quigg, Rosner; Miller and Olsson, McWilliams, etc.

Have adapted the formalism # 1 to the case of confining interaction.

Reproduce the current potentials within the uncertainties.

Method of inverse problem-2

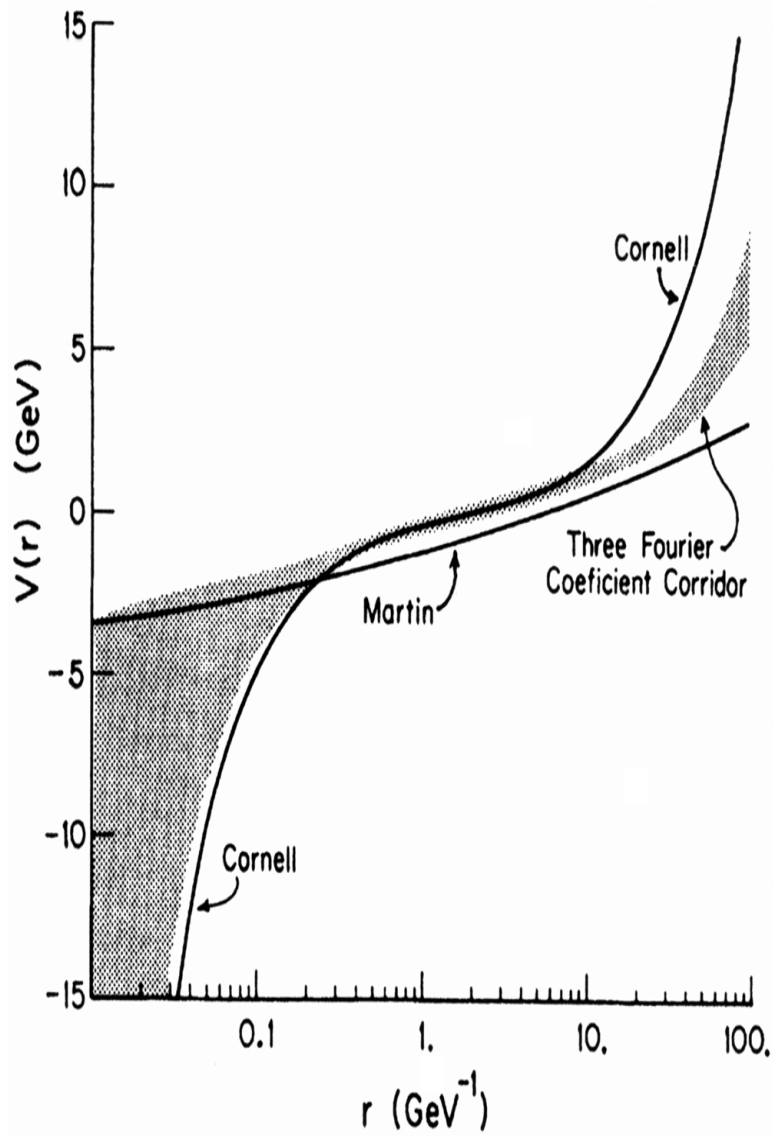


Figure 4: Results by Olsson and Miller.

