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Compliance of numerical formulations for describing superconductor/ferromagnet heterostructures

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ABSTRACT

In recent years, a number of numerical and finite-element methods in particular have been developed to investigate various properties of superconducting materials. Following discussion on how to interpret different models' results regarding similar physical phenomena, the consistency of these models has been of increased interest. In this paper the compliance of three models is under scrutiny: the adaptive resistivity, the edge-element and the electrostatic-magnetostatic analogue model. The set of test cases is two-fold: a simple superconducting strip with ferromagnetic substrate of varying permeability as is commonly used in (RABiTS produced) coated conductors and a superconducting strip flanked by ferromagnetic shields of varying dimensions at the lateral edges. Complicated field and loss functions are encountered in the latter.

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1. Introduction

Several technical applications like magnetic resonance imaging and fusion have become dependent on superconductors since classical materials could not provide field- and current-strengths high enough to sustain operation. Coated conductors, while not yet widely used due to monetary and technical constraints, have the potential to establish themselves in a range of additional technical applications like fault current limiters (FCL), transformers, motors and superconducting magnetic energy storage systems (SMES) as well as current leads and coils. In most of the growing ranges of application, hysteretic ac-loss is a key operational parameter and technical usage is dependent upon it.

Methods to calculate superconducting parts' properties are most interesting scientifically firstly due to the exigence of describing and understanding fundamental phenomena and secondly because of a greatly reduced lead time of simulations compared to constructing and preparing actual experiments and samples. Three different models are considered in this paper: the electrostaticmagnetostatic analogue model [1], the edge-element model based on direct magnetic field formulation [2] and the adaptive-resistivity model based on iterative adaption of material properties [3]. While the different models have successfully been utilised [1–3], a conclusive study of the various approaches is yet to be conducted. The models' varied approaches to formulating superconductor representations is quite fortunate since a concurrence raises the probability of an assertion being indeed correct; it is highly unlikely the same error would occur in completely different approaches.

While analytical models are not devoid of elegance in addition to being computationally convenient, they are severely hampered by their inability to describe more intricate geometries [4,5]. Hence the necessity for numerical formulation should become evident given the requirement to simulate arbitrary geometries.

2. Geometries and models

The first and conceivably simple geometry consists of a superconducting strip applied on top of a ferromagnetic substrate material as can usually be found in coated conductors fabricated using the RABiTS (Rolling Assisted Biaxially Textured Substrate) technique (see Fig. 1). The substrate material's permeability is varied while the substrate's dimensions are geared to prevalent coated conductor geometries: the thickness of the substrate is 80 µm, the superconducting layer itself is 1 µm thick. In the case of the electrostatic-magnetostatic analogue, the superconducting layer is actually a mathematical line so it has no thickness. The gap between these two layers would normally be filled with buffer layers or oxides. For simplicity reasons however it is assumed that the gap has vacuum characteristics. To reduce the degrees of freedom of the simulation considerably, the gap is assumed to be 10 µm wide which is much more than a realistic buffer layer; however, influence of the gap width turned out to be negligible as long as it remained small compared to substrate thickness. Substrate and superconductor width is 4 mm.



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Fig. 1. Cross-section of a bilayer geometry – superconductor on ferromagnetic substrate.

The second more sophisticated geometry features another superconducting strip of the same dimensions as before. No substrate is present however. Instead a horseshoe-shaped ferromagnet encloses the strip's lateral edges (see Fig. 2). The ferromagnet's coverage of the superconductor is varied as is its thickness: the former between 25% and 50% of the strip's surface and the latter between 5 μ m and 500 μ m. The general idea of this assembly is for the ferromagnets to attract magnetic flux thereby shielding the superconductor from flux-entry and supporting its endeavour to expel flux from its volume. This is supposed to sustain the superconducting state to higher fields and currents than would be possible without any kind of shielding geometry and it should reduce ac-loss as well [6–9].

Due to the geometry of the problems presented, it is sufficient to consider a two-dimensional cross section of the assembly under examination. This simplifies the analysis considerably, not only in complexity of the physical model, but also in terms of computation time needed. Both the following cases are therefore to be understood as being two-dimensional as is the mesh used in numerical simulation.

A challenge in all simulations featuring thin strips is the high aspect ratio which requires a lot of mesh points in the finite element model. Deriving a physically correct result and simultaneously keeping the mesh point count down as much as possible to reduce computation time is absolutely imperative. In our approach, we consider a superconducting strip meshed with a height to width mesh distribution of between 1:500 and 1:2000.

The edge-element model [2] is a time-dependent implementation of Maxwell's equations to simulate superconducting assemblies directly with the help of the power law $\rho = \frac{E_c}{J_c} \left| \frac{J}{J_c} \right|^{n-1}$. For each configuration, a full cycle at 100 Hz is simulated giving not only field, current and loss profiles for each time-step but also overall loss values. In all the calculations for this paper, an n-value for the exponent of the power law of 101 was used. This rather high value was chosen with the goal of approaching the critical state approximation which is used by the other two models. Apart from three-dimensional problems, the edge-element model is very versatile and able to model stacks of cables, coils and most other conceivable geometries.



