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ORIGINAL PAPER

Applicability of the Adaptive Resistivity Method to Describe the Critical State of Complex Superconducting Systems

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Abstract The adaptive resistivity method is a special algorithm that allows approaching the Bean critical state by an iterative adjustment of the material's resistivity and can be quite easily implemented in commercially available finiteelement codes. Its main advantage is that the critical-state model does not depend on time (or frequency of the applied loads) so that the critical-state description is uniquely determined by the field profiles at the peak value of the applied loads. We proved its validity in the simulation of complex superconducting systems comparing its results either with analytical descriptions, where they exist, or the well-known edge-element model based on direct magnetic field formulation.

Keywords Critical state · Ac losses · Finite element · ANSYS

1 Introduction

The magnetic flux penetration is a necessary basic knowledge for determining the electromagnetic features of type-II superconducting materials. Once established that the magnetic field is shielded by the superconductors through the generation of bulk currents with density $J_C(B)$ [1–3], the problem moves to the solution of the Maxwell equations

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F. Grilli · P.A.C. Krüger Karlsruhe Institute of Technology, Hermann-von-Helmholtz-Platz 1, Eggenstein-Leopoldshafen, Germany with the Ampère-Maxwell law modified according the expression $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_C(B)$. Even if the problem could be simplified by removing the magnetic field dependence of the critical current density [1], a relatively simple solutions can be found only for some regular shapes, e.g. slab, cylinder, sphere, or infinitely thin tape or regular tape arrays (see for example [4, 5]).

In facts the modern high temperature superconductors such as multi-filamentary tapes, possibly involving the presence of ferromagnetic materials, or multi-tape conductors, constitute a hard, if not impossible, task for analytical solutions due to the irregular shape and distribution of the tapes and to the magnetic properties of the materials.

One way around this problem is passing through the use of numerical methods, possibly based on finite-element approaches. There are several different ways to attack the problem, among them: (a) a modified eddy current approach, consisting in solving the electromagnetic problem through the introduction of an electrical resistivity of the superconducting material depending on the current according to a power law [6-8], (b) a critical-state formulation using the vector potential of the magnetic field for forcing the local current to be null or having the value J_C [9], (c) an iterative method adapting progressively the resistivity in a way that the maximum current density is J_C [10, 11]. These approaches are all generally valid but the solution of a problem could involve a considerable computation time. Some of the authors have developed the third method, based on an adaptive resistivity algorithm. This method has demonstrated to be very effective in describing the critical state of complex systems when an external magnetic field or a current is monotonically increased. In this paper a further step in the method development is done, showing how it works with complete field or current cycles and with presence of ferromagnetic materials.

The adaptive resistivity algorithm has been presented and widely described in [10, 11]. It is based on the Bean criticalstate model, simulating a superconducting material through its critical current density, J_C , which is a constant upper limit to the current density that can flow in the superconductor itself. It can be demonstrated [12, 13] that this corresponds to a flux penetration from the superconductor's edges, such that the magnetic field is null where the current density is less than J_C , and it can assume values different from zero only when $|\vec{J}| = J_C$. The main advantage of the Bean critical-state model is that it does not depend on time (or frequency of the applied load) and its results are uniquely determined by the field and current profiles at the peak value of the applied load (either current or external magnetic field). The model can be summarized in the following way:

$$\begin{cases} |\vec{B}| = 0 \\ \rho = \rho_0 \end{cases} \quad \text{if } |\vec{J}| < J_C, \qquad \begin{cases} |\vec{B}| > 0 \\ \rho > \rho_0 \end{cases} \quad \text{if } |\vec{J}| = J_C \qquad (1) \end{cases}$$

where ρ_0 is the resistivity of the fully superconducting state. Equation (1) corresponds to the well-known power law $\rho = \rho_0 (J/J_C)^n$ with $n \to \infty$.

With the aim of solving the Bean critical model through a finite-element analysis, it is possible to iteratively converge on the correct spatial distribution of the resistivity by following a very simple algorithm. First, the transient electromagnetic problem is solved for a constant resistivity ρ_0 equal in all i = 1, ..., M regions in which the wire cross section is divided. Next, the resistivity of each *i*-element is iteratively changed according to

$$\rho_i^{k+1} = \max\left\{\rho_i^k \frac{|J_i^k|}{J_C}, \rho_0\right\}$$
(2)

until:

$$\frac{|\rho_i^{k+1} - \rho_i^k|}{\rho_i^k} < \varepsilon \tag{3}$$

where k is the cycle index, ε the convergence parameter and ρ_0 the minimum value of the resistivity. This procedure corresponds to finding a resistivity distribution among elements that makes the current density to be everywhere less or equal to the critical current density, as prescribed by the critical-state model.