Method of inverse problem-3

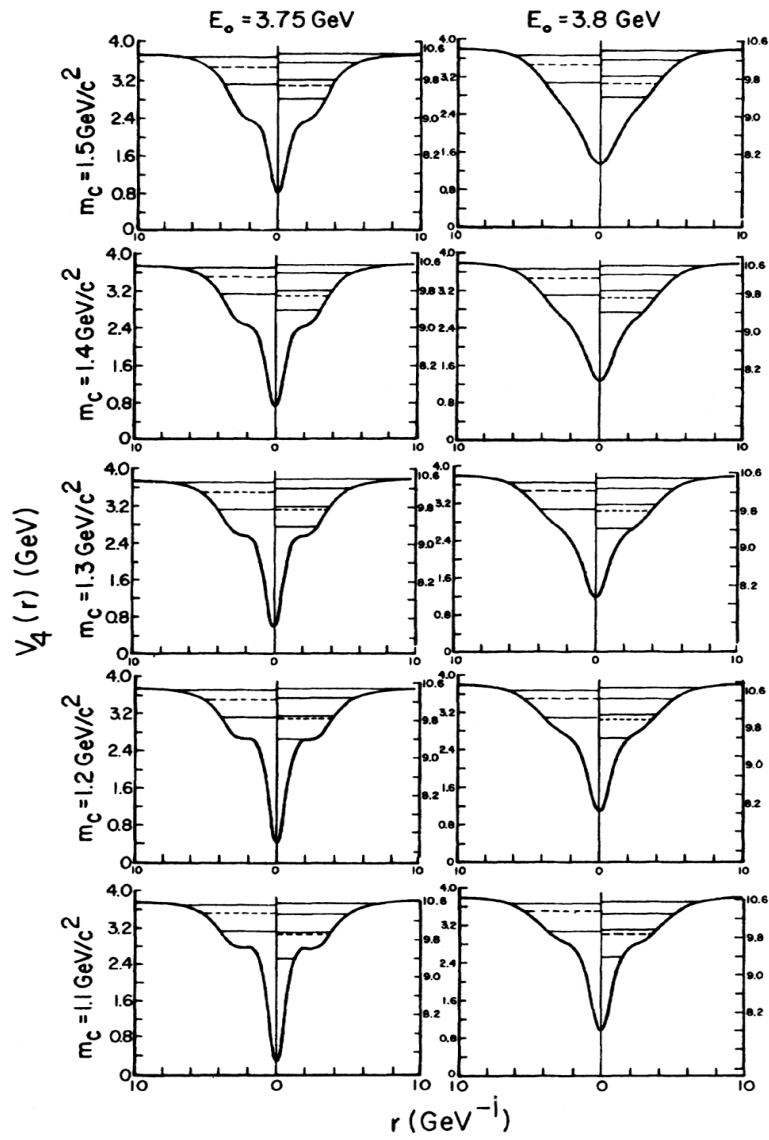


Figure 5: Results by Thacker, Quigg and Rosner.

