Modeling of Current Density Distributions in Critical State by Commercial FE Codes

Stefania Farinon, Pasquale Fabbricatore, Fedor Gömöry, Michela Greco, and Eugen Seiler

Abstract—The method of minimum magnetic energy variation has proved its merit in modeling the magnetic field interaction with hard type-II superconductors. Its main advantage is that the front of flux penetration is calculated starting from a physical principle equivalent to the Bean's critical state model. We have developed a procedure that adapts the above-mentioned method to the environment of the commercial Finite Element code ANSYS. As a step forward with respect to existing works, this allows to include magnetic materials into simulations. The case of a superconducting strip covered by a ferromagnetic sheath is reported.

Index Terms—AC losses, critical state, finite element analysis, iron sheath.

I. INTRODUCTION

▼RITICAL state model [1] allows to derive analytical • expressions for current distribution and AC loss in the case of some idealized shapes like a slab in parallel magnetic field or an infinitely thin strip in perpendicular magnetic field. For more complicated shapes, numerical methods should be developed. Here we present the results obtained when implementing the minimum energy variation procedure [2] into the environment of the commercial finite element code. In difference to the original approach [3], [4] that used analytical formulas for inductances to evaluate the energy, we utilized the FE codes ANSYS and FEMLAB for this purpose. This makes the method more general, and e.g. the magnetic field penetration into a superconducting strip covered by magnetic material with linear or nonlinear properties can be simulated. Involving the critical state distribution of current density represents a step forward with respect to previous works dealing with superconducting strips covered by a magnetic material [5]–[7]. With this approach, the AC loss of filamentary MgB_2 wire with iron matrix or YBCO coated conductor on ferromagnetic substrate can be simulated.

II. CRITICAL STATE OF AN ISOLATED STRIP

A. The Model

Let us consider an infinitely long isolated strip, whose linear dimensions in the xy plane are -a < x < a, -b < y < b,

S. Farinon, P. Fabbricatore, and M. Greco are with the Istituto Nazionale di Fisica Nucleare, Sezione di Genova, 16146 Italy (e-mail: stefania.farinon@ge.infn.it).

F. Gömöry and E. Seiler are with the Institute of Electrical Engineering, 84239 Bratislava.

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exposed to the uniform external magnetic field B_a in the y direction. It has been demonstrated [2]–[4] that the current distribution inside the strip at each applied field can be found by minimizing the total magnetic energy E_t :

$$E_t = E_{field} + E_{self} + E_{int} \tag{1}$$

where E_{self} is the energy of the current distribution in the superconducting region, E_{field} is the energy of the applied field and E_{int} is the energy due to the interaction of the current distribution with the applied field.

To evaluate the total energy as function of the applied field, we subdivide the strip in $4 \times n_x \times n_y$ sub-areas, each of them, obeying the Bean critical state model, carries either no current or the critical current density J_C . E_{field} and E_{int} can be simply calculated as:

$$E_{field} = \frac{B_a^2}{2\mu_0} S \tag{2}$$

$$E_{int} = -4 \sum_{\substack{i=1,n_x \\ j=1,n_y}} I_{ij} A_{field \ ij} = -4B_a \sum_{\substack{i=1,n_x \\ j=1,n_y}} I_{ij} x_i \ (3)$$

where S is the total section, comprising the air surrounding the superconducting strip, and I_{ij} is the current flowing in the ij sub-area, which can be either 0 or $J_C \Delta x \Delta y = J_C a/n_x b/n_y$, and $A_{field ij}$ is the average vector potential of the sub-area ij only due to the external applied field.

The E_{self} term, the energy due to the solely system of currents, is much more difficult to be analytically calculated, but it is a very simple task if introducing a finite element code for magnetic calculation like ANSYS. Modeling the strip subdivided in the $4 \times n_x \times n_y$ sub-areas and applying the proper current density distribution, the corresponding energy can be simply calculated as:

$$E_{self} = \int_{S} \frac{B^2}{2\mu_0} dS = 4 \sum_{\substack{i=1,n_x \\ j=1,n_y}} I_{ij} A_{self\ ij}$$
(4)

where $A_{self ij}$ is the average vector potential of the sub-area ij generated by the current distribution.

