

Scattering and excited meson spectroscopy with LQCD

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Hadron Spectrum Collaboration



Outline

- Scattering in lattice QCD
 - Isospin-2 $\pi\pi$
- Excited charmonium spectrum
- Summary

arXiv:1203.6041
arXiv:1204.5425

Meson Spectroscopy

Current and upcoming experimental efforts

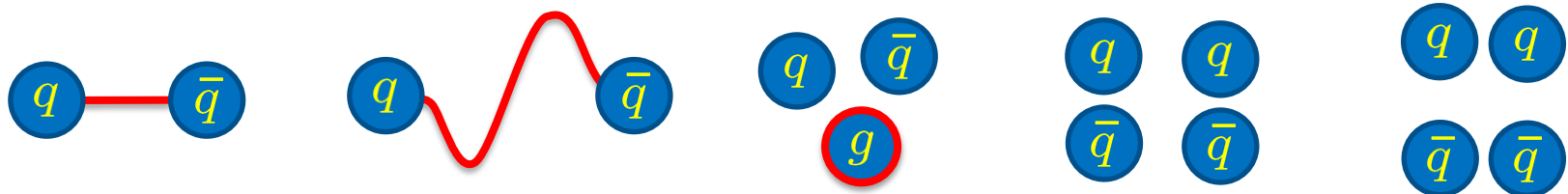


CLAS12

BESIII

KLOE2

...

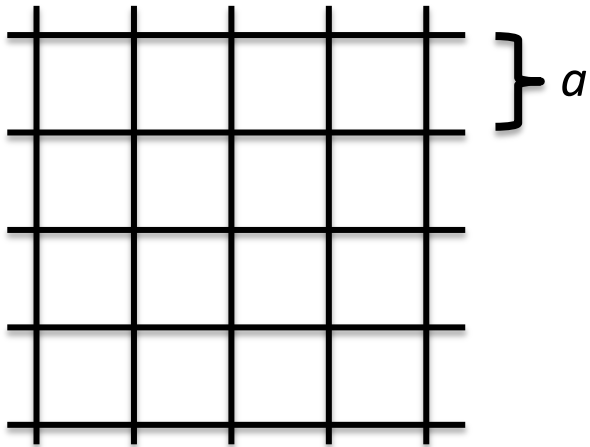


Exotics ($J^{PC} = 1^{-+}, 2^{+-}, \dots$)? – can't just be a $q\bar{q}$ pair

Probe low energy d.o.f. of QCD

e.g. hybrids, multi-mesons

QCD on a Lattice



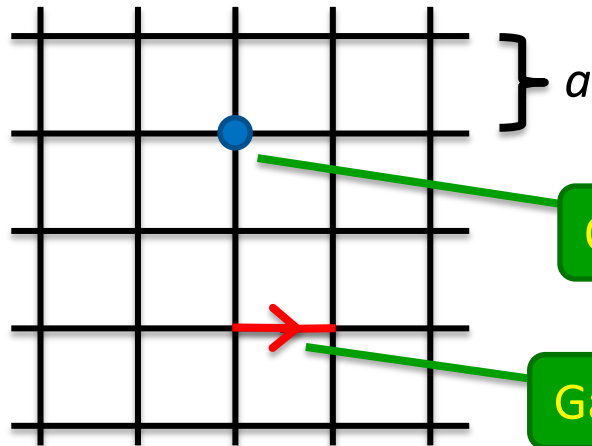
Discretise on a grid (spacing = a) – regulator

Finite volume \rightarrow finite no. of d.o.f.

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$$\psi(x) \rightarrow \psi_x$$

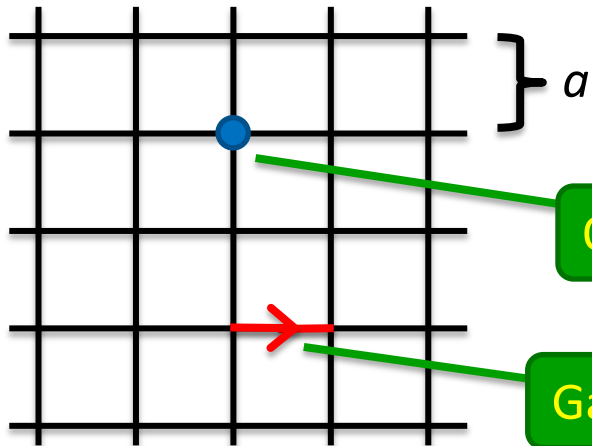
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$$A_\mu(x) \rightarrow U_{x,\mu} = e^{-aA_{x,\mu}}$$

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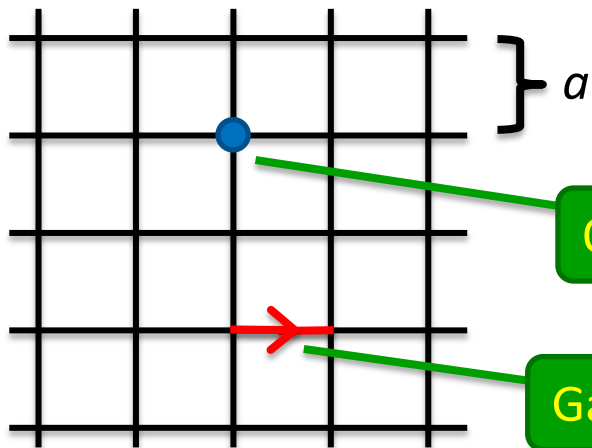
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Path integral formulation

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U f(\psi, \bar{\psi}, U) e^{iS[\psi, \bar{\psi}, U]}$$

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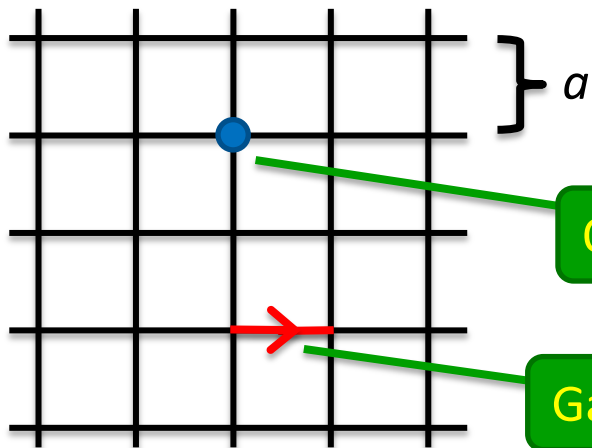
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Do fermion integral analytically then use importance sampling Monte Carlo

QCD on a Lattice



Quar

Gauge



c, μ

JLab

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Calculate **energies** and **matrix elements** (“overlaps”, Z 's)
from correlation functions of meson interpolating fields

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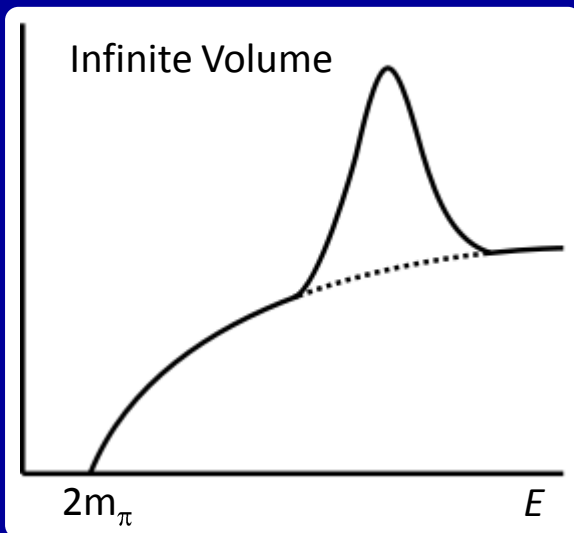
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$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2 E_n} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Multi-hadron states in a finite volume

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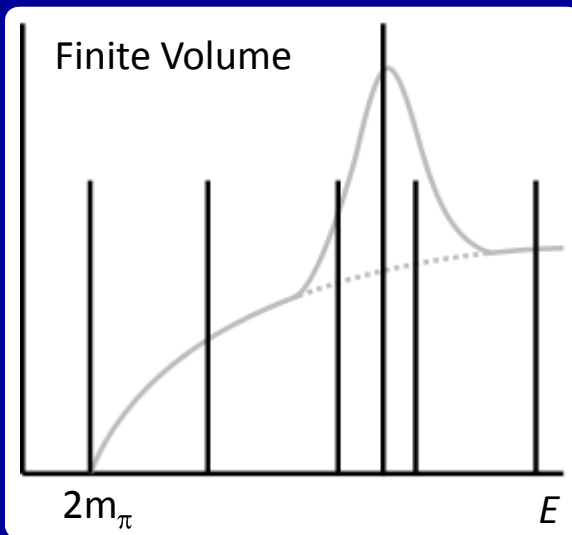


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Cubic box with periodic b.c.s:

$$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$$

→ discrete spectrum



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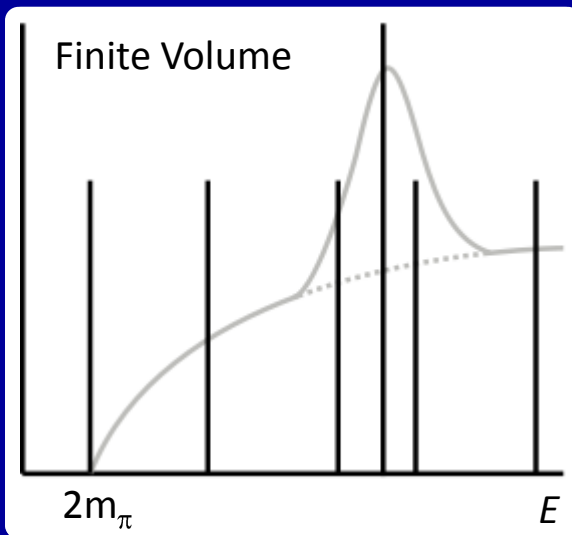
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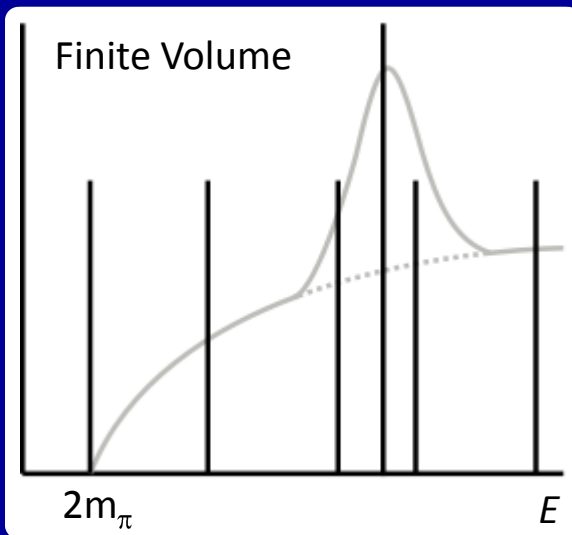
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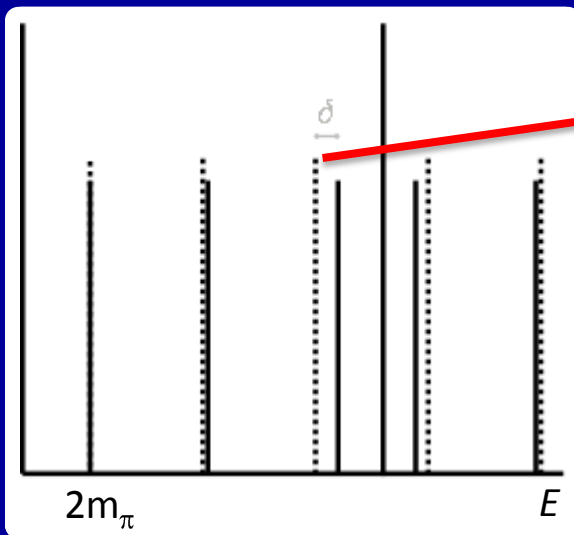
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Extract phase shift at discrete p_{cm}