Method of inverse problem-4

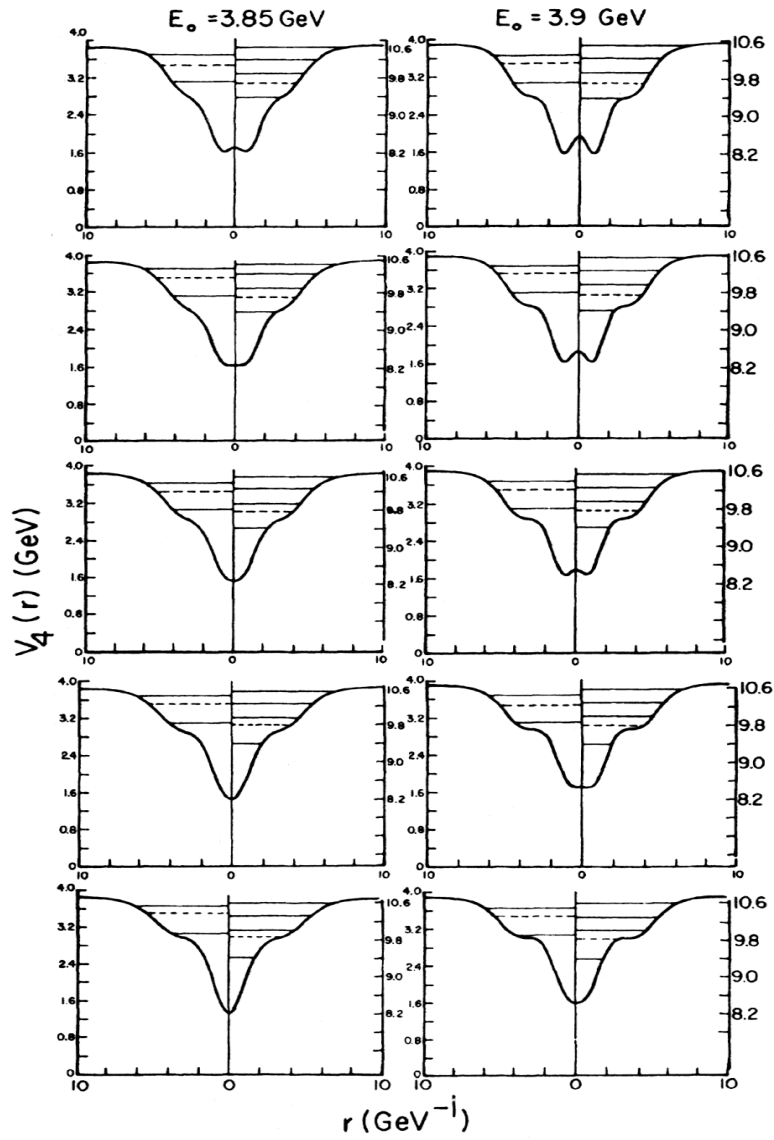


Figure 6: Results by Thacker, Quigg and Rosner (cont.).

Bottomonium-1

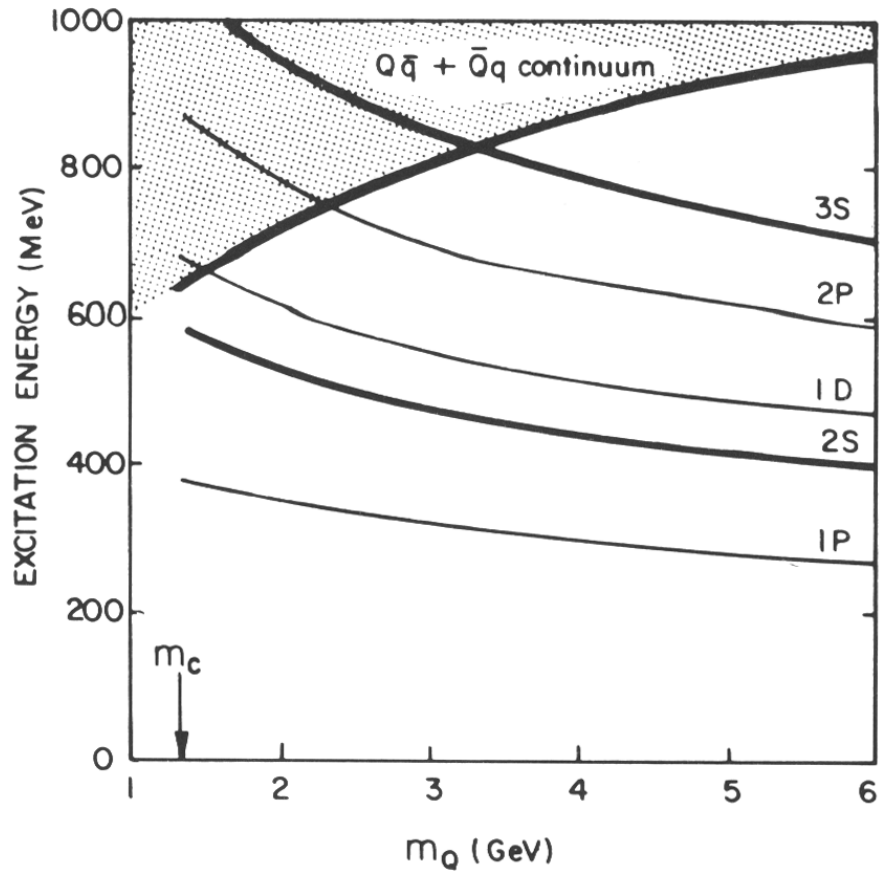


Figure 7: Levels of heavy $Q\bar{Q}$ mesons, as predicted by Eichten and Gottfried.

