Modeling Experimental Magnetization Cycles of Thin Superconducting Strips by Finite-Element Simulations

G. Iannone, S. Farinon, G. De Marzi, P. Fabbricatore, and U. Gambardella

Abstract—To broaden the use of an adaptive resistive algorithm (ARA), we present a 3-D model able to reproduce the experimental magnetic moment loop for a finite-size superconducting strip in a perpendicular field. We also develop the same model by using *H*-formulation, to strengthen the ARA results. We find that 2-D models computed with the two different approaches, i.e., ARA and *H*-formulation, provide similar results, and both well compare to approximate analytical solutions. Then, we verify that the computed magnetic moment loop by using either 3-D numerical approaches reproduces the experimental curve, i.e., the magnetic loops of a commercial YBCO-coated conductor sample measured at 5 K. Instead, the 2-D numerical data need a scaling factor to match the experimental ones. The 3-D computations provide support to the hypotheses on the current surface profile that has been used to find the scaling factor.

Index Terms—Critical state, finite-element analysis, hysteresis losses, magnetization loop.

I. INTRODUCTION

I N THE LAST FEW years, the use of finite-element methods (FEMs) became gradually more extended, thus allowing to address complex problems, such as the current distribution and the magnetic field inside the superconductors in a variety of geometries, and great attention has been also given to the study of high-temperature superconducting coils [1]–[6]. In order to optimize the design of a superconducting device, a key role is played by the precise computation of the current density and the magnetic field distribution inside the conductors induced by the magnetic fields or resulting from transport currents. In addition, to predict the performance of superconducting devices and achieve a reliable use of them, it is very important to give an accurate estimate of the energy losses induced by the irreversible motion of the magnetic flux within the superconductor.

In the past years, in order to compute the current density and magnetic field profiles within a superconducting sample, the

Manuscript received May 20, 2014; revised July 25, 2014; accepted July 28, 2014. Date of publication August 5, 2014; date of current version November 26, 2014. The work of G. Iannone was supported by the Programma Operativo Nazionale Ricerca e Competitività 2007–2013 under Grant PONa3_00007. This paper was recommended by Associate Editor M. Parizh.

G. Iannone is with the Department of Physics, University of Salerno, 84084 Salerno, Italy (e-mail: Iannone@sa.infn.it).

S. Farinon and P. Fabbricatore are with the Istituto Nazionale di Fisica Nucleare—Sezione di Genova, 16146 Genova, Italy.

G. De Marzi is with the Superconductivity Laboratory, ENEA Centro Ricerche Frascati, 00045 Rome, Italy.

U. Gambardella is with the Istituto Nazionale di Fisica Nucleare—Sezione di Napoli, 80126 Napoli, Italy.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TASC.2014.2345339

Bean critical state model (CSM) [7], [8] has been widely used. Within this framework, the computation by means of analytical expressions is usually limited to either simple geometries or reduced descriptions of the material's properties. For general shapes, analytical expressions became more complicated to solve; hence, numerical methods are often preferred, such as the ones based on variational principles [6], [9], [10], the integrodifferential numerical method [11], and the FEM.

In numerical models, a FEM is used in order to solve the Maxwell equations in two dimensions, (2-D), or three dimensions, (3-D). Three major numerical approaches, featuring different equations, have been developed so far: the A-Vformulation, based on the magnetic vector potential [12]; the $T-\Omega$ formulation, based on the current vector potential [13]; and the *H*-formulation, based on the magnetic field [14], [15]. To solve the electromagnetic problems, the relation between electric field, i.e., E, and the current density, i.e., J, is also required. For a superconducting material, a nonlinear E-Jcharacteristic can be adopted, which takes into account the behavior of a superconducting material near its critical current value. Alternative algorithms were also developed, such as the adaptive resistive algorithm (ARA) [16]-[20] to model 2-D superconducting strips, which provided good results with respect to the analytical approximate solutions.

In this paper, we extend the ARA approach in order to model an actual 3-D isotropic finite-size superconducting strip placed in an applied magnetic field perpendicular to its surface and compare a computed 2-D and 3-D magnetic moment loop with the experimental one, measured in a finite-size YBCO superconducting sample at 5 K. Furthermore, we implement, for the same model, also the H-formulation by using COM-SOL, whereas for the ARA approach, we use ANSYS. We first introduce H-formulation in Section II used to make the same comparisons between H-formulation and the analytical approximate solutions, used as a benchmark. In Section III, we compare the results of an H-formulation 2-D model in terms of magnetic field and current density profiles, hysteresis loop, and losses, with analytic solutions [21]-[23]. In Section IV, we compare the magnetization cycle numerically computed for a finite rectangular strip of superconducting material (both 2-D and 3-D models) to the actual measured magnetization loop of a commercial YBCO-coated conductor tape placed in a magnetic field perpendicular to its surface. In Section V, we conclude that both the ARA approach and the H-formulation method can reproduce all the features experimentally observed in a real thin-film configuration in 3-D numerical computation. Moreover, 2-D models, which save computation time, provide similar good results when suitably scaled to account for the finite size of the real sample.

