

Tetraquarks in a diquark-antidiquark model

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ATHOS 2012, 20-22 June 2012, Camogli, Italy

Classification of diquark states

- Our diquarks have no spatial excitations (or better: we consider their excitations higher in energy than the hadrons we are going to study)
- This limits the possible states to:

$$|[\bar{3}]_f, [\bar{3}]_c, 0\rangle, |[6]_f, [\bar{3}]_c, 1\rangle$$
$$|[\bar{3}]_f, [6]_c, 1\rangle, |[6]_f, [6]_c, 0\rangle$$

According to most models, color sextuplet are not a favored configuration (possibly not bound at all)

- Thus we will use only the **scalar** ($J^P=0^+$), **axial-vector** ($J^P=1^+$) diquarks

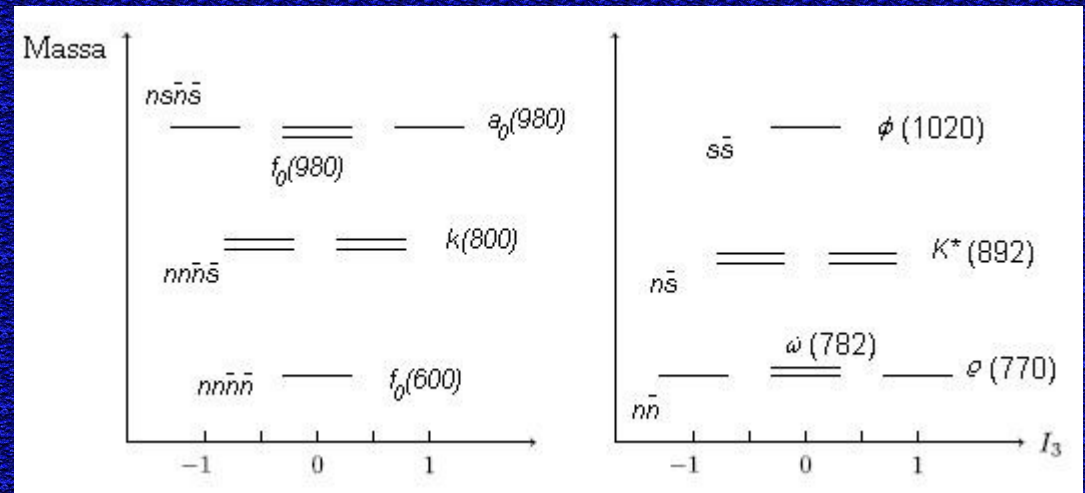
$$|[\bar{3}]_c, [\bar{3}]_f, 0 \rangle, |[\bar{3}]_c, [6]_f, 1 \rangle$$

Tetraquarks

- Tetraquarks are exotic mesons composed of two quarks and two antiquarks
- They can have explicitly exotic quantum numbers (e.g. $I=2$ or $Y=+2$) or the same quantum numbers of the normal mesons (cryptoexotics)
- No explicit exotics are known with certainty (several candidates: $\pi_1(1400)$, $Z^+(4430)$, ...)
- Possible cryptoexotics (since Jaffe in the '70): the light scalar nonet
 - very well established: $a_0(980)$, $f_0(980)$
 - not so well established: $f_0(600)$, $k(800)$

The light scalar nonet as tetraquark candidates

- The principal factors that urge the identification of the scalar nonet as tetraquarks are:
 - The reversed spectrum of the nonet:



- The OZI rule violation in the hadron decays ($f_0(600) \rightarrow \pi\pi$, $f_0(980) \rightarrow K\bar{K}$ not suppressed)
- The OZI rule violation in the radiative decays ($\phi(1020) \rightarrow a_0(980) \gamma$)

Intrinsic degrees of freedom

- The intrinsic degrees of freedom are classified through the representations of the group $SU_{sf}(6) \otimes SU_c(3)$

$$SU_{sf}(6)$$

$$[6] \otimes [\bar{6}] \otimes [6] \otimes [\bar{6}] = [1] \oplus [35] \oplus [35] \oplus [405] \oplus [35] \oplus [280] \oplus [35] \oplus [1] \oplus [280] \oplus [189]$$

$$SU_f(3)$$

$$[3] \otimes [\bar{3}] \otimes [3] \otimes [\bar{3}] = [1] \oplus [8] \oplus [8] \oplus [10] \oplus [8] \oplus [8] \oplus [1] \oplus [\bar{10}] \oplus [27]$$

