Baryon Spectroscopy in the Charm and Beauty Sectors using a Renormalization Group Improved Quark Phenomenological Model

Calvin Kalman and Ian D'Souza

June 30,1998

Realistic Model

- This model is consistent over the entire baryon spectra.
- This model is *economical* in parameters.
- There are *physical underpinnings* to the model. That is, this model is much more than a curve fitting to the measured spectra. The parameters of the model represent physical variables and minimize absorption of unknown physics into various constants.

The Underlying Assumptions

- We calculate the mass spectrum in a non-relativistic model.
- This approach has been successful in other quark models in both baryon and meson systems, and, compared to relativistic calculations of the spectrum, gives similar results with **fewer** parameters.
  - Kalman-Tran / Dong et al (BARYONS)
  - Kalman-D'Souza / Godfrey-Isgur (MESONS)

Comparison of Kalman-Tran to Dong,et al.

- Dong, Su, Wu calculate the baryons spectrum in the *u*, *d* and *s* sectors in a fully relativistic model.
- Their model contains 12 parameters compared to the 7 of Kalman-Tran.
- They obtain marginally better results, but this may simply be due to the larger number parameters used.

Kalman-Tran

- Prior to the current improvements; the most consistent quark model to-date:
- A single set of parameters is used for the whole spectra.
- All properties; masses, radii and wavefunction structures subjected to a strict test by a decay model which has only two parameters

The Underlying Assumptions
continued

- The quark interaction can be described by a short distance Coulomb potential (*De Rugula, et al*).
- Plus, a confining linear potential. Thus, we treat the quarks as interacting through a potential of the type $V(r_{ij}) = -C\alpha_s / r_{ij} + a r_{ij}$

The Underlying Assumptions
continued

- Hyperfine interactions are included.
- Spin-orbit interactions are not included. We still obtain a good fit to the spectrum and this may be due partly to a cancellation of the *LS* interactions between the long and short range potentials.

Harmonic-Oscillator Basis

H.O.; confining potential that is analytically solvable in the 3-body system, allowing a separation of the center of mass motion.

$$V'(r_{ij}) = \frac{1}{2}\mu\omega^2 r_{ij} + [V(r_{ij}) - \frac{1}{2}\mu\omega^2 br_{ij}]$$

$$= V_0(r_{ij}) + U(r_{ij})$$

$$V(r_{ij}) = -C_F\alpha_s / r_{ij} + ar_{ij} + C$$

Baryon Hamiltonian

- The total Hamiltonian is then the sum of the terms
  - Quark constituent masses
  - Quark kinetic energy
  - Baryon C.M. kinetic energy
  - H.O. interquark potential
  - Coulomb + Linear - proportional H.O. interquark potential
  - Hyperfine interaction

$$H = \sum_i \left(m_i + \frac{p_i^2}{2m_i} \right) - \frac{\left(\sum_i p_i \right)^2}{2 \sum_i m_i} + \sum_{i<j} \frac{1}{2} k r_{ij}^2 + \sum_{i<j} \left(\frac{-2\alpha_s}{3 r_{ij}} + a r_{ij} - \frac{1}{2} b k r_{ij}^2 \right) + H_{hyp}$$

Baryon Hamiltonian

continued

- The hyperfine term of the Hamiltonian is made of two parts, a *contact* term H_{FC} and a *tensor* term H_{TEN} :

$$H_{hyp} = H_{FC} + H_{TEN}$$

$$\left(H_{FC}\right)_{ij} = \frac{2\alpha_s}{3m_i m_j}\left[\frac{8\pi}{3}\mathbf{S}_i \cdot \mathbf{S}_j \delta^3(r_{ij})\right]$$

$$\left(H_{TEN}\right)_{ij} = \frac{2\alpha_s}{3m_i m_j}\frac{1}{r_{ij}^3}\left[\frac{3\left(\mathbf{S}_i \cdot \mathbf{r}_{ij}\right)\left(\mathbf{S}_j \cdot \mathbf{r}_{ij}\right)}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j\right]$$

Parameters of the Model

- Free parameters: H.O. spring constants, quark masses, α_S, a, b, and an overall constant C of the Kalman-Tran model are fixed from the u,d quark sector baryons except:
  - The constituent masses of the s, c and b quarks are fit from the spectra of baryons containing these quarks.
  - The strong coupling $\alpha_S = 1$ in the Kalman-Tran model.
- In this work, we allow the strong coupling to vary with the energy scale, and, we insist that the coupling is the physical value; the experimental value at the Z mass.