Fig. 2. Cross-section of a shielding geometry – superconductor with C-shaped ferromagnetic shields.

The adaptive resistivity model [3] calculates the distribution of current density and magnetic field according to the Bean critical state model through a specially developed algorithm, approaching the solution by an iterative adjustment of the material resistivity.

The electrostatic-magnetostatic analogue model [1] - contrary to the edge-element model – is not dependent on time and models the case of maximum penetration: only the time-step where the amplitude of an applied field is maximal is simulated. This is also the case in the adaptive resistivity model [3], but since the approach via electrostatic-magnetostatic analogies requires no iteration, the solution is inherently faster. While this does not allow time-resolved observation of field or current penetration, it is sufficient for calculating ac-losses. The well-known correlation of electrostatic and magnetostatic parameters is used to model a superconductor by representing its physical properties in electrostatic quantities. For example: a classical superconductor is a perfect diamagnet, meaning no field lines are penetrating its surface. This behaviour is modeled in electrostatics by setting the surface-potential equal to zero. In contrast an area with critical current-flow is modeled by setting a constant surface-charge. Because only properties of surfaces are used, the superconductor volume need not be considered in this model, permitting the omission of mesh points enclosed by superconductor surfaces. This effectively reduces the superconducting strip to a one-dimensional line further reducing computation time. The downside of this model at the moment is its inability to model more than one superconductor in a given geometry and the necessity for manual interaction since the iteration process has not been automated yet.

3. Discussion

Interesting to note is the models concurrence with analytical models: for a permeability constant of unity, the results coincide with the Brandt model's estimates [4], for very high permeability ($\mu_r = 1000$) with those predicted by Mawatari for infinite permeability [4,5] (see Fig. 3). The results differ only marginally for permeabilities even higher than 1000.

The edge-element and the adaptive resistivity models' ac-loss calculations may be considered to be in almost perfect agreement while the electrostatic–magnetostatic analogue model's ac-loss results are smaller by roughly 10%. However, if scaled slightly the qualitative resemblance of the electrostatic–magnetostatic analogue model's loss function is striking as well (see Fig. 4).

The ferromagnetic substrate in the bilayer geometry reduces losses in the superconductor by roughly 30% for prevalent



Fig. 3. Field profiles for varying substrate permeabilities in bilayer geometry; the dashed lines underlying the field profiles for the permeabilities of 1 and 1000 are the analytical results of Brandt's and Mawatari's formulation, respectively.



Fig. 4. Ac-losses in dependence of applied field for substrates of varying permeability as observed in bilayer geometry computed with different models: electrostatic-magnetostatic analogue model (solid), edge-element model (small dashes), adaptive resistivity model (long dashes).



Fig. 5. AC-losses in dependence of shield thickness as observed in a shielding geometry with a permeability of μ_r = 50; comparison of results obtained with the edge-element model (solid) and the adaptive-resistivity model (dashed).

substrate materials with a relative permeability constant of about 50. Generally speaking, the losses are lower the higher the relative permeability constant. This does not take into account losses in the ferromagnetic substrate itself. The influence of the ferromagnetic substrate is greatest around 25 mT; at higher applied field values, the amount of reduction stagnates. Unfortunately, a concurring effect is the induction of eddy currents and other loss phenomena in the ferromagnet.

The shielding effect of horseshoe-shaped ferromagnetic materials hindering flux penetration into the superconductor volume has been reported elsewhere so the reduction of ac-losses is not at all surprising [1,6–8,10]. The behaviour of the loss functions however is. When coverage and thickness of the ferromagnetic shields are above threshold values, an as yet unknown effect is observed: the maximum of the previously arc-shaped curve shifts ever more towards higher values of field and downwards with increasing shield thickness. The broadening continues until the formerly well



Fig. 6. Field profiles for varying shield thickness as observed in a shielding geometry.

defined peak has developed into a plateau (see Fig. 5). Looking at the field profiles, a seemingly unphysical high negative value is observed, which is strongly influenced by the permeability constant (see Fig. 6). With increasing shield thickness, a strong reduction of the field-profile is not only observed in the outer, covered parts of the superconducting strip, but also in the central, unshielded part. This is not unexpected either, but the magnitude of the effect and the resulting profiles warrant further attention.

4. Conclusion and outlook

Not only were we able to show encouraging compliance between the three models under scrutiny but we were also able to confirm the assumption that all approaches are suited to investigate specific properties of superconductor performance. We used this knowledge to investigate complex shielding geometries numerically and the unexpected results are of great interest. Further study is needed to understand the shielding behaviour in more detail. We are planning to investigate this further and also conduct studies with different geometries. Although the numerical results are in accord, we are planning to compare our calculations to samples prepared to represent our calculated geometries. Once we have a better understanding of the various effects we plan to apply our knowledge to improving actual technical applications.

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