II. NUMERICAL FORMULATIONS

The *H*-formulation meets a high level of popularity, with both 2-D and 3-D models being developed so far [24]–[27]. Among the main advantages of the *H*-formulation one can cite is its excellent ability to converge, combined with ease of imposition of the boundary conditions.

In *H*-formulation, both the Ampere law $\nabla \wedge H = J$ and the Faraday law $\nabla \wedge E = -\mu_0(\partial(\mu_r(H))/\partial t)$ are solved in terms of the three components of the magnetic field vector $H = [H_x, H_y, H_z]^T$. Since in the following no ferromagnetic materials are present, we assume $\mu_r = 1$; hence, Faraday law reduces to $\nabla \wedge E = -\mu_0(\partial H/\partial t)$.

The current density as $J = [J_x, J_y, J_z]^T$ and the electrical field vectors $E = [E_x, E_y, Ez]^T$ are related by an exponential E-J characteristic, i.e.,

$$E = E_0 \left(\frac{J}{J_c(B)}\right)^n \tag{1}$$

where *n* is a feature related to the broadening of the E-J curve (*n*-index), E_0 is the electrical field criterion used in determining J_c (usually in the range $10^{-4} \div 10^{-5}$ Vm⁻¹), and $J_c(B)$ represents the magnetic field dependence of the critical current density. In CSMs, $J_c(B)$ can be kept constant (Bean [7], [8]) or can be analytically defined (Fixed Pinning [28], Square Root [29], Kim [30], Exponential models [31]). Strictly speaking, the CSM assumption implies that $n = \infty$, but as a matter of fact, such a condition can be already achieved when n > 500 (i.e., the E-J curve assumes a very steep profile) [32].

In this paper, CSM is first used to compare the numerical model to the approximate analytical models by adopting a pure Bean model (J_c does not depend on the applied field, i.e., $J_c(B) = J_{c0}$). Then, in order to make a comparison with experimental magnetization loops, in the calculations, we use the experimental $J_c(B)$ inferred from the hysteresis cycle loops where the sample is not biased, neglecting the self-field contribution.

Substituting Ampere law and (1) into Faraday law, we obtain a set of three partial differential equations for magnetic field H, i.e.,

$$\mu_0 \frac{\partial H}{\partial t} + \nabla \wedge (\rho \nabla \wedge H) = 0 \tag{2}$$

where

$$\varrho = \frac{E_0}{J_c} \left(\frac{J}{J_c}\right)^{n-1}.$$
(3)

Equation (2) is solved by applying a time-dependent uniform magnetic field in the z-direction (i.e., along the thickness direction). Dirichlet boundary conditions are imposed on a boundary area ten times larger than the largest sample dimension: $H_x = H_y = 0$, $H_z = H_0 \sin(\omega t)$. The applied field is varied at the frequency of 1 Hz.

In the ARA approach, the solution is reached through an iterative adjustment of material resistivity [16]. In this approach, the superconductor can be subdivided into many elements with a starting value for the resistivity, i.e., ρ_0 , which is then iteratively changed according to

$$\rho_l^{k+1} = \max\left\{\rho_i^k \frac{|J_i^k|}{J_c}, \rho_0\right\}, \quad \text{until} \ \frac{|\rho_i^{k+1} - \rho_i^k}{\rho_i^k} < \varepsilon \quad (4)$$

where *i* is the element label, *k* is the cycle index, ε is the convergence parameter, and ρ_0 is the minimum value of the resistivity. The iterative solution corresponds to the resistivity distribution among the elements that satisfy $J \leq J_c$ everywhere in the superconductor, as prescribed by the CSM.

ARA calculations are carried out with a Dell Precision T5600 with a two-core Intel Xeon processor E52609 and 64-GB RAM and *H*-formulation calculation with a Lenovo ThinkStation dual six-core Intel Xeon E52630 (2.3 GHz) with 64-GB RAM.

