

Analysis tools: the heavy quark sector

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Outline

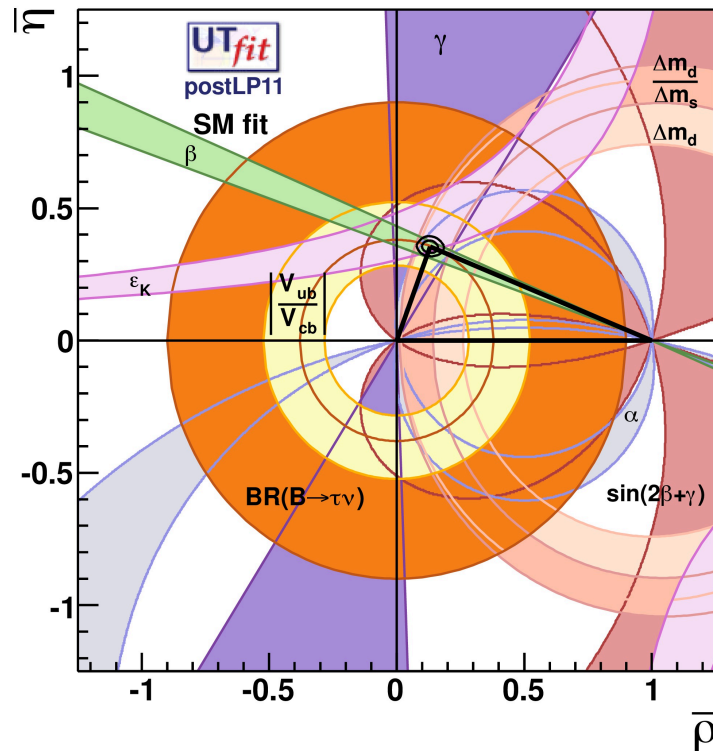
- B and D multibody decays at B factories
 - Why Dalitz analyses were/are so attractive
- Dalitz (time dependent) analysis
 - Measurement of γ
 - Measurement of α
 - D-mixing
- Some very recent highlights

Many thanks to my BaBar colleagues for figures and plots

B factory (initial) mission



Cabibbo
Kobayashi
Maskawa V_{ij}
Unitarity
triangle



- Study CKM matrix parameter
- Search for deviations from Standard Model predictions (the Holy Graal of New Physics...)

The hadronic uncertainties



B → Kπ system

Charming Penguin $\sim \lambda^2$

$V_{us} V_{ub}^* \sim \lambda^4$

Buras & Silvestrini
hep-ex/9812329

$$\begin{aligned}
 A(B^0 \rightarrow K^+ \pi^-) &= V_{ts} V_{tb}^* \times P_1 - V_{us} V_{ub}^* \times \{E_1 - P_1^{GIM}\} \\
 \sqrt{2} \cdot A(B^0 \rightarrow K^0 \pi^0) &= -V_{ts} V_{tb}^* \times P_1 - V_{us} V_{ub}^* \times \{E_2 + P_1^{GIM}\} \\
 \sqrt{2} \cdot A(B^+ \rightarrow K^+ \pi^0) &= V_{ts} V_{tb}^* \times P_1 - V_{us} V_{ub}^* \times \{E_1 + E_2 + A_1 - P_1^{GIM}\} \\
 A(B^+ \rightarrow K^0 \pi^+) &= -V_{ts} V_{tb}^* \times P_1 + V_{us} V_{ub}^* \times \{A_1 - P_1^{GIM}\}
 \end{aligned}$$

- ❖ Initially thought to be a good place to measure CKM gamma angle
- ❖ In fact a place where **hadronic interactions** played a basic role.
- ❖ Theorists argued for years they can “**predict**” hadronic interactions effects, in fact selling their “**model**”...
- ❖ An experiment driven approach (Ciuchini et al) was more solid (but not satisfactory)

***What about measuring hadronic interaction directly ?
Angular analysis in B or D multibody decays***

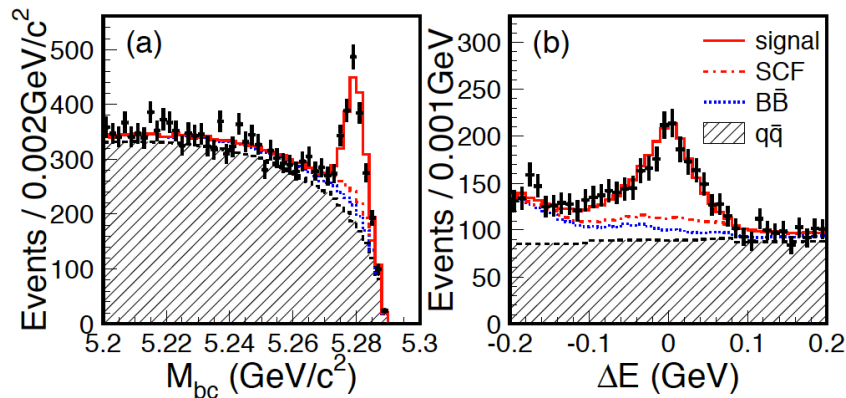
B and D meson multibody decays



Belle Coll., PRL. 98 (2007) 221602

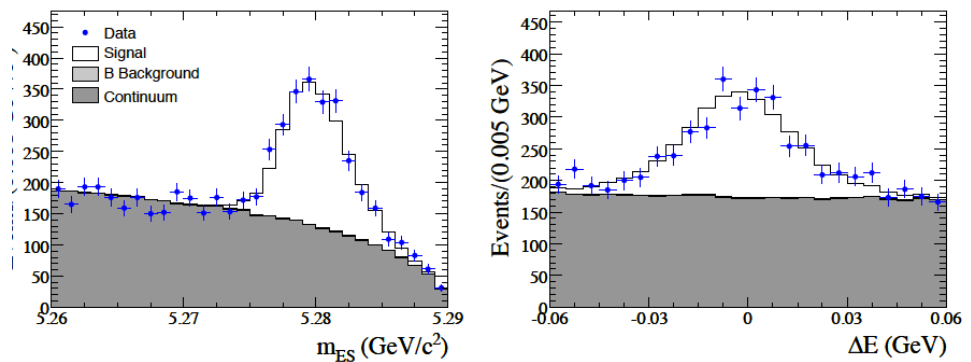
449M BB pairs.

$B^0 \rightarrow \pi^+\pi^-\pi^0$



BaBar, sub PRD, arXiv:1201.5897

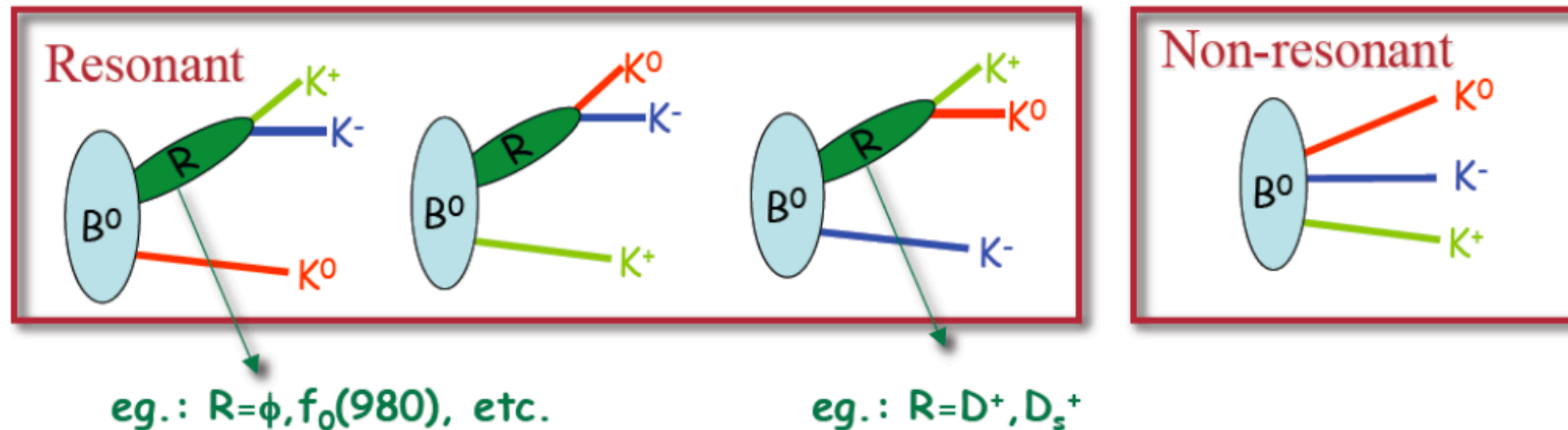
417M BB pairs. $B^0 \rightarrow K^+K^-K_S^0, K_S^0 \rightarrow \pi^+\pi^-$



- ❖ Full reconstruction of multibody final states with high efficiency
- ❖ Great particle ID (pi, K), good KS and pi0 reconstruction
- ❖ Well defined initial state kinematics and good momentum and angular (inv. mass) resolution

Dalitz plot in B and D decays

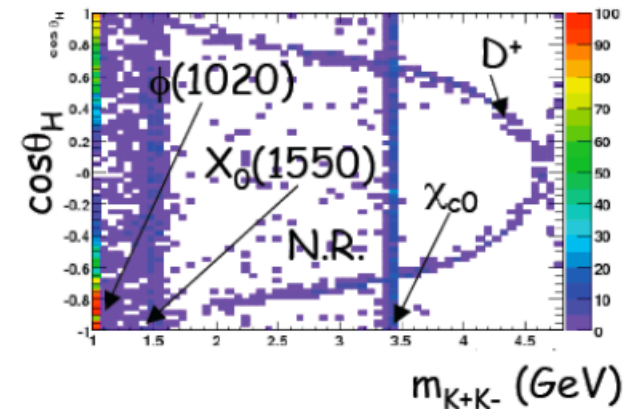
$K^+K^-K^0$ Dalitz plot composition:



$B^0 \rightarrow K^+K^-K^0$ amplitude in the **Isobar Model**:

$$A = \sum_r c_r f_r \quad \begin{array}{l} f_r = \text{resonant amplitudes, NR} \\ c_r = \text{complex isobar coefficients} \end{array}$$

$$\bar{A} = \sum_r \bar{c}_r \bar{f}_r \quad \text{the } \underline{f_r}\text{'s interfere!}$$



Three body decays Dalitz

$$d\Gamma = \frac{|\overline{\mathcal{M}}|^2}{32(2\pi)^3 M_X^3} dm_{BC}^2 dm_{AC}^2$$

X: Heavy meson

Resonance R dominated
amplitude

$$\mathcal{M}(ABC|R) = \sum_{m_\lambda} \langle AB|R_{m_\lambda} \rangle T_R(\sqrt{s_{AB}}) \langle CR|D^0 \rangle$$

$$= Z_\lambda(\vec{p}, \vec{q}) B_{D^0}^{\prime L}(p) B_R^{\prime L}(q) T_R(\sqrt{s_{AB}}).$$

Angular distribution
(Zemach/Helicity
formalism)

Form factor(s)
(Blatt-Weisskopf)

Propagator
(Breit Wigner)

Isobar model

(not overlapping resonances R)

$$\mathcal{M}(ABC) = c_0 + \sum_R c_R \mathcal{M}(ABC|R)$$



D^0 decays and Charged B decays into D^0K

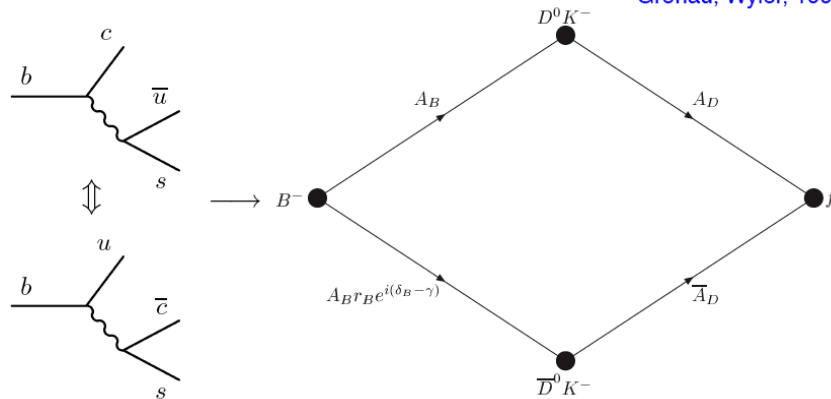
Measuring CKM γ

BaBar Coll, Phys.Rev.Lett.105:121801,2010

Weak and strong interaction



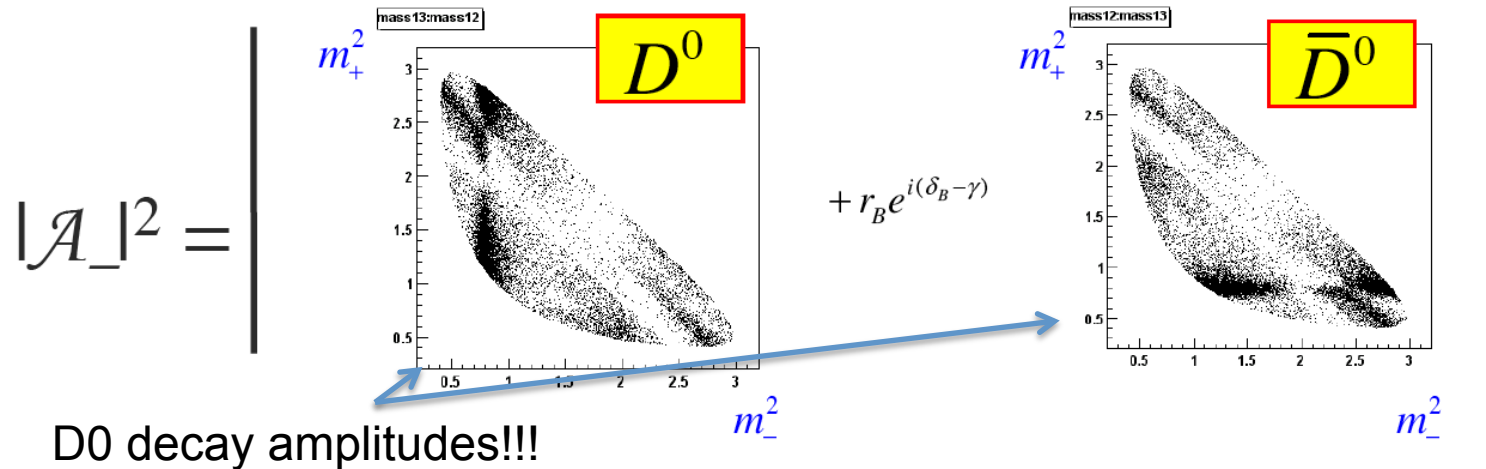
Gronau, Wyler, 1991



$$r_B = \frac{|\mathcal{A}(B^- \rightarrow \bar{D}^0 K^-)|}{|\mathcal{A}(B^- \rightarrow D^0 K^-)|} \approx 0.1 - 0.3$$

δ_B = strong phase
 γ = weak phase

Pictorial view of decay rate of $B^- \rightarrow D^0 K^-$ with $D^0 \rightarrow K_S \pi^+ \pi^-$



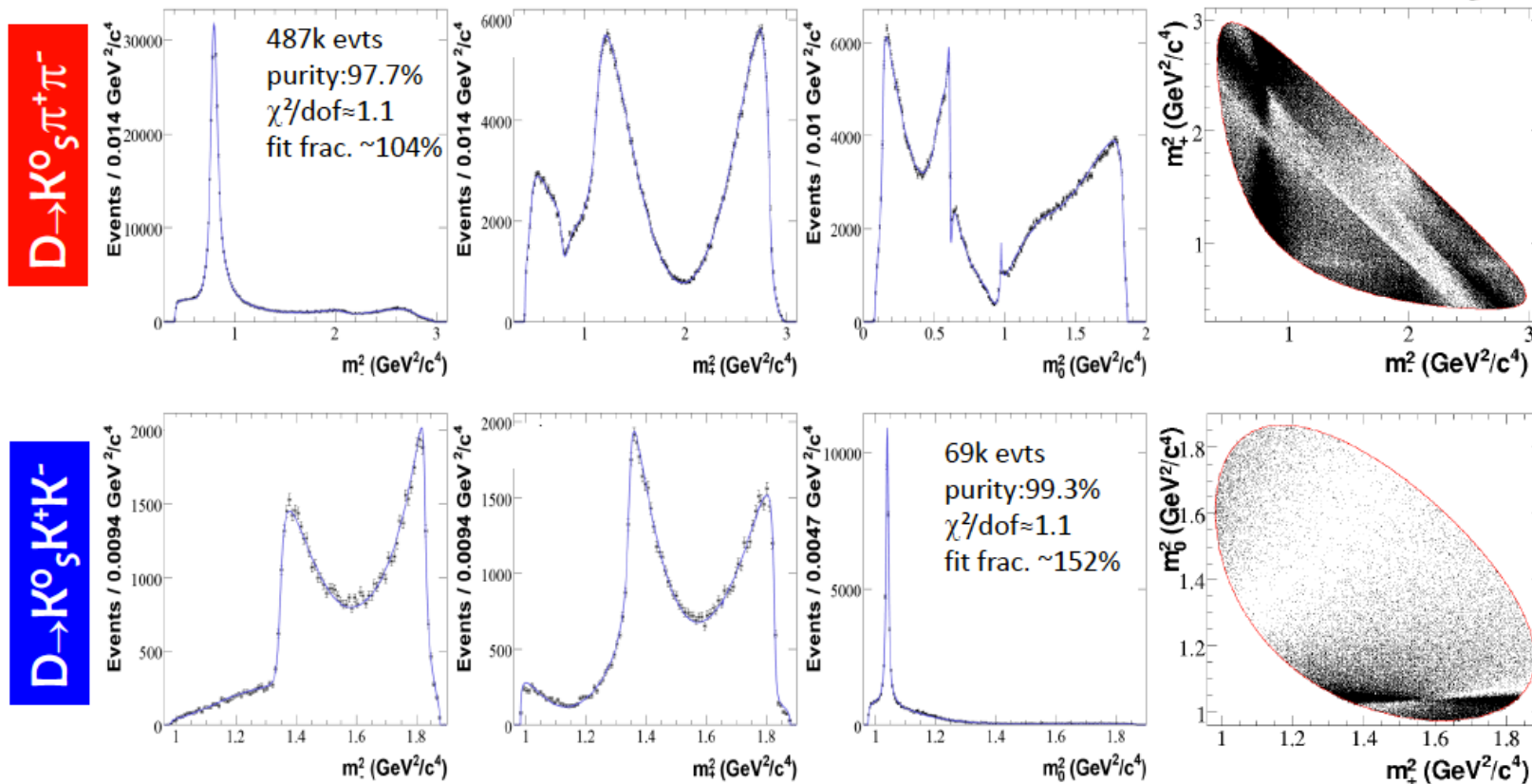
2

D0 decay amplitudes!!!