Bottomonium-2

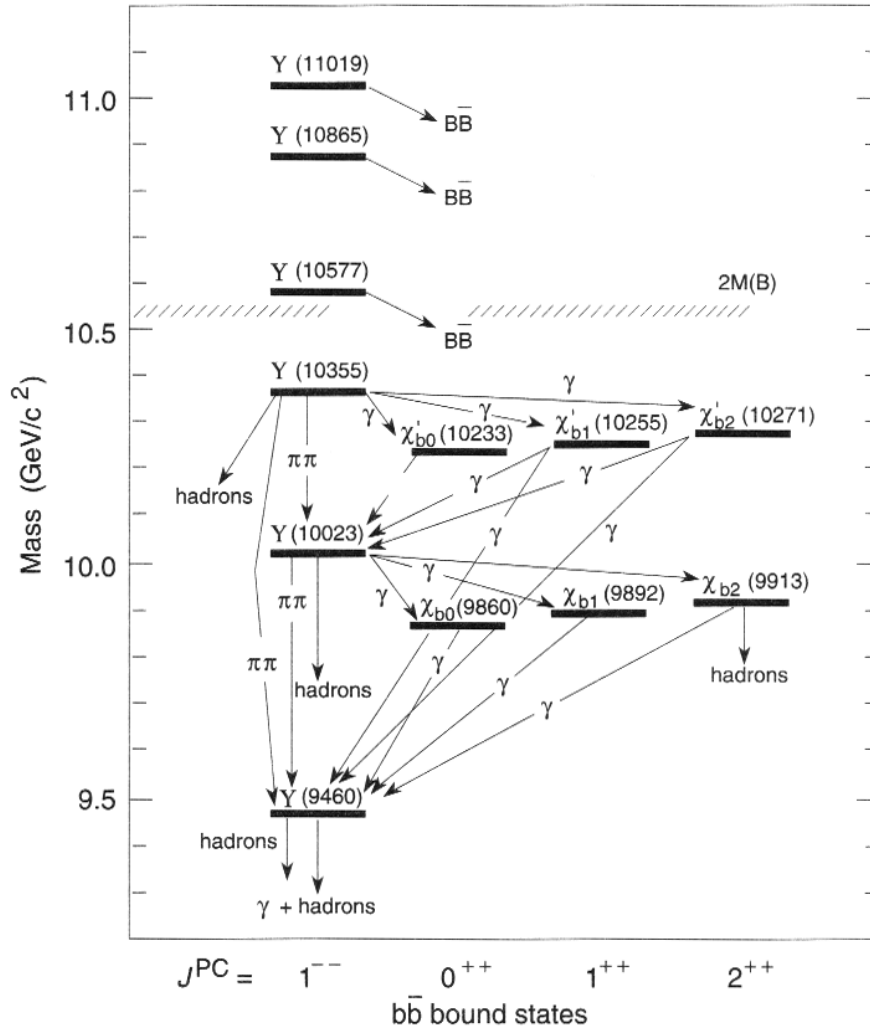


Figure 8: Experimental states of Bottomonium (Figure borrowed from the book by Martin and Grosse).

Flavour independence 1

The $Q\bar{Q}$ game becomes more interesting if the **same** potential is used with different masses.

Anticipated by Eichten and Gottfried, etc. Became hotter when Υ and excitations discovered (Lederman).

A first consequence of **flavour independence**:

In a given potential, deeper binding for heavier mass

This explains

- why first levels of $Q\bar{Q}$ below the Zweig-allowed threshold $(Q\bar{q}) + (\bar{Q}q)$
- why $b\bar{b}$ more deeply bound than $c\bar{c}$, while $b\bar{q}$ very similar to $c\bar{q}$: more $b\bar{q}$ bound states expected and seen!

Flavour independence 2

Several successful fits of **both** $c\bar{c}$ and $b\bar{b}$ spectra:
Coulomb-plus-linear, Logarithmic, power-law potentials,
etc.