B. The Algorithm

The primary target of the algorithm is to determine the order in which the sub-areas pass from 0 current density to J_C and the corresponding applied magnetic fields, by minimizing the total magnetic energy E_t . With this aim, there are two possible procedures. The first one, the most commonly followed, consists in increasing very slowly the magnetic field and calculating, at each step, the current density distribution which minimizes E_t . This method is time consuming, as the magnetic field step should

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Fig. 1. Hysteresis magnetization curve of an isolated strip ($a = 200 \,\mu\text{m}$ and $b = 20 \,\mu\text{m}$). Both M and H are normalized to $J_C a$.

be small enough to guarantee that the switching on of one only sub-area at a time is energetically convenient. As a consequence there will be a lot of redundant steps in which no new sub-area is switched on.

Our procedure consists in finding at the same time the order of turning on of the sub-areas and their corresponding magnetic fields. The first area to be switched on is the one which minimizes E_{self} . The magnetic field at which this area has to be turned on is reached when the total magnetic energy of this current distribution $E_t(1)$ equals the energy of the zero current distribution $E_t(0)$:

$$\frac{B_a^2(1)}{2\mu_0}S = \frac{B_a^2(1)}{2\mu_0}S + 4\sum_{\substack{i=1,n_x\\j=1,n_y\\j=1,n_y}}I_{ij}(1)A_{ij}(1) - 4B_a\sum_{\substack{i=1,n_x\\j=1,n_y}}I_{ij}(1)x_i$$

$$B_a(1) = \frac{\sum_{\substack{i=1,n_x\\j=1,n_y\\j=1,n_y}}I_{ij}(1)x_i}{\sum_{\substack{i=1,n_x\\j=1,n_y}}I_{ij}(1)x_i} \tag{5}$$

Then, the next sub-area to be switched on is the one which reaches the same value of total magnetic energy of the first one at the lowest value of magnetic field. Going on this way, we can reconstruct the right order of turning on of the sub-areas and in parallel the magnetic fields at which this happens.

C. Results

As an example, we show the results of a strip whose linear dimensions are $a = 200 \ \mu \text{m}$ and $b = 20 \ \mu \text{m}$. It is subdivided in $4 \times 100 \times 10$ sub-areas, and the critical current density is 10^8 A/m^2 . It is appropriate to represent the process of magnetic flux penetration by the magnetization curve.

1) Shape of the Magnetization Curve: Once the current profile is determined, the initial magnetization curve is simply given by:

$$M_i = \frac{1}{ab} \sum_{\substack{i=1,n_x\\j=1,n_y}} I_{ij} x_i \tag{6}$$



Fig. 2. Normalized external susceptibility χ'' as function of the normalized magnetic field. The continuous line is obtained by our procedure while the empty dots are corresponding to our numerical reference method [3].

and then the reversal magnetization curve after reaching a given maximum H_m [8]:

$$M(H) = M_i(H_m) - 2M_i \left[\frac{(H_m - H)}{2}\right]$$
(7)

In Fig. 1 the resulting magnetization curve is shown. M and H are both normalized to J_Ca . To verify the correctness of our analysis, there are several parameters that can be checked. First of all the normalized magnetization correctly extends from -0.5 to 0.5. Secondly, the slope of the initial magnetization at low field, as shown in Fig. 1, corresponds to the external susceptibility in conditions of perfect screening, χ_0 , numerically calculated as described in [9] for a single strip. In this case $\chi_0 = 9.963$. Finally we can compare the normalized full penetration field, that is the minimum field at which the strip is fully penetrated by the critical current density, with its analytical expression for a single strip:

$$H_{p\ norm}^{analytical} = \frac{b}{\pi a} \left[\frac{2a}{b} \arctan \frac{b}{a} + ln \left(1 + \frac{a^2}{b^2} \right) \right] \quad (8)$$

In our case the analytical value corresponding to b/a = 0.1 is $H_{p norm}^{analytical} = 0.210$ and the numerical value is $H_{p norm}^{numerical} = 0.205$, that is our method is in good agreement with the existing theory.