D^0 decay amplitude used as *input* for B decay amplitude description

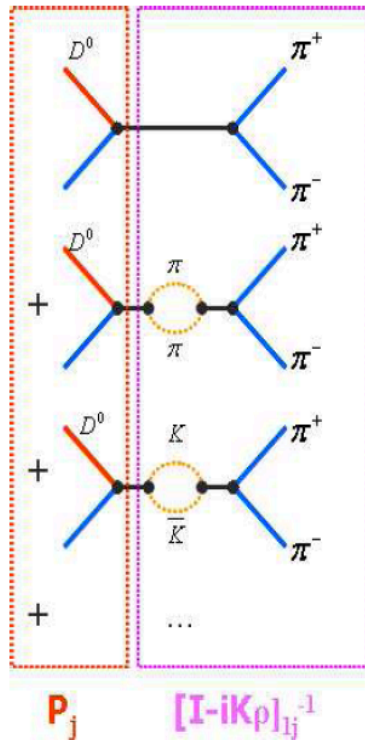
Dalitz models

- $K^0_S \pi \pi$: 10 BW resonances (isobar) for **P(dominant)-D waves** + K-matrix for $\pi\pi$ (first $>4\sigma$ evidence!) and $K\pi$ (LASS) **S-waves** (K-matrix formalism deals with broad, overlapping, multi-channels resonances).
- $K^0_S KK$: isobar model used for the first time ($K^0_S \alpha_0(980)$ S-wave and $K^0_S \phi(1020)$ P-waves dominate).



$\pi\pi$ S-wave K-matrix

No clear indication from scattering exp of BW structure
(<http://arxiv.org/abs/hep-ph/9611235>)



$$T_R \equiv \hat{F}_i = (I - i\hat{K}\rho)_{ij}^{-1} \hat{P}_j.$$

Scattering matrix

V. V. Anisovich and A. V. Sarantsev,
Eur. Phys. J. A 16 (2003) 229 [arXiv:hep-ph/0204328].

$$\hat{K}_{ij}(s) = \left(f_{ij}^{scatt} \frac{1 - s_0^{scatt}}{s - s_0^{scatt}} + \sum_{\alpha} \frac{g_i^{\alpha} g_j^{\alpha}}{m_{\alpha}^2 - s} \right) \left\{ \frac{1 - s_{A0}}{s - s_{A0}} \left(s - s_A \frac{m_{\pi}^2}{2} \right) \right\}.$$

Production vector

Parameters fitted!

$$\hat{P}_j(s) = f_{1j}^{prod} \frac{1 - s_0^{prod}}{s - s_0^{prod}} + \sum_{\alpha} \frac{\beta_{\alpha} g_j^{\alpha}}{m_{\alpha}^2 - s}.$$

j as $\pi\pi$, KK , $\eta\eta$, $\eta'\eta'$, and 4π .

31: An intuitive description of the K-matrix representation of transition amplitude.

Dalitz model details

Component	a_r	ϕ_r (deg)	Fraction (%)
$K^*(892)^-$	1.740 ± 0.010	139.0 ± 0.3	55.7 ± 2.8
$K_0^*(1430)^-$	8.2 ± 0.7	153 ± 8	10.2 ± 1.5
$K_2^*(1430)^-$	1.410 ± 0.022	138.4 ± 1.0	2.2 ± 1.6
$K^*(1680)^-$	1.46 ± 0.10	-174 ± 4	0.7 ± 1.9
$K^*(892)^+$	0.158 ± 0.003	-42.7 ± 1.2	0.46 ± 0.23
$K_0^*(1430)^+$	0.32 ± 0.06	143 ± 11	< 0.05
$K_2^*(1430)^+$	0.091 ± 0.016	85 ± 11	< 0.12
$\rho(770)^0$	1	0	21.0 ± 1.6
$\omega(782)$	0.0527 ± 0.0007	126.5 ± 0.9	0.9 ± 1.0
$f_2(1270)$	0.606 ± 0.026	157.4 ± 2.2	0.6 ± 0.7
β_1	9.3 ± 0.4	-78.7 ± 1.6	
β_2	10.89 ± 0.26	-159.1 ± 2.6	
β_3	24.2 ± 2.0	168 ± 4	
β_4	9.16 ± 0.24	90.5 ± 2.6	
f_{11}^{prod}	7.94 ± 0.26	73.9 ± 1.1	
f_{12}^{prod}	2.0 ± 0.3	-18 ± 9	
f_{13}^{prod}	5.1 ± 0.3	33 ± 3	
f_{14}^{prod}	3.23 ± 0.18	4.8 ± 2.5	
s_0^{prod}	-0.07 ± 0.03		
$\pi\pi$ S-wave			11.9 ± 2.6
M (GeV/ c^2)	1.463 ± 0.002		
Γ (GeV/ c^2)	0.233 ± 0.005		
F	0.80 ± 0.09		
ϕ_F	2.33 ± 0.13		
R	1		
ϕ_R	-5.31 ± 0.04		
a	1.07 ± 0.11		
r	-1.8 ± 0.3		

CA $K^*\pi$
DCS $K^*\pi$
 $\pi\pi$ P,D waves

$\pi\pi$ S-wave (K-matrix)

$K\pi$ S-wave

Component	a_r	ϕ_r (deg)	Fraction (%)
$K_S^0 a_0(980)^0$	1	0	55.8
$K_S^0 \phi(1020)$	0.227 ± 0.005	-56.2 ± 1.0	44.9
$K_S^0 f_0(1370)$	0.04 ± 0.06	-2 ± 80	0.1
$K_S^0 f_2(1270)$	0.261 ± 0.020	-9 ± 6	0.3
$K_S^0 a_0(1450)^0$	0.65 ± 0.09	-95 ± 10	12.6
$K^- a_0(980)^+$	0.562 ± 0.015	179 ± 3	16.0
$K^- a_0(1450)^+$	0.84 ± 0.04	97 ± 4	21.8
$K^+ a_0(980)^-$	0.118 ± 0.015	138 ± 7	0.7



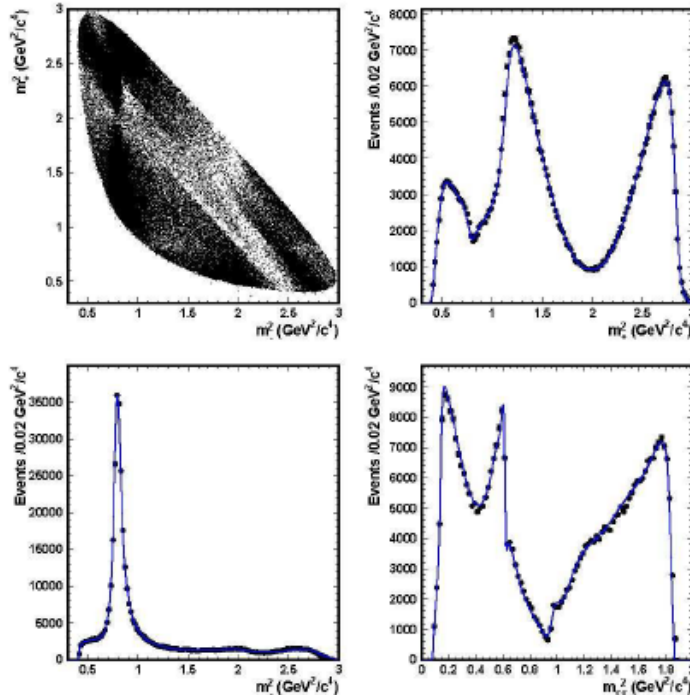
... or alternatively



Dalitz fit

Belle, 540 fb^{-1}

Very pure sample (95%)

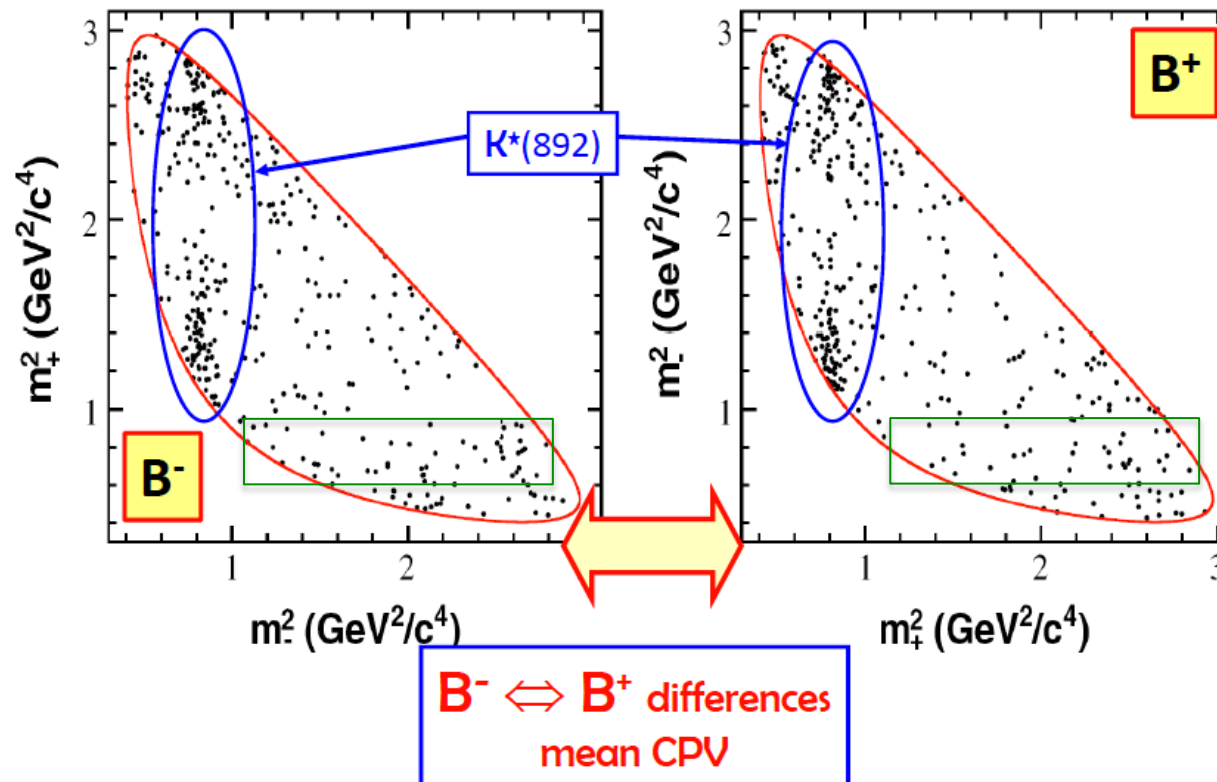


Resonance	Amplitude	Phase (deg)	Fit fraction
$K^*(892)^-$	1.629 ± 0.005	134.3 ± 0.3	0.6227
$K_0^*(1430)^-$	2.12 ± 0.02	-0.9 ± 0.5	0.0724
$K_2^*(1430)^-$	0.87 ± 0.01	-47.3 ± 0.7	0.0133
$K^*(1410)^-$	0.65 ± 0.02	111 ± 2	0.0048
$K^*(1680)^-$	0.60 ± 0.05	147 ± 5	0.0002
$K^*(892)^+$	0.152 ± 0.003	-37.5 ± 1.1	0.0054
$K_0^*(1430)^+$	0.541 ± 0.013	91.8 ± 1.5	0.0047
$K_2^*(1430)^+$	0.276 ± 0.010	-106 ± 3	0.0013
$K^*(1410)^+$	0.333 ± 0.016	-102 ± 2	0.0013
$K^*(1680)^+$	0.73 ± 0.10	103 ± 6	0.0004
$\rho(770)$	1 (fixed)	0 (fixed)	0.2111
$\omega(782)$	0.0380 ± 0.0006	115.1 ± 0.9	0.0063
$f_0(980)$	0.380 ± 0.002	-147.1 ± 0.9	0.0452
$f_0(1370)$	1.46 ± 0.04	98.6 ± 1.4	0.0162
$f_2(1270)$	1.43 ± 0.02	-13.6 ± 1.1	0.0180
$\rho(1450)$	0.72 ± 0.02	40.9 ± 1.9	0.0024
σ_1	1.387 ± 0.018	-147 ± 1	0.0914
σ_2	0.267 ± 0.009	-157 ± 3	0.0088
NR	2.36 ± 0.05	155 ± 2	0.0615

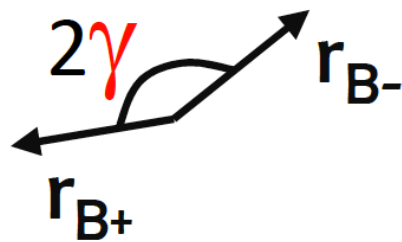
- ◆ Dalitz model: 13 different (BW) resonances and a non-resonant contribution
- ◆ Results with this refined model consistent with the analysis performed for the Belle ϕ_3 measurement, PRD73, 112009 (2006)
- ◆ To test the scalar $\pi\pi$ contributions, K-matrix formalism is also used

CP violation and γ

$$600 \pm 31 \tilde{D}^0 K^-$$



Opposite sign K^*
either Doubly
Cabibbo suppressed
decay
OR
Interference with
 $b \rightarrow u$ transition

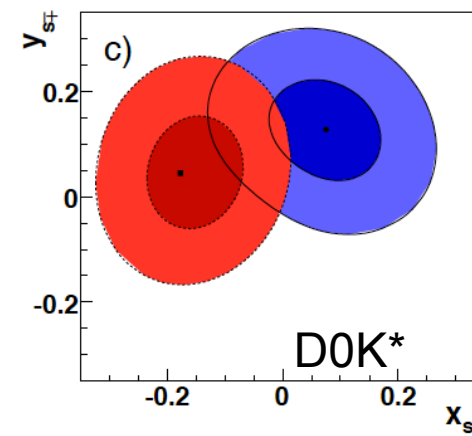
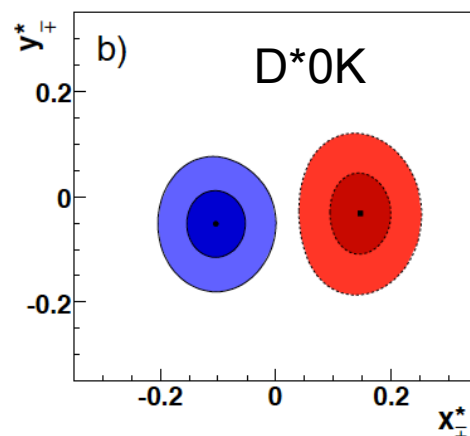
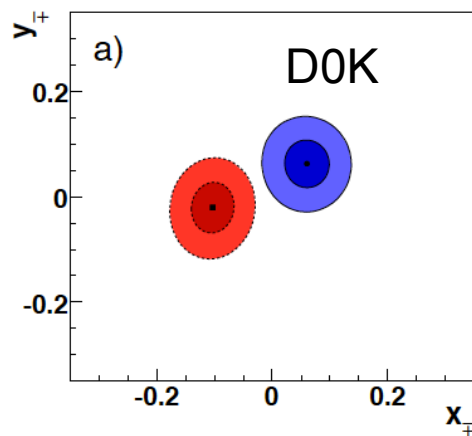


γ extraction

Fit results: 12 (3x4) Cartesian coordinates

$$z^{(*)}_{(s)\pm} \equiv (x^{(*)}_{(s)\pm}, y^{(*)}_{(s)\pm}) \\ = (\text{Re}, \text{Im}) \{ r^{(*)}_{sB} e^{i(\delta^{(*)}_{(s)B} \pm \gamma)} \}$$

→ Almost Gaussian behavior near physical bounds ($r_B \approx 0$), better than the 7 physics params.



Babar obtains:

$$\gamma = (68^{+15}_{-14} \pm 4 \pm 3)^\circ$$

CP violation at 3.5sigmas!

Model systematics

$B^0 \rightarrow \pi^+ \pi^- \pi^0$ time-dependent Dalitz analysis

The best place to measure alpha
with no hadronic hassles...