This shows that data do not contradict flavour
independence.

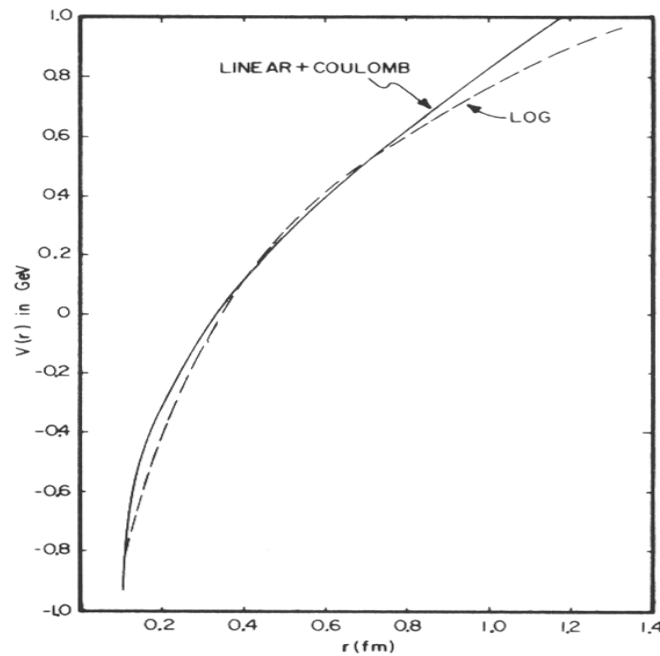


Figure 9: Comparison of a logarithmic and a Coulomb-plus-linear potential (borrowed from a paper by the Cornell group).