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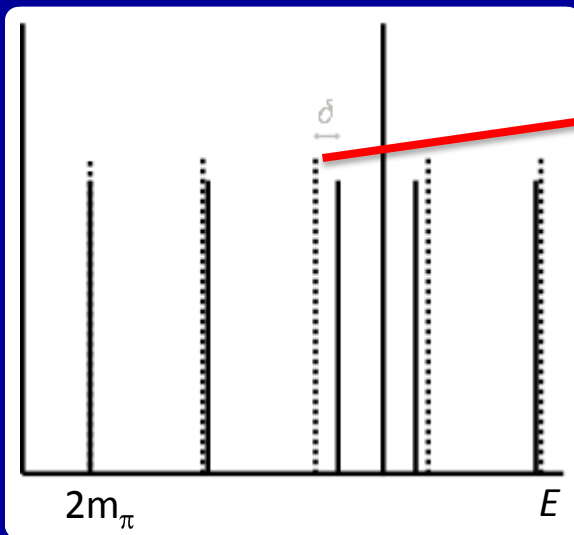
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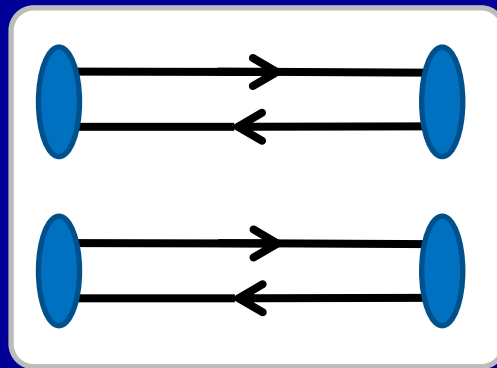
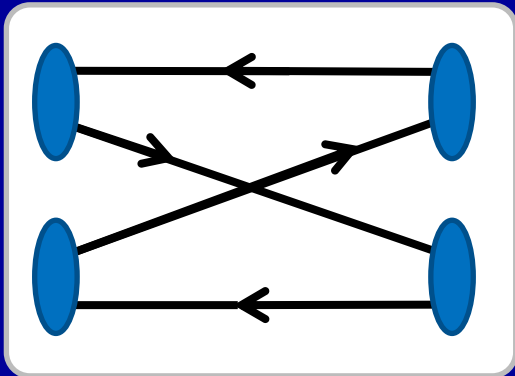
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Map out phase shift → resonance parameters (mass, width)

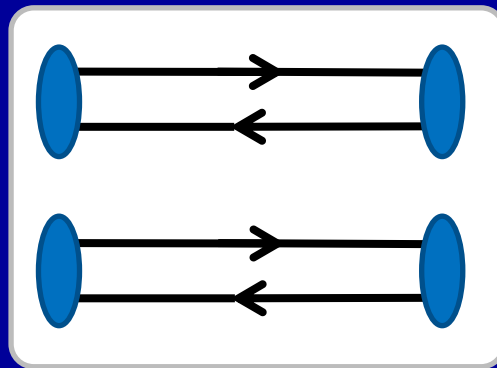
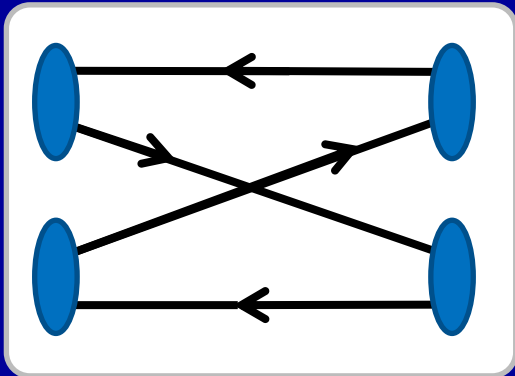
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+ similar diagrams

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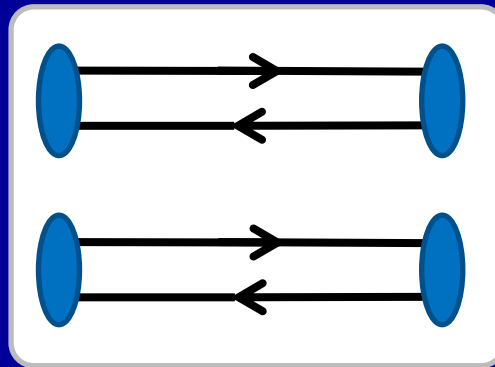
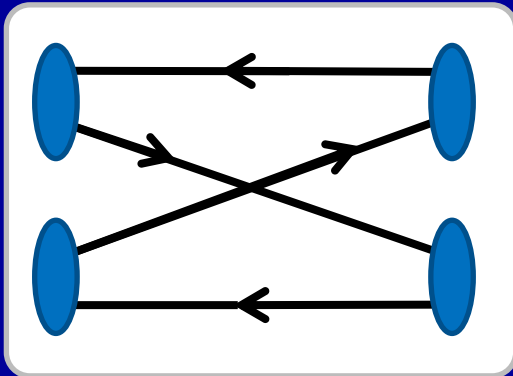
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Large basis of ops

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

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Dynamical (unquenched) [$N_f = 2+1$]

$M_\pi \approx 400$ MeV

Three volumes: $16^3, 20^3, 24^3$ ($L_s \approx 1.9 - 2.9$ fm)

$a_s \approx 0.12$ fm, finer in t ($a_s/a_t \approx 3.5$)

arXiv:1203.6041 – Jo Dudek, Robert Edwards, CT

Lüscher Method and $\pi\pi$ $l=2$

(Elastic) energy shifts in finite volume \rightarrow phase shift

Nucl. Phys. B354, 531

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Matrices in space of l contributing to irrep

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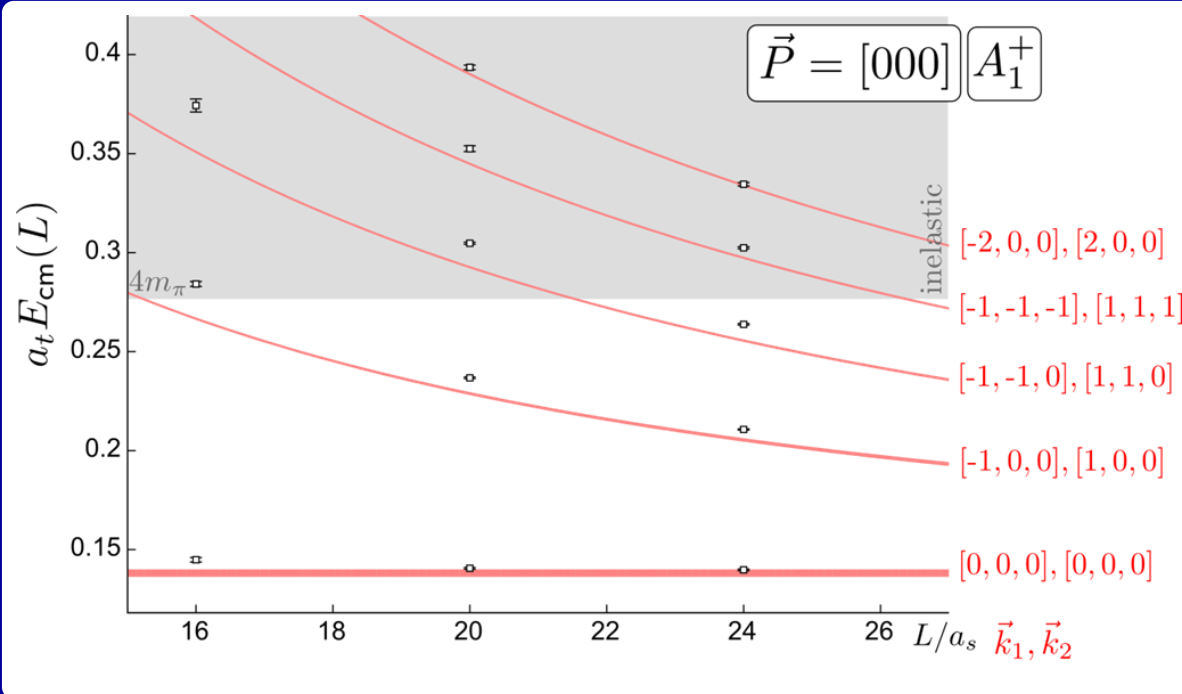
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More p_{cm} points **BUT** reduced sym so more mixing