[BaBar Coll, PRD 76 \(2007\) 012004](#)

Time-dependence

Time-dependent Dalitz analysis

$$d\Gamma(B^0 \rightarrow \pi^+ \pi^- \pi^0) = \frac{1}{(2\pi)^3} \frac{|A_{3\pi}|^2}{8m_{B^0}^3} ds_+ ds_-$$

Mandelstam variables $s_+ = (p_+ + p_0)^2$
 $s_- = (p_- + p_0)^2$

$$|\mathcal{A}_{3\pi}^{\pm}(\Delta t)|^2 = \left[|A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2 \mp (|A_{3\pi}|^2 - |\bar{A}_{3\pi}|^2) \cos(\Delta m_d \Delta t) \right. \\ \left. \pm 2\text{Im} [\bar{A}_{3\pi} A_{3\pi}^*] \sin(\Delta m_d \Delta t) \right]$$

+ (-)
 B^0 bar (B^0)
 decaying

$$A_{3\pi} = f_+ A^+ + f_- A^- + f_0 A^0$$

$$\bar{A}_{3\pi} = f_+ \bar{A}^+ + f_- \bar{A}^- + f_0 \bar{A}^0$$

charge of
 the ρ

Isobar
 model: $f(s_i, s_j)$

Weak and
 strong phase

$$A^\kappa = T^\kappa e^{-i\alpha} + P^\kappa$$

Bilinear coefficients, U and I



$$|A_{3\pi}|^2 \pm |\bar{A}_{3\pi}|^2$$

$$\sum_{\kappa \in \{+, -, 0\}} |f_\kappa|^2 U_\kappa^\pm + 2 \sum_{\kappa < \sigma \in \{+, -, 0\}} (\text{Re}[f_\kappa f_\sigma^*] U_{\kappa\sigma}^{\pm, \text{Re}} - \text{Im}[f_\kappa f_\sigma^*] U_{\kappa\sigma}^{\pm, \text{Im}})$$

$$\text{Im}(\bar{A}_{3\pi} A_{3\pi}^*)$$

$$\sum_{\kappa \in \{+, -, 0\}} |f_\kappa|^2 I_\kappa + \sum_{\kappa < \sigma \in \{+, -, 0\}} (\text{Re}[f_\kappa f_\sigma^*] I_{\kappa\sigma}^{\text{Im}} + \text{Im}[f_\kappa f_\sigma^*] I_{\kappa\sigma}^{\text{Re}})$$

27 real parameters

- Normalized to U_+^+ determined from the fit
- Unique solution, gaussian!!
- Easy combination among exp.'s

κ, σ :
charge of
the ρ

$$\begin{aligned} U_\kappa^\pm &= |A^\kappa|^2 \pm |\bar{A}^\kappa|^2, \\ U_{\kappa\sigma}^{\pm, \text{Re(Im)}} &= \text{Re(Im)} [A^\kappa A^{\sigma*} \pm \bar{A}^\kappa \bar{A}^{\sigma*}], \\ I_\kappa &= \text{Im} [\bar{A}^\kappa A^{\kappa*}], \\ I_{\kappa\sigma}^{\text{Re}} &= \text{Re} [\bar{A}^\kappa A^{\sigma*} - \bar{A}^\sigma A^{\kappa*}], \\ I_{\kappa\sigma}^{\text{Im}} &= \text{Im} [\bar{A}^\kappa A^{\sigma*} + \bar{A}^\sigma A^{\kappa*}]. \end{aligned}$$

Branching
fractions and
direct CPV

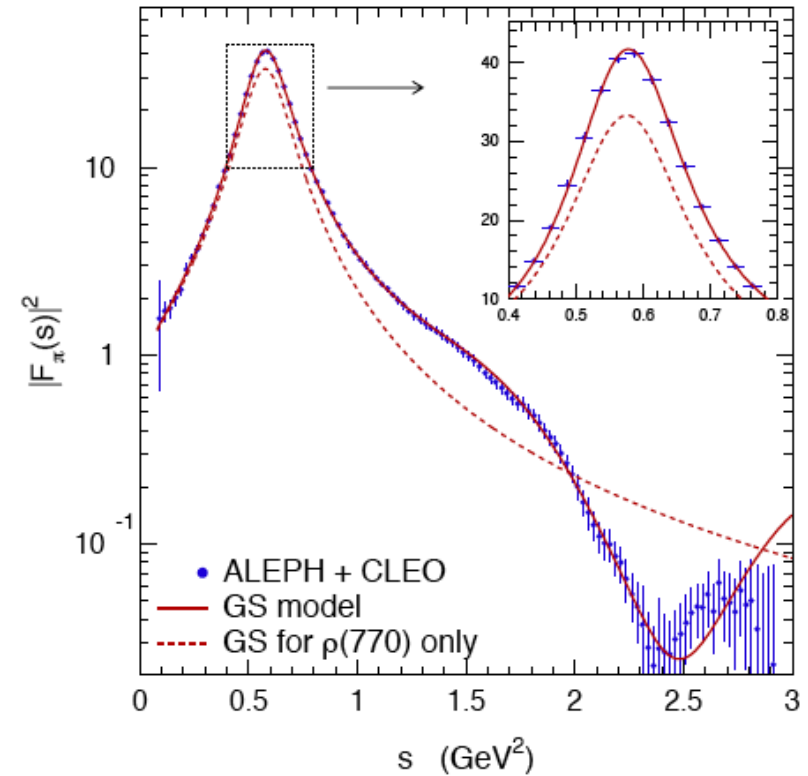
Mixing induced
CPV

Sensitive to
interference
pattern!

Dalitz Model



- ρ dominated
 - Rel. BW Gounaris-Sakurai
 - » $\rho(770)$, $\rho(1450)$, $\rho(1700)$ mass and width from τ and e^+e^- data (Alep & CMD-2)
 - » $\pi^+\pi^-$ scalar allowed in systematics evaluation
 - » Crosscheck relative phase on signal data



Square Dalitz Plot



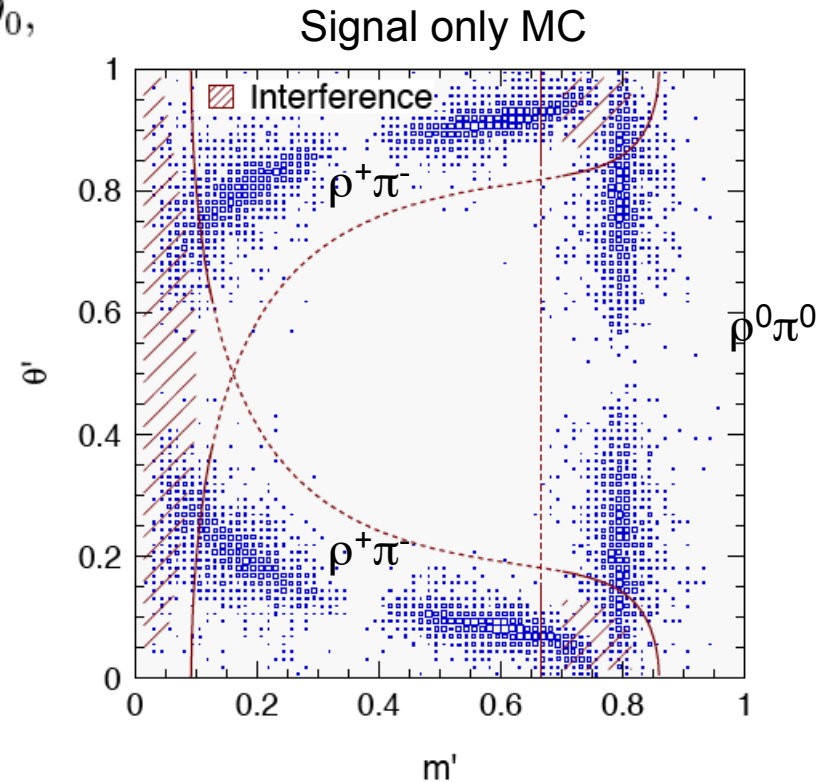
$$m' \equiv \frac{1}{\pi} \arccos \left(2 \frac{m_0 - m_0^{\min}}{m_0^{\max} - m_0^{\min}} - 1 \right), \quad \theta' \equiv \frac{1}{\pi} \theta_0,$$

Helicity
angle

Most of sensitivity from
interference region!

*Measure terms like \cos
 $(2\alpha)\sin(\Delta m_d \Delta t)$*

*Extract strong phases
directly*



Dashed lines: isocontour for $\sqrt{s_{+,-,0}} = 1.5 \text{ GeV}/c^2$

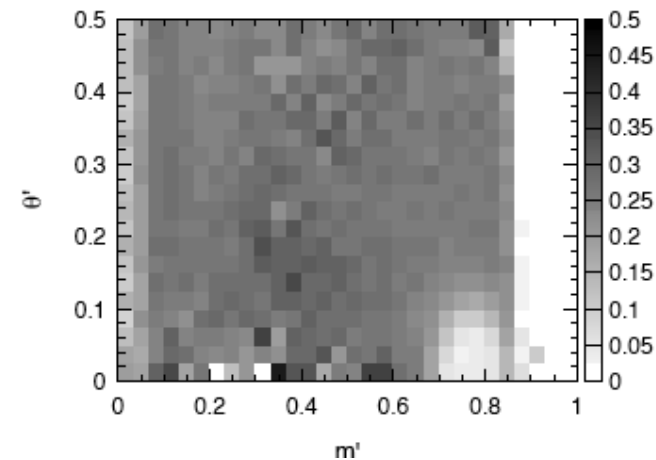
Signal description

- 22% $\rho^+\pi^-$ and 13% $\rho^0\pi^0$ are misreconstructed (track or cluster from other B)
 - » Self-Cross Feed events
 - » Concentrate in corners: where interference sits...

- Detailed parametrization on Dalitz

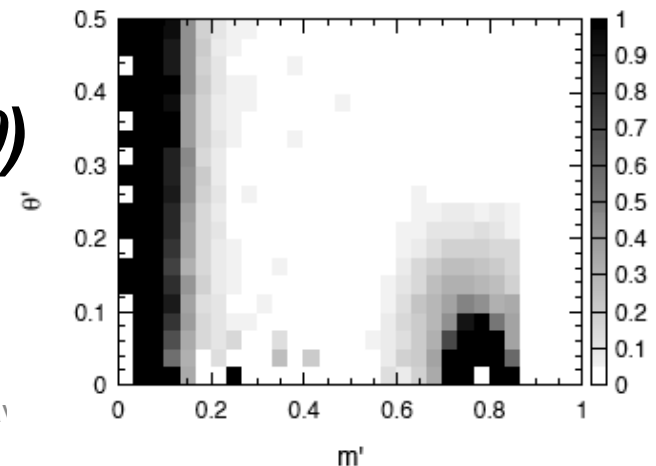
- efficiency $\varepsilon(m', \theta)$
- SCF rate $f_{SCF}(m', \theta)$ and
- SCF resolution function on Dalitz

$$\varepsilon(m', \theta)$$



- Δt resolution

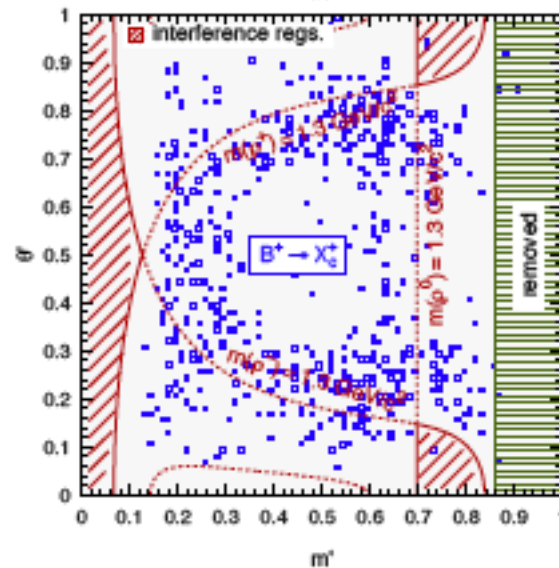
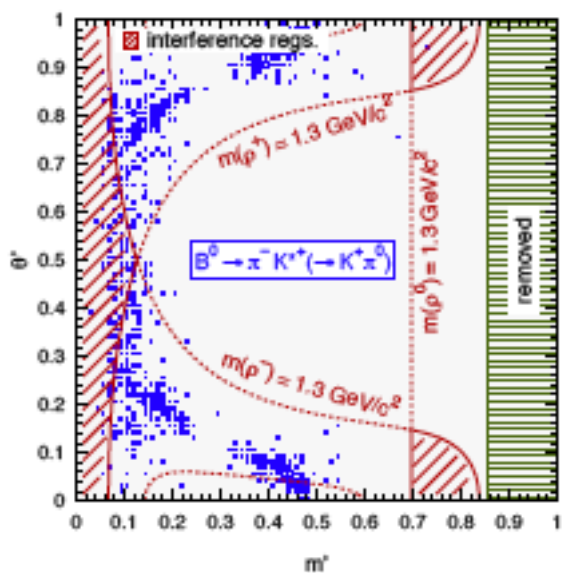
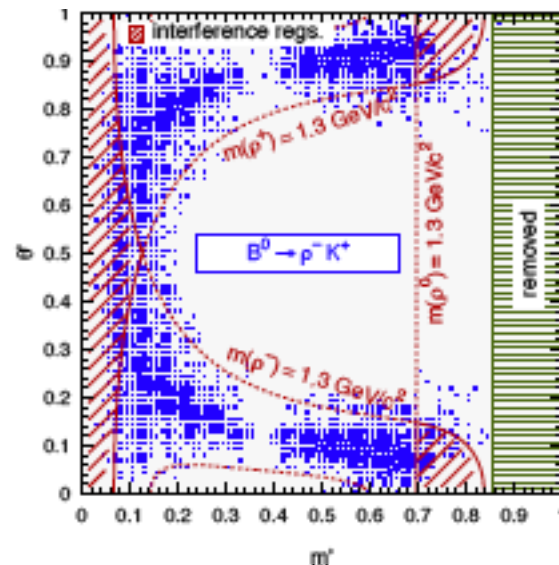
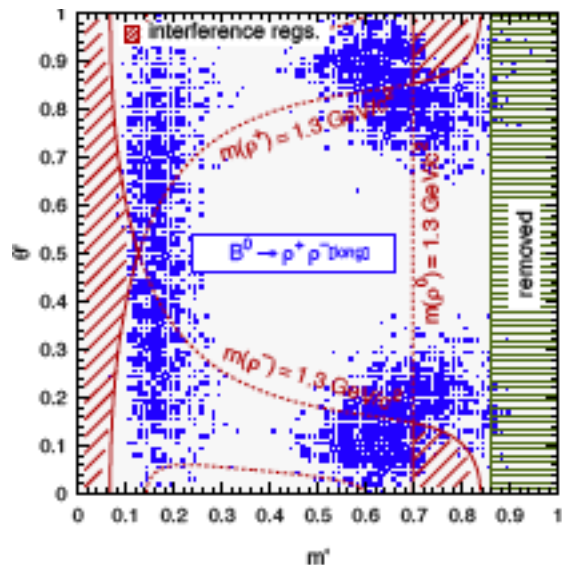
$$f_{SCF}(m', \theta)$$



B-background Dalitz distributions



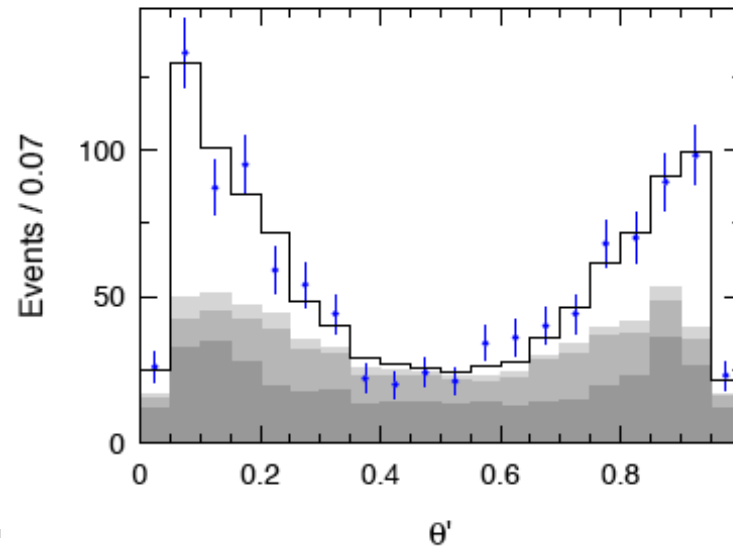
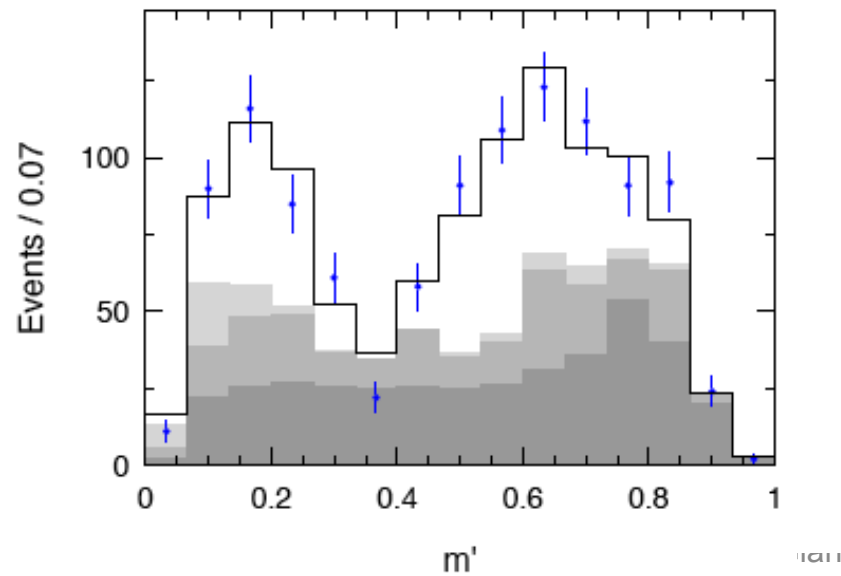
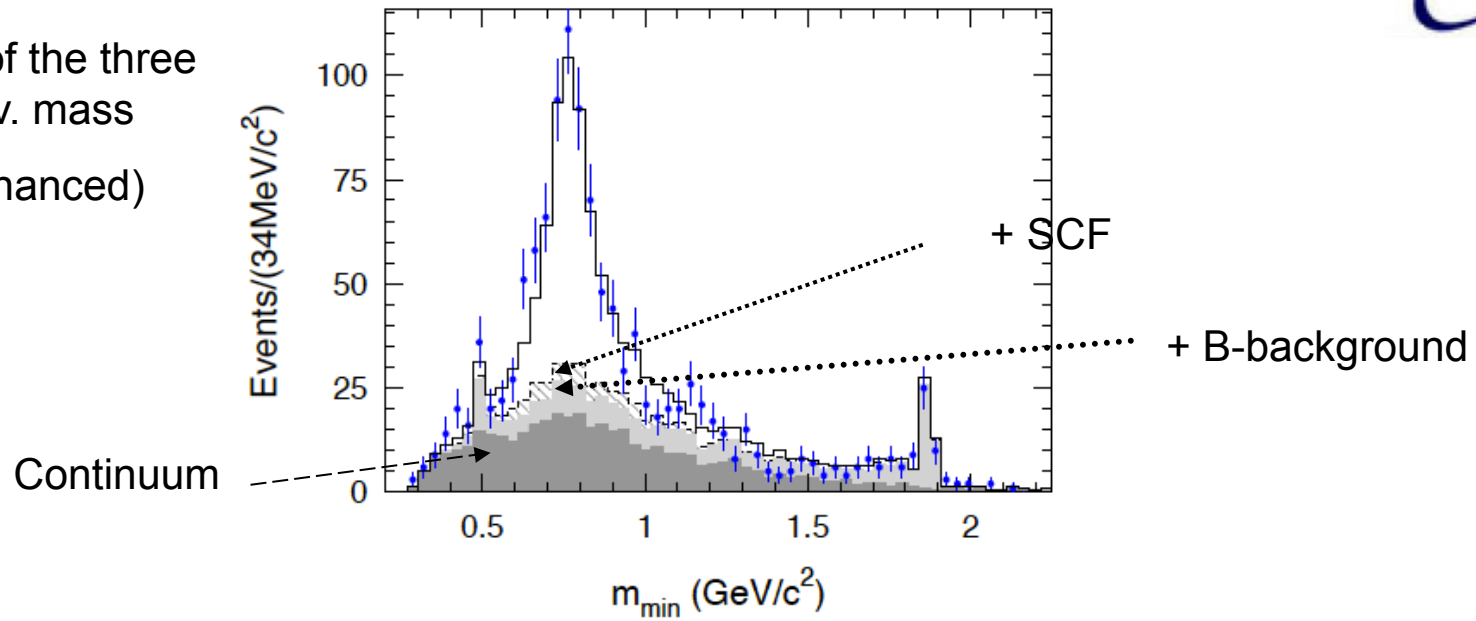
MC simulation



