# Generation of Higher Harmonics in Voltage on Superconducting Wire Carrying Cosine-Like AC Current

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Abstract-Hard superconductors are known to exhibit nonlinear and hysteretic magnetic properties. Then, the electrical field on the surface of a wire made from such material carrying AC current, will be nonharmonic also in the case when the excitation current is purely cosine-like. We show the harmonic spectra of this signal computed in both numerical and analytical way, following the critical state model with constant  $j_c$ . Odd harmonics only are present in the signal. Assuming the cosine-like transport current, the sine components are negligible in wide range of current amplitudes, except the fundamental sine component. The ratio between harmonic components remains constant up to the current amplitudes close to the critical current. Theoretical results are compared with experiments carried out on two samples: multifilamentary tape from HTS Bi-2223 in metallic matrix, and sintered tubes of Bi-2223. Measured content of higher harmonics laid below theoretical results for both the samples, indicating that the simple model with constant  $j_c$  does not sufficiently describe the flux pinning in these materials.

*Index Terms*—Critical state model, hard superconconductors, harmonic generation by nonlinear circuit element, transport of AC current by superconductor.

## I. INTRODUCTION

T HE voltage on a superconductor carrying AC transport current reflects the dynamics of magnetic flux penetration into its bulk [1]–[3]. Therefore one can expect that the mechanisms controlling the flux dynamics and determining important characteristics like e.g., the AC loss can be better understood from analysis of the voltage signal. We decided to perform such analysis with the help of signal representation by its harmonic spectrum. This would be particularly convenient when the variation of flux dynamics should be investigated [4], [5]. Surprisingly, the formulas for higher harmonics in AC transport regime of hard superconductors have not been derived yet, in contrast to AC magnetic experiments when this approach was used e.g., to detect deviations from the critical state model [6]–[8].

Investigation of higher harmonics represents an extension of transport AC loss measurement, that is based on separation of the in-phase fundamental harmonic component only.

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State-of-the art lock-in amplifiers offer the possibility to perform a more complex harmonic analysis. In this paper we first derive the formulas for higher harmonics theoretically, and then compare the predictions with experimental results.

# II. THEORY

Consider a round wire with radius R made from hard superconductor, i.e., type II superconductor with flux pinning in its volume. The distribution of transport current in the section of the wire [9] can be derived following the principles of the critical state model [10] that requires the critical current density,  $j_c$ , as the only parameter characterizing the superconductors properties. For the AC current amplitudes,  $I_0$ , lower than the wire's critical current  $I_c = \pi R^2 j_c$ , the so called neutral zone remains near the wire axis that is current-free and where also the electrical field is zero. With the help of auxiliary variable  $F = I_0/I_c$ one can express the vector potential generated by the harmonic current  $I(t) = I_0 \cos \omega t$  on the wire surface (r = R) in the first half of the cycle as

 $A_s(t)$ 

$$= -\frac{\mu_0 I_0}{4\pi} \begin{bmatrix} \cos \omega t + \frac{1-F}{F} \ln(1-F) + \\ -\frac{2-F+F\cos \omega t}{F} \ln\left(1-\frac{F}{2} + \frac{F}{2}\cos \omega t\right) \end{bmatrix}$$

and a similar expression can be found also for the second half of the cycle [11].

Due to the existence of the neutral zone, the electrical field on the wire surface is entirely given by the time derivative of the vector potential,  $E_s(t) = -\partial A_s(t)/\partial t$ . Thus for the first half of the cycle  $(0 \le \omega t \le \pi)$  we find that

$$E_s(t) = \frac{\mu_0 I_0}{4\pi} \omega \sin \omega t \cdot \ln \left( 1 - \frac{F}{2} + \frac{F}{2} \cos \omega t \right)$$
(1a)

while for  $\pi \leq \omega t \leq 2\pi$  the surface electrical field will be

$$E_s(t) = \frac{\mu_0 I_0}{4\pi} \omega \sin \omega t \cdot \ln \left( 1 - \frac{F}{2} - \frac{F}{2} \cos \omega t \right).$$
(1b)

The voltage waveform is given by the time-dependent dimensionless quantity

$$e_s(t) = E_s(t) \frac{4\pi}{\mu_0 I_0 \omega} \tag{2}$$

that can be developed in a Fourier series

$$e_s(t) = \sum_{n=1}^{\infty} \left[ e_n'' \cos\left(n\omega t\right) + e_n' \sin\left(n\omega t\right) \right].$$
(3)