$\pi\pi$ $l=2$ spectra

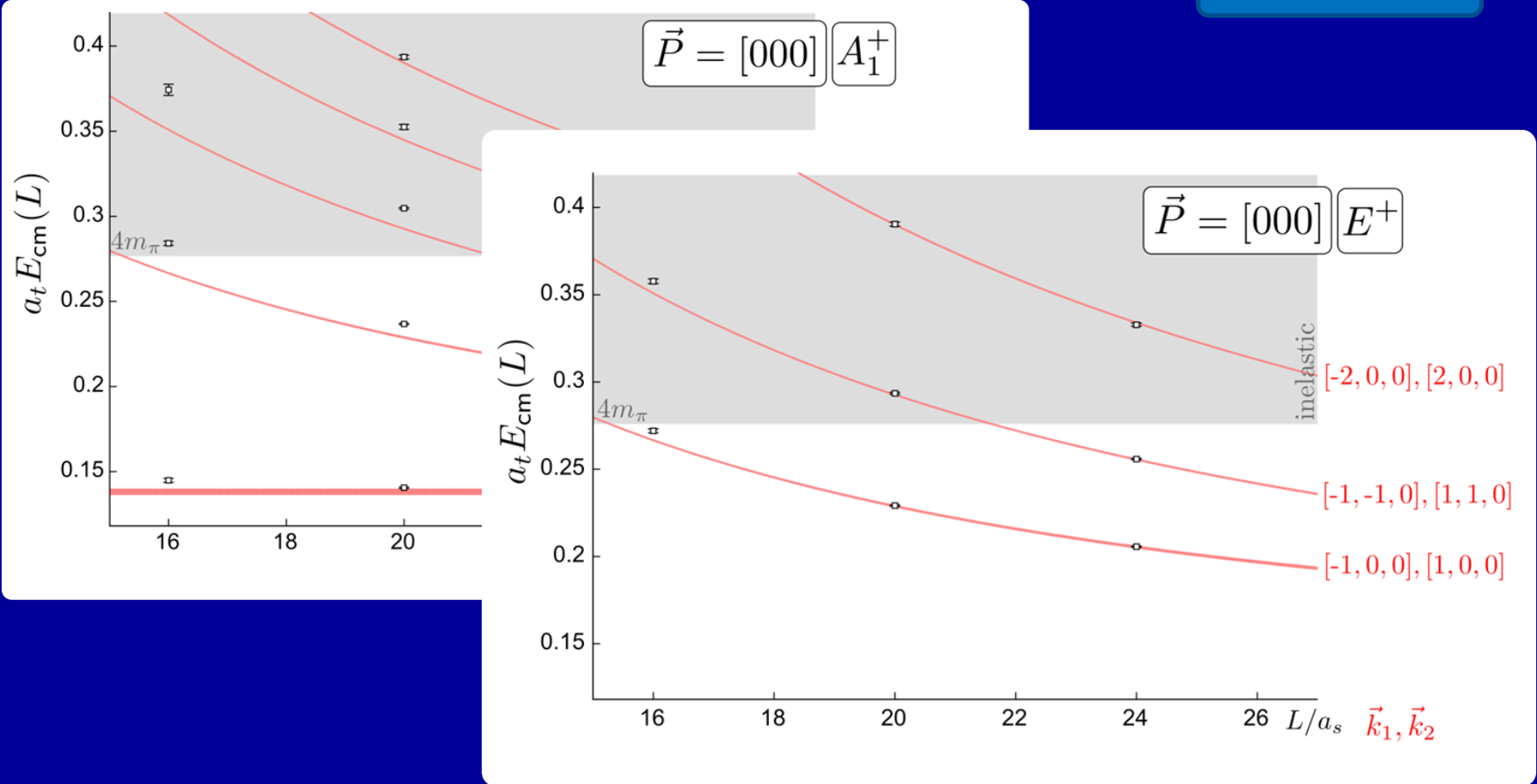
arXiv:1203.6041



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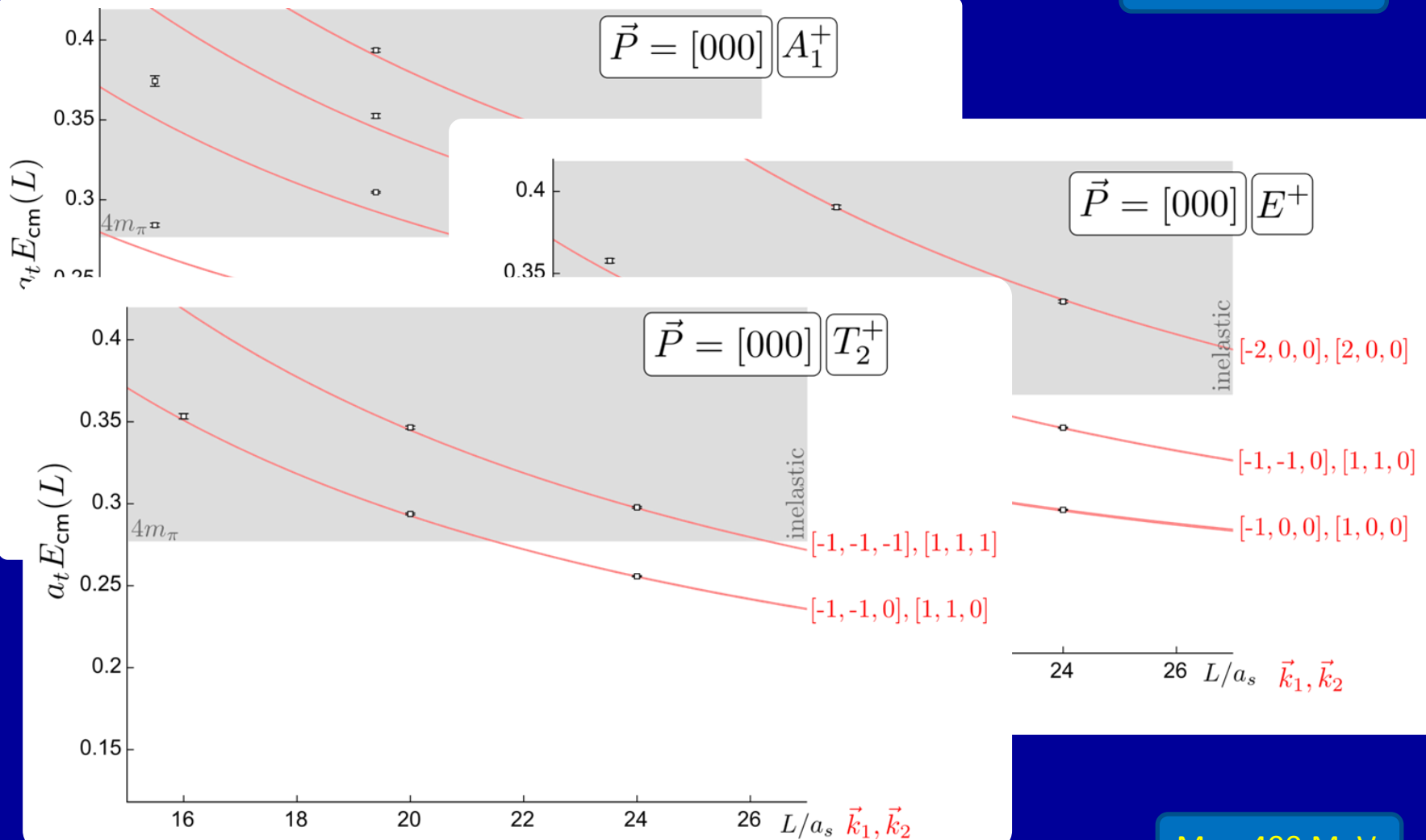
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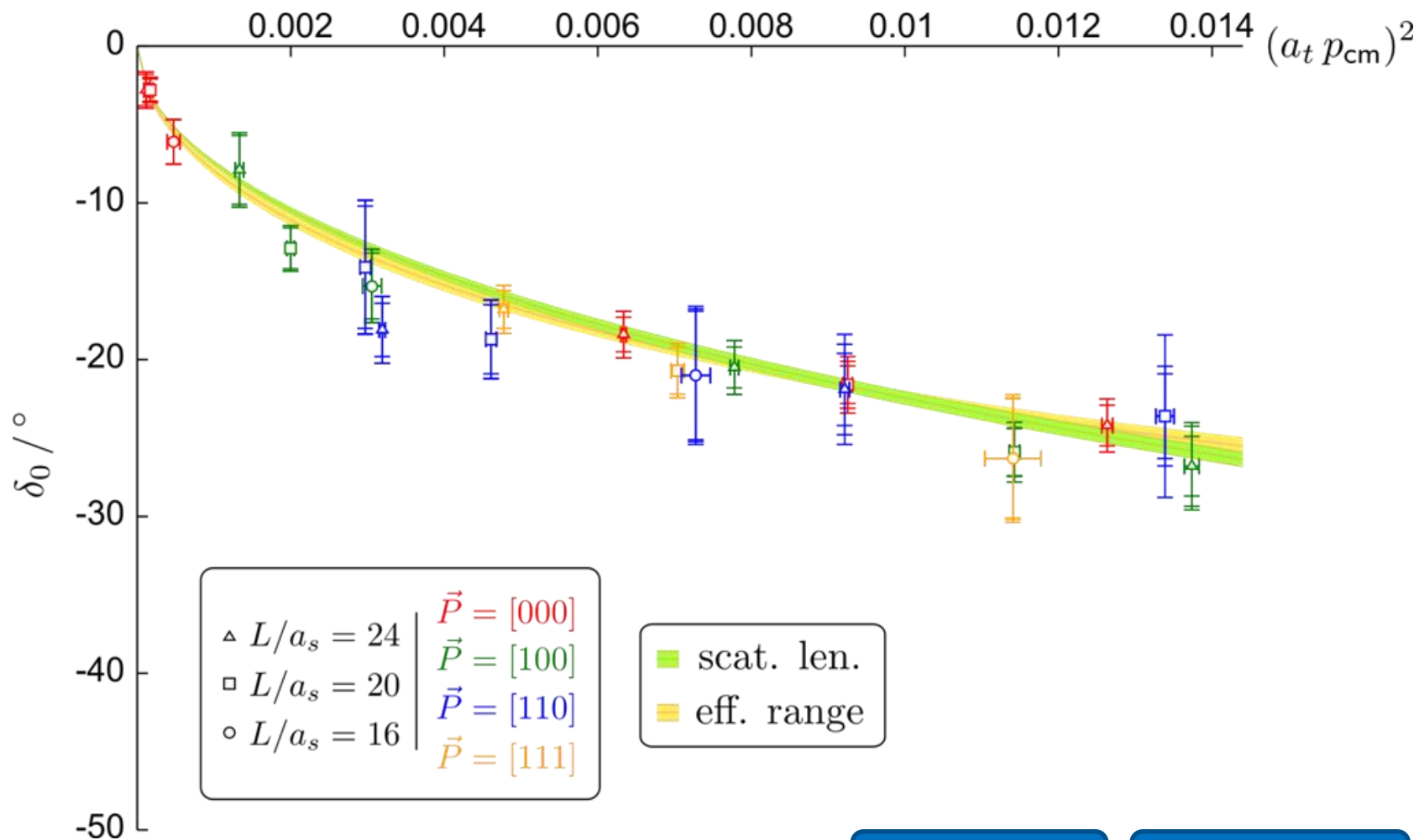
