

# Towards a unified description of $\gamma\gamma \rightarrow \pi^0\pi^0$ and $e^+e^- \rightarrow \gamma^* \rightarrow \gamma\pi^0\pi^0$

Work in progress with Diogo Boito

# Plan

- 1) Motivations
- 2) Standard Muskhelishvili-Omnès (reminder)
- 3) “left-hand” cut in  $\gamma^* \rightarrow \gamma\pi^0\pi^0$
- 4) Correctly analytic resonance propagator
- 5)  $V\pi$  electromagnetic form factor

## Preamble

- Difficulties of (exotic) meson spectroscopy:

No (low energy)  $\pi^\pm$ ,  $\pi^0$ ,  $\eta$ ,  $\eta'$  beams available  
(otherwise e.g. exotic  $1^{-+}$  easily decided from  $\pi\eta$   
or  $\pi\eta'$   $J = 1$  phase-shift)

- Importance of **production** experiments  
(+ proper treatment)

$$\gamma\gamma \rightarrow \pi\pi \text{ to } \gamma\gamma^* \rightarrow \pi\pi$$

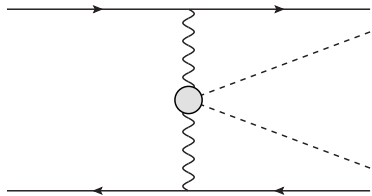
- $\gamma\gamma \rightarrow \pi\pi$ : Typical case for application of Muskhelishvili - Omnès (MO) [*Gourdin, Martin (1960)*]

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- $\gamma\gamma \rightarrow \pi\pi$ : Typical case for application of **Muskhelishvili - Omnès (MO)** [*Gourdin, Martin (1960)*]

→ Also **measurements**:

from  $e^+e^- \rightarrow e^+e^-\pi\pi$   
[*MARKII (1990), CELLO (1992), Crystal Ball (1990), Belle(2007,2008)* ]



- Analyticity + exp. data + **chiral symmetry**  
[*Morgan, Pennington (1988,1991); Donoghue, Holstein (1993)* ]

■ Why MO useful ?

→ FSI disentangled in universal way (also for  $l = 2$ )

→ Analyticity: (consequence of confinement in QCD) allows energy extrapolations:

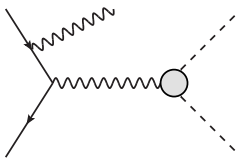
1) Subthreshold extrapolations: polarizabilities

2) Complex plane extrapolations : resonances

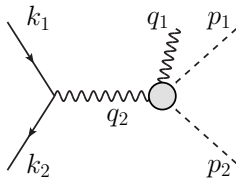
– Same poles as in elastic amplitudes

– Residues: coupling constants (e.g.  $\sigma\gamma\gamma$   
[Pennington (2006)] )

■  $e^+e^- \rightarrow \gamma^* \rightarrow \gamma\pi\pi$  :

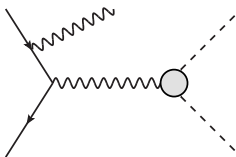


(A)

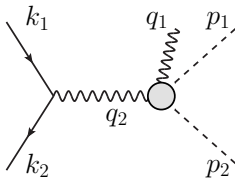


(B)

■  $e^+e^- \rightarrow \gamma^* \rightarrow \gamma\pi\pi$  :



(A)



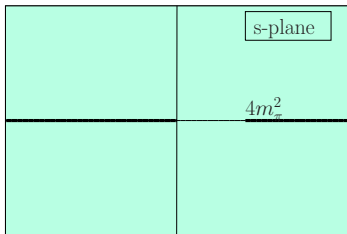
(B)

- $\gamma\gamma$  becomes  $\gamma\gamma^*(q^2)$
- Access to generalized polarizabilities
- Access to couplings:  $\sigma\omega\gamma$ ,  $f_0(980)\omega\gamma$ ,  $\sigma\phi\gamma$ ,  $f_0(980)\phi\gamma$  [Achasov, Ivanchenko (1989)]



## Standard MO (quick reminder)

- Partial-wave amplitude  $A_j(s)$ : analytic, two cuts



- Basic consequence:  $A_j(s) = A_j^L(s) + A_j^R(s)$
- Assume FS scattering elastic:  $S_j = \exp 2i\delta_j(s)$

- MO: dependence on FSI made fully explicit

$$A_I(s) = A_I^L(s) + \Omega_I(s) \left[ \alpha + \beta s + s^2 \int_{4m_\pi^2}^{\infty} ds' \Omega(s, s') A_I^L(s') \right]$$

$$\rightarrow \Omega_I(s) = \exp \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s' - s)}$$

$$\rightarrow \Omega(s, s') = \frac{\sin \delta_I(s')}{\pi(s' - s)(s')^2 |\Omega_I(s')|}$$

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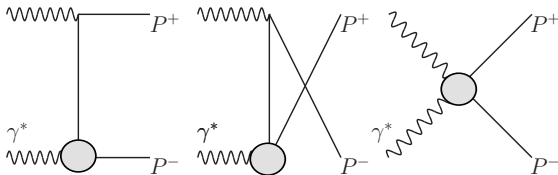
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$$\rightarrow \text{Watson theorem: } \text{Phase}(A_I(s)) = \delta_I(s) \pmod{\pi}$$