In a previous paper [10] we assessed the applicability of the adaptive resistivity method by comparing its results with specific cases, for which there exist analytical solutions. In particular, we focused on the isolated thin strip in either transport current or perpendicular field, concentrating on the information acquired when the load is applied to the "virgin state", i.e. where I or H are increased from zero. Simulating the thin strip using the finite-element code ANSYS [14], we compared the adaptive resistivity method results with the Brandt theory [15] in terms of field and current density profiles and ac losses, proving that the proposed algorithm follows the Bean critical-state model. Moreover, we were able to simulate with good accuracy the magnetization loss of a Bi-2223/Ag wire with a complex filament structure after heavy deformation processing [16–20] by using the critical current density J_C as a fitting parameter.

Since the adaptive resistivity algorithm does not descend from any physical equation or theory, if we want to conclusively assess its validity we still need to check its ability to describe the full hysteretic cycle, not only the monotonic increase of the load from the "virgin state", then to simulate a superconductor both carrying a transport current and exposed to a perpendicular magnetic field and finally to describe more complex systems, possibly including ferromagnetic structures. The results coming from the adaptive resistivity algorithm will be compared with either analytical descriptions, where they exist, or the well-known and assessed edge-element model based on direct magnetic field formulation [6–8].

To this aim, in this paper we will analyze the following examples:

- thin strip with transport current and exposed to transverse magnetic field, compared to the analytical solution by Brandt [15];
- thin strip on ferromagnetic substrate, compared to the analytical solution by Mawatary for an ideal soft magnetic substrate $(\mu_r \rightarrow \infty)$ [21];
- z-stack of thin strips compared to the analytical solution of an equivalent isolated thin strip [22];
- strip with C-shaped nonlinear ferromagnetic shielding compared to the numerical solutions from the equivalent edge-element model based on direct magnetic field formulation.

3 Thin Strip with Transport Current Exposed to a Transverse Magnetic Field

Let us consider a thin strip, 2a in width and d in thickness $(d \ll a)$, having J_C (A m⁻²) as critical current density, carrying a transport current I_a in a perpendicular magnetic field H_a , both of them fully cycled at the same frequency and with the same phase. It can be demonstrated [15] that the field and current profiles when both I_a and H_a are increased from zero simultaneously are given by

$$J(x) = J_C [j(x + w, a + w, b) + pj(-x - w, a - w, b)]$$
(4)
$$H(x) = H_C [h(x + w, a + w, b)]$$

$$= n_{C}[n(x + w, u + w, b)] - ph(-x - w, a - w, b)]$$
(5)

where





Fig. 1 Normalized current density (**a**) and magnetic field (**b**) profiles of a thin strip (a = 6 mm, $d = 1 \mu \text{m}$, $J_C = 2.5 \cdot 10^{10} \text{ A/m}^2$) with transport current ($I_a = 0.7 \cdot I_C = 210 \text{ A}$) and exposed to a perpendicular

magnetic field ($B_a = B_c = 0.01$ T) as function of the normalized tape width at various percentages of the cycle as calculated by the adaptive resistivity algorithm (*lines*) and the Brandt thin strip theory (*dots*)

$$j(x, a, b) = \begin{cases} 1 & b \le x \le a \\ \frac{1}{2} - \frac{1}{\pi} \arctan[\frac{b^2 - ax}{\sqrt{|x^2 - b^2|(a^2 - b^2)}}] & |x| < b \\ 0 & x \le -b \end{cases}$$

$$h(x, a, b) = \begin{cases} 0 & |x| \le b \\ \frac{x}{2|x|} \operatorname{arctanh}[\frac{\sqrt{|x^2 - b^2|(a^2 - b^2)}}{ax - b^2}] & |x| > b \end{cases}$$
(7)

with $b = a\sqrt{1 - f^2}\sqrt{1 - c^2}$, w = afc, p = sign(f - c), $f = I_a/I_C = I_a/(2adJ_C)$, $c = \tanh(B_a/B_C) = \tanh(\pi B_a/\mu_0 J_C d)$.