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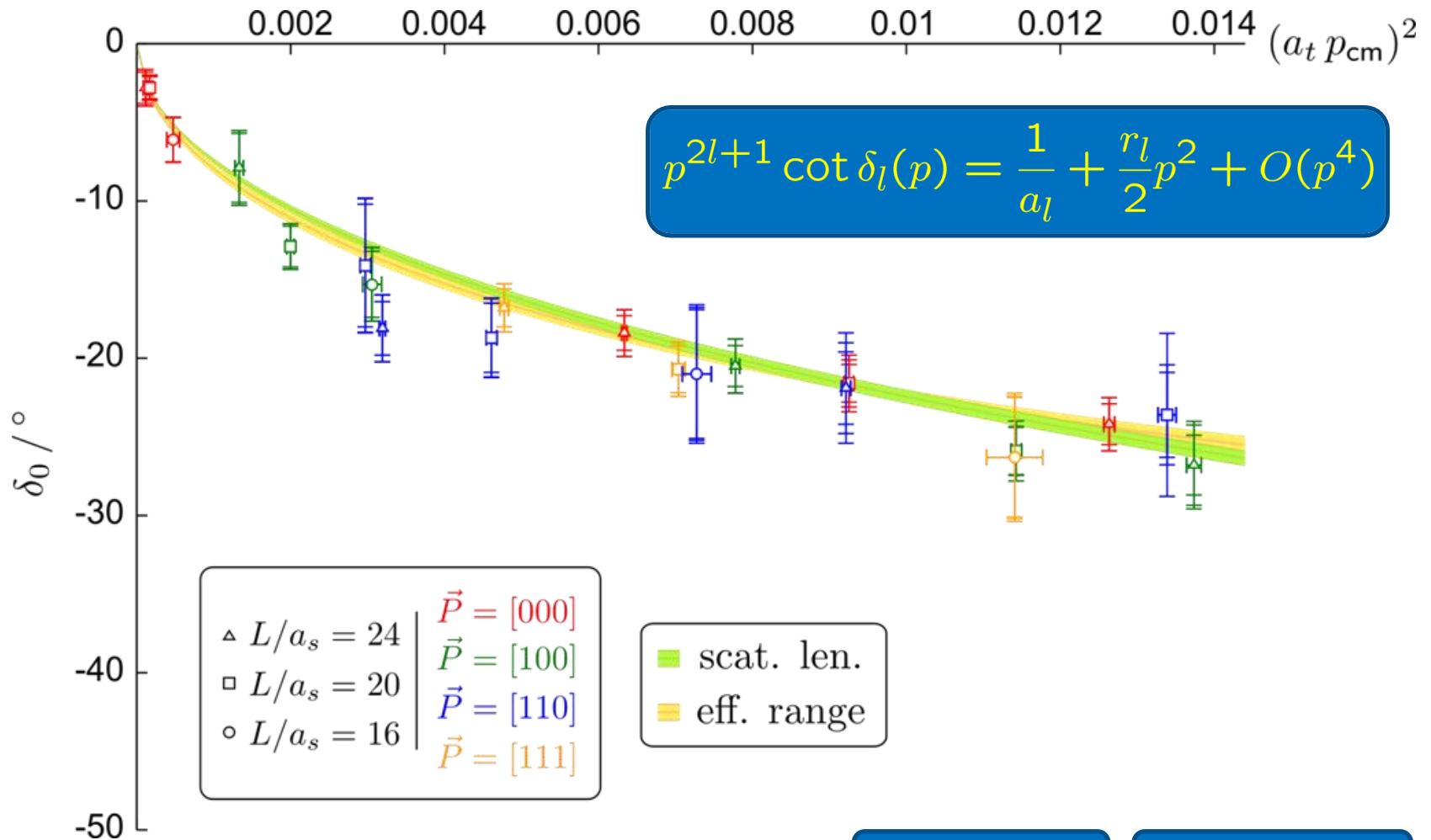
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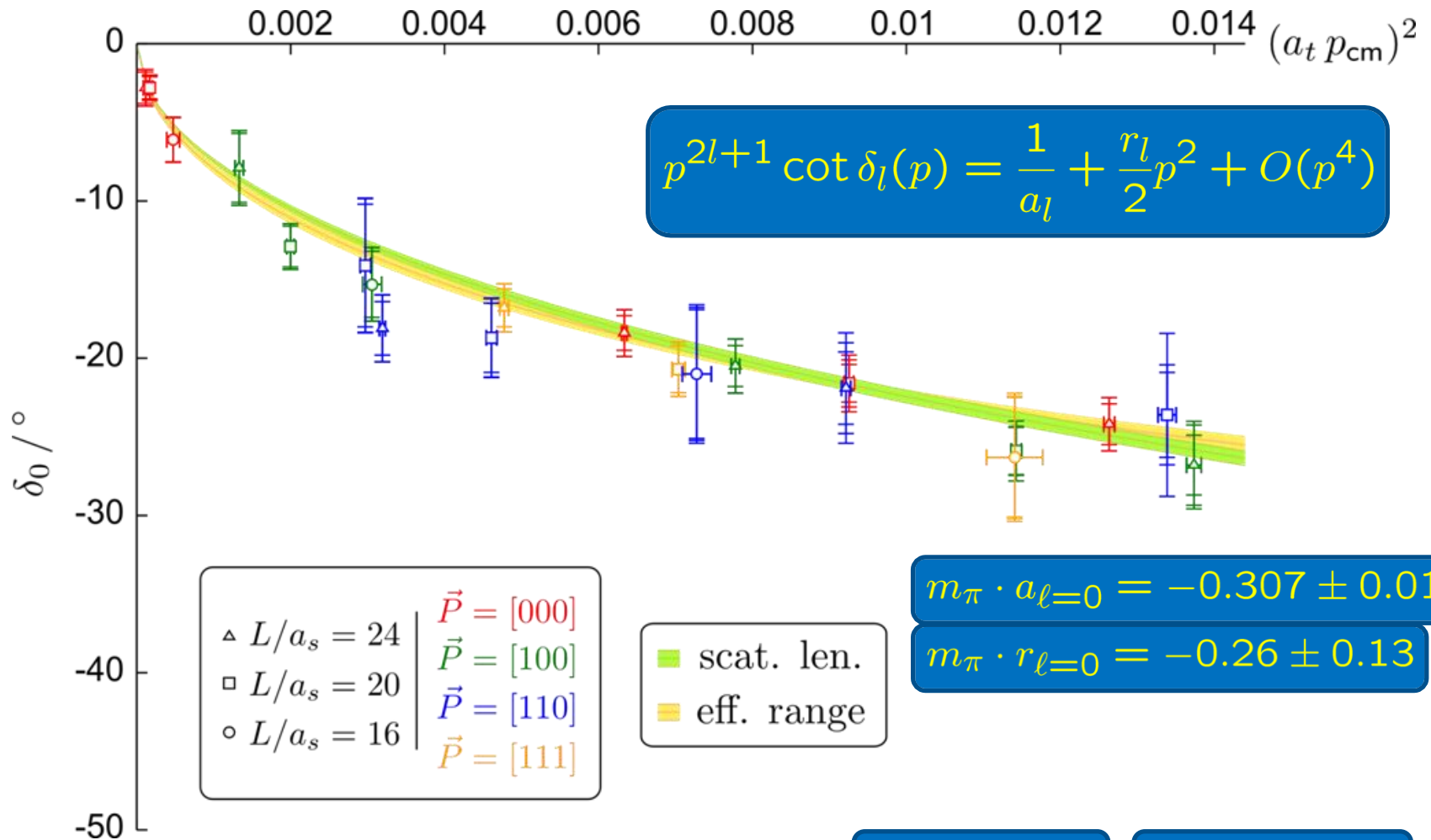
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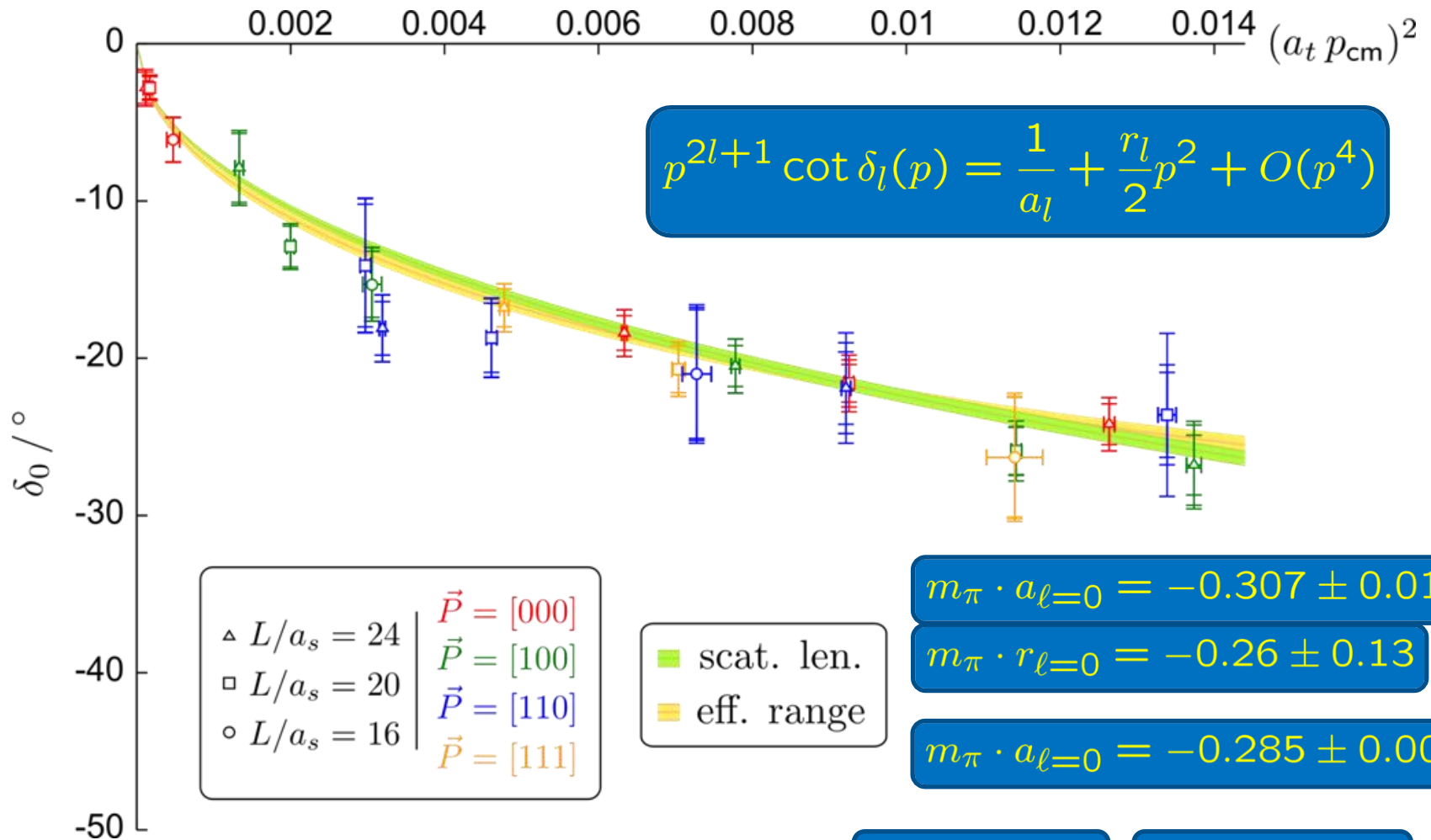