Flavour independence 3

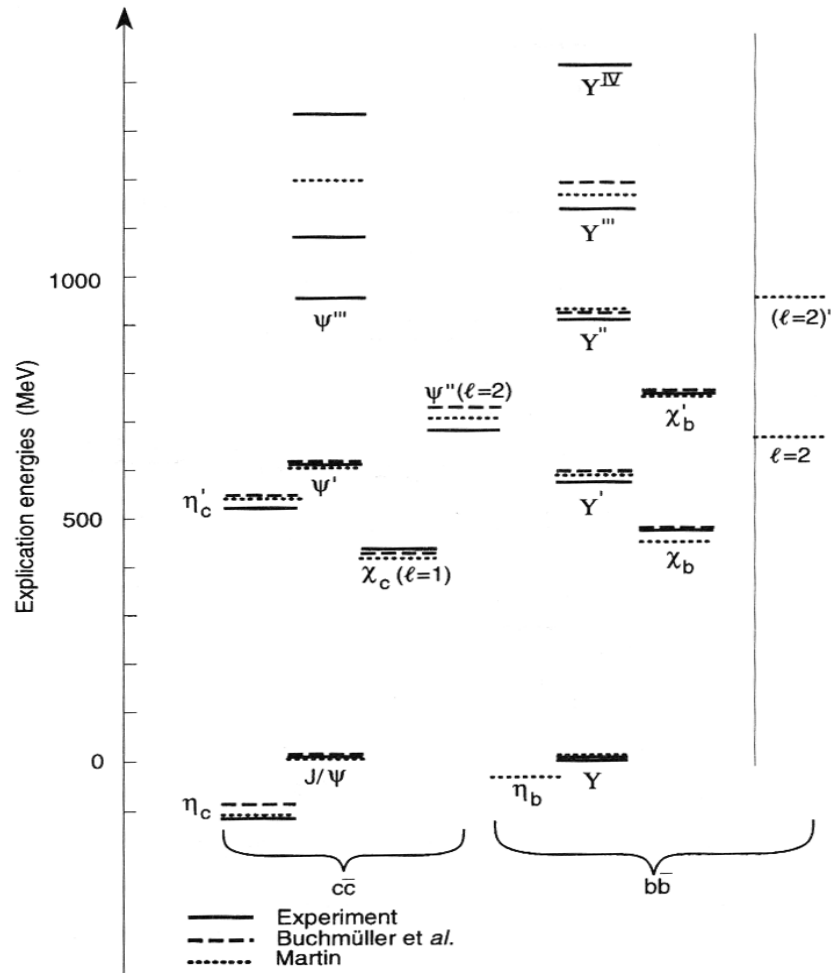


Figure 10: Fits to Charmonium and Upsilon spectra with the Martin and Buchmüller potentials (borrowed from the book by Grosse and Martin).

Flavour independence 4

Also E concave function of the inverse reduced mass

$$(b\bar{c}) > (b\bar{b} + c\bar{c})/2$$

Upper bound also with mild restrictions on $V(r)$.

$dE/d\mu^{-1} = \langle \mathbf{p}^2 \rangle / 2$ can be controlled along its way from $(b\bar{b})$ or $(c\bar{c})$ to $(b\bar{c})$

Thus

$$(b\bar{c}) \leq \frac{(c\bar{c}) + (b\bar{b})}{2} - \frac{9}{8} \delta E(c\bar{c}) \left[1 - \left(\frac{m_b + m_c}{2m_b} \right)^{1/3} \right] \\ + 2\delta E(b\bar{b}) \left[\left(\frac{m_b + m_c}{2m_c} \right)^{1/3} - 1 \right],$$

resulting into

$$6.26 < (b\bar{c}) < 6.43 \text{ GeV}$$

Simpler upper bound

$$(b\bar{c}) > (b\bar{s} + (c\bar{c}) - (c\bar{s})) \simeq 6.52 \text{ GeV},$$

while experiment not yet conclusive

$$(b\bar{c}) = 6.4 \pm 0.4 \text{ GeV}$$

Short-range correlations

$$\delta_{12} = \langle \Psi | \delta^{(3)}(\mathbf{r}_{12}) | \Psi \rangle,$$

needed, but its computation is difficult.

Possible way out: use a crude wave function with an improved matrix element. **Schwinger rule** and generalisations.

Idea:

A and $A_B = A + [H, B]$ have same expectation value within any eigenstate of H .

A suitable choice of B might make $\langle \Psi^M | A_B | \Psi^M \rangle$ converging faster than $\langle \Psi^M | A | \Psi^M \rangle$ to the exact $\langle \Psi | A | \Psi \rangle$ as the size M of the basis is increased.

2-body:

$$A = \delta^{(3)}(\mathbf{r}) \rightarrow A_B \propto dV/dr$$

For $v(r) = r$, perfect. Explains why δ_{12}^n is the same for all S -states, independent of the radial number n . More general: δ_{12}^n increases (decreases) with n if $V'' > 0$ (< 0).

Generalization: 3-body or $N > 3$

Additional (small) centrifugal term due to internal excitations. Used for parity-violation in atoms, annihilation of exotic molecules with positrons, and short-range correlations in baryons (hyperfine effects in Breit–Fermi models, decay properties). See: J. Sucher, Hoffmann-Ostenoff et al., Fleck + JMR, H.J. Lipkin, etc.