Dalitz distributions



Smallest of the three
di-pion inv. mass
(signal enhanced)



Fit results



Parameter	Description	Result
U_+^+	Coefficient of $ f_+ ^2$	1.0 (fixed)
U_0^+	Coefficient of $ f_0 ^2$	$0.28 \pm 0.07 \pm 0.04$
U_-^+	Coefficient of $ f_- ^2$	$1.32 \pm 0.12 \pm 0.05$
U_0^-	Coefficient of $ f_0 ^2 \cos(\Delta m_d \Delta t)$	$-0.03 \pm 0.11 \pm 0.09$
U_-^-	Coefficient of $ f_- ^2 \cos(\Delta m_d \Delta t)$	$-0.32 \pm 0.14 \pm 0.05$
U_+^-	Coefficient of $ f_+ ^2 \cos(\Delta m_d \Delta t)$	$0.54 \pm 0.15 \pm 0.05$
I_0	Coefficient of $ f_0 ^2 \sin(\Delta m_d \Delta t)$	$0.01 \pm 0.06 \pm 0.01$
I_-	Coefficient of $ f_- ^2 \sin(\Delta m_d \Delta t)$	$-0.01 \pm 0.10 \pm 0.02$
I_+	Coefficient of $ f_+ ^2 \sin(\Delta m_d \Delta t)$	$-0.02 \pm 0.10 \pm 0.03$
$U_{+-}^{+,Im}$	Coefficient of $\text{Im}[f_+ f_-^*]$	$-0.07 \pm 0.71 \pm 0.73$
$U_{+-}^{+,Re}$	Coefficient of $\text{Re}[f_+ f_-^*]$	$0.17 \pm 0.49 \pm 0.31$
$U_{+-}^{-,Im}$	Coefficient of $\text{Im}[f_+ f_-^*] \cos(\Delta m_d \Delta t)$	$-0.38 \pm 1.06 \pm 0.36$
$U_{+-}^{-,Re}$	Coefficient of $\text{Re}[f_+ f_-^*] \cos(\Delta m_d \Delta t)$	$2.23 \pm 1.00 \pm 0.43$
I_{+-}^{Im}	Coefficient of $\text{Im}[f_+ f_-^*] \sin(\Delta m_d \Delta t)$	$-1.99 \pm 1.25 \pm 0.34$
I_{+-}^{Re}	Coefficient of $\text{Re}[f_+ f_-^*] \sin(\Delta m_d \Delta t)$	$1.90 \pm 2.03 \pm 0.65$
$U_{+0}^{+,Im}$	Coefficient of $\text{Im}[f_+ f_0^*]$	$-0.16 \pm 0.57 \pm 0.14$
$U_{+0}^{+,Re}$	Coefficient of $\text{Re}[f_+ f_0^*]$	$-1.08 \pm 0.48 \pm 0.20$
$U_{+0}^{-,Im}$	Coefficient of $\text{Im}[f_+ f_0^*] \cos(\Delta m_d \Delta t)$	$-1.66 \pm 0.94 \pm 0.25$
$U_{+0}^{-,Re}$	Coefficient of $\text{Re}[f_+ f_0^*] \cos(\Delta m_d \Delta t)$	$-0.18 \pm 0.88 \pm 0.35$
I_{+0}^{Im}	Coefficient of $\text{Im}[f_+ f_0^*] \sin(\Delta m_d \Delta t)$	$-0.21 \pm 1.06 \pm 0.25$
I_{+0}^{Re}	Coefficient of $\text{Re}[f_+ f_0^*] \sin(\Delta m_d \Delta t)$	$0.41 \pm 1.30 \pm 0.41$
$U_{-0}^{+,Im}$	Coefficient of $\text{Im}[f_- f_0^*]$	$-0.17 \pm 0.50 \pm 0.23$
$U_{-0}^{+,Re}$	Coefficient of $\text{Re}[f_- f_0^*]$	$-0.36 \pm 0.38 \pm 0.08$
$U_{-0}^{-,Im}$	Coefficient of $\text{Im}[f_- f_0^*] \cos(\Delta m_d \Delta t)$	$0.12 \pm 0.75 \pm 0.22$
$U_{-0}^{-,Re}$	Coefficient of $\text{Re}[f_- f_0^*] \cos(\Delta m_d \Delta t)$	$-0.63 \pm 0.72 \pm 0.32$
I_{-0}^{Im}	Coefficient of $\text{Im}[f_- f_0^*] \sin(\Delta m_d \Delta t)$	$1.23 \pm 1.07 \pm 0.29$
I_{-0}^{Re}	Coefficient of $\text{Re}[f_- f_0^*] \sin(\Delta m_d \Delta t)$	$0.41 \pm 1.30 \pm 0.21$

“Q-2body”

“interference”

U_+^+ average BF($\rho^+\pi^-$)

Excess of $\rho^0\pi^0$

U_k^-
Direct CPV $\rho^+\pi^-$

Mixing induced CPV

Statistical and systematic errors

Full covariance Matrix available

(both statistical and systematics)

Finally, α and δ

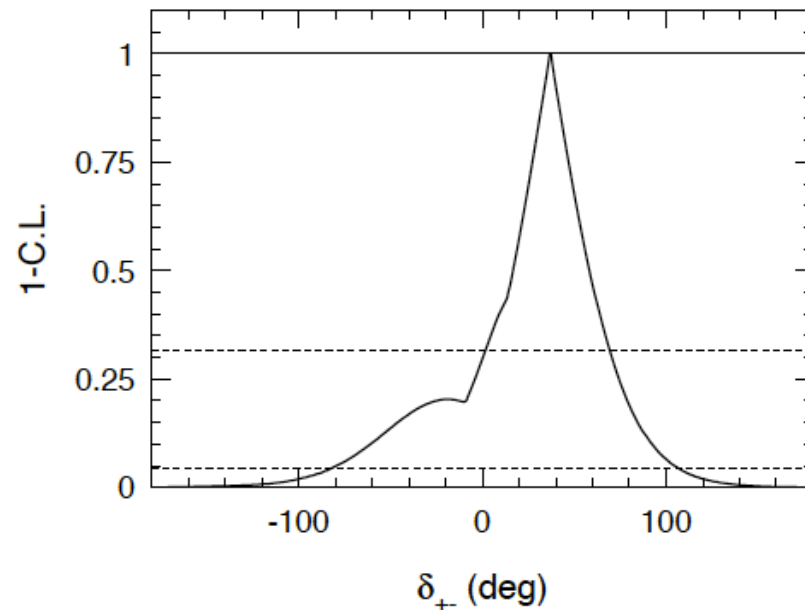
Relative phase $B^0 \rightarrow \rho^- \pi^+$ and $B^0 \rightarrow \rho^+ \pi^-$

$$\delta_{+-} = \arg(A^{+*} A^-)$$

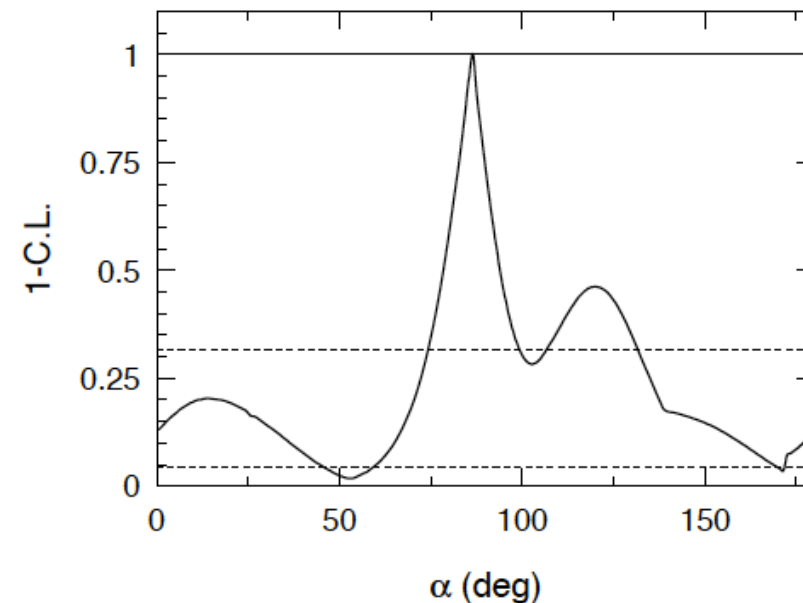
$$A^\kappa = T^\kappa e^{-i\alpha} + P^\kappa$$

$$\bar{A}^\kappa = T^{\bar{\kappa}} e^{+i\alpha} + P^{\bar{\kappa}}$$

Strong isospin: $P^0 = -(P^+ + P^-)/2$



$$\delta_{+-} = (34 \pm 29)^\circ$$



No constraint on α at 2σ



$D^0 \rightarrow K^+ \pi^- \pi^0$ time-dependent Dalitz analysis

A constraint on D^0 mixing parameters

Neutral mesons oscillations

Time evolution for meson of *known flavor at t=0*

“Short-range” $x = \frac{m_2 - m_1}{\Gamma}$ $\Gamma = \frac{\Gamma_2 + \Gamma_1}{2}$

“Long-range” $y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$

$$|M^0(t)\rangle = e^{-\bar{\gamma}t/2} \left(\cosh(\Delta\gamma t/2) |M^0\rangle - \frac{q}{p} \sinh(\Delta\gamma t/2) |\bar{M}^0\rangle \right)$$

Where $\Delta\gamma = (y + ix)\Gamma$ $\bar{\gamma} = (\Gamma_1 + \Gamma_2)/2 - i(m_1 + m_2)$

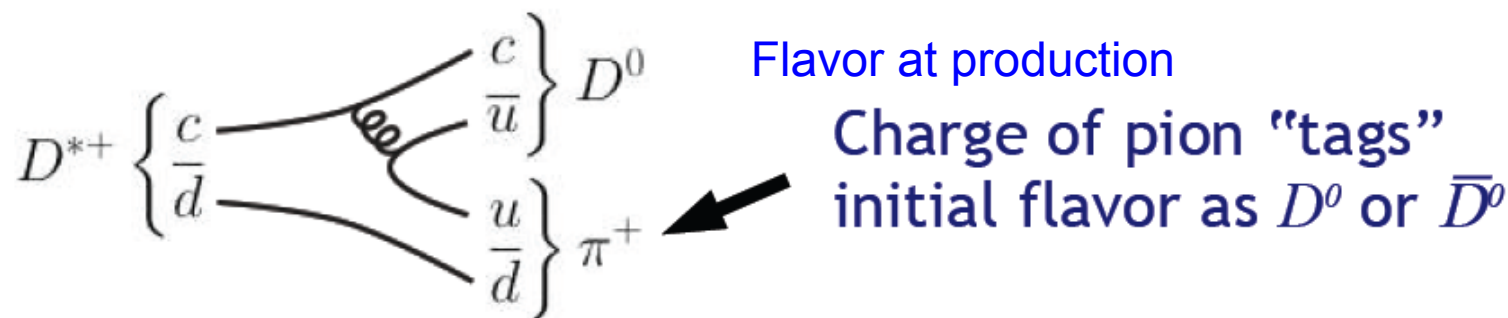
M^0 “oscillates” into \bar{M}^0 !
(also dubbed “mixing”)

An opposite flavor component appears after a while!

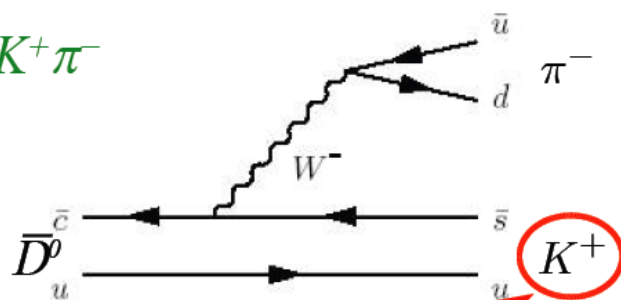


Flavor tagging

Use D^0 from $D^{*+} \rightarrow D^0 \pi^+$ decays:



$\bar{D}^0 \rightarrow K^+ \pi^-$



Flavor at decay

- Same flavour: Wrong-Sign (WS) mixing *may have occurred*
- Opposite flavour: Right-Sign (RS) unmixed events

Charge of K identifies decay flavor

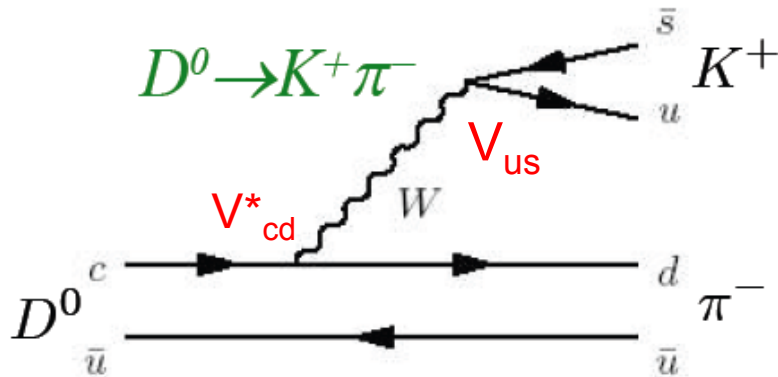
$$\bar{A}_f \equiv \langle f | H | \bar{D}^0 \rangle$$



Double-Cabibbo Suppressed Decays

Hadronic decays do not uniquely identify decay flavor
 Get unmixed wrong-sign decays from DCS decays

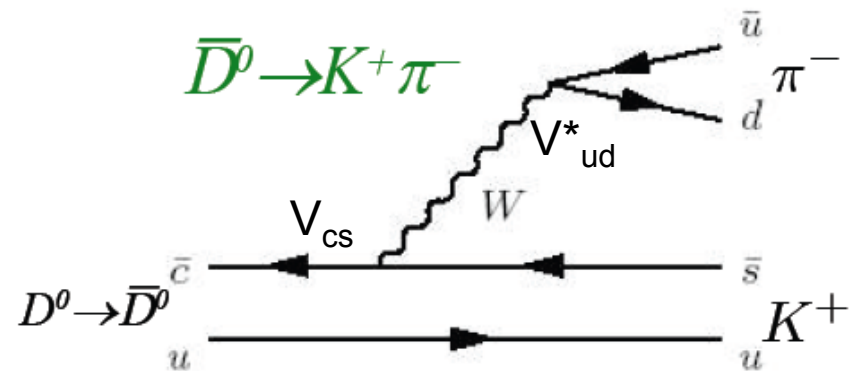
DCS decay:



Relative rate ~0.3%

$$A_f \equiv \langle f | H | D^0 \rangle$$

Mixed decay:



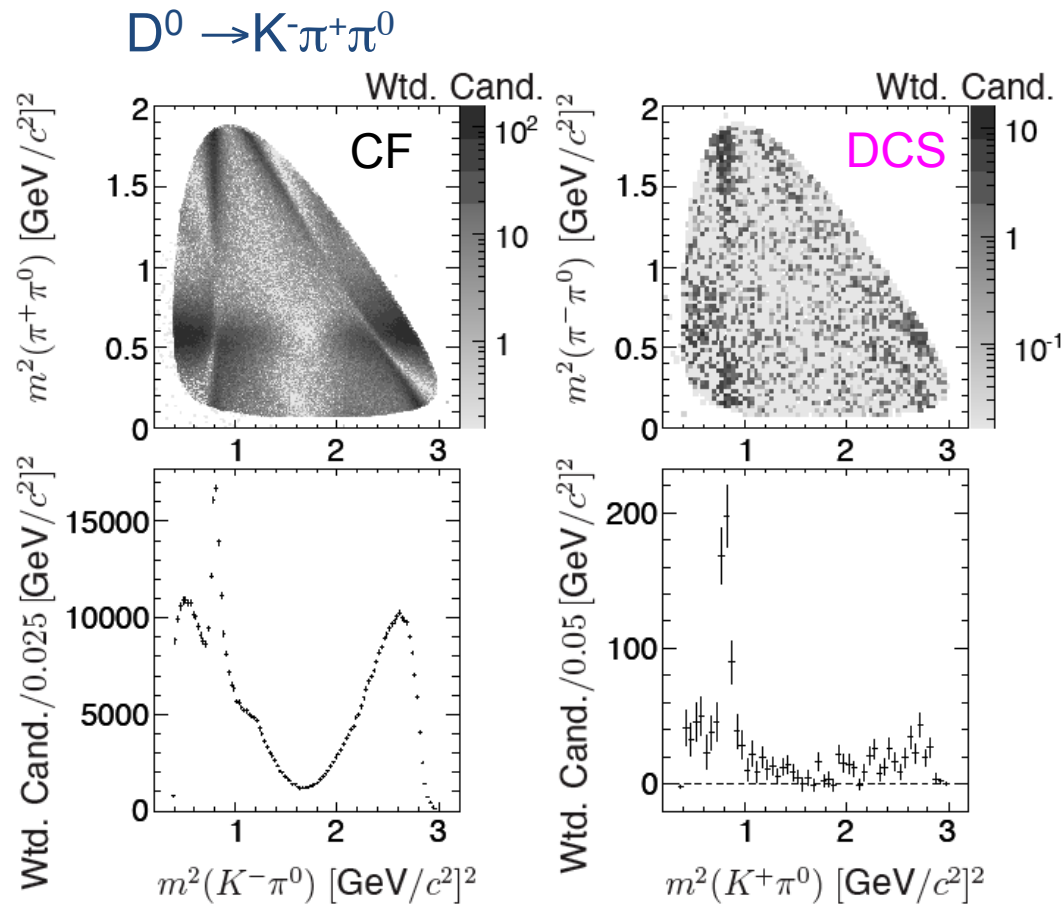
Relative rate: 0.005% (for $x=0.01$)



Separating x and y

First evidence *Phys.Rev.Lett.* 98 (2007) 211802 in $D^0 \rightarrow K\pi$
BUT $K\pi$ only cannot separate x and y

Need info on **strong phases**



DCS decays proceed primarily through $K^{*+}\pi^-$ while **CF** through $K\rho^+$



Mixing in $D^0 \rightarrow K^+ \pi^- \pi^0$

$$D^0 \rightarrow K^+ \pi^- \pi^0$$

Two ways to reach same final state \Rightarrow interference!

$$D^0 \xrightarrow{\text{mix}} \bar{D}^0 \rightarrow K^+ \pi^- \pi^0$$

Time dependent WS rate

$$\Gamma_{\bar{f}}(s_{12}, s_{13}, t) = e^{-\Gamma t} \left\{ |A_{\bar{f}}|^2 \quad \leftarrow \boxed{\text{DCS}} \right. \\ \left. + |A_{\bar{f}}| |\bar{A}_{\bar{f}}| [y'' \cos \delta_{\bar{f}} - x'' \sin \delta_{\bar{f}}] (\Gamma t) \quad \leftarrow \boxed{\text{Interference}} \right. \\ \left. + \frac{x''^2 + y''^2}{4} |\bar{A}_{\bar{f}}|^2 (\Gamma t)^2 \right\} \quad \leftarrow \boxed{\text{Mixing}}$$

$$\bar{f} = K^+ \pi^- \pi^0 \quad A_{\bar{f}} = \langle \bar{f} | \mathcal{H} | D^0 \rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f} | \mathcal{H} | \bar{D}^0 \rangle$$

$$y'' = y \cos \delta_{K\pi\pi^0} - x \sin \delta_{K\pi\pi^0}$$

$$x'' = x \cos \delta_{K\pi\pi^0} + y \sin \delta_{K\pi\pi^0}$$

$\delta_{K\pi\pi^0}$: strong phase difference between CF and DCS decay amplitudes

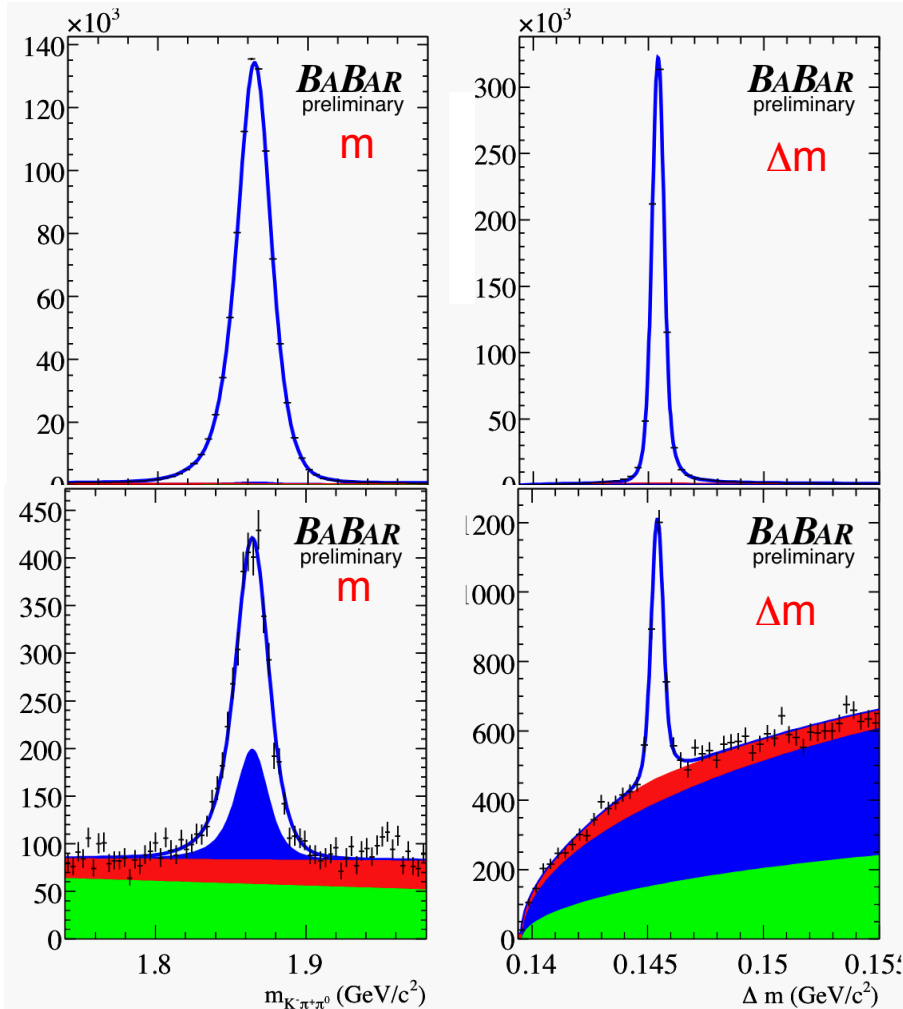


RS and WS ($m_{K\pi\pi^0}$, Δm) fits

Determine signal and background yields in subsequent Dalitz analyses.

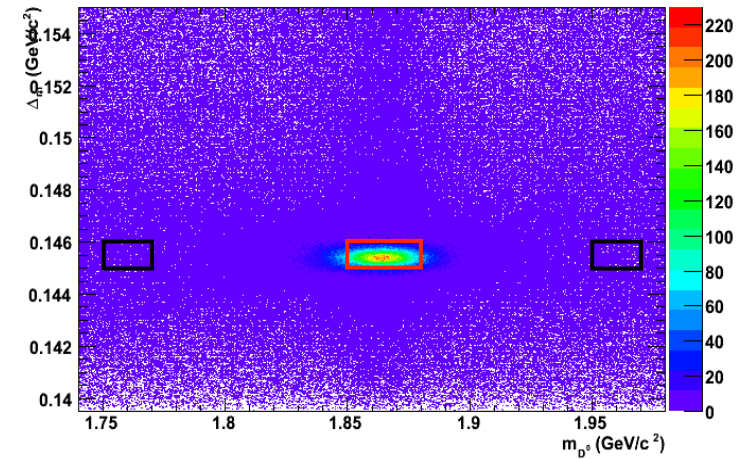
$$0.145 < \Delta m < 0.146 \text{ GeV}/c^2$$

$$1.85 < m_{K\pi\pi^0} < 1.88 \text{ GeV}/c^2$$



- signal
- mis-tagged D^0
- mis-reconstructed D^0
- combinatoric

signal and sideband regions



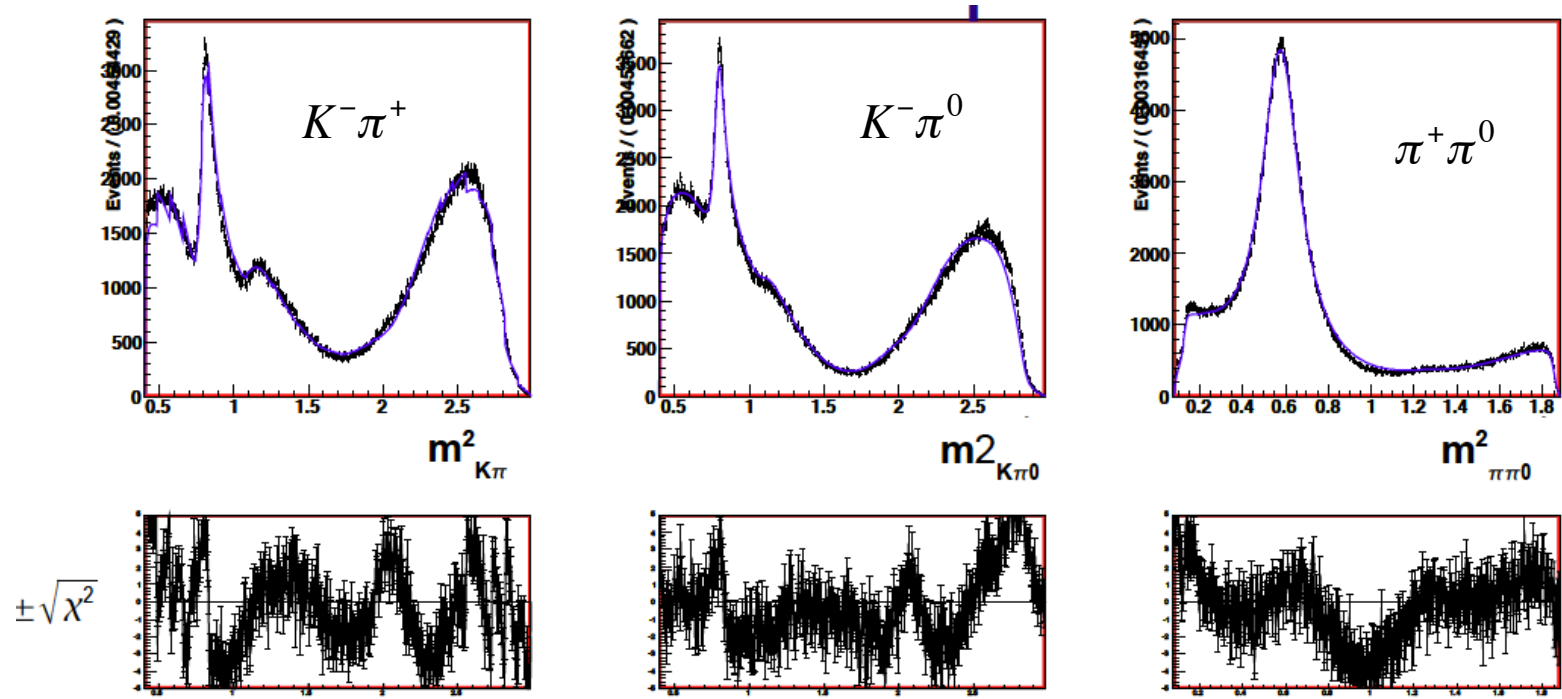
signal box yields:

Category	N events (RS)	N events (WS)
Signal	639802 ± 1538	1483 ± 56
Combinatoric	1537 ± 57	499 ± 57
Mistag	2384 ± 57	765 ± 29
Misreconstructed D^0	3117 ± 93	227 ± 75



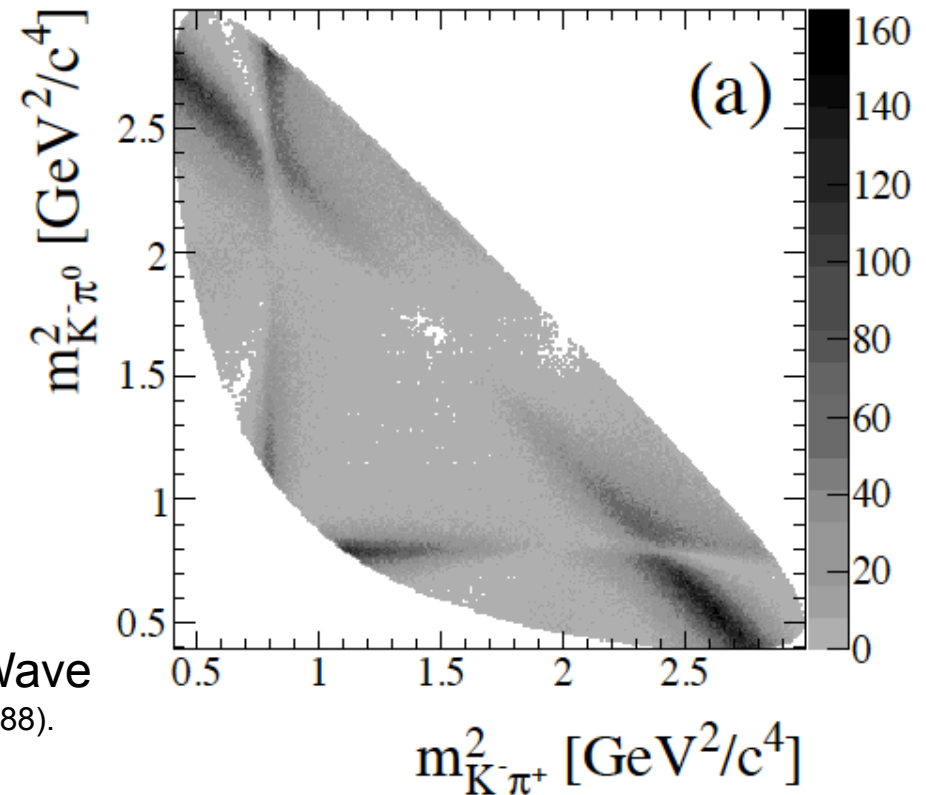
$D^0 \rightarrow K^- \pi^+ \pi^0$ RS Dalitz fit

Time-integrated analysis to determine CF amplitudes, $\bar{A}_{\bar{f}}$



RS Dalitz fit results

Resonance	Amplitude	Phase (degrees)	Fit Fraction (%)
$\rho(770)$	1 (fixed)	0 (fixed)	65.2 ± 4.5
$K^{*-}(1680)$	1.52 ± 0.06	144.3 ± 3.2	0.39 ± 0.04
$K_2^{*-}(1430)$	0.030 ± 0.001	-167.5 ± 2.5	0.31 ± 0.03
$K_2^{*0}(1430)$	0.0431 ± 0.0007	13.4 ± 0.9	0.73 ± 0.06
$K^{*-}(1410)$	0.24 ± 0.01	39.1 ± 4.1	0.17 ± 0.02
$K_0^{*-}(1430)$	2.95 ± 0.05	183.7 ± 0.9	3.6 ± 0.3
$K^{*-}(892)$	0.382 ± 0.001	163.3 ± 0.2	10.3 ± 0.7
$K^{*0}(1410)$	0.17 ± 0.01	-221.0 ± 3.5	0.009 ± 0.0001
$K_0^{*0}(1430)$	2.53 ± 0.01	91.6 ± 0.3	8.3 ± 0.6
$K^{*0}(1680)$	2.74 ± 0.07	-17.0 ± 1.5	1.4 ± 0.1
$K^{*0}(892)$	0.400 ± 0.001	3.4 ± 0.3	11.1 ± 0.7
$\rho(1700)$	6.06 ± 0.09	136.3 ± 0.7	4.1 ± 0.3
Total fit fraction = 106%			