Scale Dependence

One could try to fit each sector separately, but that would lead to a very large number of parameters and offer little physical insight.

Incorporating the c and b Spectra

- The strong coupling α_S is not a constant but depends on the energy of the interaction, that is, on the separation between the quarks.
- Is it possible to describe the c, b spectra by incorporating a scale dependent coupling, while *at the same time* connecting to the excellent results of the Kalman-Tran model in the u, d and s sector?

Scale Dependent Coupling

- The strength of the coupling α_s depends on the relative momentum or separation between **two** quarks.
- Unfortunately, we do not know what the inter-quark spacing is for a given excited state of the (3-body) baryon.

  # Capstick and Isgur:

    – Perform a transform from p-space to x-space, but cutting off the pole in α_s at p=0 (large x) by fitting with a parameter.

    – No different from parametrizing the strong coupling.

Internal Quasi Particles

- Although not normally viewed as physical objects, here, the internal basis excitations are taken as **quasi 2-particle** states.
- We then estimate the energy of the excitations, and obtain the inter-quark separation for a particular excited state.
- This energy is then used to calculate the (energy dependent) strong coupling α_S for the two excitations for each baryon state.

Decoupled H.O. Basis

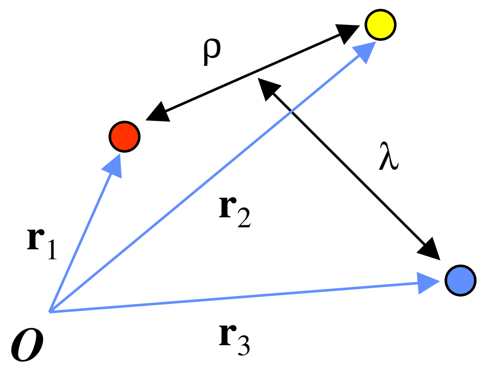
- The Hamiltonian is transformed to a decoupled basis by the transformation:

$$\rho = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\lambda = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)$$

- The diagram shows the mode oscillators in this new basis (ρ,λ).

- The mode decoupling will form the basis of the calculation of the scale dependent strong coupling constant.

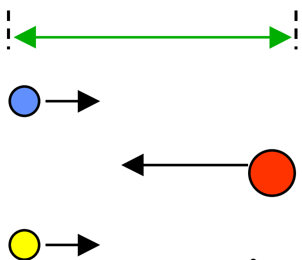


If all three quarks have different masses

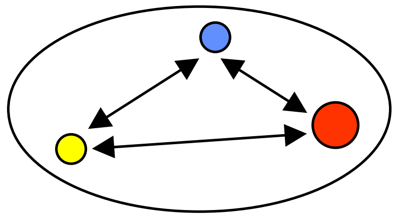
$$\lambda = \sqrt{\frac{2}{3}}\left(\frac{m_1 r_1}{m_1 + m_2} + \frac{m_2 r_2}{m_1 + m_2} - r_3\right)$$

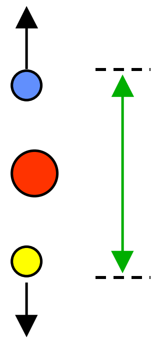
Baryon Inter-quark Coupling

- *What is* the separation between quarks in a baryon?
- For a given value of the quantum number *n*, each of the two excitations will have a two-body energy for the quasi-particles they represent.
- These energies represent the **size** of the quasi-particles.



λ mode





ρ mode

Energy of the Quasi-Particles

Energy of the quasi-particle with mode quantum number n is:

$$\sum_i m_i + (n + 3/2)\omega_{\rho,\lambda}$$

Where i=1,2 for the ρ, and i=1,2,3 for the λ

Determining α_S

The value of the strong coupling is fixed at the *mass of the Z* = 91 GeV by convention:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln\left(\mu^2 / \Lambda^2\right)}\left[1 - \frac{2\beta_1}{\beta_0}\frac{\ln\left[\ln\left(\mu^2/\Lambda^2\right)\right]}{\ln\left(\mu^2/\Lambda^2\right)} + \frac{4\beta_1^{\,2}}{\beta_0^{\,4}\ln^2\left(\mu^2/\Lambda^2\right)} \times \left\{\left(\ln\left[\ln\left(\mu^2/\Lambda^2\right)\right] - 1/2\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^{\,2}} - \frac{5}{4}\right\}\right]$$

Setting the H.O. Couplings

- A variational method is used to obtain α_ρ, and α_λ , constants dependent on *k* the H.O. spring constant.
- These constants are further corrected by Kalman-Tran to align their decay-width calculations.