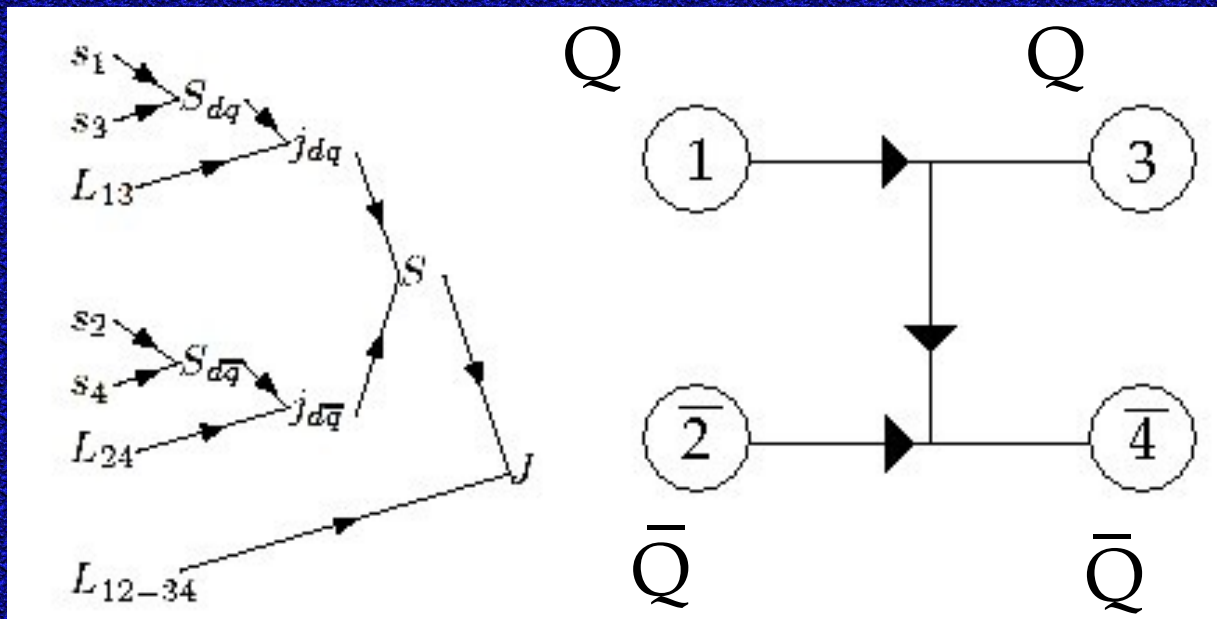
$SU_c(3)$ only the (two) singlets are allowed physical states

$$SU_s(2)$$

$$[2] \otimes [2] \otimes [2] \otimes [2] = [1] \oplus [3] \oplus [1] \oplus [3] \oplus [3] \oplus [5]$$

Spatial degrees of freedom

- The orbital momenta and the spins are combined together to give the total angular momentum following the scheme:



$$P = (-1)^{L_{13} + L_{24} + L_{12-34}} \quad C = (-1)^{L_{12-34} + S} \quad G = (-1)^{L_{12-34} + S + I}$$

Tetraquark states

- Applying the Pauli principle we can write all the possible multiplets of the tetraquarks

Scalar nonet

TABLE III. Color, flavor, spin (S_{tot}), and J^{PC} for tetraquarks with $L_{13} = L_{24} = L_{12-34} = 0$.

Color	Flavor	S_{dq}	$S_{d\bar{q}}$	j_{dq}	$j_{d\bar{q}}$	S_{tot}	S	J^{PC}
$[\bar{3}] \otimes [3]$	$[1] \oplus [8]$	0	0	0	0	0	0	0^{++}
$[\bar{3}] \otimes [3]$	$[8] \oplus [\bar{10}]$	0	1	0	1	1	1	1^+
$[\bar{3}] \otimes [3]$	$[8] \oplus [10]$	1	0	1	0	1	1	1^+
$[\bar{3}] \otimes [3]$	$[1] \oplus [8] \oplus [27]$	1	1	1	1	0	0	0^{++}
						1	1	1^{+-}
						2	2	2^{++}
$[6] \otimes [\bar{6}]$	$[1] \oplus [8] \oplus [27]$	0	0	0	0	0	0	0^{++}
$[6] \otimes [\bar{6}]$	$[8] \oplus [10]$	0	1	0	1	1	1	1^+
$[6] \otimes [\bar{6}]$	$[8] \oplus [\bar{10}]$	1	0	1	0	1	1	1^+
$[6] \otimes [\bar{6}]$	$[1] \oplus [8]$	1	1	1	1	0	0	0^{++}
						1	1	1^{+-}
						2	2	2^{++}

TABLE VI. Color, flavor, spin (S_{tot}), and J^{PC} for tetraquarks with $L_{24} = L_{13} = 0$ and $L_{12-34} = 1$.