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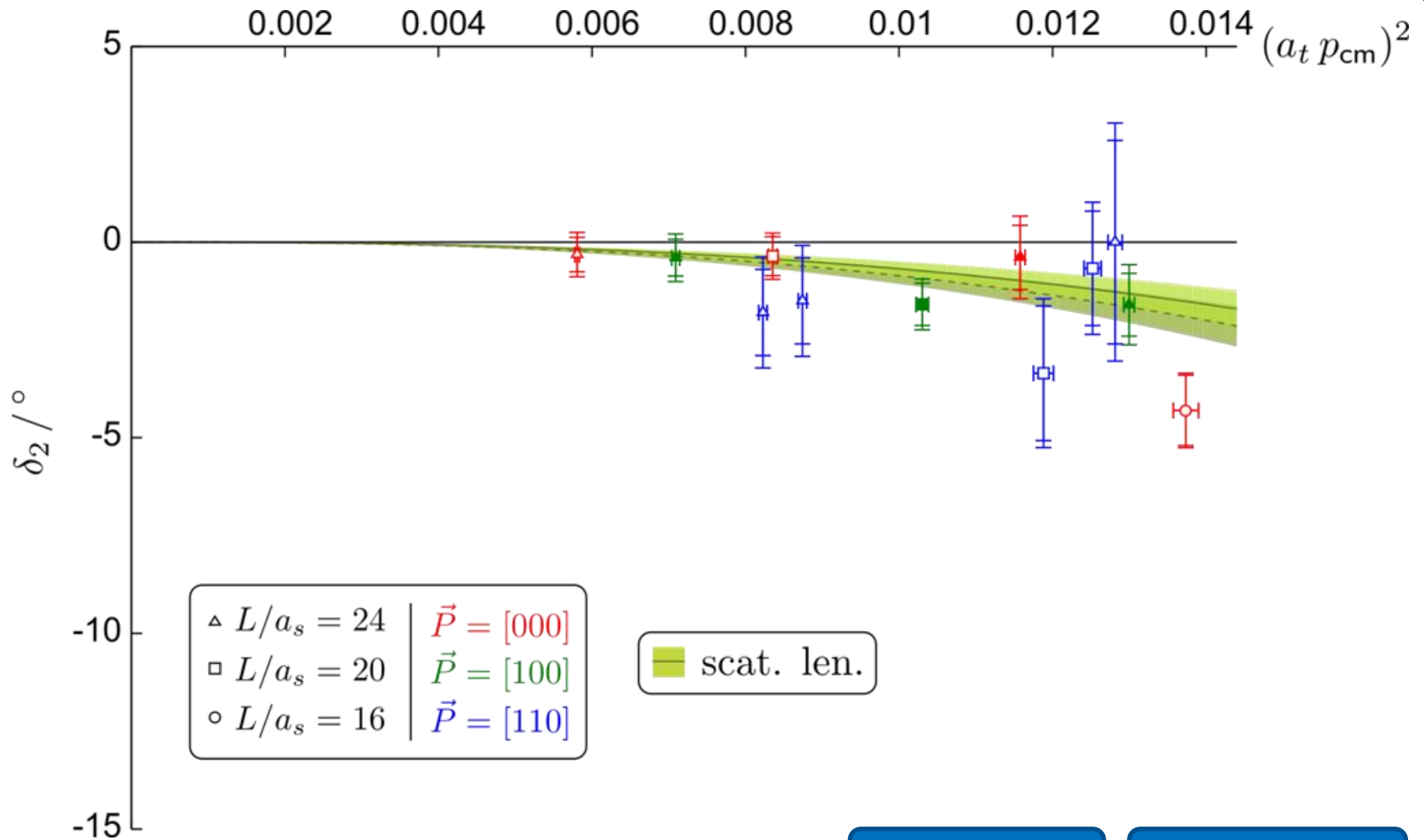
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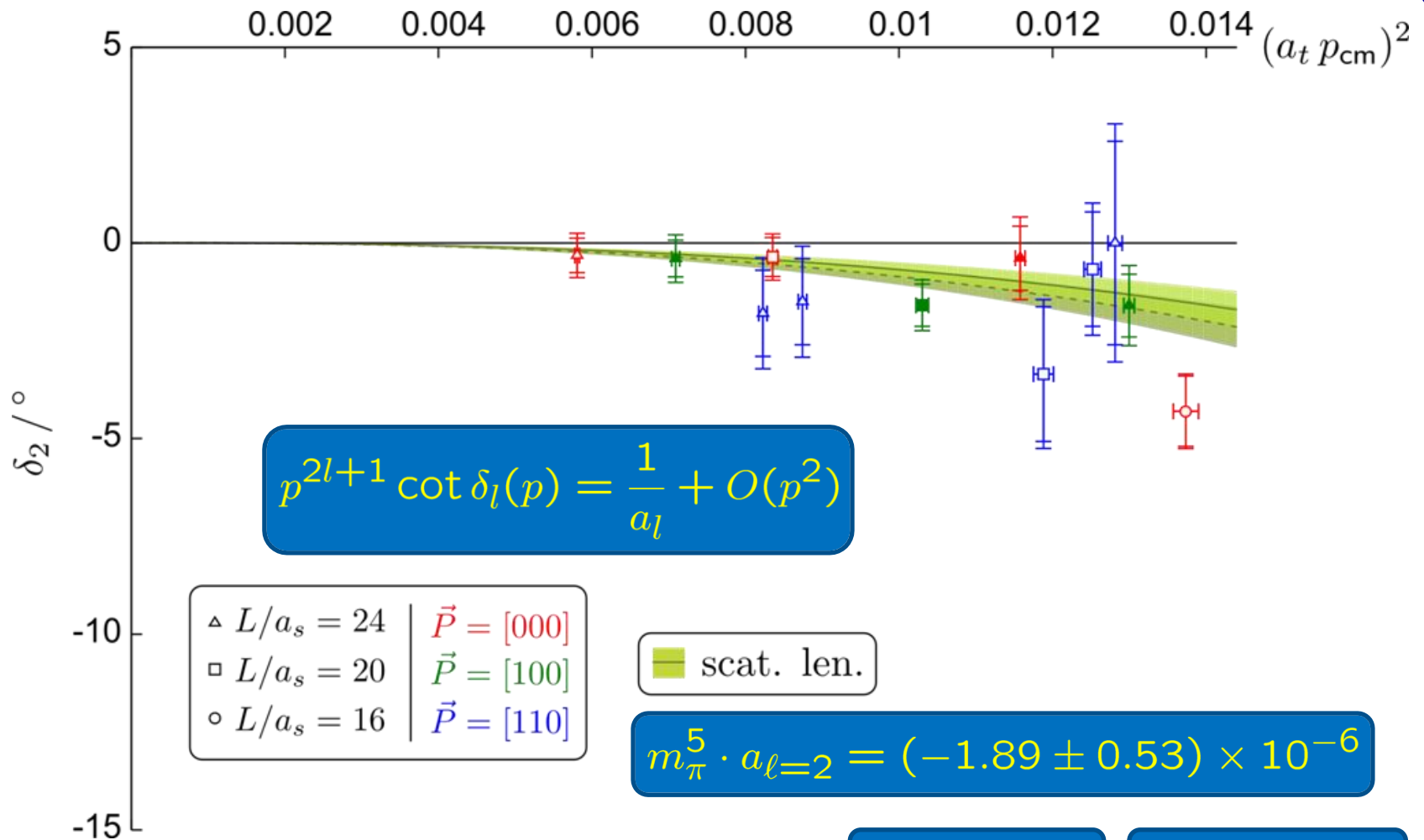
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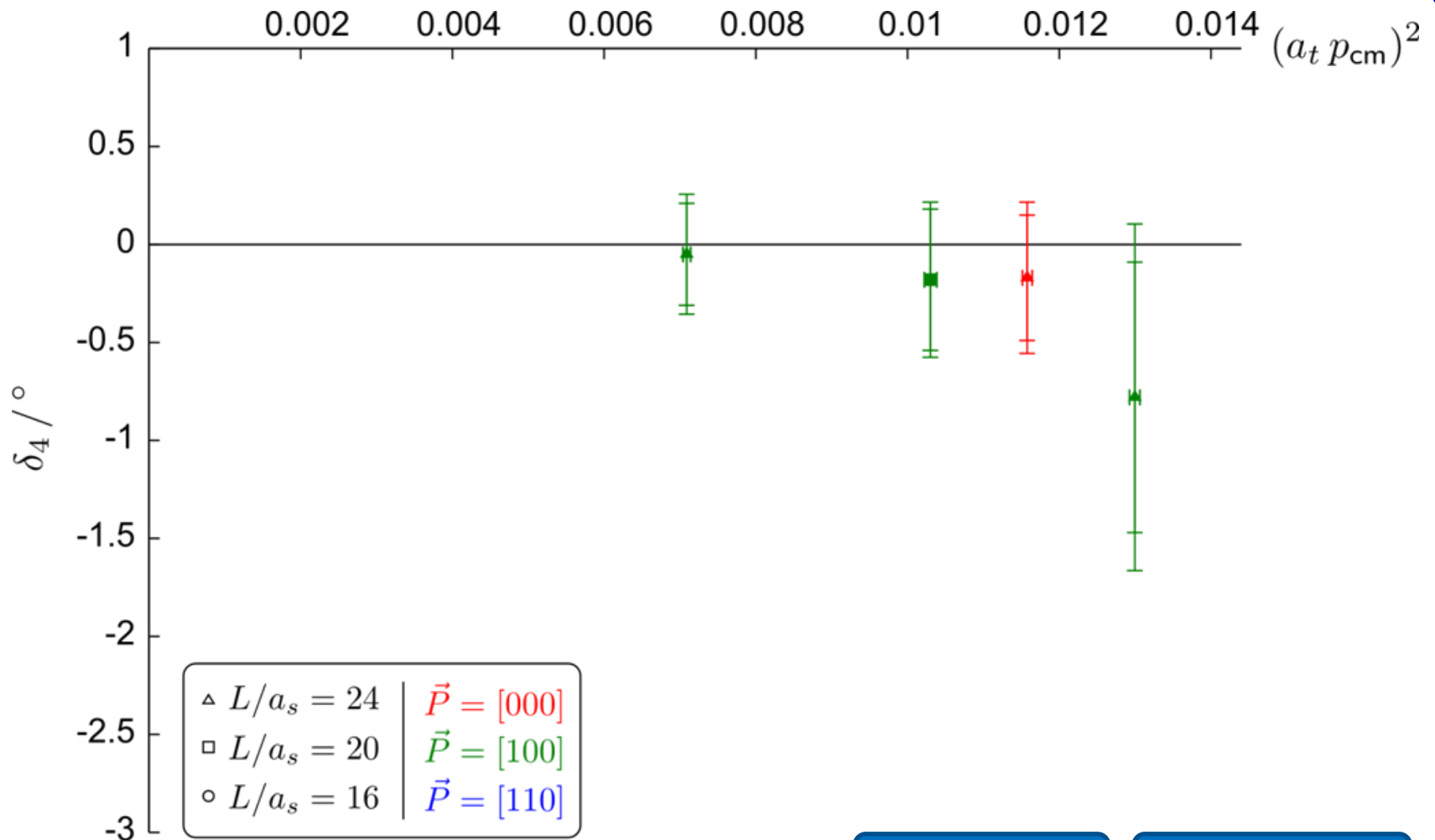