III. INFINITE THIN STRIP WITH 2-D SIMULATIONS

As we are dealing with a rectangular strip of superconducting film, we first set 2-D simulations as they offer a fast response compared with the 3-D ones. In fact, even if the 2-D model just considers only an infinite length of the conductors, we can compare it with the results of analytical models, and then, the infinite length may be scaled out for taking into account the finite-size sample. The critical state of a thin superconducting strip having width 2a, thickness $d \ll a$, and infinite length along the y-direction has been previously analyzed with a 2-D ARA approach [16]. Here, we perform the same computation using H-formulation in 2-D, considering the same thin strip with a constant critical current density J_c exposed to a magnetic flux density $B_a(t) = B_0 \sin(\omega t)$ perpendicular to its width, i.e., along the z-direction, as sketched in Fig. 1. We remind that both the current density and magnetic field profiles inside the infinite-length strip have been analytically computed as [21]–[23]

$$J(x) = \begin{cases} \frac{xJc}{|x|}, & b < |x| < a\\ \frac{2Jc}{\pi} \operatorname{atan}\left[\frac{c_x}{\sqrt{b^2 - x^2}}\right], & |x| < b \end{cases}$$
(5a)

$$B(x) = \begin{cases} B_c \operatorname{atanh}\left[\sqrt{\frac{x^2 - b^2}{c|x|}}\right], & b < |x| < a \\ 0, & |x| < b \end{cases}$$
(5b)

where the following relations hold:

$$B_{c} = \mu_{0} J_{c} d/\pi$$

$$b = a/\cosh\left(\frac{B_{a}}{B_{c}}\right)$$

$$c = \tanh\left(\frac{B_{a}}{B_{c}}\right).$$
(6)

Here, B_c is the critical field, whereas b and c are model parameters. Similar to that in [16], we compare the analytical solutions to the results of H-formulation. We study the same model case of a thin strip with $a = 200 \ \mu\text{m}$ along the x-direction, $d = 2 \ \mu\text{m}$ along the z-direction (aspect ratio $\alpha = d/a = 0.01$), $J_c = 10^7 \text{ A/m}^2$, so that $I_c = 2ad \ J_c = 8 \text{ mA}$.

A comparison of the magnetic field profiles and current density inside the strip, as calculated by the FEM analysis and by the corresponding analytical expressions by Brandt and Indebom [21], is shown in Fig. 2(a) and (b), respectively. In



Fig. 1. Infinite-length thin strip along the y-direction. The x origin is in the middle of the strip, and the external magnetic field H is perpendicular to the wide surface.



Fig. 2. (a) Magnetic field profile and (b) current density of a thin strip $(a = 200 \ \mu\text{m}, d = 2 \ \mu\text{m}, J_c = 10^7 \ \text{A/m}^2)$ in a perpendicular magnetic field computed by *H*-formulation, for various values of the ratio B_a/B_c . Moreover, the analytical solutions, according by Brandt and Indebom [21], are reported.

Fig. 2(a), the magnetic field profile computed inside the sample decreases following the atanh(x) shape, as expected for this aspect ratio, and in Fig. 2(b), the current profiles also reproduce the atanh(x) shape, inverting its direction at the center of the sample. In Fig. 3, the 2-D current distribution across the strip section (one fourth of the geometry) is reported by using a different number of mesh elements across the strip thickness.

Thus, a good agreement is found with the analytical solution, as it occurred when using the ARA approach on the same model.



Fig. 3. Critical current distribution in one fourth of the geometry computed at $B_a = 8 \mu$ TG. The number of horizontal elements is 200. From top to bottom, the number of elements across the thickness is increased: 1, 2, 4, 8, 100. The *y*-scale is magnified by 10.

To further analyze the *H*-formulation capabilities, we also consider the same geometry, carrying a transport current in zero applied magnetic field $I_a = I_a(t)$ varying from 0 to I_c . Moreover, this situation has been analytically solved by Brandt and Indebom [21] and Norris [33], and the solution for the current density and field profiles inside the strip when increasing the current from 0 to I_a is the following:

$$J(x) = \begin{cases} J_c, & b < |x| < a \\ \frac{2J_c}{\pi} \operatorname{atan} \left[\frac{\sqrt{a^2 - b^2}}{\sqrt{b^2 - x^2}} \right], & |x| < b \end{cases}$$
(7a)

$$B(x) = \begin{cases} \frac{xB_C}{|x|} \operatorname{atanh}\left[\frac{\sqrt{x^2 - b^2}}{\sqrt{a^2 - b^2}}\right], & b < |x| < a \\ 0, & |x| < b \end{cases}$$
(7b)

where

$$B_c = \mu_0 J_c d/\pi$$

$$I_a = 2J_c d\sqrt{a^2 - b^2}$$

$$b = a\sqrt{1 - \frac{I_a^2}{I_{\max}^2}}.$$
(8)

In this occurrence, the Dirichelet boundary condition on the circular external domain for the *H*-formulation are the field values H_x and H_z , which, according to Ampere's law, induce the required current on the strip. In Fig. 4(a), the magnetic field profile computed inside the sample goes to zero toward the center of the strip according to the $\operatorname{atanh}(x)$ behavior, while having opposed directions at the sides. Fig. 4(a) and (b) shows a good agreement with the analytical solution, as it occurred when using the ARA approach [16]. Thus, we can conclude that both FEMs provide the same results.