2) External Susceptibility χ'' : Starting from the reversal magnetization curve, the external susceptibility can be calculated as:

$$\chi'' = \frac{2}{\pi H_m} \int_0^{\pi} M[H(\vartheta)] \sin \vartheta d\vartheta \tag{9}$$

In Fig. 2. we compare our results with the ones found in tabular form in [4], which we consider our numerical reference method for the critical state of a single strip. It shows clearly the excellent agreement between the two results.

III. STRIP WITH IRON SHEATH COVER

In order to investigate the influence of iron sheaths on the critical state of the strip, let us consider the simple configuration shown in Fig. 3, i.e. a single strip with an iron cover on its top and bottom. Here we present the results obtained for the case of



Fig. 3. Basic configuration to understand the influence of iron sheaths on the critical state of the strip.



Fig. 4. Flux lines at the magnetic field corresponding to the χ'' peak for an isolated strip (a) and a strip with iron sheaths (b). The grey zones correspond to the portions filled with the critical current density. Due to symmetry reasons, only half the model is shown.

constant magnetic permeability. Such simplification is plausible for magnetic fields in the mT range, where the very initial part of the B(H) characteristic is involved. In fact, the simulations performed in the environment of the FE code FEMLAB have confirmed that an artificial shift of the nonlinear B(H) curve toward lower fields is necessary to observe a difference with respect to the linear case.

To this configuration, we can apply the minimum magnetic energy variation procedure described in the previous paragraphs. The three terms in (1) have the same meaning as before, but the energy calculations have to be performed in presence of the iron sheaths:

$$E_{field} = \int_{S} \frac{B^2}{2\mu_0} dS \text{ (no currents)}$$
(10)

$$E_{self} = \int_{S} \frac{B^2}{2\mu_0} dS \quad \text{(no applied field)} \tag{11}$$

$$E_{int} = -4 \sum_{\substack{i=1,n_x\\j=1,n_y}} I_{ij} A_{field\ ij} \text{ (currents+applied field) (12)}$$

Due to the presence of the iron, this time we have to use the finite element analysis to calculate all these three terms, and not only for the second one. Nevertheless, to use the same fast algorithm as before, we need to factorize the dependence on the applied field of the first and third term. Taking advantage of the linear behavior of the iron, it can be easily demonstrated that



Fig. 5. Normalized initial magnetization as function of the iron sheath permeability and corresponding to a sheath thickness ti = b = $20 \ \mu m$. The initial magnetizations of the single strip and slab are also shown.

 E_{field} depends on B_a^2 . For what concerns the third term, we can set, with a high grade of accuracy, that:

$$A_{field\ ij} = k_{ij} x_i B_a \tag{13}$$

that is the vector potential in the sub-area ij due to the applied field and in presence of the iron sheaths, can be obtained multiplying the corresponding vector potential of the strip without the iron sheaths by a suitable coefficient k_{ij} , which does not depend on the external field. Once the coefficient matrix k_{ij} has been calculated, the optimization loop is exactly the same as for the single strip without iron.

A. Magnetization Curves

To understand the influence of the iron, let's consider the usual strip, $200 \times 20 \ \mu m^2$, covered with iron sheaths of thickness ti = 20 $\ \mu m$ and let's analyze its behavior by varying the iron relative permeability.

The first important result regards χ_0 , which significantly decreases while increasing the relative permeability. It saturates for $\mu_r \gg 1$ to the value 6.4, corresponding to the 65% of its original value in absence of any iron.

Another important consideration concerns the shape of the current density penetration (Fig. 4). In fact, due to the presence of the iron sheaths, the front of the penetration is always parallel to the short side of the strip, behaving exactly as it would be in a slab, and this starts to be true at low values of the relative permeability ($\mu_r > 100$). Nevertheless, the overall behavior is different: in presence of the iron sheaths, the initial magnetization (see Fig. 5) is well fitted by a polynomial form with the fourth power of the magnetic field, as it is for the isolated strip, whilst for the slab it is proportional to a polynomial form with the second power of the field. Moreover, the field of full penetration increases with the permeability: it ranges from 5.2 mT of the single strip, up to 13.1 mT in presence of iron sheaths with $\mu_{\rm r} > 1$. We can say that the iron acts only partially as a mirror, in the sense that the strip with the iron sheaths behaves as it would be thicker (but not infinite), having higher values of χ_0 and H_p . Finally, it must be said that all the results saturate at $\mu_{\rm r} = 1000$: higher values of the relative permeability do not correspond to distinguished results.