Calculation of the series coefficients is rather awkward. First of all, because of the waveform symmetry

$$e_s\left(t+\frac{\pi}{\omega}\right) = -e_s(t)$$

and only the odd harmonics will survive. We succeeded to resolve in closed form the formulas for the first 7 cosine components [12], with the following results: For the fundamental harmonic:

$$e_1'' = \frac{4}{\pi F^2} \left[ F\left(1 - \frac{F}{2}\right) + (1 - F)\ln(1 - F) \right].$$

This expression could be also easily derived from the classical paper of Norris [9]. The cosine components for n = 3, 5 and 7 were found as

$$\begin{split} e_3'' &= \frac{2}{3\pi F^4} \left[ F(F-2)(F^2+24F-24) + \\ &-6(F-1)(F^2-8F+8)\ln(1-F) \right] \\ e_5'' &= \frac{2}{45\pi F^6} \left[ F(F-2)(7F^4+440F^3-4280F^2 + \\ &+7680F-3840) - 30(F-1)(3F^4-72F^3 + \\ &+328F^2-512F+256)\ln(1-F) \right] \\ e_7'' &= \frac{4}{630\pi F^8} \times \\ & \left[ F(F-2)(29F^6+3472F^5-84\,112F^4+483\,840F^3 + \\ &-1\,048\,320F^2+967\,680F-3\,225\,600) + \\ &-210(F-1) \times \ln(1-F) \times \\ & \left( 3F^6-144F^5+1424F^4-5632F^3 + \\ &+10\,496F^2-9216F+3072) \right] . \end{split}$$

For the first sine component we found

$$e_1' = \log\left(1 - \frac{F}{2}\right) - \sum_{k=1}^{\infty} \frac{(2k-1)!!}{2^{k+1}k(k+1)!} \left(\frac{F}{2-F}\right)^{2k}.$$

This result is a simpler form of our previous formula [11]. The sine components for n = 3, 5 and 7 were found as

$$\begin{aligned} e_3' &= -3\sum_{k=1}^{\infty} \frac{(2k-1)!!}{2^{k+1}k\,(k+2)!} \left(\frac{F}{2-F}\right)^{2k} \\ e_5' &= -5\sum_{k=1}^{\infty} \frac{(k-1)\,(2k-1)!!}{2^{k+1}k\,(k+3)!} \left(\frac{F}{2-F}\right)^{2k} \\ e_7' &= -7\sum_{k=1}^{\infty} \frac{(k-1)\,(k-2)\,(2k-1)!!}{2^{k+1}k\,(k+2)!} \left(\frac{F}{2-F}\right)^{2k}. \end{aligned}$$

Let us briefly discuss these theoretical results. First, it is to be underlined that they do not depend on the wire dimensions. Both the angle frequency of AC current,  $\omega$ , as well as the amplitude of the current,  $I_0$ , appear as pre-factors in the formulas (1a) and (1b) for electrical field. This means that the waveform of electrical field,  $e_s$ , is a function of only one independent variable: the ratio F between the AC amplitude and the wire's critical current,  $I_c$ .

The analysis of higher harmonics in the voltage on a superconducting element carrying transport current could be helpful in understanding the AC behavior. For example, for the loss mechanisms that are of linear eddy-current like nature, no contribution to higher harmonics of the voltage signal is expected



Fig. 1. Dependence of the Fourier coefficients on normalized AC current.

[13]. There is also a strong implication regarding the experimental technique of studying the AC transport properties: the nonharmonic voltage drop ((1a), (1b)) must be present at the current leads to secure that the current in the wire is harmonic. Therefore, checking of the harmonic purity of the current should be highly recommended in these conditions.

Plot of theoretical prediction for the first 7 Fourier coefficients is given in Fig. 1. Except the cosine component of the first harmonic, all bear the negative sign. There is only one component that is difficult to be established in the experiment: the sine component of the first harmonic. This is because all the voltage induced in the signal leads adds to it. All the remaining sine components are rather insignificant with respect to the cosine ones.

#### **III. EXPERIMENTS**

We performed the analysis of voltage signals from two different superconducting conductors. The apparatus used in these experiments utilizes a cold core toroidal transformer to supply AC current to the sample that forms a part of the secondary circuit. [14] Advantage of this arrangement is that the voltage signal does not suffer from the common mode problem often encountered in this kind of experiments. All the measurements were performed in liquid nitrogen bath at atmospheric pressure.