It is worth noting that $J(x = a) = J_C$ and $J(x = -a) = pJ_C$, i.e. depending on the sign of (f - c) we could have a "current-like" state, with both edges carrying J_C , and a "field-like" state, with one edge carrying J_C and the other $-J_C$.

The full cycle can be obtained by a linear superposition of solutions of the form (4) and (5). For instance, when both loads are decreased from I_a and H_a to, respectively, $-I_a$ and $-H_a$, the resulting current and field profiles are given by

$$J_{\downarrow}(x, I, B, J_C) = J(x, I_a, B_a, J_C) - J(x, I_a - I, B_a - B, 2J_C)$$
(8)
$$H_{\downarrow}(x, I, B, J_C)$$

$$= H(x, I_a, B_a, J_C) - H(x, I_a - I, B_a - B, 2J_C)$$
(9)

As an example, we compare the Brandt theory with the adaptive resistivity method for a thin strip 12 mm in width (a = 6 mm) and 1 µm in thickness (d = 1 µm), with $J_C = 2.5 \cdot 10^{10} \text{ A/m}^2$. The two parameters governing the algorithm are set to $\rho_0 = 1 \cdot 10^{-16} \Omega \text{ m}$ and $\varepsilon = 10^{-4}$. The value

of ρ_0 is chosen in such a way that a further reduction would affect no more the final result within the desired accuracy of the convergence parameter ε . The strip is meshed with 1000 elements of the same size.

In Fig. 1 the normalized current density and magnetic field profiles are shown as functions of the normalized tape width when the transport current and the applied field are cycled from zero with amplitudes $I_a = 0.7 \cdot I_C = 210$ A and $B_a = B_C = 0.01$ T. The agreement with the Brandt theory is excellent, which confirms the validity of the proposed algorithm to simulate mixed loading conditions.

The ac losses in a thin strip exposed to ac transport current and magnetic field have been analytically calculated by Schönborg in [23]. Since our numerical data did not agree with his analytical expression, we performed again the integration for the ac losses:

$$P = 4d \int_{-a}^{a} J(x)\phi(x) dx,$$

$$\phi(x) = \begin{cases} \mu_0 \int_{b-w}^{x} H(u) du & x \ge b - w \\ 0 & -b - w < x < b - w \\ \mu_0 \int_{x}^{-b-w} H(u) du & x \le -b - w \end{cases}$$
(10)

The resulting energy per unit length per cycle dissipated in the strip is given by

$$P = \frac{\mu_0 I_C^2}{4\pi} \left(\frac{b}{a}\right)^2 (P_1 - p P_2)$$

$$P_1 = \alpha A \operatorname{arcosh} \alpha - \alpha^2 + \beta B \operatorname{arcosh} \beta - \beta^2 + 2$$

$$P_2 = -A(\alpha + 2\beta) \operatorname{arcosh} \beta - B(2\alpha + \beta) \operatorname{arcosh} \alpha$$

$$+ 2(\alpha + \beta)^2 \operatorname{arctanh} \frac{AB}{\alpha\beta + 1} - 2AB$$
(11)

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where
$$\alpha = (a + w)/b$$
, $\beta = (a - w)/b$, $A = \sqrt{\alpha^2 - 1}$ and $B = \sqrt{\beta^2 - 1}$.

As shown in [10], due to the particular characteristic of the Bean critical-state model, the ac losses are uniquely determined by the field profile at peak values of the applied loads. This property can be extended to the adaptive resistivity method, so that if \vec{B} is the magnetic field at peak values of the applied loads, the ac losses can be calculated as

$$P = 4J_C \sum_{\text{all element with } J=J_C} \left(\int \vec{B} \cdot \vec{n} \, d\ell \right) \Delta S \tag{12}$$

If, using the Norris terminology, we define the "kernel" as a location where $J \neq J_C$ and B = 0, it follows that $\phi(x) =$ $\int \vec{B} \cdot \vec{n} d\ell$ is the magnetic flux calculated with respect to the kernel, where ℓ is a line connecting the generic Cartesian coordinates (x, y) to the kernel, \vec{n} the unit vector normal to this line and ΔS the cross-sectional area of each element. In general, we can select any location between the two fronts of the field penetration, -b - w and b - w, as a kernel to perform the summation in Eq. (12). In order to choose a single location valid for the whole range of variation of H_a and I_a , we have to determine the position where the two fronts collapse: it happens when $I_a \to I_C$ $(f \to 1)$ or $H_a \to \infty$ $(c \to 1)$, i.e. when $b \rightarrow 0$. Under these conditions, the two fronts of penetration converge to x = -w = -afc, which is then the kernel for the integration in (12). This location can be numerically determined by running a simulation at full saturation.