Fig. 6. Normalized initial magnetization of the strip as function of d, the iron sheath distance from the strip, and corresponding to a sheath thickness ti = $20 \ \mu m$ and an iron permeability $\mu r = 1000$.



Fig. 7. Normalized external susceptibility as function of the iron sheath permeability, for the sheath thickness ti = $b = 20 \ \mu m$.

Another important parameter we took into account is the iron sheath thickness. What we found is that there is no significant dependence of the magnetization on this parameter, at least up to values as low as 1 μ m.

The last parameter we analyzed is the distance of the iron sheaths from the strip. The results, in terms of initial magnetization curves, are shown in Fig. 6. The behavior is the one expected: detaching the iron from the strip, the magnetization curves moves from the one of the strip in contact with the iron sheaths, to the one of the isolated strip. Anyway, the effect due to the presence of iron is strongly evident even at relatively large distance, as 10 μ m.

B. External Susceptibility χ''

If we try to apply to the external susceptibility the same reasoning made for the magnetization curves, i.e. that the iron sheaths have the same effect of a thicker strip, we should expect that the χ'' peak would decrease, because the absolute value of external susceptibility is proportional to χ_0 , and move toward higher values of the magnetic field, because the field at which it occurs is nearly proportional to the b/a ratio. We have found the decrease in the peak height, however [see Fig. 7] its position, at least for high values of the relative permeability ($\mu_r > 100$), moves toward lower values of the magnetic field instead of higher fields. This demonstrates that any qualitative analysis should be taken with precaution in explaining the behavior of a strip with iron sheaths. Further simulations will be necessary to find a rigorous explanation of these results.

IV. CONCLUSION

The use of commercial finite element codes allows describing of the magnetic field penetration in iron sheathed superconducting strips exposed to transverse magnetic fields. This description is compatible with the critical state model because the superconducting region shields the field with the proper critical current density distribution. The field penetration in the strip is substantially different when iron plates are coupled to the strip itself: the total effect is the reduction of χ_0 and of the ac losses in the superconductors.

Next step will be the analysis of the parallel field configuration, which represents the real case of a tape in the superconducting magnets.

REFERENCES

- C. P. Bean, "Magnetization of hard superconductors," *Phys. Rev. Letters*, vol. 8, pp. 250–253, Mar. 1962.
- [2] A. Sanchez and C. Navau, "Magnetic properties of finite superconducting cylinder," *Phys. Rev. B*, vol. 64, p. 214 506, 2001.
- [3] E. Pardo, A. Sanchez, and C. Navau, "Magnetic properties of arrays of superconducting strips in a perpendicular field," *Phys. Rev. B*, vol. 67, p. 104 517, 2003.
- [4] E. Pardo, D.-X. Chen, A. Sanchez, and C. Navau, "The transverse critical-state susceptibility of rectangular bars," *Supercond. Sci. Technol.*, vol. 17, pp. 537–544, 2004.
- [5] M. Majoros, B. A. Glowacki, and A. M. Campbell, "Transport ac losses and screening properties of Bi-2223 multifilamentary tapes covered with magnetic materials," *Physica C*, vol. 338, p. 251, 2000.
- [6] —, "Magnetic screening as a possible way to decrease transport ac losses in multifilamentary superconductors—basic theoretical considerations," *Physica C*, vol. 334, p. 129, 2000.
- [7] Y. A. Genenko, A. V. Snezhko, and A. Usoskin, "Finite magnetic shields effect on the transport current in superconducting strips," *Physica C*, vol. 401, p. 236, 2004.
- [8] E. H. Brandt, "Superconductor disks and cylinders in an axial magnetic field: II. Nonlinear and linear ac susceptibilities," *Phys. Rev. B*, vol. 58, p. 6523, 1998.
- [9] P. Fabbricatore, S. Farinon, S. Innocenti, and F. Gömöry, "Magnetic flux shielding in superconducting strip arrays," *Phys. Rev. B*, vol. 61, p. 6413, 2000.
- [10] A. Forkl, "Magnetic-flux distribution in single-crystalline, ceramis and thin-film high-Tc superconductors," *Physica Scripta*, vol. T49A, p. 148, 1993.