The Kalman-Tran Model

- Kalman-Tran perturb the parameters α_ρ and α_λ from their original values (calculated in the spectral determination) to obtain decay rates consistent with experiment. These corrected values, if re-applied to the mass spectrum calculation, do not restore the original spectrum due to the sensitivity of the spectral calculation. The problem is more pronounced in the c,b sector and is tamer in the u,d,s sector.
- Good experimental data was not available to K-T at the time for good fitting of the heavier quark sectors. With the data of today, the heavy sector is not consistent in the K-T model.

Connecting to the The Kalman-Tran Model

The final values of α_ρ and α_λ no longer fit the baryon spectrum as well as the original set. Thus comparison is made in this work to the original set determined by the variational calculation. The variation in α_s, realigns the spectrum especially in the heavy sector. In the s sector this coupling is approaching the order of 1.0 the value used by Kalman–Tran and the evolution must stop as we are approaching the non-perturbative region. No especial attempt was made in this work to join smoothly to the Kalman–Tran model, however it turns out that the predicted data is indeed close to the Kalman–Tran predictions in the s–sector. Thus we can allow the parameters to revert to the original Kalman-Tran parameters in the u, d sector.

Tables of Selected Results

Spectrum of Λ_b. Composition

State	**Mass**	Expt	Capstick							
$^2S_{\frac{1}{2}}{}^+$	5640	5641	5585	.95	.08	-.33	-.008	.005	0	.001
$^2S_{\lambda\lambda}\,\frac{1}{2}^+$	5898		6085	-.06	-.96	-.38	-.003	-.001	-.001	0
$^2S_{\rho\rho}\,\frac{1}{2}^+$	6372		6280	-.33	.38	-.87	.02	-.006	0	0
$^2D_{\lambda\lambda}\,\frac{3}{2}^+$	5894		6145	-.93	.39	0	-.005	.01	-.01	0
$^2D_{\rho\rho}\,\frac{3}{2}^+$	5898		6305	.41	.90	-.02	-.001	-.006	-.006	0
$^2D_{\lambda\lambda}\,\frac{5}{2}^+$	5918		6355	-.94	0	.40	0	.003		

Note that for Σ_b, the spin 3/2 state (5905 GeV) and the 5/2 state(5906 GeV) lie below the 1/2 state (5960 GeV).

Tables of Selected Results

Spectrum of Λ_c. Composition

State	Mass	Expt	Capstick							
$^2S\,\frac{1}{2}^+$	2280	2285	2265	.95	.06	-.33	-.02	.02	0	0
$^2S_{\lambda\lambda}\,\frac{1}{2}^+$	2752	2625	2775	-.08	-.91	-.39	-.01	-.004	0	0
$^2S_{\rho\rho}\,\frac{1}{2}^+$	3126			-.33	.39	-.89	.03	-.01	0	0
$^2S_{\rho\lambda}\,\frac{1}{2}^+$	3453			.03	-.03	.04	.95	-.36	-.04	-.05
$^2D_{\lambda\lambda}\,\frac{3}{2}^+$	2740			-.86	.57	-.005	-.005	.04	-.01	.001
$^2D_{\rho\rho}\,\frac{3}{2}^+$	2752			.55	.82	-.04	-.002	-.02	-.02	0
$^2D_{\rho\lambda}\,\frac{3}{2}^+$	3306			-.01	.07	.8	-.2	-.45	.32	.01
$^2D_{\lambda\lambda}\,\frac{5}{2}^+$	2792			-.86	-.01	.5	.001	.01		
$^2D_{\rho\lambda}\,\frac{5}{2}^+$	3306			-.01	-.95	-.03	-.31	.16		
$^2D_{\rho\rho}\,\frac{5}{2}^+$	3495			-.55	.05	-.87	-.001	-.001		

Tables of Selected Results

Spectrum of Σ_c.