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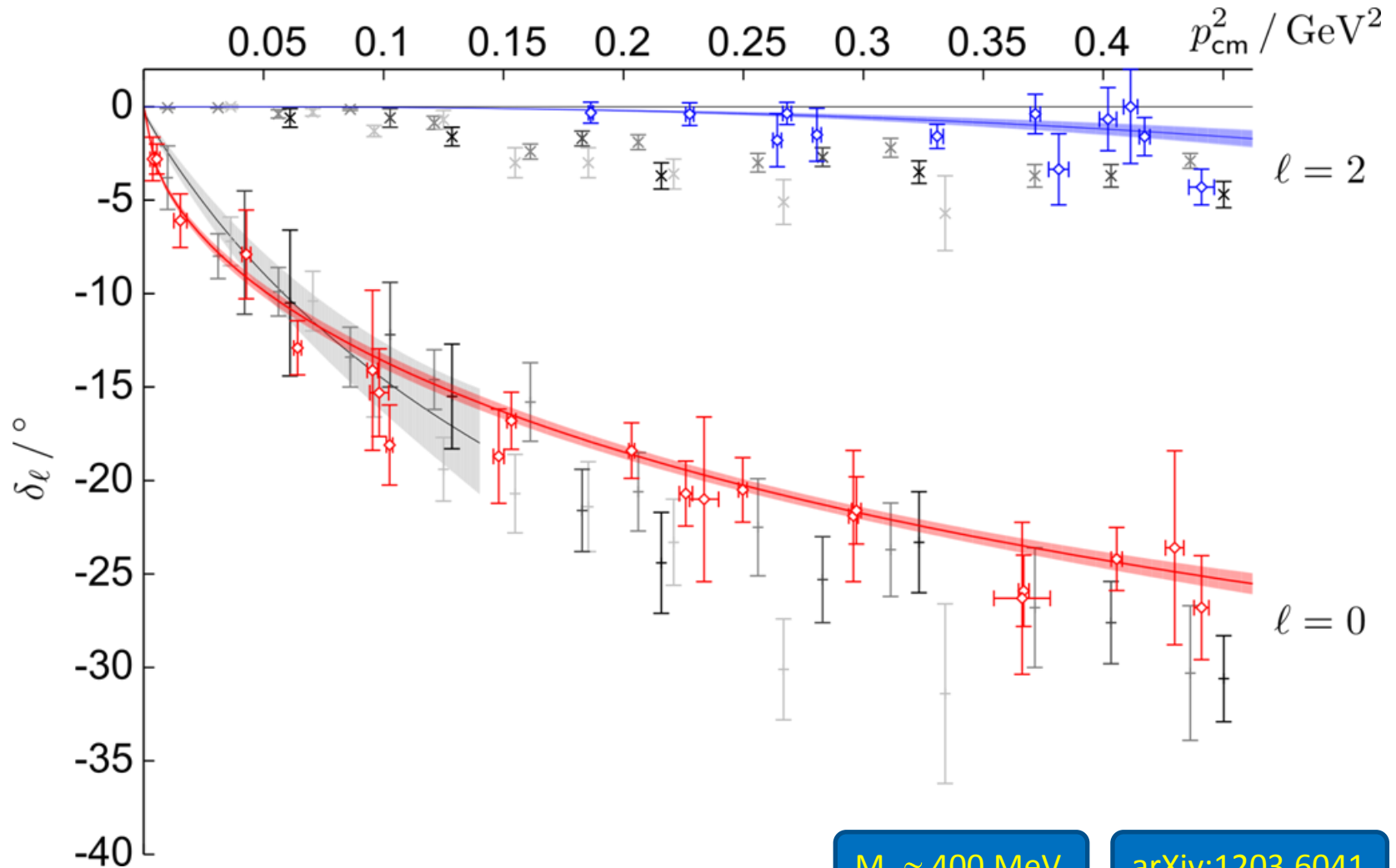
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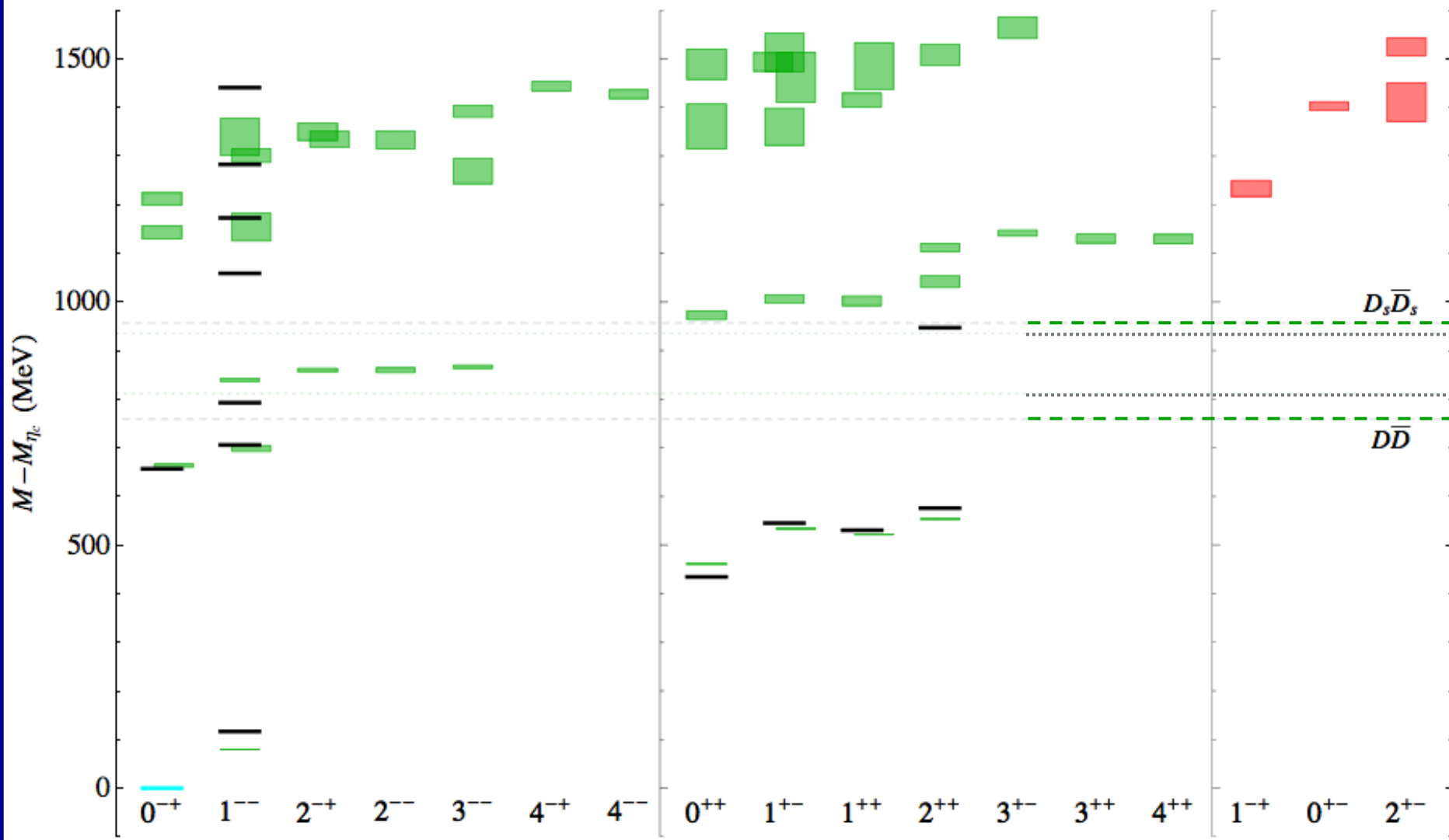
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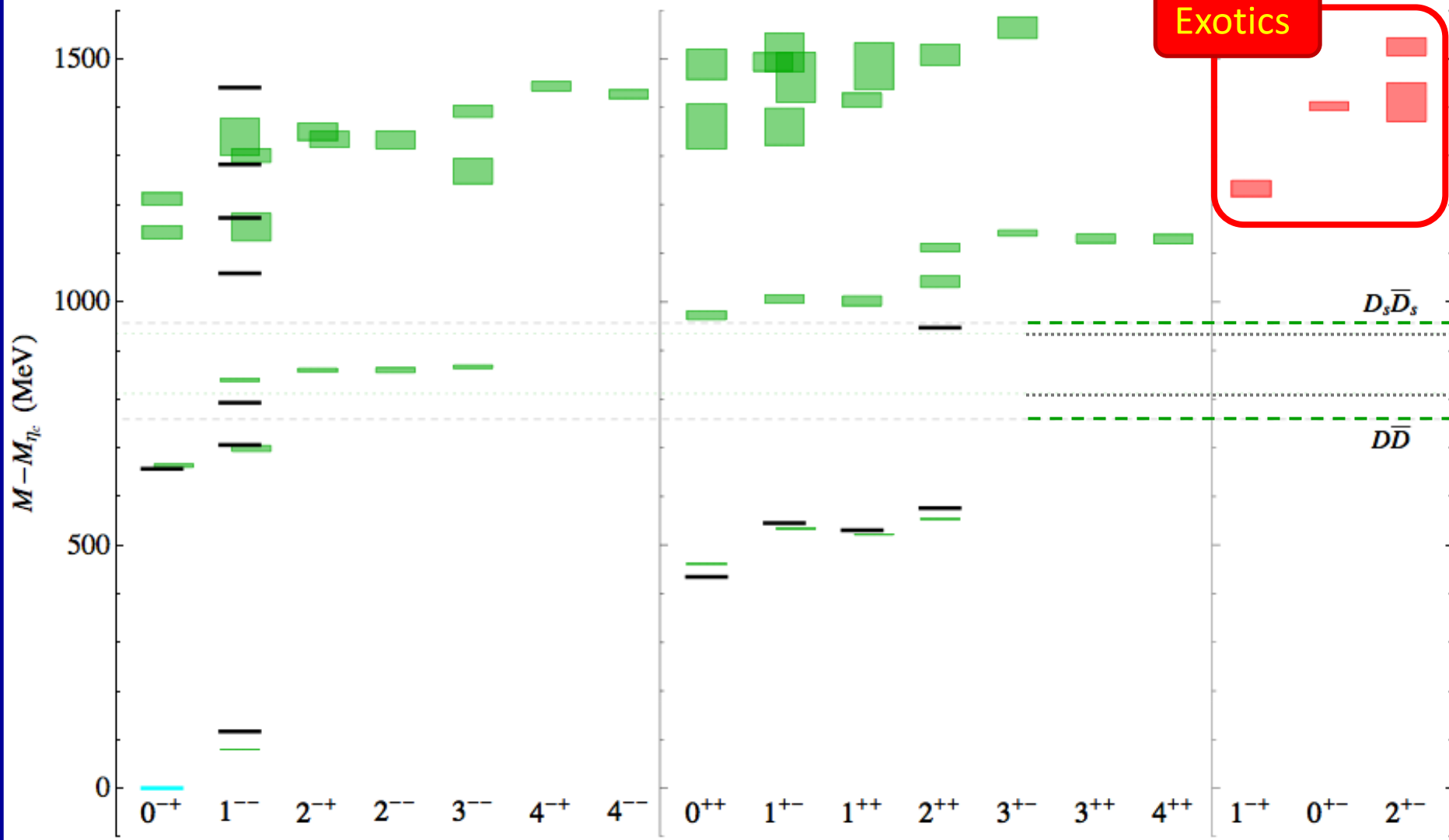
Excited charmonium spectrum