Effective range parametrization for $K\pi$ S-Wave

D. Aston et al., LASS Collaboration, Nucl. Phys. B 296, 493 (1988).

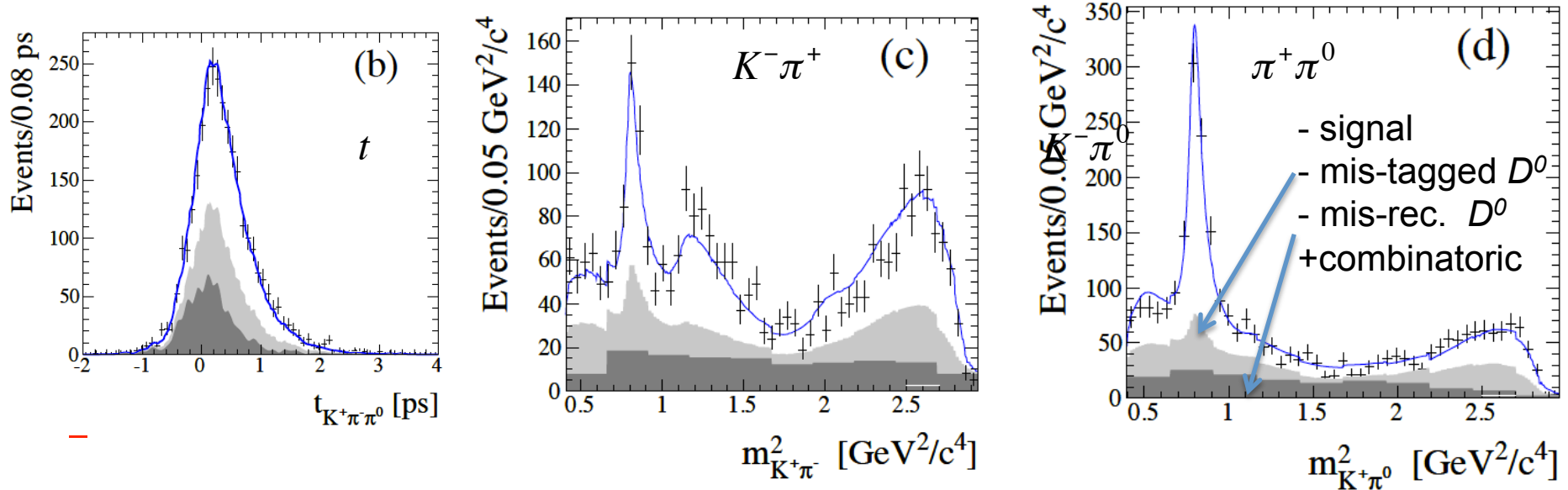
Scattering amplitude

$$T = \underbrace{\sin \delta_B e^{i\delta_B}}_{\text{Non-res}} + \underbrace{\sin \delta_R e^{i\delta_R} e^{2i\delta_B}}_{\text{Resonant}}$$

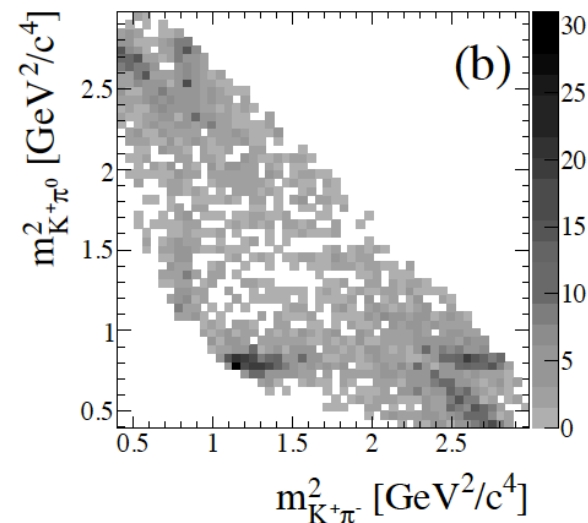
$D^0(t) \rightarrow K^+ \pi^- \pi^0$ WS Dalitz



Through t-dependence, distinguish DCS amplitudes from the CF amplitudes arising from mixing.



Resonance	Amplitude	Phase (degrees)	Fit Fraction (%)
$\rho(770)$	1 (fixed)	0 (fixed)	39.8 ± 6.5
$K_2^{*0}(1430)$	0.088 ± 0.017	-17.2 ± 12.9	2.0 ± 0.7
$K_0^{*+}(1430)$	6.78 ± 1.00	69.1 ± 10.9	13.1 ± 3.3
$K^{*+}(892)$	0.899 ± 0.005	-171.0 ± 5.9	35.6 ± 5.5
$K_0^{*0}(1430)$	1.65 ± 0.59	-44.4 ± 18.5	2.8 ± 1.5
$K^{*0}(892)$	0.398 ± 0.038	24.1 ± 9.8	6.5 ± 1.4
$\rho(1700)$	5.4 ± 1.6	157.4 ± 20.3	2.0 ± 1.1
$\chi^2/ndof = 188/215 = 0.876$			
Total fit fraction = 102%			



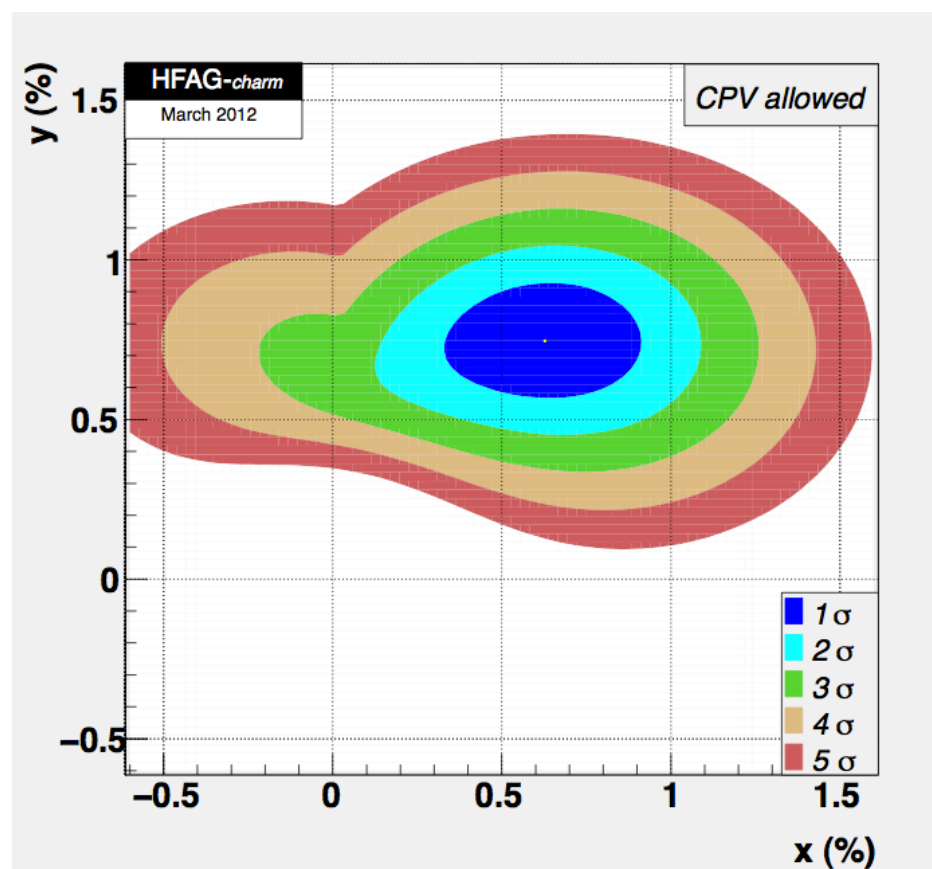
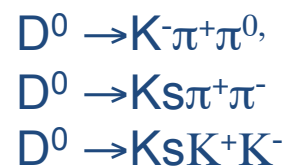


D0 mixing World Average



x and y must be measured independently to understand nature of D0 mixing

Multibody decays play a fundamental role in this



Summary

- Multibody heavy mesons decays amplitude analysis have played an important role in weak interactions (CKM paradigm) investigation
 - Measurements of α and γ
- Multibody essential to understand short range interaction.
 - D^0 mixing
- This experimental program need a detector able to reconstruct with good resolution and high efficiency several different multibody final states (π^0 rec and particle ID are must)

And today, what is at the cutting edge?



- LHCb (and CDF) is adding new information on B and charm physics.
- Charm physics seems to provide excitement
- CP violation (CHARM12)

$$\mathcal{A}_{CP}(D \rightarrow f) \equiv \frac{\Gamma(D \rightarrow f) - \Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(\bar{D} \rightarrow \bar{f})}$$

The data:

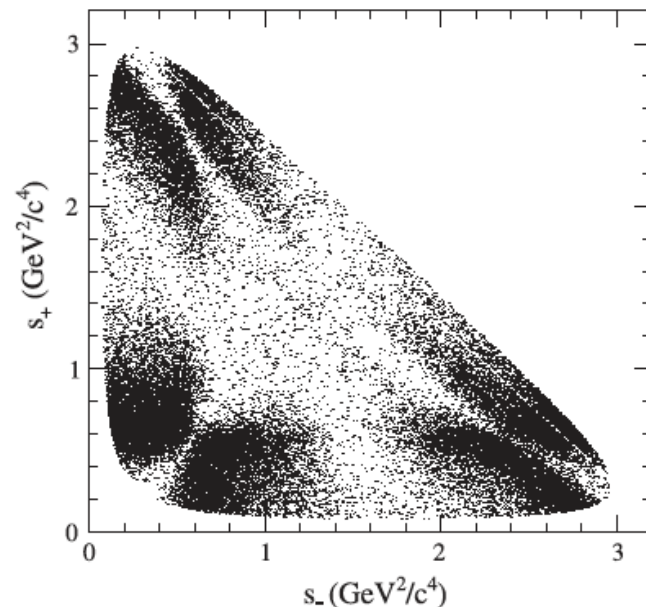
$$\begin{aligned} \Delta\mathcal{A}_{CP} &\equiv \mathcal{A}_{CP}(D \rightarrow K^+K^-) - \mathcal{A}_{CP}(D \rightarrow \pi^+\pi^-) \\ &= (-0.656 \pm 0.154)\% \quad \text{World average} \end{aligned}$$

$\sim 4\sigma$ from zero

Is this New Physics or only some (tricky) Standard Model long-range effect?

Back to multibody!

$D^0 \rightarrow \pi^+ \pi^- \pi^0$



PHYSICAL REVIEW D 78, 014015 (2008)
Dominated by I=0 amplitude (WHY??)

Y.Grossmann, et al.,

<http://arxiv.org/abs/1204.3557>

Measure amplitudes in D \rightarrow PV, VV decays

Use isospin relations between amplitude

Identify $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitude

$$\begin{aligned} & (A_{\rho^+ \pi^-} + A_{\rho^- \pi^+} + 2A_{\rho^0 \pi^0}) - \\ & (\bar{A}_{\rho^- \pi^+} + \bar{A}_{\rho^+ \pi^-} + 2\bar{A}_{\rho^0 \pi^0}) = \\ & 6(\mathcal{A}_3 - \bar{\mathcal{A}}_3) = -12ia_3 e^{i\delta_3^a} \sin \phi_3^a, \end{aligned}$$

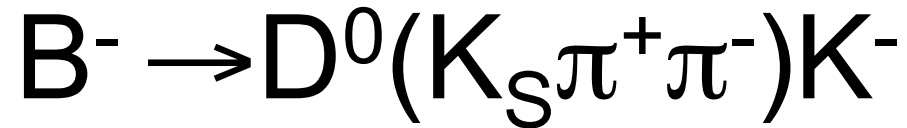
SM $\Delta I = 3/2$ \mathcal{A}_3 carries no weak phase.

If this difference is not zero, NP!

A Dalitz analysis to discover New Physics at high scale ?

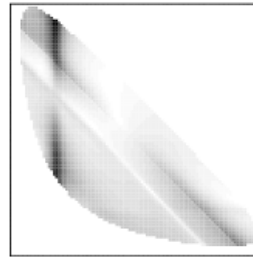


Back up

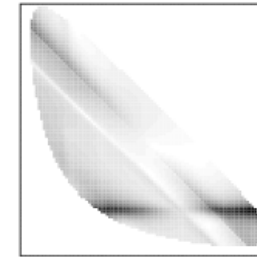


Pictorially:

$$|\mathbf{A}(m_{K_S \pi^+}^2, m_{K_S \pi^-}^2)|^2 =$$



$$+ r e^{i\delta \pm i\phi_3}$$



2

(assuming CP-conservation in D^0 decays)

Amplitude ratio:

$$r_B = \left| \mathbf{A}(B^- \rightarrow \bar{D}^0 K^-) / \mathbf{A}(B^- \rightarrow D^0 K^-) \right| \approx \frac{|V_{ub}^* V_{cs}|}{|V_{cb}^* V_{us}|} \times [\text{color supp}] \sim 0.1$$

- Higher r_B higher sensitivity
 - Extract from data
- Need input for D decay amplitude
 - High D^0 flavour tagged sample: Dalitz model

Events selection



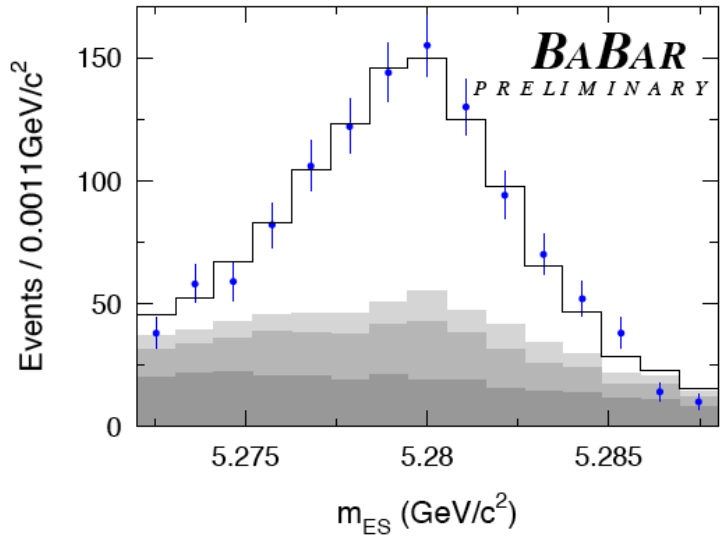
- Kinematics and topology
 - » To suppress continuum background

$$\begin{aligned}
 c_+ &= 0.080 \text{ GeV} \\
 c_- &= -0.140 \text{ GeV} \\
 c &= 0.045 \text{ GeV}
 \end{aligned}$$

$$\Delta E_{\pm}(m_0) = c_{\pm} - (c_{\pm} \mp \bar{c}) (m_0/m_0^{\max})^2$$

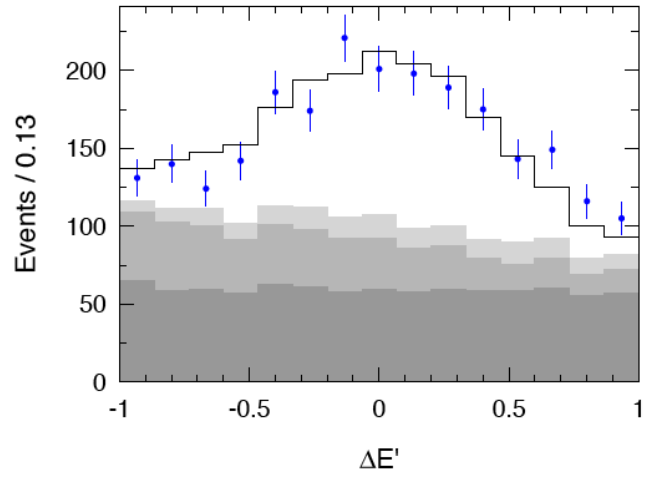
$$\Delta E' = (2\Delta E - \Delta E_+ - \Delta E_-) / (\Delta E_+ - \Delta E_-)$$

Remove
 ΔE dependence on
 π^0 momentum



$$5.272 < m_{ES} < 5.288 \text{ GeV}/c^2$$

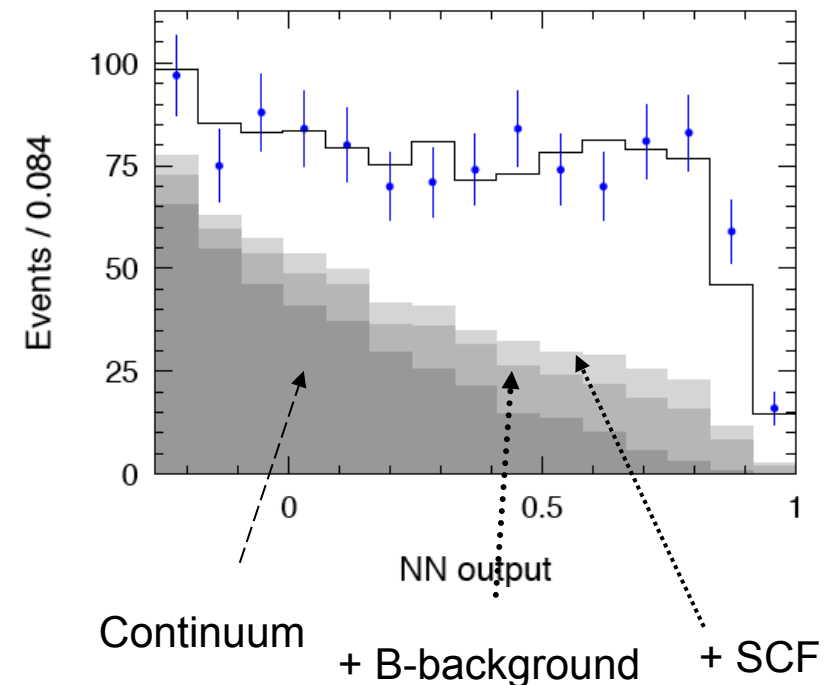
$$\begin{aligned}
 N_{\pi^+\pi^-\pi^0} &= 1847 \pm 69 \\
 \varepsilon_{\rho^+\pi^-} &= 24\%
 \end{aligned}$$