In Fig. 2 the ac losses per cycle per unit length of the investigated strip are shown as function of the applied magnetic field when the transport current is $I_a = 0.7 \cdot I_C = 210$ A. The maximum difference between the adaptive resistivity algorithm and Eq. (11) is found to be 1.5 %. For comparison the expression proposed by Schönborg in [23] is also shown. The evidenced discrepancy near magnetic field saturation (25 % at $B_a = 0.1$ T) can also be found in another paper [25], even if it is attributed to the fact that the current density formulas for thin strips are only valid for monotonic penetration of current fronts, which appears only for the low-field high-current regime.

4 Thin Strip on Ferromagnetic Substrate

The response to a perpendicular ac magnetic field of an infinitely thin strip (width 2*a* and thickness *d*) deposited on an ideal soft ferromagnetic substrate (width 2*a* and thickness d_m) with $\mu_r \rightarrow \infty$ using the critical-state model with constant J_C has been determined by Mawatari in [21] by introducing the complex magnetic field:

$$B(\zeta) = 2B_C \left[\operatorname{arctanh} \sqrt{\frac{\beta_0(\eta + a)}{a(\eta + \beta_0)}} \right]$$



Fig. 2 Ac losses per cycle per unit length of a thin strip $(a = 6 \text{ mm}, d = 1 \text{ µm}, J_C = 2.5 \cdot 10^{10} \text{ A/m}^2)$ with transport current $(I_a = 0.7 \cdot I_C = 210 \text{ A})$ as function of the applied magnetic field as calculated by the adaptive resistivity algorithm (*dots*) and the analytical expression (11) (*continuous line*). For comparison the expression proposed by Schönborg in [15] is also shown (*dotted line*)

$$-\frac{\sqrt{a\beta_0(\eta+a)(\eta+\beta_0)}}{\eta(a+\beta_0)}\right]$$
(13)

with $\zeta = x + i\epsilon$, $\epsilon = \max\{d, d_m\} \to 0$, $\eta = i\sqrt{\zeta^2 - a^2}$ and $B_C = \mu_0 J_C d/\pi$. If B_a is the amplitude of the applied magnetic field, β_0 is the solution of the equation:

$$\operatorname{arctanh}\sqrt{\frac{\beta_0}{a} - \frac{\sqrt{a\beta_0}}{a+\beta_0}} = \frac{B_a}{2B_C}$$
(14)

and the front of the penetration is $b_0 = \sqrt{a^2 - \beta_0^2}$. With $\epsilon \to 0$, the field and current density profiles when the external field is increased from zero to H_a are given by

$$B(x) = \operatorname{Re}[B(x+i\epsilon)], \qquad J(x) = -\operatorname{Im}[B(x+i\epsilon)]/\mu_0$$
(15)

It must be pointed out that Mawatari calculates the field profiles for the dc case (rather than ac) and then uses those results to compute the losses for the ac case as well. This is possible because, as mentioned in the previous paragraphs, the Bean critical-state model does not depend on time (or frequency of the applied load) and its results are uniquely determined by the field and current profiles at the peak value of the applied magnetic field.