arXiv:1204.5425 – Liuming Liu, Graham Moir, Mike Peardon, Sinéad Ryan, CT, Pol Vilaseca;
Jo Dudek, Robert Edwards, Bálint Joó, David Richards



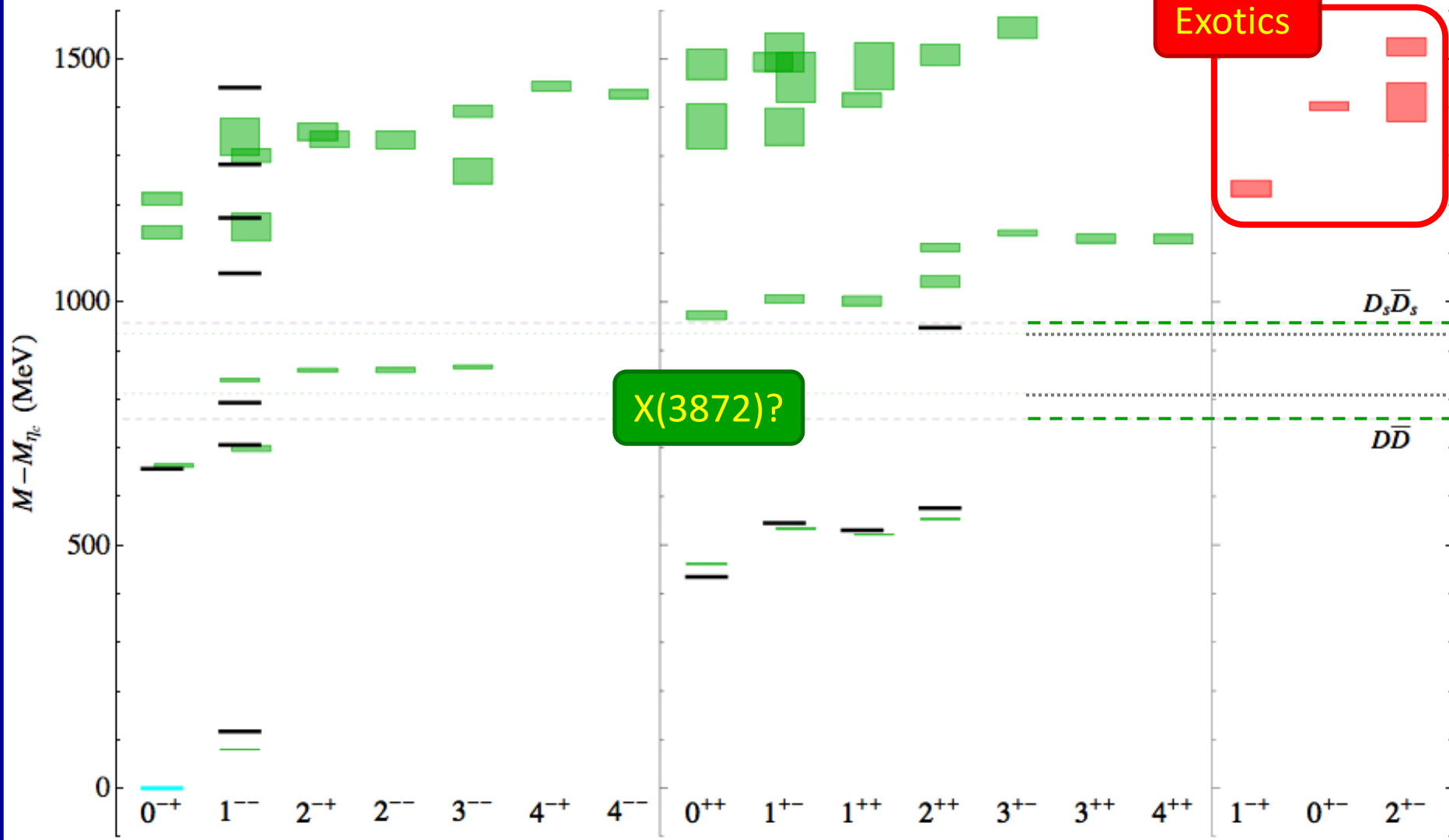
24^3 , $M_\pi \approx 400$ MeV

arXiv:1204.5425



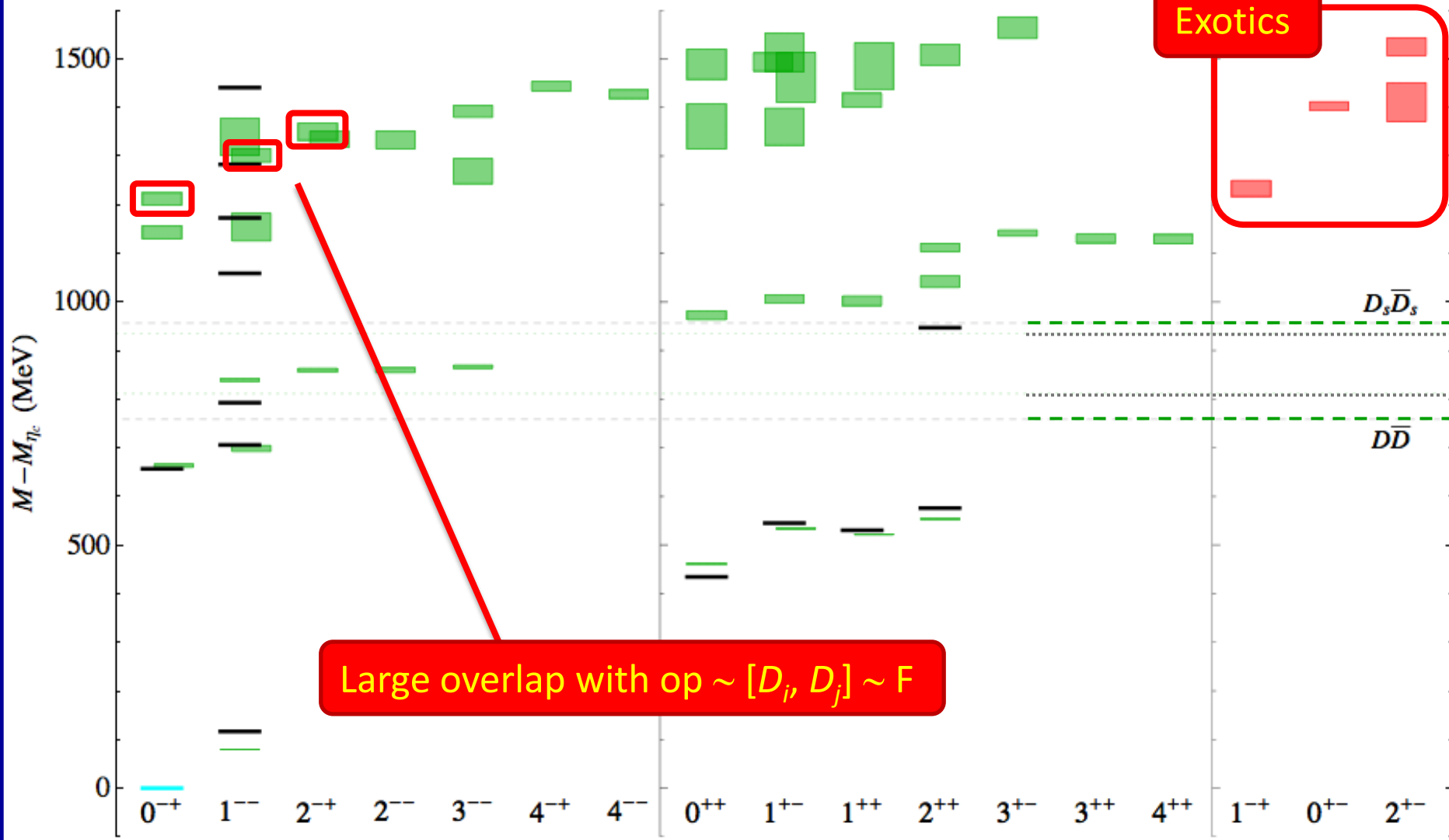
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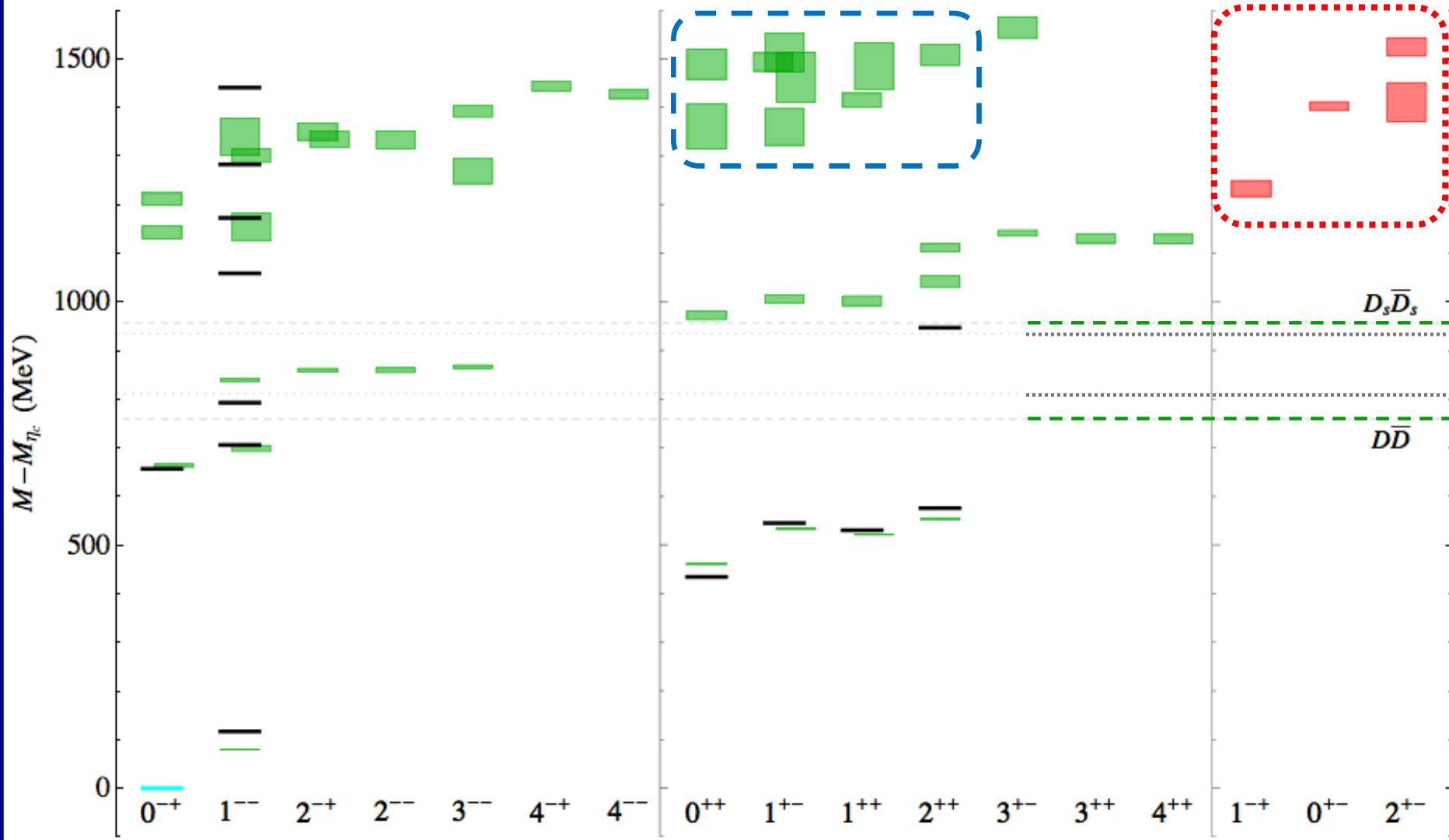
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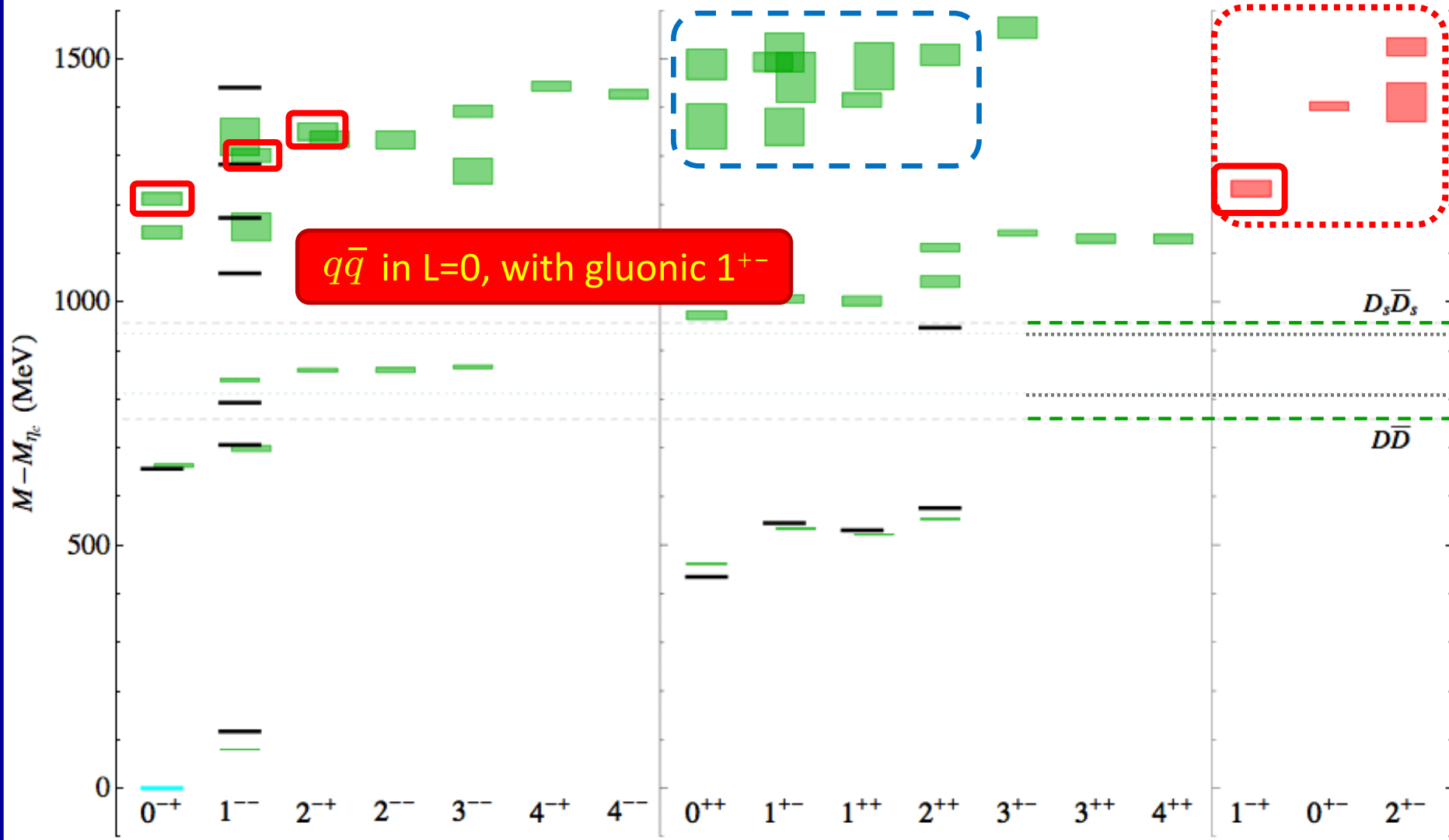
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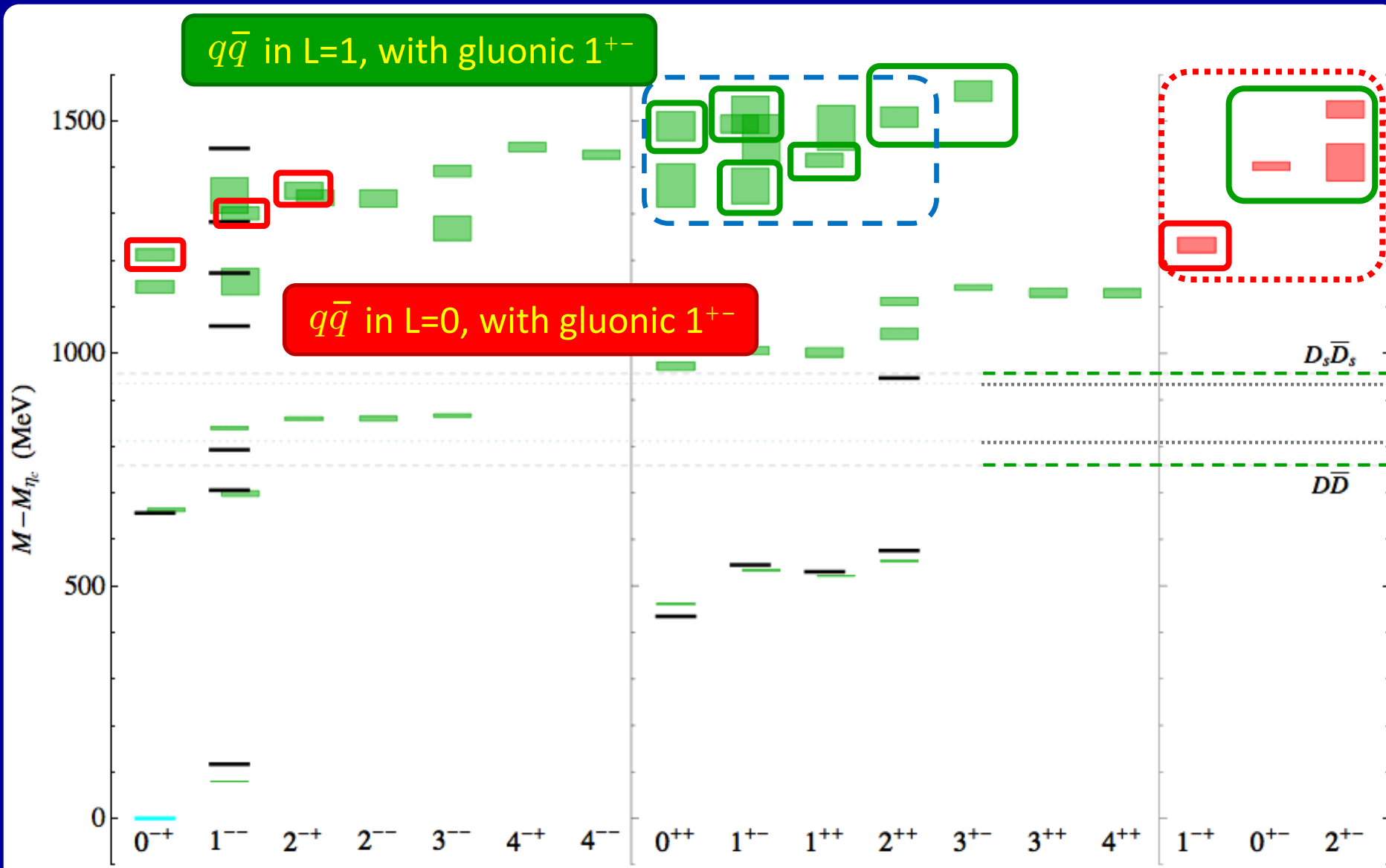