Col	Flavor	S_{dq}	$S_{d\bar{q}}$	j_{dq}	$j_{d\bar{q}}$	S_{tot}	S	J^{PC}
$[\bar{3}] \otimes [3]$	$[1] \oplus [8]$	0	0	0	0	0	0	1^{--}
$[\bar{3}] \otimes [3]$	$[8] \oplus [\bar{10}]$	0	1	0	1	1	1	0^-
								1^-
								2^-
$[\bar{3}] \otimes [3]$	$[8] \oplus [10]$	1	0	1	0	1	1	0^-
								1^-
								2^-
$[\bar{3}] \otimes [3]$	$[1] \oplus [8] \oplus [27]$	1	1	1	1	0	0	1^{--}
						1	1	$0^{++}, 1^{++}, 2^{++}$
						2	2	$1^{--}, 2^{++}, 3^{--}$
$[6] \otimes [\bar{6}]$	$[1] \oplus [8] \oplus [27]$	0	0	0	0	0	0	1^{--}
$[6] \otimes [\bar{6}]$	$[8] \oplus [10]$	0	1	0	1	1	1	0^-
								1^-
								2^-
$[6] \otimes [\bar{6}]$	$[8] \oplus [\bar{10}]$	1	0	1	0	1	1	0^-
								1^-
								2^-
$[6] \otimes [\bar{6}]$	$[1] \oplus [8]$	1	1	1	1	0	0	1^{--}
						1	1	$0^{++}, 1^{++}, 2^{++}$
						2	2	$1^{--}, 2^{++}, 3^{--}$

Algebraic mass formula

- In order to calculate the tetraquark spectrum, we need now a mass formula
- Iachello et al. (Phys.Rev.D44,898 (1991)) developed an algebraic mass formula for the normal mesons based on a SGA= $U(4) \otimes SU(3)_F \otimes SU(2)_S \otimes SU(3)_C$ stringlike model
- The spatial part is described in terms of the Casimir operators of the chain: $U(4) \supset SO(4) \supset SO(3) \supset SO(2)$

The mass
formula is:

$$M^2 = (N_n M_n + N_s M_s)^2 + av + bL + cS + dJ +$$
$$+ h \langle M'^2 \rangle_{ij,i'j'} + i \langle M''^2 \rangle_{ij,i'j'}$$

(Regge trajectories: linear **vibrational** and **rotational** slope)

Diquark-antidiquark limit

- For the tetraquark case one should introduce a new SGA for the spatial part, for example $U(10)$ or $U(4) \otimes U(4) \otimes U(4)$
- This is still work in progress
- We have developed also a diquark-antidiquark model for tetraquarks.
- In our model we freeze the spatial internal degrees of freedom inside the diquark.
- With these conditions, the diquark-antidiquark states are a subset of the states of the compact tetraquark, in particular the subset with $L_{13}=L_{24}=0$.

Tetraquark nonet spectrum in the diquark-antidiquark limit

- We can use a mass formula similar to the one used for the normal mesons:

$$M^2 = (M_{qq} + M_{\bar{q}\bar{q}})^2 + a \cdot n + b \cdot L + c \cdot S + d \cdot J$$

- We determine the diquark masses fitting the mass formula with the candidate tetraquark nonet masses. We obtain

$$M_{[n,n]} = 0.275 \text{ GeV}$$

$$M_{[n,s]} = 0.492 \text{ GeV}$$

$$M_{a_0(980)} = M_{f_0(980)} = 0.984 \text{ GeV}$$

$$M_{f_0(600)} = 0.550 \text{ GeV}$$

$$M_{k(800)} = 0.767 \text{ GeV}$$

Santopinto, Galatà Phys. Rev. C75, 045206 (2007)

Tetraquark spectrum

With the previous mass formula and the diquark masses we can determine the spectrum of all diquark-antidiquark states (preliminary extract of the first states, **still necessary to calculate decays**)

$n^{2S+1}L_J(J^{PC})$	Diquark type[50]	Mass (GeV)
$0^1S_0(0^{++})$	$[n, n][n, n]$	0.550
	$[n, n][n, s]$	0.767
	$[n, s][n, s]$	0.984
	$\{n, n\}\{n, n\}$	1.000
	$\{n, n\}\{n, s\}$	1.120
	$\{n, s\}\{n, s\}$	1.240
	$\{n, n\}\{s, s\}$	1.240
	$\{n, s\}\{s, s\}$	1.360
	$\{s, s\}\{s, s\}$	1.480

$0^3P_1(1^{-+})$	$\{n, n\}\{n, n\}$	1.482
	$\{n, n\}\{n, s\}$	1.565
	$\{n, s\}\{n, s\}$	1.653
	$\{n, n\}\{s, s\}$	1.653
	$\{n, s\}\{s, s\}$	1.745
	$\{s, s\}\{s, s\}$	1.840

$\pi_1(1400)?$

$\pi_1(1600)?$

Greetings

Thank you for
your attention!