The FEM simulations also allow to evaluate the hysteresis losses per cycle, for example, by integrating the instantaneous power $\int_V \vec{E} \cdot \vec{J} dV$ over one period or by calculating the integral $Q = \oint m(B) dB$, provided that the magnetic moment m(B) is known. In general, for a conductor, m(B) is induced by the presence of an external applied magnetic field and may be evaluated by the following volume integration [34]:

$$\vec{m} = -\frac{1}{2} \int\limits_{V} \vec{r} \times \vec{J} \, dV. \tag{9}$$

For a thin superconducting strip, the currents can only flow onto the x-y plane and $J(x,y) = [J_x, J_y]^T$ being the y-direction of the infinite length. In this case, it is easy to



Fig. 4. (a) Magnetic field profile and (b) current density of a thin strip ($a = 200 \ \mu\text{m}, d = 2 \ \mu\text{m}, J_c = 10^7 \ \text{A/m}^2$) under a transport current I_a , computed by *H*-formulation, for various values of the ratio I_a/I_c . Black smaller points represent the analytical solution reported in [21].

prove that the magnetic moment has only one component along the z-direction: $m_z = -(1/2) \iiint (xJ_y - yJ_x) dx dy dz$.

We have verified that the magnetic moment, being integrated across the strip thickness, is not really affected by the number of mesh elements along the thickness. Thus, in the following simulation, we used one element along the thickness to save computation time.

In the limit of infinite y length, both terms in the integral give the same contribution to m_z [35], [36], since the condition $\nabla \cdot J = 0$ must be fulfilled. Therefore, in the limit of an infinite long thin strip, the magnetic moment per unit length can be written as

$$m_z = -\iint x J_y \, dx \, dz. \tag{10}$$

The analytical solution for the virgin magnetic moment, per unit length, in a perpendicular field is [37]

$$m(B_a) = J_c da^2 \tanh\left(\frac{B_a \pi}{J_c d}\right) \tag{11}$$

whereas the magnetic moments in ramp-up and ramp-down (m_{\uparrow} and m_{\downarrow}) are

$$m_{\downarrow}(B_a, B_0) = m(B_a) - 2m\left(\frac{B_a - B_0}{2}\right)$$
$$m_{\uparrow}(B_a, B_0) = -m(B_a) - 2m\left(\frac{B_a + B_0}{2}\right).$$
(12)



Fig. 5. Hysteresis moment per unit length for an infinitely long strip as a function of the applied magnetic field as calculated by H-formulation at different n values (20, 50, and 500) and by using the analytical solution [21], [23]. The frequency of the applied magnetic field is 1 Hz.

The initial slope $m'(B_a = 0)$ and the saturated moment m_{sat} , for unit length, are respectively

$$m'(B_a = 0) = -\pi a^2$$

$$m_{\text{sat}} = (B_a \gg B_c) = -J_c da^2.$$
(13)

In Fig. 5, we show how the m(B) curves are affected by the n-index in the H-formulation method. The solid line corresponds to the analytical solution, whereas solid markers are the curves calculated with n = 20, n = 50, and n = 500, respectively. The computed magnetization cycles attain to the analytical curve of the Bean model as the *n*-index increases, according to the fact that E-J curves became steeper and steeper. Formally, the Bean model would require $n = \infty$, but at n = 500, we observed a good agreement. Once fixed n = 500, we computed losses Q per cycle and unit length as a function of the maximum applied magnetic field B_0 , integrating $Q = \oint m(B) dB$. Instead, the ARA approach computes $Q = \int_{V} \vec{E} \cdot \vec{J} dV$, as described in [16]. In Fig. 6, the $Q(B_a)$ points calculated using ARA and H-formulation are shown, in the B_0 range (0.5 \div 3.0) B_c . In the same plot, the analytical curve $Q(B_0) = (\mu_0/\pi)I_C^2(2\ln(\cosh(B_0/B_c))) (B_0/B_c) \tanh(B_0/B_c)$ is also reported. As shown in Fig. 6, we found an excellent agreement between the analytical solution and the numerical 2-D models, both ARA and Hformulation.