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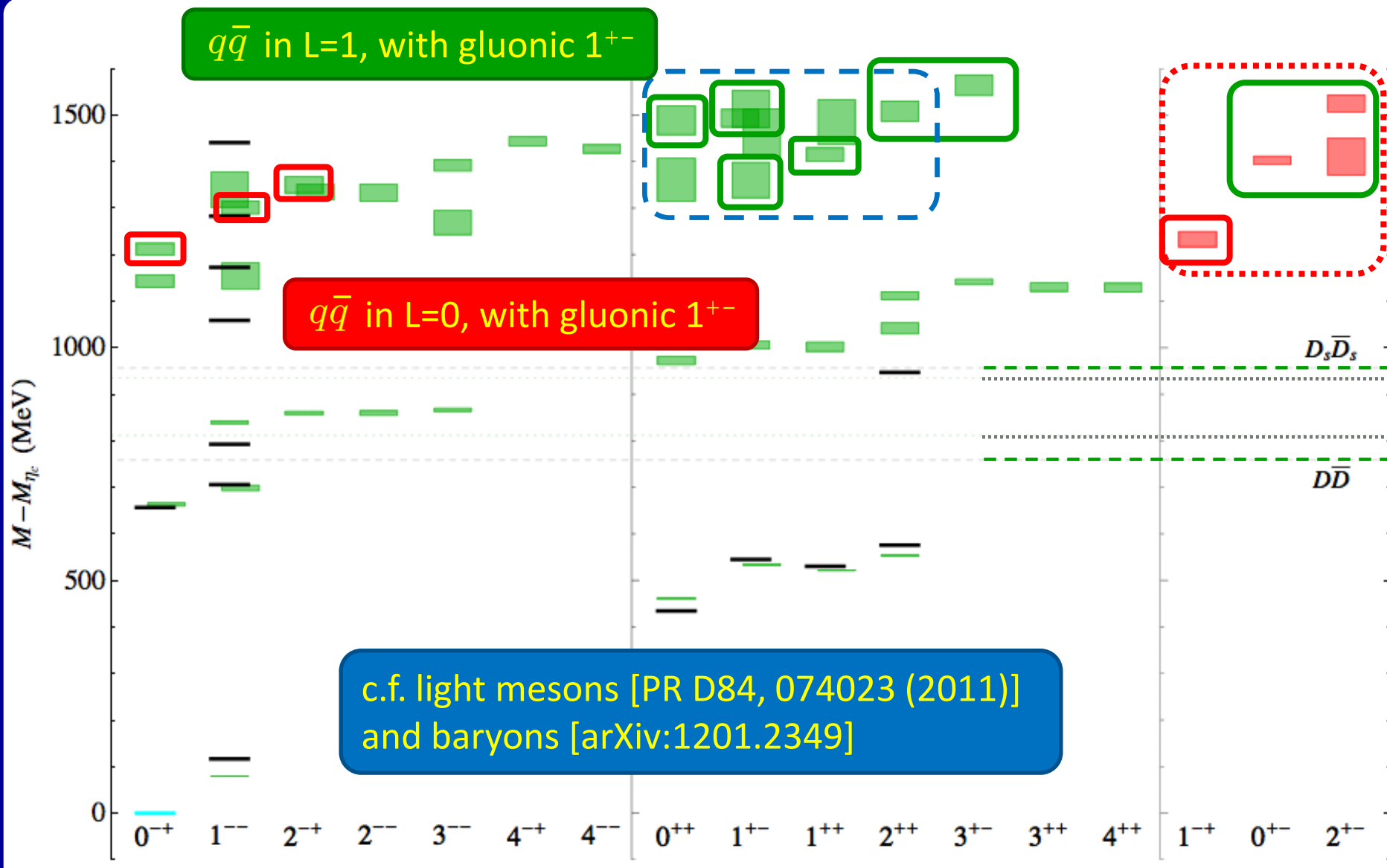
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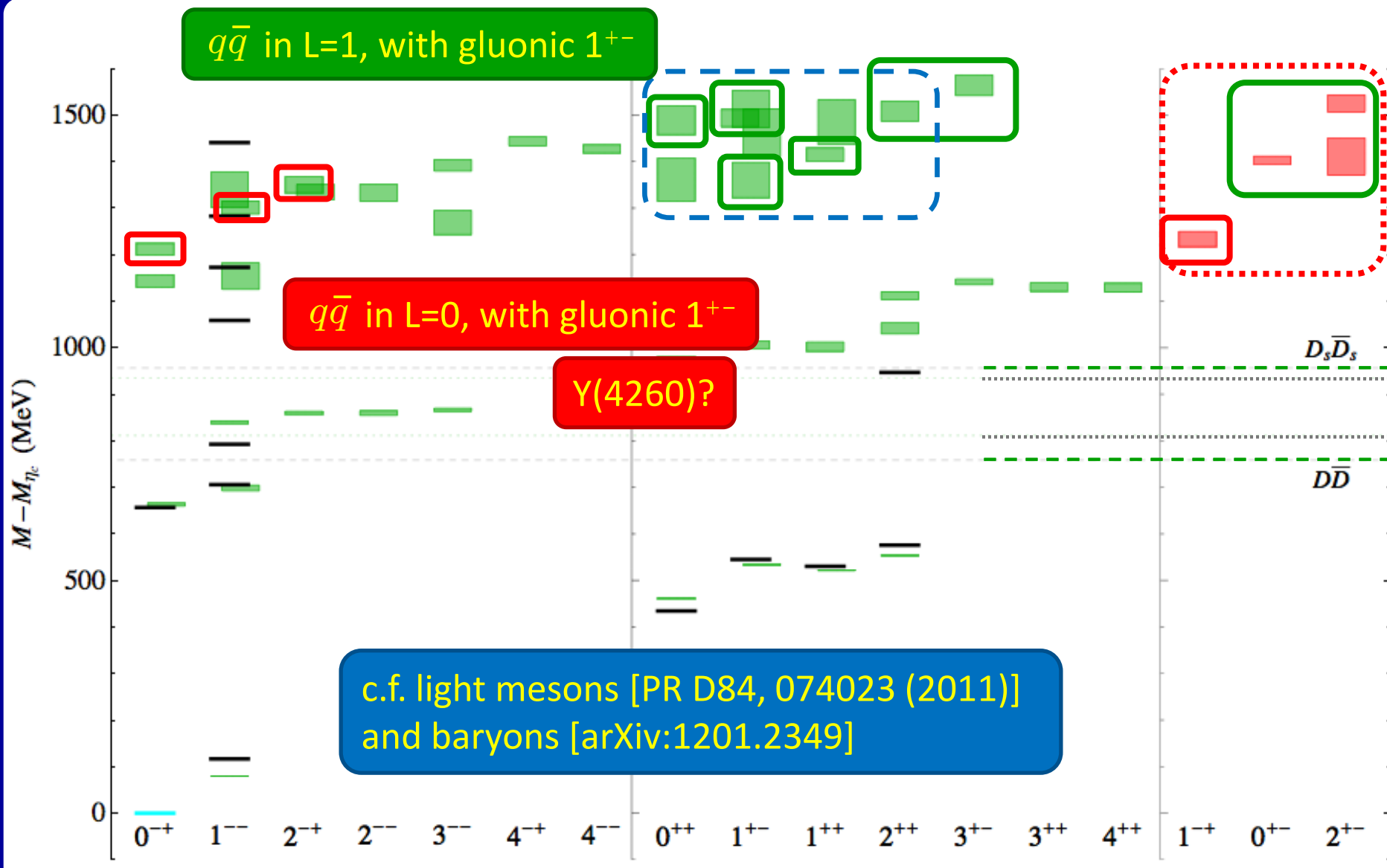
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 - Baryons, ...

Outlook

- Other **scattering** channels – **resonances**, decays, ...
- Isoscalars mixing with **glueballs**; 0^{++} channel
- Radiative transitions
- Lighter pion masses, larger volumes, lattice spacing, ...