Spin forces 1: χ states

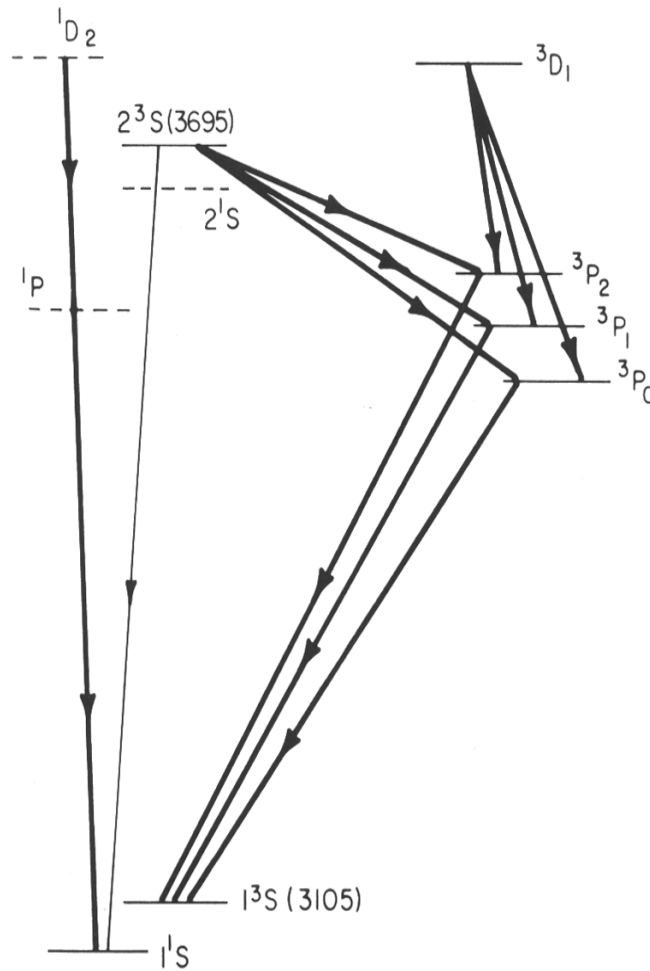


Figure 11: First levels of Charmonium, with possible transitions, as predicted in the pioneering paper of the Cornell group.

Spin forces 2: χ states

Latest data

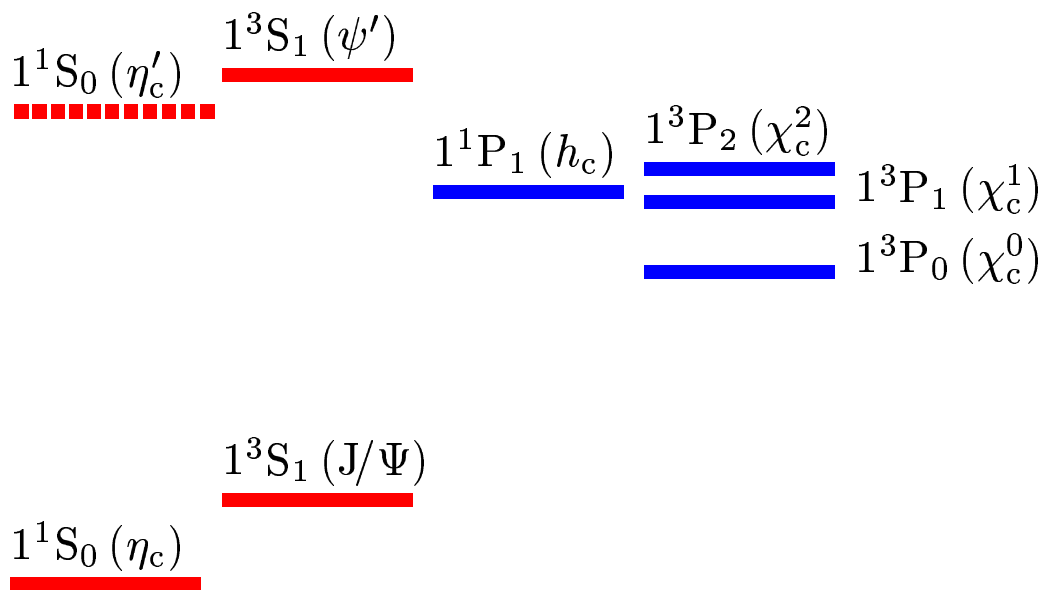


Figure 12: Charmonium spectrum, for the first levels: 1S and 2S (red), 1P (blue) and 1D (green), with fine and hyper-fine structure.