IV. 2-D AND 3-D ANALYSES COMPARED WITH EXPERIMENTAL DATA

One of the goal of our analyses is to extend the numerical computation to real samples, which are finite and where the critical current density is affected by the magnetic field. Thus, we first introduce the $J_c(B)$ dependence in our models evaluated from experimental data measured in thin-film superconducting samples. We have measured an YBCO sample of rectangular shape (dimensions: 2a = 4 mm l = 5 mm), cut from a 4-mm-wide tape manufactured by SuNAM. In this coated conductor,



Fig. 6. Hysteresis losses per unit length as a function of the ratio B_a/B_c as calculated by FEM software Comsol (red closed rhombs) ANSYS (cyan open circles) and by using the analytical solution (line) by Brandt and Indebom [21].



Fig. 7. Sketch of the simplified current path in a superconducting strip with dimensions $2a \times l$ under perpendicular magnetic field.

the $d = 1 \ \mu m$ thick superconducting YBCO layer is deposited onto a metallic nonmagnetic substrate by a novel reactive coevaporation by deposition and reaction method [38], [39]. The magnetic moment is measured by means of an Oxford MAGLAB Vibrating Sample Magnetometer system with a magnetic field perpendicular to the film surface (up to 12 T with a ramp rate varying from 0.4 to 0.2 T/min), whereas the sample temperature is kept stable at T = 5 K. Measured curves show a small paramagnetic contribution, which can be ascribed to the sample holder.

In order to estimate $J_c(B)$ from the m(B) loop, Bean's critical state model [7], [8] is assumed for rectangular geometries at each field value.

We consider a simplified J_c distribution as sketched in Fig. 7 (actually just one fourth of the total geometry), where the components of J belong only to x and y in the two regions delimited by a 90° turn of the currents.

This assumption is also supported by analytical computations [11], [40]. In this case, m has just one component along the z-direction, i.e., m_z , and (9) becomes

$$m_z = \frac{1}{2} \int_{V} (xJ_y - yJ_x) \, dV.$$
(14)



Fig. 8. Critical current behavior as determined by the experimental measurement of the YBCO rectangular sample magnetic moment loop (inset) at 5 K in perpendicular field.

This can be integrated in one fourth of the geometry, to get the final relation for the full geometry, i.e.,

$$m_z = -2d \iint (xJ_y - yJ_x) \, dx \, dy. \tag{15}$$

Assuming a full penetration of the magnetic field inside the sample and using the proper integration limits, we obtain

$$m_{\rm sat} = -dJ_c(B)a^2l\left(1 - \frac{2a}{3l}\right).$$
 (16)

Introducing the magnetization M as m/V for the $J_c(B)$ (A/m²) is

$$J_c(B) = \frac{2M_{\text{sat}}(B)}{a\left(1 - \frac{2a}{3l}\right)} \tag{17}$$

where $2M_{\rm sat}(B)$ (A/m) is the width of the magnetization hysteresis loop for increasing and decreasing fields at a given field value B, whereas 2a and l (with a < l) are the sample dimensions and d its thickness. The same result has previously been obtained in [41] with a different approach. It is worth noting that (16) reduces to (13) in the limit of infinite length. We also point out that (16) is strictly valid only if the superconductor is completely saturated by the shielding currents. In the scientific literature, different formulas than (17) are sometimes reported [42]–[44]. In particular, $M_{\rm sat}$ is usually replaced by ΔM , i.e., the differences of the magnetization between the field-up and field-down branches.

Therefore, from the experimental amplitude ΔM of the hysteresis loop measured on a finite-size superconducting strip, the critical current density behavior as a function of applied magnetic field can be extracted according to (17). This behavior is shown in Fig. 8, together with the experimental magnetic moment loop. We actually use this $J_c(B)$ computed from the experimental M(B) in the numerical model. As in the 2-D simulation, the current only flows in the infinite y-direction, the use of the $J_c(B)$ curve leads to a higher-than-expected magnetic moment. Thus, the computed m(B) curve has to be scaled by the factor (1 - 2a/3l), which takes into account the finite size of the sample. Fig. 9 shows the computed m(B) using the H-formulation in 2-D (circles) according to (10),



Fig. 9. Comparison between the magnetic moment loop measured in a YBCO rectangular sample ($a = 2000 \ \mu\text{m}$, $l = 5000 \ \mu\text{m}$, $d = 1 \ \mu\text{m}$) at $T = 5 \ \text{K}$ (solid dots) and the 2-D computations: *H*-formulation (red open rhombs) and ARA (open green circles). Either numerical result has been scaled by the factor (1 - 2a/3l). The low-field regions are magnified in the upper and lower insets.



