

Analysis tools: Theory discussion

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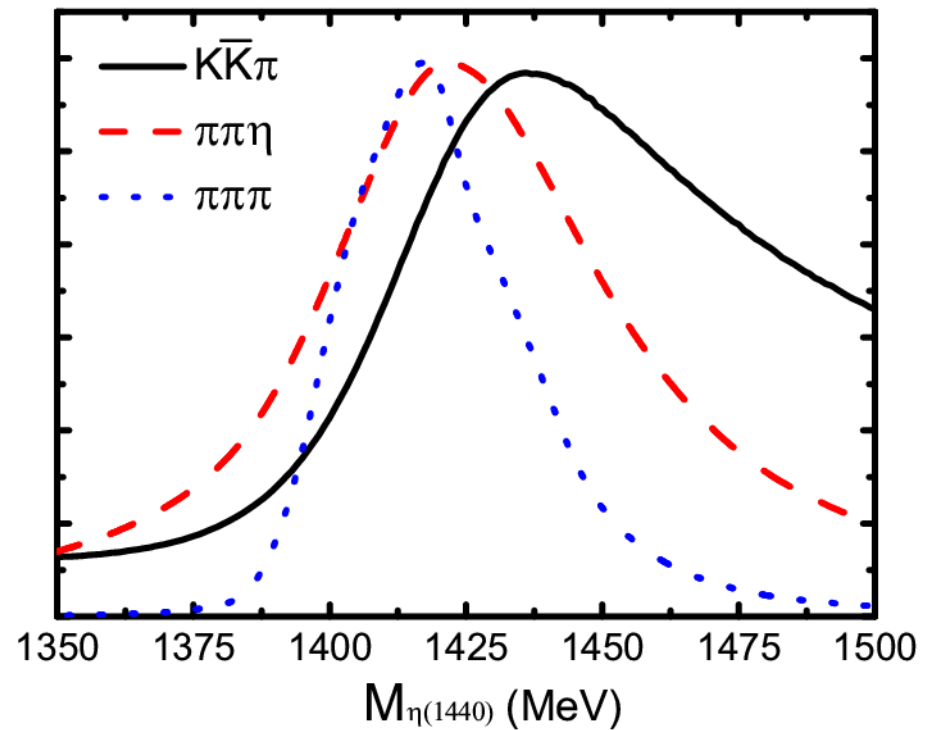
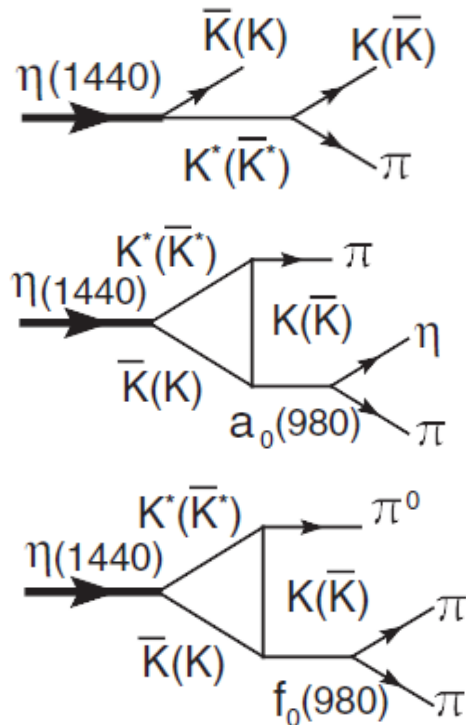
Issues:

- 1) **Model dependence for coupled-channel 2-body scatterings**
SAID, MAID, Bonn-Gachina -- K-matrix
EBAC, Juelich -- DCC integrals
unitarity: how many channels ? only 2-body ones ?
pole values: model independent ? BRs ?
- 2) **Connection between various isobar results from production processes and coupled-channel two-body scattering results**
BW fit or K-matrix fit ? how to deal with energy-dependence of production vertices and decay dynamics ?
- 3) **What PWA approaches experiments are supposed to use ?**