$$-1 < \Delta E' < 1$$

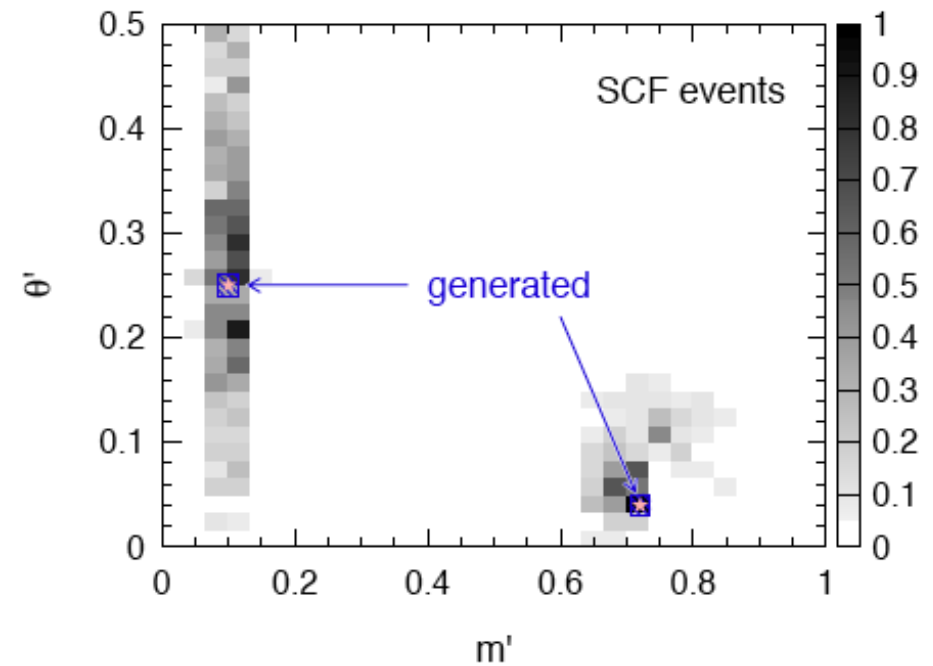
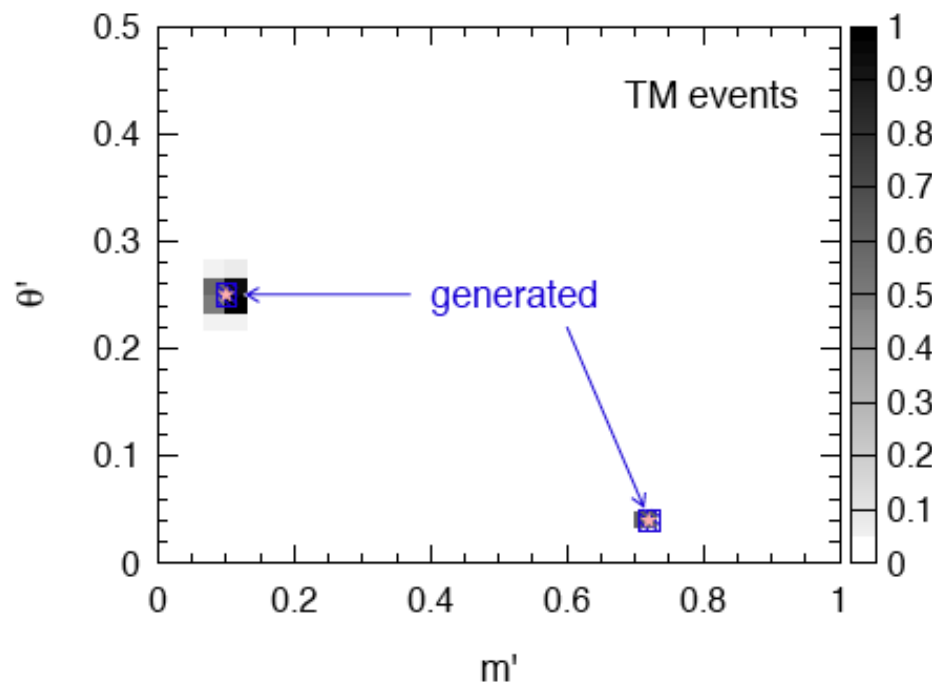
Backgrounds

- Mainly continuum
 - » Neural Network with several inputs
- Neutral and charged B
 - » Hundred exclusive final states analyzed
 - » 18 categories
- 6 B^0 charmless decays
- 8 B^+ charmless decays
- 2 $b \rightarrow c$ decays
 - » $(\rho\pi)^+, \rho\rho, \rho K, K\pi\pi, a_1\pi, D\pi, \dots$
- Flavor- Dalitz correlation accounted for

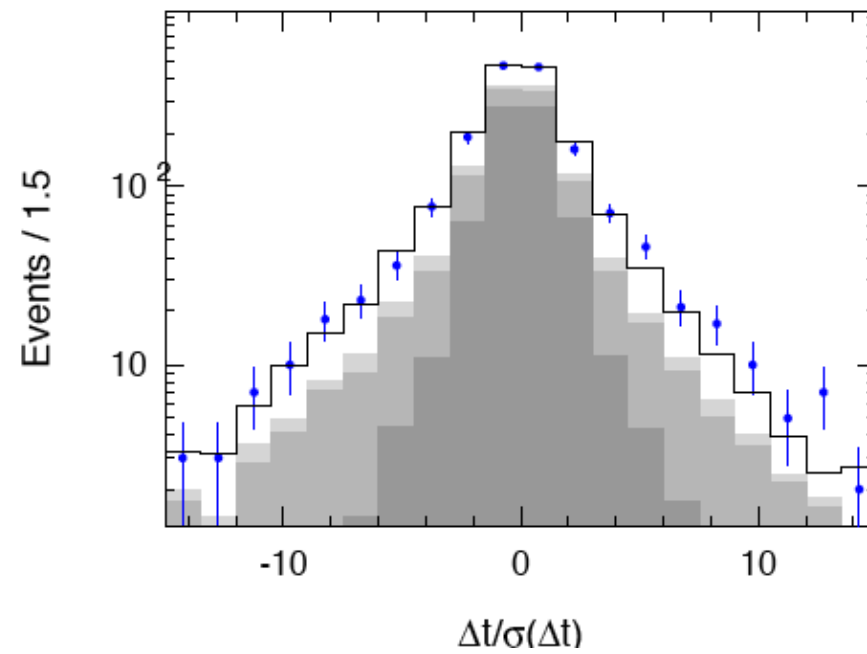


Maximum likelihood fit

Resolution effects



Δt distribution



Systematics studies



- Dalitz plot model:
 - toyMC: Generate with alternative models:
 - Nonresonant (uniform) component
 - Scalar $\pi^+\pi^-$ component
 - » [low mass scalar, $f_0(980), f_2(1270)$]
- ρ lineshape
 - Check relative amplitude assumption
 - Vary mass and width
- B-background
 - Assumption on CP contents
- And more
 - Fit bias (negligible)
 - Parameters other PDF (background Dalitz, f_{SCF})

*Relevant for
interference
parameters*

*Relevant for
Q-2b
parameters*

Quasi-2body parameters



$$\mathcal{A}(\rho^\pm \pi^\mp) \sim (S \pm \Delta S) \sin \Delta m \Delta t - (C \pm \Delta C) \cos \Delta m \Delta t$$

$$C^+ = \frac{U_+^-}{U_+^+}, \quad C^- = \frac{U_-^-}{U_-^+}, \quad S^+ = \frac{2I_+}{U_+^+}, \quad S^- = \frac{2I_-}{U_-^+}, \quad \mathcal{A}_{\rho\pi} = \frac{U_+^+ - U_-^+}{U_+^+ + U_-^+},$$

$\rho^+\pi^-$
 $\rho^-\pi^+$
 $\rho^+\pi^-$
 $\rho^-\pi^+$

- » They include interference effects
- » Averaging S and C

$$\begin{aligned} \mathcal{A}_{\rho\pi} &= -0.142 \pm 0.041 \pm 0.015, \\ C &= 0.154 \pm 0.090 \pm 0.037, \\ S &= 0.01 \pm 0.12 \pm 0.028, \end{aligned}$$

$$\begin{aligned} C &= (C^+ + C^-)/2, \quad \Delta C = (C^+ - C^-)/2, \\ S &= (S^+ + S^-)/2, \quad \text{and } \Delta S = (S^+ - S^-)/2 \end{aligned}$$

$$\begin{aligned} \Delta C &= 0.377 \pm 0.091 \pm 0.021, \\ \Delta S &= 0.06 \pm 0.13 \pm 0.029. \end{aligned}$$

*Systematics dominated by
B-background CP content*

Q-2b interpretation

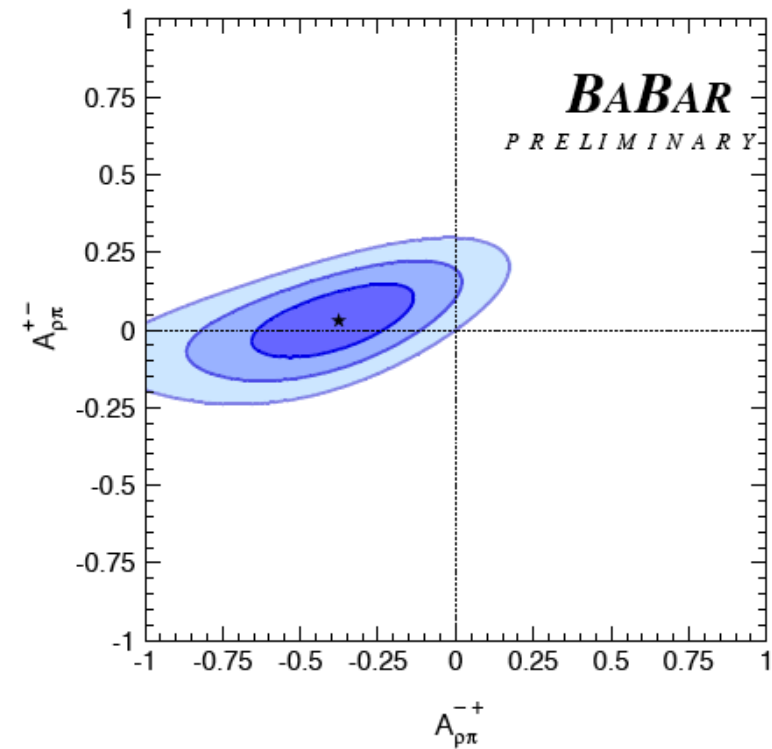
Compare B^0/ρ^+ and \bar{B}^0/ρ^-
more intuitively:

$$\kappa^{+-} = (q/p)(\bar{A}^-/A^+)$$

$$A_{\rho\pi}^{+-} = \frac{|\kappa^{+-}|^2 - 1}{|\kappa^{+-}|^2 + 1}$$

$$A_{\rho\pi}^{+-} = 0.03 \pm 0.07 \pm 0.03$$

$$A_{\rho\pi}^{-+} = -0.38_{-0.16}^{+0.15} \pm 0.07$$



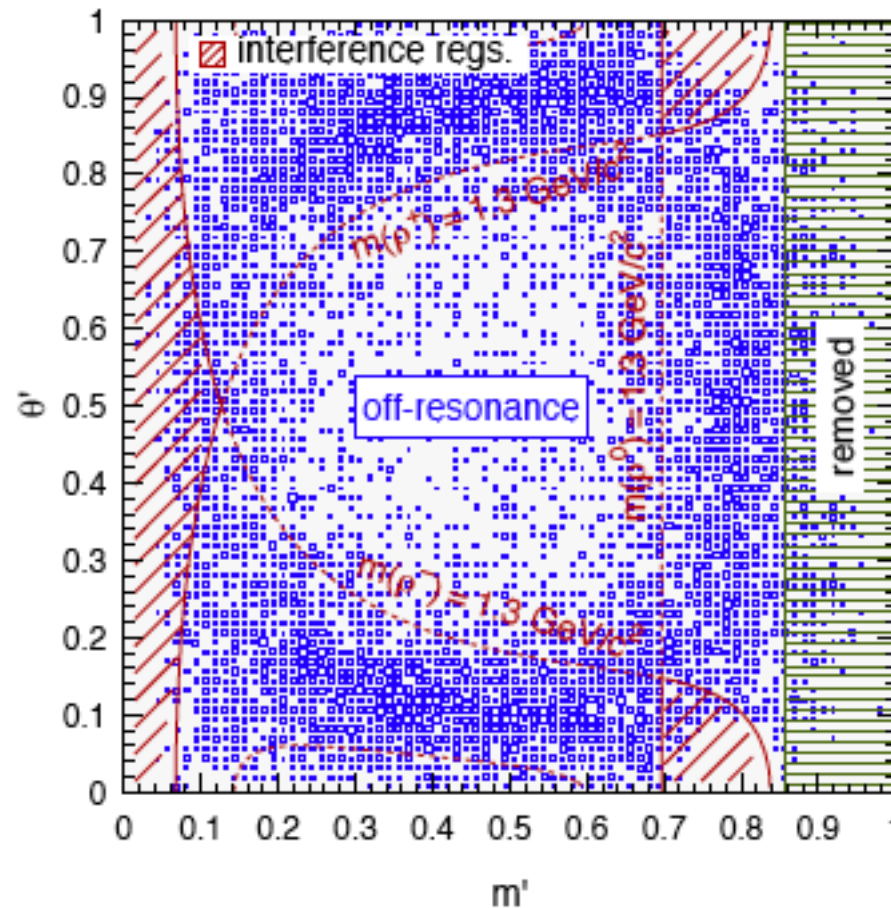
3σ effect
(syst. included)

Summary of B background



Class	Mode	BR [10^{-6}]	Expected number of events
0	$B^+ \rightarrow \rho^+ \rho_{\text{[long]}}^0$	19.1 ± 3.5	52 ± 10
0	$B^+ \rightarrow a_1^+ (\rightarrow (\rho\pi)^+) \pi^0$	20.0 ± 15.0	32 ± 24
0	$B^+ \rightarrow a_1^0 (\rightarrow \rho^{+-} \pi^{-+}) \pi^+$	20.0 ± 15.0	19 ± 14
1	$B^+ \rightarrow \pi^+ \rho^0$	8.7 ± 1.0	73 ± 8
1	$B^+ \rightarrow \rho^0 K^+$	4.3 ± 0.6	6 ± 1
2	$B^+ \rightarrow \pi^+ K_S^0 (\rightarrow \pi^+ \pi^-)$	8.3 ± 0.4	10 ± 1
3	$B^+ \rightarrow \pi^0 \rho^+$	10.8 ± 1.4	63 ± 8
3	$B^+ \rightarrow \pi^+ K_S^0 (\rightarrow \pi^0 \pi^0)$	3.7 ± 0.2	15 ± 2
4	$B^+ \rightarrow \pi^+ \pi^0$	5.5 ± 0.6	14 ± 2
4	$B^+ \rightarrow K^+ \pi^0$	12.1 ± 0.8	8 ± 1
5	$B^+ \rightarrow (K^{(**)}(1430)\pi)^+ \rightarrow (K^+ \pi \pi)^+$	29.0 ± 5.4	38 ± 5
6	$B^0 \rightarrow \pi^- K^{*+} (\rightarrow K_S^0 \pi^+)$	3.3 ± 0.4	2 ± 1
7	$B^0 \rightarrow \rho^+ \rho_{\text{[long]}}^-$	25.2 ± 3.7	67 ± 10
7	$B^0 \rightarrow (a_1 \pi)^0$	39.7 ± 3.7	39 ± 4
8	$B^0 \rightarrow K^+ \pi^-$	18.9 ± 0.7	12 ± 0
9	$B^0 \rightarrow \pi^- K^{*+} (\rightarrow K^+ \pi^0)$	3.3 ± 0.4	20 ± 2
9	$B^0 \rightarrow K^{(**)}(1430)\pi \rightarrow K \pi \pi^0$	11.2 ± 2.2	212 ± 34
10	$B^0 \rightarrow \gamma K^{*0}(892, 1430) (\rightarrow (K^+ \pi^-)^0)$	27.4 ± 1.5	14 ± 1
10	$B^0 \rightarrow \pi^0 K^{*0} (\rightarrow K^+ \pi^-)$	1.3 ± 0.5	9 ± 4
10	$B^0 \rightarrow \eta' (\rightarrow \rho^0 \gamma) \pi^0$	0.4 ± 0.2	3 ± 2
11	$B^0 \rightarrow \rho^- K^+$	9.9 ± 1.6	103 ± 17
12	$B^0 \rightarrow K^+ \pi^- \pi^0_{\text{[nonres]}}$	4.6 ± 4.6	38 ± 38
13	$B^0 \rightarrow \pi^0 K_S^0 (\rightarrow \pi^+ \pi^-)$	5.8 ± 0.5	50 ± 4
14	$B^0 \rightarrow D^- (\rightarrow \pi^- \pi^0) \pi^+$	7.5 ± 2.3	599 ± 184
15	$B^0 \rightarrow \bar{D}^0 (\rightarrow K^+ \pi^-) \pi^0$	11.0 ± 3.2	100 ± 29
16	$B^0 \rightarrow \bar{D}^0 (\rightarrow \pi^+ \pi^-) \pi^0$	0.4 ± 0.1	35 ± 9
17	$B^0 \rightarrow J/\psi (\rightarrow e^+ e^-, \mu^+ \mu^-) \pi^0$	2.6 ± 0.5	77 ± 15
18	$B^0 \rightarrow \{\text{neutral generic } b \rightarrow c \text{ decays}\}$	–	173 ± 15
19	$B^+ \rightarrow \{\text{charged generic } b \rightarrow c \text{ decays}\}$	–	396 ± 20

Off-resonance Dalitz distr.