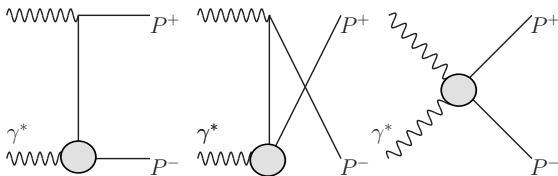
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→ S-wave projection  $q^2 = 0$ :

$$h^B(s) = \frac{4m_\pi^2}{s\beta_\pi(s)} \log \frac{1 + \beta_\pi(s)}{1 - \beta_\pi(s)}, \quad \beta_\pi(s) = \sqrt{1 - 4m_\pi^2/s}$$

→ Now when  $q^2 \neq 0$

$$h^B(s, q^2) = F_\pi(q^2) \left( \frac{sh^B(s) - 2q^2}{s - q^2} \right)$$

- Pole at  $s = q^2$  (infrared singularity)
- MO integral:  $s' = q^2$  pole well defined with  $q^2 + i\epsilon$  prescription

■ MO integral

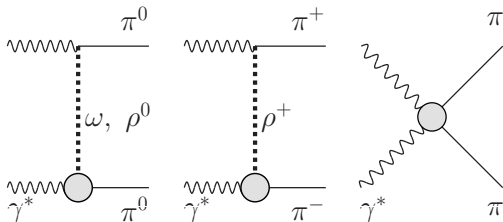
$$= F_\pi(q^2) s^2 \frac{K^l(s, q^2) - K^l(q^2, q^2)}{s - q^2}$$

with

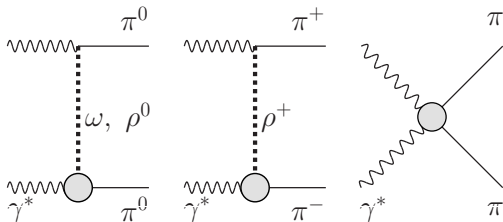
$$K^l(s, q^2) = -\sqrt{\frac{4-l}{3}} \int_{4m_\pi^2}^{\infty} ds' \Omega^l(s, s') (s' h_B(s') - 2q^2)$$

■ No Watson theorem now !

- Vector mesons poles ( $\Gamma_V = 0$  for the moment):



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- Helicity amplitude  $H_{++}^V(s, q^2, z) =$

$$\check{C}_V F_{VP}(q^2) \frac{(s - q^2)[-4t + q^2(1 + \beta_\pi(s)z)^2]}{2(m_V^2 - t)} + (z \rightarrow -z)$$

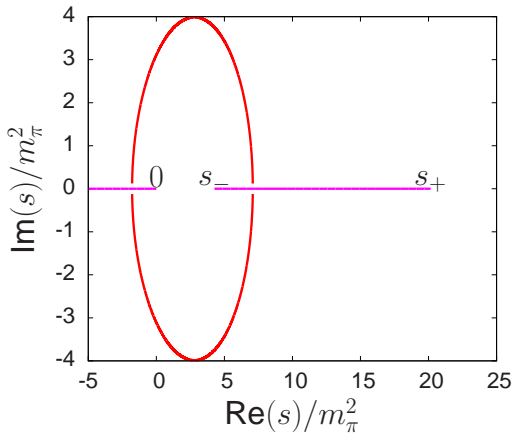
- PW projection involves

$$L_V(s, q^2) = \log \frac{2(m_V^2 - m_\pi^2) + (q^2 - s)(1 + \beta_\pi(s))}{2(m_V^2 - m_\pi^2) + (q^2 - s)(1 - \beta_\pi(s))}$$

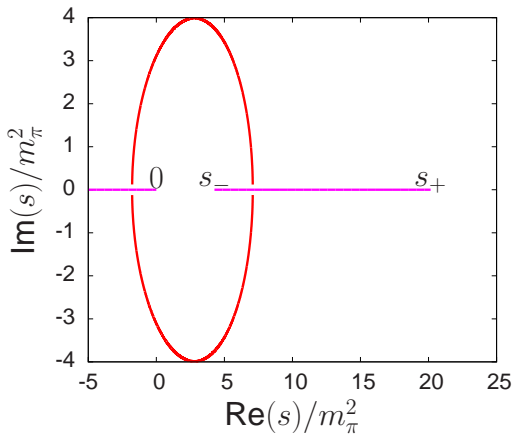


- When  $q^2 = 0$ , cut:  $[-\infty, -m_V^2 (1 - \frac{m_\pi^2}{m_V^2})^2]$
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- Naive ansatz:  $m_V \rightarrow m_V - i\Gamma_V/2$  (not correct!)