1. Coupled channel dynamics

$J/\psi \rightarrow \gamma \eta(1440) \rightarrow \gamma (\bar{K}K\pi, \pi\pi\eta, \pi\pi\pi)$ as an example

J.J.Wu, X.H.Liu, Q.Zhao, B.S.Zou, PRL108 (2012) 081803



**Simple BW fit to mass spectra of different final states
→ very different mass and width for the same resonance !**

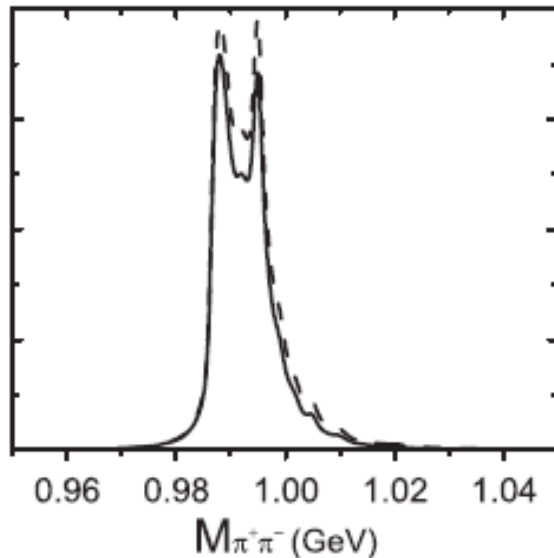
Triangle singularity \rightarrow natural explanation of BRs

$$J/\psi \rightarrow \gamma \eta(1440) \rightarrow \gamma \bar{K} K \pi \quad (2.8 \pm 0.6) \times 10^{-3}$$

$$J/\psi \rightarrow \gamma \eta(1440) \rightarrow \gamma \pi^+ \pi^- \eta \quad (3.0 \pm 0.5) \times 10^{-4}$$

$$J/\psi \rightarrow \gamma \eta(1440) \rightarrow \gamma \pi^+ \pi^- \pi^0 \quad (1.5 \pm 0.2) \times 10^{-5}$$

**& narrow $f_0(980)$ width $\Gamma = 9.5 \pm 1.1$ MeV in $J/\psi \rightarrow \gamma \pi^+ \pi^- \pi^0$
BESIII coll., PRL108 (2012) 182001**



$$\eta(1440) = \eta(1405 \sim 1475)$$

**J.J.Wu, X.H.Liu, Q.Zhao, B.S.Zou,
PRL108 (2012) 081803**

2. Breit-Wigner (BW) fit to a resonance

For a single resonance production:

$$T_{ab}^P(s) = \frac{N_{ab}(s)}{M_R^2 - s - iM_R \Gamma_R}$$

$N_{ab}(s)$ is process dependent ! Taking it as a constant may or may not be a good approximation.

Coupled channel dynamics is important:

eg. $J/\psi \rightarrow \pi^+ \pi^- \pi^0 \rightarrow \bar{K} K \pi$

Guo, Mitchell, Shepherd, Szczepaniak, PRD85 (2012) 056003

$J/\psi \rightarrow a_2 \rho \rightarrow \pi^+ \pi^- \pi^0,$

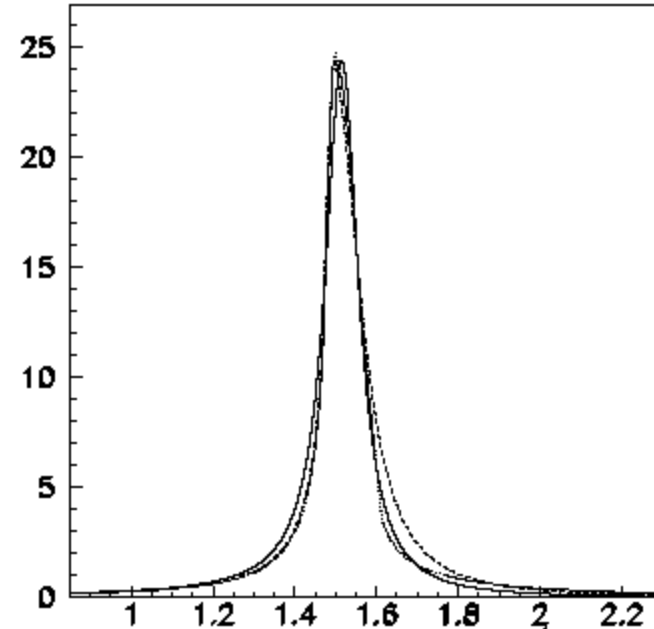
Li, Bugg, Zou, PRD55 (1997) 1421

Many channels are strongly coupled with $N_{ab}(s)$ hardly predictable

Energy dependent width in BW fit: eg. $N^*(1535)$

B.C.Liu, B.S.Zou, PRL 96(2006)042002

$$BW(p_{N^*}) = \frac{1}{M_{N^*}^2 - s - iM_{N^*}\Gamma_{N^*}(s)}$$



(1) $\Gamma_{N^*}(s) = 98 MeV$
 $M_{N^*} = 1515 MeV$

(2) $\Gamma_{N^*}(s) = \Gamma_{N^*}^0 \left(0.5 \frac{\rho_{\pi N}(s)}{\rho_{\pi N}(M_{N^*}^2)} + 0.5 \frac{\rho_{\eta N}(s)}{\rho_{\eta N}(M_{N^*}^2)} \right) = \Gamma_{N^*}^0 [0.8\rho_{\pi N}(s) + 2.1\rho_{\eta N}(s)]$

$M_{N^*} = 1535 MeV$ and $\Gamma_{N^*}^0 = 150 MeV$

(3) $\Gamma_{N^*}(s) = \Gamma_{N^*}^0 [0.8\rho_{\pi N}(s) + 2.1\rho_{\eta N}(s) + 3.5\rho_{\Lambda K}(s)]$ $M_{N^*} \approx 1400 MeV$
 $\Gamma_{N^*}^0 = 270 MeV$

For people doing PWA with simple BW fit to data:

Please keep in mind uncertainties of $N_{ab}(s)$ and $D(s)$ for each single resonance and more complicated interfering overlapping resonances, do not claim too much from the fit.

3. Unitarity and K-matrix approach

Relations among S-matrix, T-matrix, **K-matrix**

$$S = I + 2i \rho T, \quad T = \frac{\mathbf{K}}{1 - i\rho\mathbf{K}}$$

Unitarity relation: $S^+ S = I \rightarrow \text{Im} T = T^+ \rho T, \quad \text{Im} \frac{1}{T} = -\rho$

$$\rightarrow \frac{1}{T} = \frac{1}{\mathbf{K}} - i\rho, \quad \mathbf{K} \sim \text{real}$$

For a single channel BW resonance:

$$S = \frac{M_R^2 - s + ig^2 \rho(s)}{M_R^2 - s - ig^2 \rho(s)}, \quad T = \frac{g^2}{M_R^2 - s - ig^2 \rho(s)}, \quad \mathbf{K} = \frac{g^2}{M_R^2 - s}$$

How to add two resonances for a single channel 2-body scattering ?

$$1) \quad T = \frac{g_1^2}{M_1^2 - s - ig_1^2 \rho(s)} + \frac{g_2^2}{M_2^2 - s - ig_2^2 \rho(s)} \quad \text{violates unitarity}$$

$$2) \quad K = \frac{g_1^2}{M_1^2 - s} + \frac{g_2^2}{M_2^2 - s} \rightarrow T = \frac{g_1^2(M_2^2 - s) + g_2^2(M_1^2 - s)}{(M_1^2 - s)(M_2^2 - s) - i\rho(s)[g_1^2(M_2^2 - s) + g_2^2(M_1^2 - s)]}$$

$$3) \quad S = \frac{M_1^2 - s + ig_1^2 \rho(s)}{M_1^2 - s - ig_1^2 \rho(s)} \times \frac{M_2^2 - s + ig_2^2 \rho(s)}{M_2^2 - s - ig_2^2 \rho(s)}$$