State	**Mass**	**Expt**	**Capstick**	Composition								
$^{2}S_{\frac{1}{2}}^{+}$	2454	2455	2440	-.99	-.05	-.05	.03	.03	-.007	0		
$^{2}S_{\lambda\lambda\,\frac{1}{2}}^{+}$	3069		2890	.08	-.95	-.36	-.05	.006	0	0		
$^{2}S_{\rho\rho\,\frac{1}{2}}^{+}$	3303		3005	.05	.36	-.92	.15	.05	.02	.001		
$^{2}S_{\rho\lambda\,\frac{1}{2}}^{+}$	3338		3035	.04	-.05	.15	.99	.007	.06	.001		
$^{4}D_{\rho\rho\,\frac{1}{2}}^{+}$	3490		3080	.05	-.01	.08	-.01	.99	-.01	-.01		
$^{4}S_{\frac{3}{2}}^{+}$	2518	2530	2985	-.99	-.09	-.17	.05	.01	.005	-.04	-.07	0
$^{4}S_{\rho\rho\,\frac{3}{2}}^{+}$	2777		3060	.05	-.97	.24	-.1	-.01	-.01	-.07	-.01	-.001
$^{4}S_{\lambda\lambda\,\frac{3}{2}}^{+}$	2946		3065	-.21	.24	.95	.02	.001	0	-.01	-.01	0
$^{4}D_{\rho\rho\,\frac{3}{2}}^{+}$	3377		3130	-.05	.1	.007	-.94	-.06	-.04	-.35	.05	-.01
$^{4}D_{\lambda\lambda\,\frac{3}{2}}^{+}$	3432		3140	.001	-.01	.001	-.06	.73	-.68	.04	-.08	-.09
$^{2}D_{\lambda\lambda\,\frac{3}{2}}^{+}$	3490		3200	.01	-.03	.01	-.05	.67	.73	-.08	.1	-.03
$^{2}D_{\lambda\lambda\,\frac{5}{2}}^{+}$	2789		3065	.96	-.08	.1	.02	-.27				
$^{2}D_{\rho\rho\,\frac{5}{2}}^{+}$	2948		3080	-.09	-.98	.05	-.15	-.01				
$^{2}D_{\rho\lambda\,\frac{5}{2}}^{+}$	3432		3155	-.1	.06	.99	-.006	0				
$^{4}D_{\lambda\lambda\,\frac{7}{2}}^{+}$	3365		3090	-.99	.08							

Tables of Selected Results

Spectrum of Ξ_c. Composition

State	Mass	Expt									
$^2S\frac{1}{2}^+$	2461	2465	-.9	-.45	-.02	.04	.008	-.008	0		
$^2S_{\lambda\lambda}\frac{1}{2}^+$	3009		.51	-.88	.13	-.1	-.03	.01	0		
$^2S_{\rho\rho}\frac{1}{2}^+$	3246		.16	-.1	-.98	.04	-.05	.008	0		
$^2S_{\rho\lambda}\frac{1}{2}^+$	3394		.07	-.08	.04	.99	.001	.01	0		
$^4D_{\rho\rho}\frac{1}{2}^+$	3499		.04	-.02	-.03	-.01	.99	-.009	-.016		
$^4S\frac{3}{2}^+$	2682	2645	.89	.18	.5	0	-.01	-.01	.01	.008	0
$^4S_{\rho\rho}\frac{3}{2}^+$	2867		.09	-.8	.01	.07	-.001	-.01	-.64	.01	0
$^4S_{\lambda\lambda}\frac{3}{2}^+$	2963		-.5	-.02	.89	-.02	.02	.019	-.01	-.09	-.001
$^2D_{\lambda\lambda}\frac{5}{2}^+$	2874		-.97	.04	-.009	-.008	.31				
$^2D_{\rho\rho}\frac{5}{2}^+$	3000		-.04	-.99	.06	.06	0				
$^2D_{\rho\lambda}\frac{5}{2}^+$	3469		.009	-.04	-.99	.006	.006				

Tables of Results

- More experimental data needed in the b sector.
- The c sector fits well.

Parameters

Capstick-Isgur
13 parameters
for u,d,s sector

Dong,Su,Wu
12 parameters
u,d,s sector

Kalman–Tran
6 parameters
u,d,s sector

Kalman–D'Souza one less parameter than Kalman–Tran

Achievements / Results

- The c and b sectors give a good fit to the experimental data with this method/model.
- The s sector is almost as good as the K-T strange sector.   There is a problem with the low lying 1/2+ state, which shows larger mixings.
- The strong coupling of the model is now no longer a lumped parameter, but the physical coupling.