First we present the results obtained for a Bi-2223 ceramic tube with inner diameter  $D_{int} = 5.8$  mm, outer diameter  $D_{out} = 7.2 \text{ mm}$  and total length of 73 mm produced by CAN Superconductors. Self-field critical current at 77 K is  $I_c = 85$  A. For such a hollow cylinder in self-field conditions, the formulas (1a) and (1b) for the surface voltage still apply, provided that now the definition of the auxiliary variable is  $F = hI_a/I_c$  where  $h = 1 - (D_{int}/D_{out})^2$  [15]. In Fig. 2, a typical voltage waveform registered at the AC current amplitude equal to 67% of  $I_c$  is presented. As one can see, the waveform deviates from the theoretical one in the part where the vortices with opposite polarity start to penetrate into the sample. This causes a smoothing of the waveform, that is equivalent to the lowering of the content of higher harmonics. This is indeed what can be observed in Fig. 3, where the magnitude of 3rd, 5th and 7th harmonic normalized with respect to the loss (cosine) component of the first harmonic,  $mag_n = \sqrt{(e'_n)^2 + (e''_n)^2/e''_1}$  is plotted. We found this kind of presentation the most adequate, because it is not sensitive



Fig. 2. Typical voltage waveform from the sintered Bi-2223 tube. R.m.s. value of AC current was 47 A, frequency 72 Hz.



Fig. 3. Relative magnitude of higher harmonics (with respect to the loss component) for a hollow tube from Bi-2223 ceramics. With AC r.m.s. current of 60 A the critical current is reached at the amplitude value. The harmonic content that resembles that predicted by the critical state model at lower currents, changed abruptly after surpassing the  $I_c$ .

to an error in phase adjustment for higher harmonics. Only the cosine component of the fundamental harmonic was used in the denominator because the sine component often contains signals induced in the signal leads by AC current in other parts of the test circuit.

Another object tested for the generation of higher harmonics was a single-layer coil made from a tape with critical current of 38 A, produced by Australian Superconductors and containing 37 Bi-2223 filaments in silver matrix. Seven turns were placed on nonmetallic mandrel of 95 mm diameter, with pitch of 10 mm. In this way, total length of the used tape was 210 cm. We observed that in this configuration, the AC loss was quite similar to that registered in ideal self-field conditions on a straight piece of tape [14]. Therefore we deduce that the penetration of magnetic flux inside that tape should be described by the self-field model [9] and expect that the formulas for the higher harmonics given in the part II are still valid. The shape of voltage waveform acquired at 72 Hz is plotted in Fig. 4. One could observe a strong similarity with the waveform form Fig. 2, including the discrepancy with respect to the theoretical prediction for the part of the cycle when opposite



Fig. 4. Typical voltage waveform from multifilamentary Bi-2223/Ag tape. R.m.s. value of AC current was 15.4 A, frequency 72 Hz.



Fig. 5. Relative magnitude of higher harmonics (with respect to the loss component) measured on a Bi-2223/Ag tape. No significant change in harmonic content was observed in frequency range 36 Hz - 144 Hz.

vortices start to penetrate. In this case, the suppression of higher harmonics with respect to the theoretical prediction was even more pronounced than in the case of ceramic tube. One possible explanation is offered by the presence of metallic matrix - as a linear medium it could add some losses without producing higher harmonics. However, this explanation was ruled out by the observed independence of harmonic content on AC frequency - see Fig. 5. It is more plausible to assume that the dependence of critical current density on magnetic field, that is disregarded in the simplest version of the critical state model used in our theoretical calculations, should lead to the effect observed in the registered waveforms and thus in the measured higher harmonics. Indeed, the most prominent depression of the local critical current density should happen on the wire's surface and at the moment when the current reaches the maximum. Lowering the critical current density on the wire surface at that moment would lead to more massive penetration of magnetic flux inside the wire. Linked to this process would be the appearance of higher voltage than expected for the case of critical current density independent on magnetic field. Another explanation of lower content of higher harmonics is the presence of flux creep that results is smooth current-voltage curve, different from the original critical state

model assumption. This is equivalent to a modification of the assumed flux penetration regime toward the linear one [16].

### **IV. CONCLUSIONS**

We have derived theoretical expressions for the higher harmonics of the voltage that appears on a wire from hard type II superconductor. The simplest critical state model with critical current density independent on magnetic field was utilized. Only odd harmonics may be present due to the symmetry of the flux penetration in self-field AC cycle. For practical purposes, it is sufficient to consider the harmonics up to n = 7. Experimental investigation on two superoconducting wires carrying transport AC current has shown weaker generation of higher harmonic than expected. As one of possible explanations, the dependence of local critical current density on magnetic field could be offered. More thorough modeling of the AC transport cycle is necessary to understand the observed discrepancies.

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