Introducing again a thin strip defined by a = 6 mm, $d = d_m = 1 \ \mu\text{m}$, $J_C = 2.5 \cdot 10^{10} \ \text{A/m}^2$, using the adaptive resistivity algorithm we can calculate the field and current density profiles as function of the relative permeability of the ferromagnetic substrate, as shown in Fig. 3. The applied field is $B_a = 2B_C = 0.02$ T, so that the solution of



Fig. 3 Normalized current density (a) and magnetic field (b) profiles of a thin strip with ferromagnetic substrate (a = 6 mm, d = 1 µm, $J_C = 2.5 \cdot 10^{10}$ A/m², $d_m = 1$ µm) exposed to a perpendicular magnetic field ($B_a = 2B_C = 0.02$ T) as function of the normalized tape



Fig. 4 Ac losses per cycle per unit length of a thin strip with ferromagnetic substrate (a = 6 mm, $d = d_m = 1 \mu \text{m}$, $J_C = 2.5 \cdot 10^{10} \text{ A/m}^2$) as function of the applied magnetic field as calculated by the adaptive resistivity algorithm (*dots*) and the analytical expression (18) (*continuous line*). Ac losses are in the superconductor only



width as calculated by the Mawatari formulation (*full symbols*) and the adaptive resistivity algorithm for different values of the substrate relative permeability (*lines*)



Fig. 5 Ac losses per cycle per unit length of a stack of 15 thin strips $(a = 6 \text{ mm}, d = 1 \text{ µm}, D = 2, 10 \text{ and } 100 \text{ µm}, J_C = 2.5 \cdot 10^{10} \text{ A/m}^2)$ as function of the normalized applied transport current as calculated by the adaptive resistivity algorithm (*lines*) and the analytical expression (11) for the equivalent strip (*dots*)

Eq. (14) is $\beta_0/a = 0.8185$, and then the front of the penetration is $b_0/a = 0.575$. As expected, with $\mu_r \rightarrow \infty$, the finite-element analysis tends to converge on the Mawatari modelization, and for $\mu_r = 100000$, we nearly get superposition. In facts, the Mawatari analytical solution is only valid in the $\mu_r \to \infty$ limit, no analytical solutions are available for a finite relative permeability.

The dissipated energy per cycle per unit length in the superconductor can be calculated introducing the ac susceptibility $\chi = \chi' + i\chi$ [24]:

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Fig. 6 (a) Normalized integral current density distribution at $I_a = 0.7 \cdot I_C = 210$ A as function of the normalized tape width of a stack of 15 thin strips (a = 6 mm, d = 1 µm, D = 2,10 and 100 µm, $J_C = 2.5 \cdot 10^{10} \text{ A/m}^2$) as calculated by the adaptive resistivity algorithm (lines) and the analytical expression for an equivalent strip (dots); (**b**) current density map in 1/4symmetry of the $D = 10 \,\mu\text{m}$ configuration (z-axis is 30 times stretched)



$$P = \pi \frac{B_a^2}{\mu_0} \chi'' \tag{16}$$

where from [21]

$$\chi'' = 4a^2 \frac{B_C}{B_0} \int_0^{\beta_0/a} \frac{\sqrt{t}(3+t)}{(1+t)^2} \\ \times \left[1 - 4\frac{B_C}{B_0} \left(\operatorname{arctanh} \sqrt{t} - \frac{\sqrt{t}}{1+t} \right) \right] dt$$
(17)

Equation (17) can be integrated, so that, if $t_0 = \beta_0/a$:

$$P = 4\pi a^2 \frac{B_C B_0}{\mu_0} \left[\frac{2t_0^{3/2}}{1+t_0} + 4\frac{B_C}{B_0} \left(\operatorname{arctanh} t_0 - \frac{t_0}{(1+t_0)^2} - \frac{2t_0\sqrt{t_0}\operatorname{arctanh}\sqrt{t_0}}{1+t_0} \right) \right]$$
(18)

Figure 4 shows the ac losses per cycle per unit length of the proposed thin strip with ferromagnetic substrate as function of the applied magnetic field as calculated by the adaptive resistivity algorithm with $\mu_r = 100000$ and the analytical expression (18). The agreement is very good, evidencing that the ac losses are still uniquely determined by

the field profile along the strip at peak values of the applied loads, so that they can be calculated using (12), where \vec{B} is the field profile at the peak value of the applied field. The maximum difference between the adaptive resistivity algorithm and Eq. (18) is found to be 5 %. This difference, larger than in the previous case the mesh being equal, is partly due to the fact that the finite-element model with $\mu_r = 100000$ is not an exact description of an infinitely thin strip deposed on an ideal soft ferromagnetic substrate (see also Fig. 3).