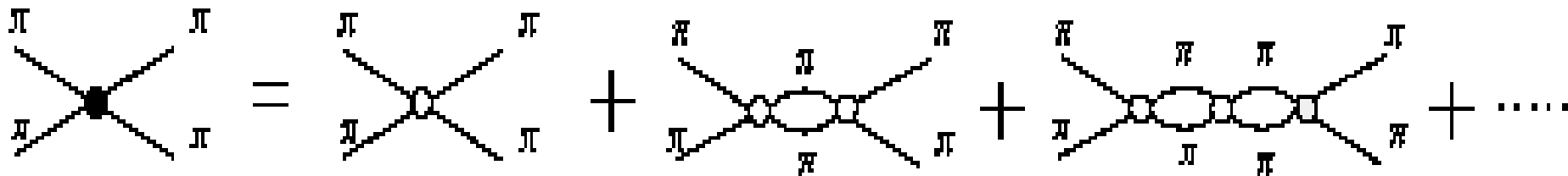
$$\rightarrow T = \frac{g_1^2(M_2^2 - s) + g_2^2(M_1^2 - s)}{[M_1^2 - s - ig_1^2 \rho(s)][M_2^2 - s - ig_2^2 \rho(s)]}$$

For a multi-hadron production final state

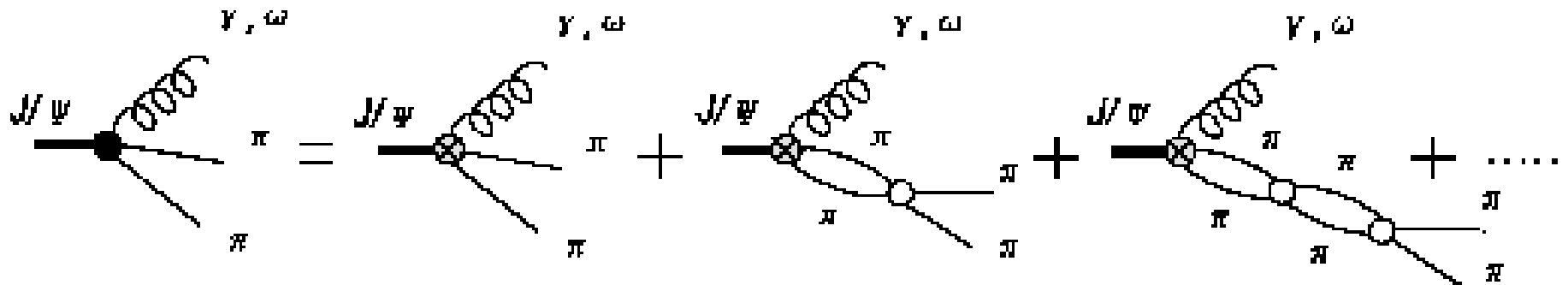
$$T_p = \frac{C_1(s)}{M_1^2 - s - ig_1^2 \rho(s)} + \frac{C_2(s)}{M_2^2 - s - ig_2^2 \rho(s)} \quad \text{not violating unitarity}$$