Fig. 10. Sheet current distribution computed from 3-D simulation: (a) one fourth of the model geometry in the ARA approach and (b) full geometry in H-formulation.

together with the analogous computation performed with the ARA approach (squares) and superimposed to the experimental m(B) cycle measured on the YBCO film (solid line). The computed data follow the experimental one within 1% except near the peak value in the virgin magnetization branch where there is a maximum 8% discrepancy among curves. It is worth noting that (17) is based on the CSM framework; thus, it is valid for n approaching ∞ . Although an n value for an actual YBCO sample is depending on the temperature and applied magnetic field and it has a value much less than the n = 500 used in the numerical computation, we had to use the high n value to keep the CSM approach.

In order to support the hypothesis, which leads to (17), we also perform 3-D simulations, which are more time consuming but can provide stronger support to our assumptions. Thus, in this 3-D model, we just put the actual finite dimensions of the measured sample and the $J_c(B)$ in Fig. 8. In Fig. 10(a) and (b), we show the surface current density distribution computed by both the ARA approach and the *H*-formulation, respectively. As expected, we observe a 90° turn of the surface current density as sketched in Fig. 7, in agreement with the assumptions that lead to (17). In addition, the models give out also the m(B) curve, which implicitly include the features of a finite sample.



Fig. 11. Comparison between the magnetic moment measured in a YBCO rectangular sample ($a = 2000 \ \mu m$, $l = 5000 \ \mu m$, $d = 1 \ \mu m$) at 5 K (solid dots) and the 3-D computations: *H*-formulation (red open rhombs) and ARA (open green circles). The low-field regions are magnified in the upper and lower insets.

In Fig. 11, the ARA and the H-formulation computations of the magnetic moment loop are shown, without any geometrical correction factor, superimposed to the experimental curve as measured before. Moreover, in this case, we have an excellent agreement with experimental data.

V. CONCLUSION

Following previous works on the ARA algorithm [16]–[18], in this paper, we have presented a further step of this approach with the challenge of reproducing the experimental magnetization hysteresis curve of a real YBCO sample. To add value to this numerical result, we also used the *H*-formulation algorithm for the same model case. As the ARA approach has been validated, its previous results against the analytical solution [21]–[23] for the transport current and magnetization cases, we performed the same steps to validate also the *H*-formulation method. Instead of ARA, the *H*-formulation is affected by the *n*-index; hence, we also studied the dependence of computed magnetization loops on *n*, finding that by using n = 500, the Brandt behavior of Bean CSM is well reproduced.

To model the experimental magnetic moment measurements, we first computed in 2-D the magnetization curve of a thin superconducting strip using the $J_c(B)$ obtained from experimental data. We have shown that to achieve a good matching among the measured and the 2-D computed m(B), a scaling factor (1 - 2a/3l) must be considered, in order to take into account the finite size of the sample. We additionally found that 3-D models for the same geometry do not require any scaling factor to reproduce the experimental m(B) magnetic moment loop for the YBCO sample. In addition, 3-D models also provided the current density distribution, which confirmed the surface current path onto the finite-size strip, very similar to the one used to derive the scaling factor in the 2-D models.

Finally, we can conclude that we observed a good agreement between ARA and *H*-formulation in 2-D and 3-D numerical simulations of superconducting finite-size rectangular samples and that both models can accurately reproduce the experimental measurements.

ACKNOWLEDGMENT

The authors would like to thank G. Messina (ENEA) for fruitful discussions and A. Ferrentino (University of Salerno) for technical assistance.