Q-2b parameters cov. matrix



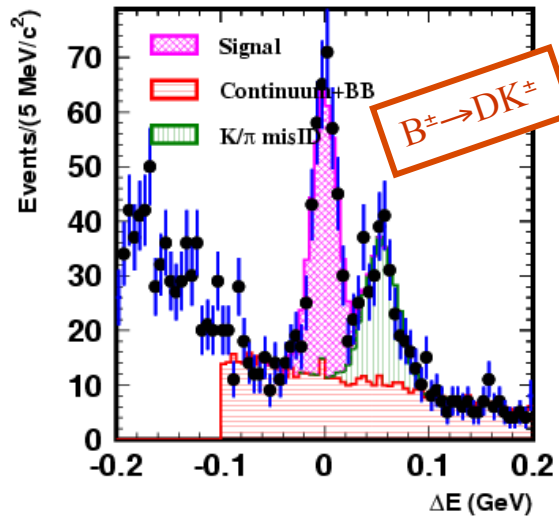
	$\mathcal{A}_{\rho\pi}$	C	ΔC	S	ΔS
$\mathcal{A}_{\rho\pi}$	1.93				
C	-0.71	9.68			
ΔC	-0.55	2.63	8.93		
S	-0.03	-0.71	-0.13	15.3	
ΔS	-0.03	-0.57	-0.07	3.93	17.08

Multiplied by 1000

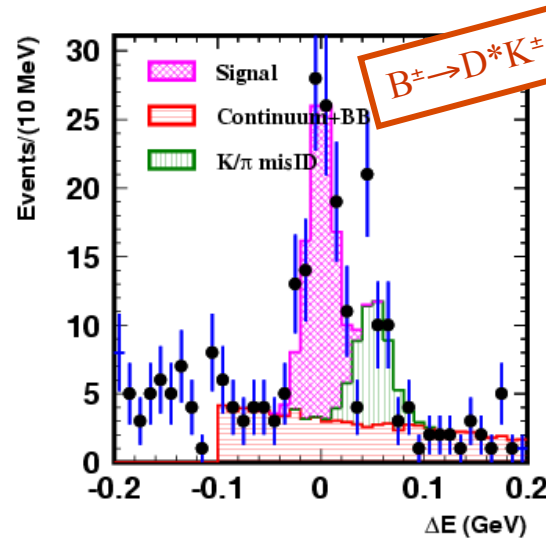
Belle data set

Belle result (357 fb⁻¹)

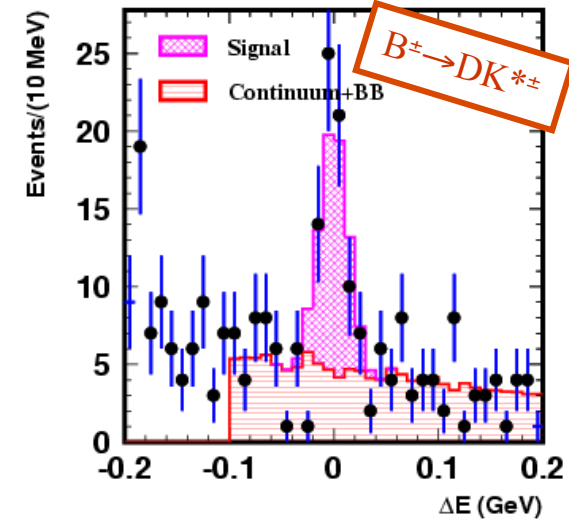
[hep-ex/0604054, PRD **73**, 112009 (2006)]



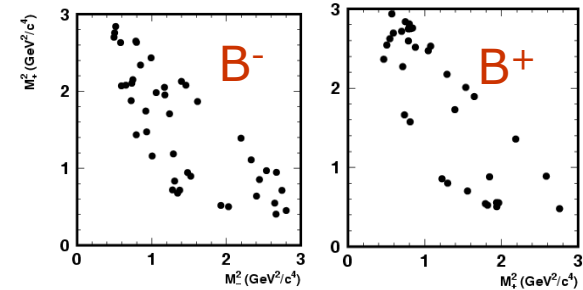
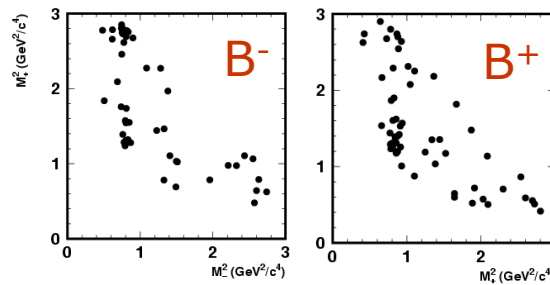
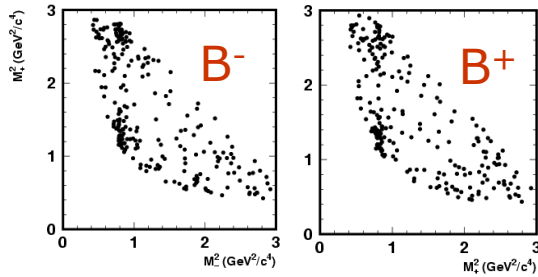
331 ± 17 events



81 ± 8 events



54 ± 8 events

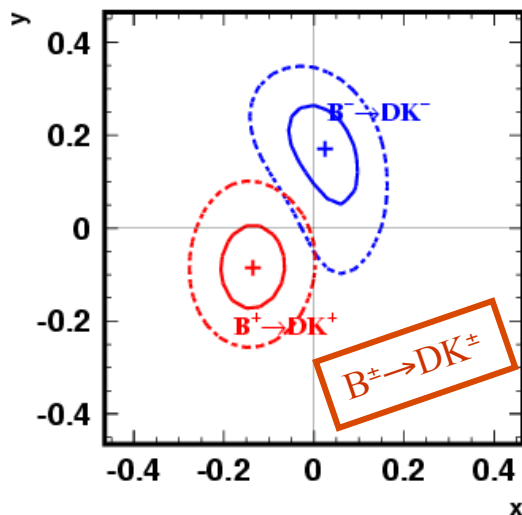


Belle fit results

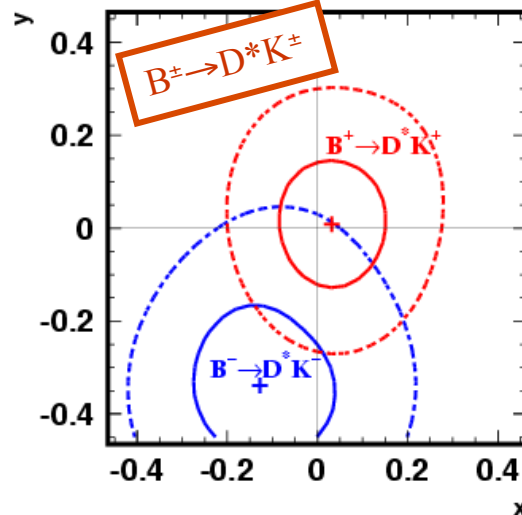


Fit parameters are $x_{\pm} = r_B \cos(\pm\varphi_3 + \delta)$ and $y_{\pm} = r_B \sin(\pm\varphi_3 + \delta)$
 (better behaved statistically than r_B, δ, φ_3)

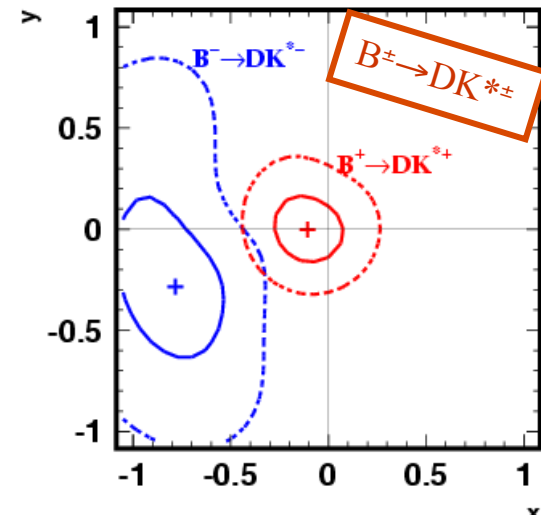
Unbinned maximum likelihood fit with event-by-event background
 treatment ($\Delta E - M_{bc}$ included into likelihood)



$x_- = 0.025^{+0.072}_{-0.080}$
 $y_- = 0.170^{+0.093}_{-0.117}$
 $x_+ = -0.135^{+0.069}_{-0.070}$
 $y_+ = -0.085^{+0.090}_{-0.086}$



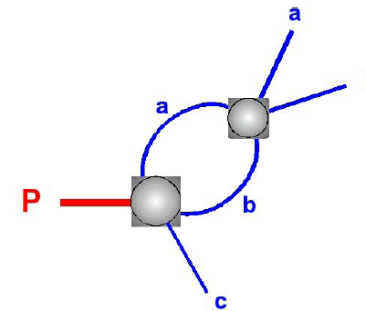
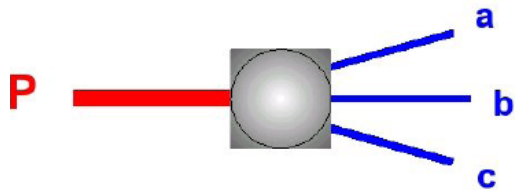
$x_- = -0.128^{+0.167}_{-0.146}$
 $y_- = -0.339^{+0.172}_{-0.158}$
 $x_+ = 0.032^{+0.120}_{-0.116}$
 $y_+ = 0.008^{+0.137}_{-0.136}$



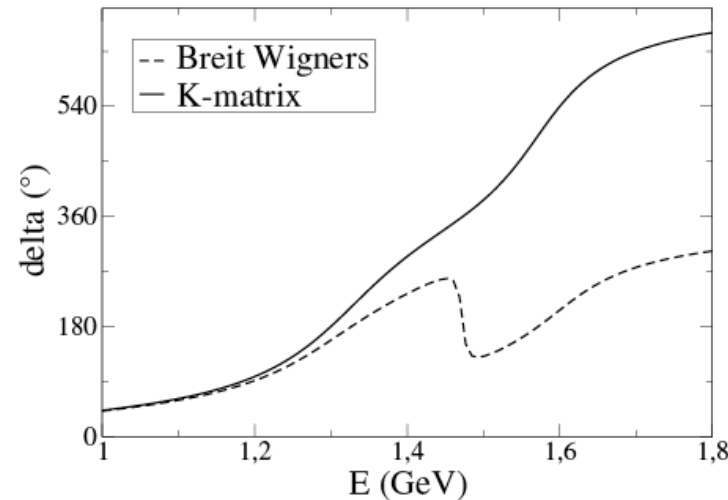
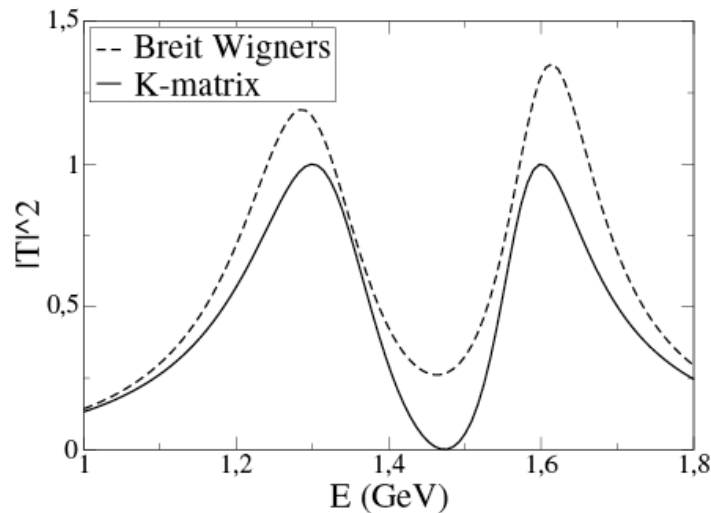
$x_- = -0.784^{+0.249}_{-0.295}$
 $y_- = -0.281^{+0.440}_{-0.335}$
 $x_+ = -0.105^{+0.177}_{-0.167}$
 $y_+ = -0.004^{+0.164}_{-0.156}$

Dalitz plot, resonances, K-matrix

Isobar Approximation



– K-matrix parametrization for overlapping resonances



● BW not unitary if resonances are close !

– Better for $\pi\pi$ S-wave description

Theory helps

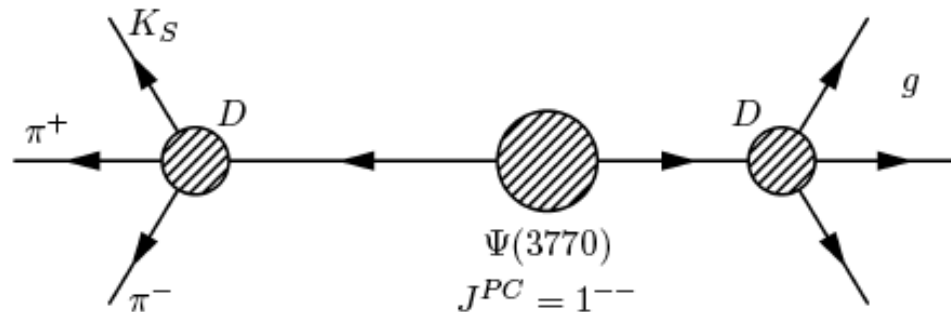
- Model independent

Unknowns:

$$c_i \equiv \int dp A_{12,13} A_{13,12} \cos(\delta_{12,13} - \delta_{13,12})$$

$$s_i \equiv \int dp A_{12,13} A_{13,12} \sin(\delta_{12,13} - \delta_{13,12})$$

- entangled states: consider the following decay



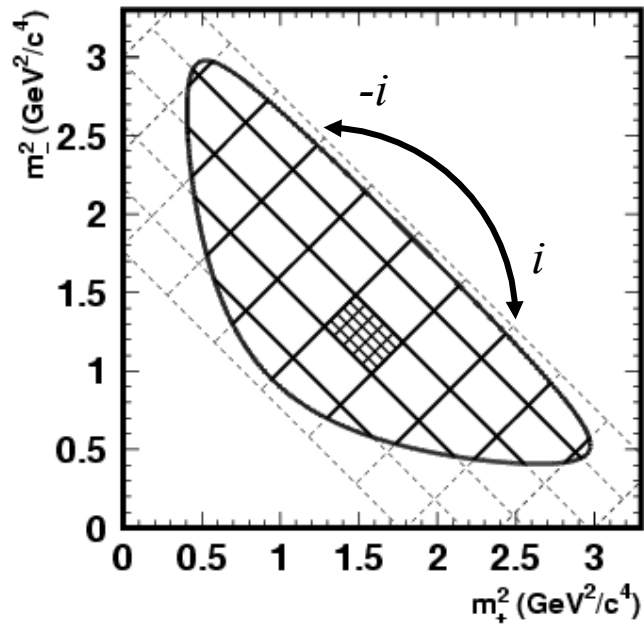
CLEO-c
CP tagged
Sample!

i th bin of $K_S\pi^+\pi^-$ and j th bin of g

$$\Gamma_{i,j} \propto T_i T_j^g + T_{\bar{i}} T_{\bar{j}}^g - 2(c_i c_j^g + s_i s_j^g)$$

- if g a CP even (odd) eigenstate, $s_j^g = 0$, $T_j^g = T_{\bar{j}}^g = \pm c_j^g$,
no sensitivity to s_i
- if $g = K_S\pi^+\pi^-$ and $j = i$ ($j = \bar{i}$) one measures s_i^2

Binned analysis of CLEO-c data



Number of events in **D^0 -plot**: K_i

Number of events in **B -plot**

$$M_i = h_B [K_i + r_B^2 K_{-i} + \sqrt{K_i K_{-i}} (xC_i + yS_i)]$$

Number of events in **D_{CP} -plot**

$$N_i = h_{CP} [K_i + K_{-i} \pm 2\sqrt{K_i K_{-i}} C_i]$$

$$C_i = \langle \cos \Delta\delta(D) \rangle_D \quad S_i = \langle \sin \Delta\delta(D) \rangle_D$$

$$C_i^2 + S_i^2 \leq 1 \quad (=1 \text{ if bin size is small enough})$$

- C_i, S_i can be obtained from B data (M_i) only \Rightarrow Very poor sensitivity
- C_i from D_{CP} , S_i from B data \Rightarrow Poor sensitivity for y
- S_i constrained as $S_i = \pm\sqrt{1 - C_i^2}$ \Rightarrow Bias if bin size is large

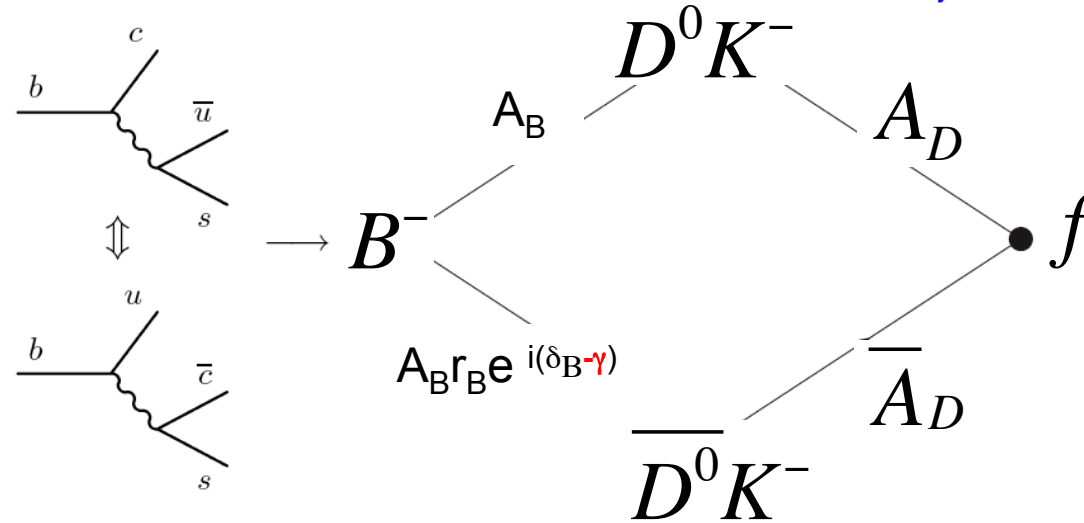
Measurements of γ in $B \rightarrow D^0 K$



Basic idea

Gronau, Wyler, 1991

Tree level only



interference between $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$

- Multiple incarnations: D decay to CP eigen., flavour eigen., multibody decays
- Use of D^{*0} (both $D^0\pi^0$ and $D^0\gamma$!)
– and K^*
- Neutral and charged B decays



Charm physics with B-factory



BaBar is a B-factory: $e^+e^- \rightarrow \Upsilon(4S) \rightarrow b\bar{b}$
 $\sigma_{\text{eff}}(b\bar{b}) = 1.1 \text{ nb}$, but
 $\sigma(c\bar{c}) = 1.3 \text{ nb}$ **→**
Millions of reconstructed charm hadrons
BaBar is also a charm factory