Unitarity for two-body scattering:

$$T_{el} = K / (1 - i \rho K) = K + K i \rho K + K i \rho K i \rho K + \dots$$



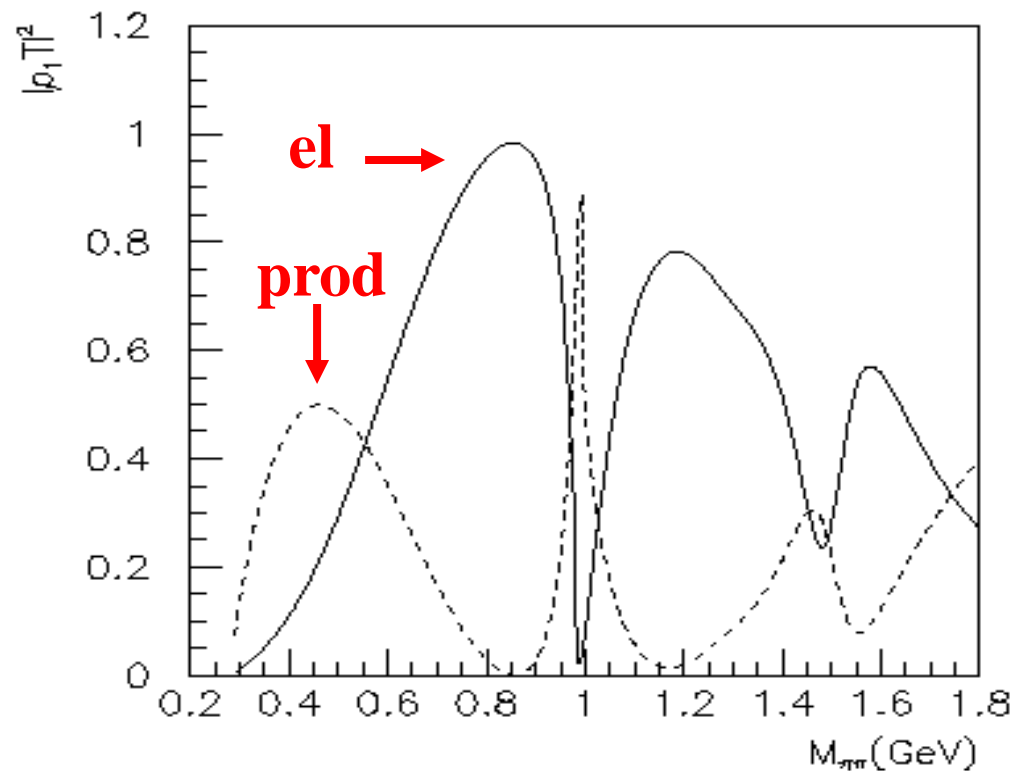
$$T_{prod} = P / (1 - i \rho K) = P + P i \rho K + P i \rho K i \rho K + \dots$$



T_{el} from Bugg, Sarantsev, Zou Nucl. Phys. B471 (1996) 59
with σ pole at $(0.571 - i 0.420)$ GeV

$$T_{prod} = T_{el} * P / K, \quad K = \tan \delta / \rho \quad \text{for } M < 0.6 \text{ GeV}$$

assuming $P=1$



Note: P is usually complex and energy-dependent

3 ways for application of K-matrix to production processes:

1) Fit production data together with data of 2-body scatterings

B.S.Zou, D.V.Bugg, PRD48 (1993) R3948

A.Sarantsev et al.

2) Fix K-matrix from 2-body scatterings, adjust P-matrix to fit the production data

Focus coll. (J.Link et al.) PLB585 (2004) 200: “**Dalitz plot analysis of $D(s)^+$ and D^+ decay to $\pi^+ \pi^- \pi^+$ using the K matrix formalism**”

3) Fit production data alone by adjusting K-matrix parameters freely.

Problem: the obtained K-matrix is very likely inconsistent with 2-body scattering data.

4. Concluding remarks

- 1) **Both BW and K matrix fits have their pros and cons**
- 2) **Try both if possible, to get feelings on uncertainties**
- 3) **One fit is better than none**
- 4) **Keep in mind model dependence of extracted mass and width,
Do not claim new resonance too easily**
- 5) **For charmonium radiative decays to two pseudoscalars,
it is better to give model independent bin-by-bin fit results**