REFERENCES

- M. Zhang and T. A. Coombs, "3D modeling of high-Tc superconductors by finite element software," *Supercond. Sci. Technol.*, vol. 25, no. 1, 2012, Art. ID. 015009.
- [2] F. Sirois, M. Dione, F. Roy, F. Grilli, and B. Dutiot, "Evaluation of two commercial finite element packages for calculating AC losses in 2-D high temperature superconducting strips," *J. Phys. Conf. Ser.*, vol. 97, 2008, Art. ID. 012030.
- [3] D. N. Nguyen, S. P. Ashworth, J. O. Willis, F. Sirois, and F. Grilli, "A new finite-element method simulation model for computing AC loss in roll assisted biaxially textured substrate YBCO tapes," *Supercond. Sci. Technol.*, vol. 23, no. 2, Feb. 2010, Art. ID. 025001.
- [4] E. Pardo and F. Grilli, "Numerical simulations of the angular dependence of magnetization AC losses: Coated conductors, Roebel cables and double pancake coils," *Supercond. Sci. Technol.*, vol. 25, no. 1, Jan. 2012, Art. ID. 014008.
- [5] F. Sirois and F. Grilli, "Numerical considerations about using finiteelement methods to compute ac losses in HTS," *IEEE Trans. Appl. Supercond.*, vol. 18, no. 3, pp. 1733–1742, Sep. 2008.
- [6] E. Pardo, "Modeling of coated conductor pancake coils with a large number of turns," *Supercond. Sci. Technol.*, vol. 21, no. 6, Jun. 2008, Art. ID. 065014.
- [7] C. P. Bean, "Magnetization of hard superconductors," *Phys. Rev. Lett.*, vol. 8, no. 6, pp. 250–253, Mar. 1962.
- [8] C. P. Bean, "Magnetization of high-field superconductors," *Rev. Mod. Phys.*, vol. 36, no. 1, pp. 31–39, Jan. 1964.
- [9] L. Prigozhin, "Analysis of critical state problems in type-II superconductivity," *IEEE Trans. Appl. Supercond.*, vol. 7, no. 4, pp. 3866–3873, Dec. 1997.
- [10] A. Badia-Majos and C. Lopez, "Electromagnetics close beyond the critical state: Thermodynamic prospect," *Supercond. Sci. Tech.*, vol. 25, no. 10, Oct. 2012, Art. ID. 104004.
- [11] E. H. Brandt, "Square and rectangular thin superconductors in transverse magnetic field," *Phys. Rev. Lett.*, vol. 74, no. 15, pp. 3025–3028, Apr. 1995.
- [12] G. Barnes, M. McCulloch, and D. Dew-Hughes, "Computer modelling of type II superconductors in applications," *Supercond. Sci. Technol.*, vol. 12, no. 8, pp. 518–522, Aug. 1999.
- [13] N. Amemiya, S. Murasawa, N. Banno, and K. Miyamoto, "Numerical modelings of superconducting wires for AC loss calculations," *Phys. C*, vol. 310, no. 1, pp. 16–29, Dec. 1998.
- [14] R. Brambilla, F. Grilli, and L. Martini, "Development of an edge-element model for AC loss computation of high-temperature superconductors," *Supercond. Sci. Technol.*, vol. 20, no. 1, pp. 16–24, Jan. 2007.
- [15] Z. Hong, A. M. Campbell, and T. A. Coombs, "Numerical solution of critical state in superconductivity by finite element software," *Supercond. Sci. Technol.*, vol. 19, no. 12, pp. 1246–1252, Dec. 2006.
- [16] S. Farinon, P. Fabbricatore, and F. Gomory, "Critical state and magnetization loss in multifilamentary superconducting wire solved through the commercial finite element code ANSYS," *Supercond. Sci. Technol.*, vol. 23, no. 11, Nov. 2010, Art. ID. 115004.
- [17] S. Farinon, P. Fabbricatore, F. Grilli, and P. A. C. Krüger, "Applicability of the adaptive resistivity method to describe the critical state of complex superconducting systems," *J. Supercond. Novel Magn.*, vol. 25, no. 7, pp. 2243–2350, Oct. 2012.
- [18] P. A. C. Krüger, F. Grilli, and S. Farinon, "Compliance of numerical formulations for describing superconductor/ferromagnet heterostructures," *Phys. C*, vol. 471, no. 21/22, pp. 1083–1085, Nov. 2011.
- [19] A. O. Hauser, "Calculation of superconducting magnetic bearings using a commercial FE-program (ANSYS)," *IEEE Trans. Magn.*, vol. 33, no. 2, pp. 1572–1575, Mar. 1997.
- [20] C. Gu and Z. Han, "Calculation of AC loss in HTS tape with FEA program ANSYS," *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 2859–2862, Jun. 2005.