5 Z-stack of thin strips

This configuration, consisting in several thin strips stacked along the axis perpendicular to the strip wide face, is particularly interesting because it can be interpreted as the 2D cross section of a racetrack or pancake winding in the case their parallel legs are enough distant one to each other to ignore the corresponding electromagnetic interactions. If D is the stacking interval and I_a the total current in the winding, it has been shown [22] that, in the limit of small D/a **Fig. 7** Sketch of the ferromagnetic shielding: $d_1 = 3.076$ mm, $d_2 = 1$ µm, $t_1 = 75$ µm, $t_2 = 3$ µm, $t_3 = 52$ µm



ratios, ac losses in a thin stack of N thin strips are close to the losses in an equivalent isolated thin strip having NJ_C as critical current density and carrying the transport current NI_a . The z-stack arrangement can be modeled and solved through the adaptive resistivity algorithm: each strip of the stack is allowed a different resistivity distribution and is decoupled from all the other strips in such a way that each turn carries exactly the applied current I_a .

As an example, let us consider 15 thin strips (a = 6 mm, $d = 1 \,\mu\text{m}, J_C = 2.5 \cdot 10^{10} \,\text{A/m}^2$) stacked with different spacings D = 2, 10 and 100 µm. Figure 5 shows the resulting ac losses per cycle per unit length as a function of the applied transport current as calculated by the adaptive resistivity algorithm compared to the equivalent isolated thin strip. As expected, the ac losses of the stack tend to converge on the losses of the equivalent strip as the stacking interval decreases, confirming the validity of the adaptive resistivity algorithm to describe multiple conductor arrangements. In order to assess that the current density penetration is also matched, Fig. 6 shows a comparison of the current density distribution at $I_a = 0.7 \cdot I_C = 210$ A of the equivalent strip with the integral current density calculated through the adaptive resistivity algorithm, together with a map of the current density distribution of the $D = 10 \ \mu m$ configuration. The integral current density is defined as $J^{\Sigma}(x) = \sum_{i=1}^{N} J_i(x)$, where $J_i(x)$ is the current density distribution of the *i*strip.

6 Nonlinear ferromagnetic shielding

The crucial test of the present work consists of the description through the adaptive resistivity method of a thin strip with a C-shaped ferromagnetic shielding enclosing its lateral edges as shown in Fig. 7. As shown in [26], this configuration is potentially interesting for ac loss reduction. The ferromagnetic material is supposed to be nonlinear, with its permeability described as

$$\mu(H) = 48.770 e^{-1.849 \cdot 10^{-4} \cdot H^{8.267 \cdot 10^{-1}}} + 69.969 e^{-4.214 \cdot 10^{-5} \cdot H^{1.186}} + 1$$
(19)

Equation (19) fits measured data from [27]. The strip ($a = 6 \text{ mm}, d = 1 \text{ } \mu\text{m}, J_C = 2.5 \cdot 10^{10} \text{ A/m}^2$) is supposed to carry



Fig. 8 Normalized current density (**a**) and magnetic field (**b**) profiles of a thin strip (a = 6 mm, d = 1 µm, $J_C = 2.5 \cdot 10^{10}$ A/m²) with C-shaped ferromagnetic shielding with transport current ($I_a = 0.7 \cdot I_C = 210$ A) and exposed to a perpendicular magnetic field ($B_a = B_C = 0.01$ T) as function of the normalized tape width at various percentages of the cycle as calculated by adaptive resistivity algorithm (*lines*) and the edge-element model based on direct magnetic field formulation (*dots*). Ac losses are in the superconductor only

a transport current $I_a = 0.7I_C$ in an applied field $B_a = B_C = 0.01$ T, both cycled at the same frequency.

Despite the complexity of this highly nonlinear arrangement, Fig. 8 shows a rather good agreement between the two numerical finite-element approaches. This allows the conclusion that the adaptive resistivity algorithm is qualified to describe the Bean critical state of complex systems, in presence of contemporary transport current and perpendicular field, even if nonlinear ferromagnetic structures are present.

7 Conclusions

We have proved that the proposed adaptive resistivity method, based on the commercial finite-element code AN-SYS, is qualified to describe the Bean critical state of complex systems, in presence of contemporary transport current and perpendicular field, even if nonlinear ferromagnetic structures are present. Its results have been successfully compared to the analytical expression for the isolated thin strip, with or without soft ferromagnetic substrate, and the edge-element solutions based on direct magnetic field formulation, crucial when no analytical expressions were available.

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