Spin forces 3: LS and T

Reasonably explained by spin-orbit and tensor forces.

$$V = V_{\text{ct}} + V_{\text{LS}} \mathbf{L} \cdot \mathbf{S} + V_{\text{T}} S_{12},$$

with V_{ct} being the central-triplet potential, and

$S_{12} = 3 \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}} \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, one gets

$${}^3P_m^{\text{pert}} = \frac{{}^3P_0 + 3 {}^3P_1 + 5 {}^3P_2}{9}$$

$$\langle V_{\text{LS}} \rangle = \frac{-2 {}^3P_0 - 3 {}^3P_1 + 5 {}^3P_2}{12}$$

$$\langle V_{\text{T}} \rangle = \frac{5}{72} (-2 {}^3P_0 + 3 {}^3P_1 - {}^3P_2)$$

Table 1: Expectation values (in MeV) of spin-orbit and tensor potentials (assumed to be treated at first order) for the $\ell = 1$ levels of $c\bar{c}$ and $b\bar{b}$.

State	χ_c	χ_b	χ'_b
$\langle V_{\text{LS}} \rangle$	35	13	9
$\langle V_{\text{T}} \rangle$	10	3	2

Spin forces 4: $J/\Psi - \eta_c$

Earlier 300 MeV disturbing. Better understood at 117 MeV.

Extrapolation to $b\bar{b}$: small, explains why η_b not yet seen.

Puzzling η'_c

Other problems with radial excitations: missing modes of ψ' , S-D orbital mixing, etc.

Perhaps $\psi' - \eta'_c$ very small, if

- ★ $J/\Psi - \eta_c$ larger than pure chromomagnetic splitting, due to mixing with a glueball (Anselmino)
- ★ $n = 2$ chromomagnetic splitting only about 60% of $n = 1$.
- ★ Mixing to $D\bar{D}, D\bar{D}^* + \text{c.c.}$ unfavourable. e.g., in the Cornell model, as completed by Martin and J.M.R., $\psi' - \eta'_c$ receives about -20 MeV .

A word about **S-D mixing in ψ''** . “Closest neighbour” argument: 1D mixed mainly with 2S. May be also some 1S, due to the favourable node structure.

Spin forces 5: 1P_1 state of $c\bar{c}$

A naive $\sigma_1 \cdot \sigma_2 V_{SS}$ associated with $V_{SS} \propto \delta^{(3)}(\mathbf{r})$ gives $^3P_m - ^1P_1 = 0$.

Experimental masses: $h_c = 3526.1$, $^3P_m = 3525.3$ MeV.
See, however, other talks: h_c remains to be confirmed.

At this level, everything can contribute: details in non-relativistic reduction, higher-order QCD effect, etc.
Also what is meant by **centre of gravity** of triplet states?

The usual $^3P_m^{\text{pert}} = (^3P_0 + 3 ^3P_1 + 5 ^3P_2)/9$ assumes a **perturbative** treatment of V_{LS} and V_T .

A non-perturbative treatment starts from

$$H(^3P_m) = \frac{H(^3P_0) + 3 H(^3P_1) + 5 H(^3P_2)}{9}.$$

From the variational principle

$$^3P_m^{\text{non pert}} \geq \frac{^3P_0 + 3 ^3P_1 + 5 ^3P_2}{9}$$

by a few MeV. Then my reading of data is

$$^3P_m^{\text{non pert}} - ^1P_1 \simeq +\text{a few MeV}, \quad (2)$$

Conclusions

Many developments on [few-body quantum systems](#) stimulated by problems raised in quark models studies of hadron properties.

In particular:

- Level pattern of alkalin atoms
- Level pattern of muonic atoms
- Spectroscopy of Antiproton-Helium atomcules
- Systematics of short-range correlations
- Stability of 3-charge systems (m_1^+, m_2^-, m_3^-)
- Stability of 4-charge molecules $(m_1^+, m_2^+, m_3^-, m_4^-)$
- Relations between N -body and $(N - 1)$ -body energies (Hall–Post inequalities)
- etc.