- [21] E. H. Brandt and M. Indebom, "Type II superconductor strip with current in a perpendicular magnetic field," *Phys. Rev. B*, vol. 48, no. 17, pp. 12 893–12 906, Nov. 1993.
- [22] E. Zeldov, J. R. Clem, M. McElfresh, and M. Darwin, "Magnetization and transport currents in thin superconducting films," *Phys. Rev. B*, vol. 49, no. 14, pp. 9802–9822, Apr. 1994.
- [23] M. R. Halse, "AC face field losses in a type II superconductor," J. Phys. D: Appl. Phys., vol. 3, no. 5, pp. 717–720, May 1970.
- [24] F. Grilli, R. Brambilla, F. Sirois, A. Stenvall, and S. Memiaghe, "Development of a three-dimensional finite-element model for high-temperature superconductors based on the *H*-formulation," *Cryogenics*, vol. 53, pp. 142–147, Jan. 2013.
- [25] V. M. R. Zermeno, A. B. Abrahamsen, N. Mijatovic, B. B. Jensen, and M. P. Soerensen, arXiv:1308.2568v1 [cond-mat.supr-con], 2013.
- [26] F. Grilli *et al.*, "Self-field effects and AC losses in pancake coils assembled from coated conductor Roebel cable," *IEEE Trans. Appl. Supercond.*, vol. 24, no. 3, Jun. 2014, Art. ID. 4801005.
- [27] M. D. Ainslie *et al.*, "An improved FEM model for computing transport AC loss in coils made of RABITS YBCO coated conductors for electric machines," *Supercond. Sci. Technol.*, vol. 24, no. 4, Apr. 2011, Art. ID. 045005.
- [28] L. Ji, R. H. Sohn, G. C. Spalding, C. J. Lobb, and M. Tinkham, "Criticalstate model for harmonic generation in high-temperature superconductor," *Phys. Rev. B*, vol. 40, no. 16, pp. 10936–10945, Dec. 1989.
- [29] D. Leblanc and M. A. LeBlanc, "AC-loss valley in type-II superconductor," *Phys. Rev. B*, vol. 45, no. 10, pp. 5443–5449, Mar. 1992.
- [30] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, "Critical persistent currents in hard superconductors," *Phys. Rev. Lett.*, vol. 9, no. 7, pp. 306–309, Oct. 1962.
- [31] W. A. Fietz, M. R. Beasley, J. Silcox, and W. W. Webb, "Magnetization of superconducting Nb-25%Zr wire," *Phys. Rev.*, vol. 136, no. 2A, pp. A335–A345, Oct. 1964.
- [32] Z. Hong *et al.*, "A two-dimensional finite element method to calculate the AC loss in superconducting cables, wires and coated conductors," *J. Phys. Conf. Ser.*, vol. 97, no. 1, Feb. 2008, Art. ID. 012117.
- [33] W. T. Norris, "Calculation of hysteresis losses in hard superconductors carrying ac: Isolated conductors and edges of thin sheets," J. Phys. D: Appl. Phys, vol. 3, no. 4, pp. 489–507, Apr. 1970.
- [34] J. D. Jackson, *Classical Electrodynamics*. New York, NY, USA: Wiley, 1962, p. 147.
- [35] M. M. Doria, E. H. Brandt, and F. M. Peeters, "Magnetization of a superconducting film in a perpendicular magnetic field," *Phys. Rev. B*, vol. 78, no. 5, Aug. 2008, Art. ID. 054407.
- [36] E. H. Brandt, "Superconductors of finite thickness in a perpendicular magnetic field: Strips and slabs," *Phys. Rev. B*, vol. 54, no. 6, pp. 4246– 4264, Aug. 1996.
- [37] E. H. Brandt, "Superconductor disks and cylinders in an axial magnetic field. I. Flux penetration and magnetization curves," *Phys. Rev. B*, vol. 58, no. 10, pp. 6506–6522, Sep. 1998.
- [38] V. Matias and R. H. Hammond, "YBCO superconductor wire based on IBAD-textured templates and RCE of YBCO," *Phys. Proc.*, vol. 36, pp. 1440–1444, 2012.
- [39] J. H. Lee *et al.*, "RCE-DR, a novel process for coated conductor fabrication with high throughput," *Supercond. Sci. Technol.*, vol. 27, no. 4, Apr. 2014, Art. ID. 044018.
- [40] G. P. Mikitik and E. H. Brandt, "Critical state in thin anisotropic superconductors of arbitrary shape," *Phys. Rev. B*, vol. 62, no. 10, pp. 6800–6811, Sep. 2000.
- [41] D. X. Chen and R. B. Goldfarb, "Kim model for magnetization of type-II superconductors," J. Appl. Phys., vol. 66, no. 6, pp. 2489–2500, Sep. 1989.
- [42] N. D. Zhigadlo *et al.*, "High-pressure flux growth, structural, and superconducting properties of LnFeAsO single crystals," *Phys. Rev. B*, vol. 86, no. 21, Dec. 2012, Art. ID. 214509.
- [43] Y. Zhao, M. Ionescu, J. Horvat, A. H. Li, and S. X. Dou, "Si doping on MgB2 thin films by pulsed laser deposition," *Supercond. Sci. Technol.*, vol. 17, p. 1247, 2004.
- [44] N. T. Mua, J. C. Grivel, and T. D. Hien, "Effect of fabrication conditions on properties of BI-2212 thin films," in *Proc. SRF*, Chicago, IL, USA, Jul. 21–29, 2011, pp. 905–907.

Authors' biographies not available at the time of publication.