# Correctly analytic resonance propagator

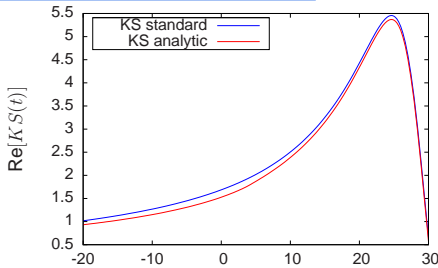
- Usual  $\rho$  propagators (Kühn - Santamaria, Gounaris - Sakurai,...) **bad** analytic properties

$$KS(t) = \frac{1}{m_V^2 - t - i\theta(t - 4m_\pi^2)m_V\Gamma_V \frac{(t - 4m_\pi^2)\beta_\pi(t)}{(m_V^2 - 4m_\pi^2)\beta_\pi(m_V^2)}}$$

- Easy to construct a **good** propagator

$$\widetilde{KS}(t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dt' \frac{\text{Im}[KS(t')]}{t' - t}$$

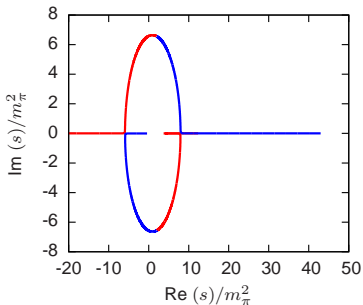
- Slightly different real part



- Partial-wave projection

$$\widetilde{L}_V(s, q^2) = \int_{4m_\pi^2}^{\infty} dt' \text{Im} [KS(t')] \log \frac{t' - t^+(s, q^2)}{t' - t^-(s, q^2)}$$

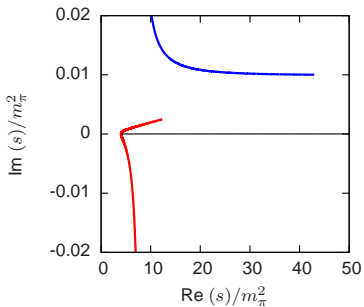
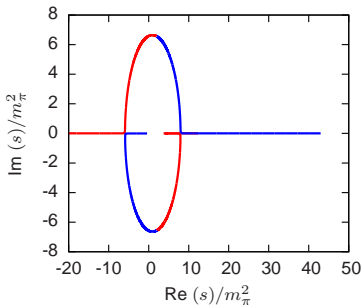
- Cut  $s$ :  $t^\pm(s, q^2) = t'$ ,



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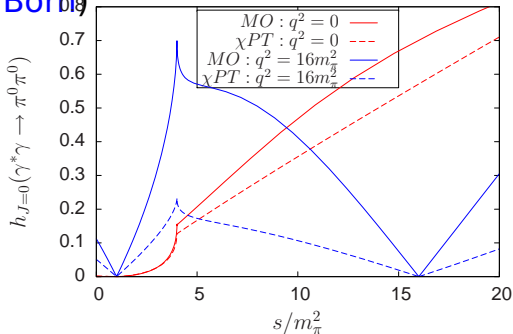
- Cut  $s$ :  $t^\pm(s, q^2) = t'$ , with  $q^2 + i\epsilon$  prescription



- Intersection w. real axis:  $s_c = 4m_\pi^2 \left( 1 - \frac{\epsilon^2}{q^2 - 4m_\pi^2} \right)$
- No overlap with unitarity cut: **MO applies !**

# Outlook

- Zero parameter prediction? Soft photon zero + Adler zero (soft pion) determine two subtractions
- Illustration (Only Born)



- Yet to be included: Vector mesons, axials ?
- When  $q = m_\phi$ : coupled-channel MO necessary

## Extra Slides

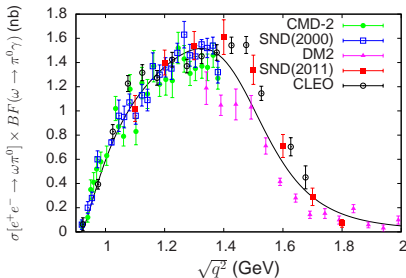


## $F_{\omega\pi}$ puzzle:

→ Expected  $\rho$  dominated, small admixture of  $\rho'$

$$F_{\omega\pi}(t) = \frac{1}{1+\beta} [GS_{\rho}(t) + \beta GS_{\rho'}(t)]$$

→ Fair agreement with expt. in  $s > (m_{\omega} + m_{\pi})^2$

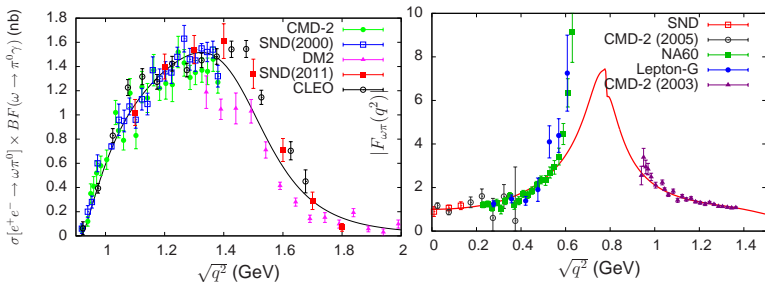


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→ But problem in range  $0.55 \text{ GeV}^2 \lesssim s < (m_{\omega} + m_{\pi})^2$

■ Also see talk by S. Schneider !