

Introduzione alle teorie supersimmetriche

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Lay-out:

1. Why supersymmetry
2. General features of a supersymmetric theory
3. The minimal supersymmetric Standard Model
4. Breaking supersymmetry
5. Present experimental status
6. Conclusions and outlook

In many respects

we are happy with the Standard Model

- a unitary, renormalizable relativistic quantum field theory
- amazingly consistent with data.

The problem of **spontaneous gauge symmetry breaking** (short range of weak interactions) is solved in a simple way by the Higgs mechanism.

This also allows **breaking of the flavour symmetry** in a phenomenologically consistent way (flavour breaking confined in the charged-current interaction sector).

Achieving the same results without the Higgs mechanism is extremely difficult.

At the same time,

we are unhappy with the Standard Model.

Many unsatisfactory aspects. Among others:

- What is the origin of flavour symmetry breaking?
- Is there a grand unification?
- Where is gravitation?

The last two points raise further problems:

- **Hierarchy**
- **Naturalness**

Two facts:

- The masses of all known particles (including the minimal Standard Model Higgs boson) are not far from the weak scale, ~ 200 GeV.
- Much larger energy scales become relevant at some point

A first question arises: why is the weak scale so much smaller than the Planck scale, $M_P \sim 10^{19}$ GeV, or the unification scale, $M_{GUT} \sim 10^{16}$ GeV?

This is usually referred to as the **hierarchy problem**.

Even worse: The Higgs mass (masses of scalar particles, in general) is strongly sensitive to any large energy scale unless a *fine tuning* of parameters is performed.

This is the so-called **naturalness problem**.

There is a very simple reason for this: scalar masses are not **naturally small**, in the sense that no symmetry is recovered when they are let go to zero.

Fermion and vector boson masses *are* naturally small: radiative corrections are proportional to the masses themselves.

Naturalness and fine tuning in a simple example

Consider a theory of two real scalars fields:

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \frac{1}{2} \partial^\mu \Phi \partial_\mu \Phi - V(\phi, \Phi)$$

with

$$V(\phi, \Phi) = \frac{m^2}{2} \phi^2 + \frac{M^2}{2} \Phi^2 + \frac{\lambda}{4!} \phi^4 + \frac{\sigma}{4!} \Phi^4 + \frac{\delta}{4} \phi^2 \Phi^2$$

Assume λ, σ, δ are all positive, small and comparable in magnitude, and assume $M^2 \gg m^2 > 0$.

Is the mass hierarchy $m^2 \ll M^2$ conserved at the quantum level?

Compute one-loop radiative corrections to m^2 (e.g. by taking the second derivatives of the effective potential at the minimum $\phi = \Phi = 0$):

$$m_{\text{one loop}}^2 = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2} \left(\log \frac{m^2}{\mu^2} - 1 \right) + \frac{\delta M^2}{32\pi^2} \left(\log \frac{M^2}{\mu^2} - 1 \right)$$
$$\mu^2 \frac{\partial m^2}{\partial \mu^2} = \frac{1}{32\pi^2} (\lambda m^2 + \delta M^2)$$

Corrections proportional to M^2 appear at one loop. One can choose $\mu^2 \sim M^2$ in order to get rid of them, but they reappear through the running of $m^2(\mu^2)$.

The mass hierarchy is preserved only if the parameters are such that

$$\lambda m^2 \sim \delta M^2 \rightarrow \frac{\delta}{\lambda} \sim \frac{m^2}{M^2}$$

This is what we usually call a *fine tuning* of the parameters.

The same thing happens if $m^2 < 0$, $M^2 \gg |m^2| > 0$. In this case the tree-level potential has a minimum at

$$\Phi = 0, \quad \phi^2 = -6m^2/\lambda \equiv v^2$$

and the symmetry $\phi \rightarrow -\phi$ is **spontaneously broken**. The degrees of freedom in this case are Φ and $\phi' \equiv \phi - v$, with

$$m_{\Phi}^2 = M^2 \quad m_{\phi'}^2 = -2m^2 = \lambda v^2/3$$

At one loop, the minimization condition $m^2 + \lambda v^2/6 = 0$ is replaced by

$$m^2 + \frac{\lambda v^2}{6} = -\frac{\lambda}{32\pi^2} \left(m^2 + \frac{\lambda v^2}{2} \right) \left(\log \frac{m^2 + \frac{\lambda v^2}{2}}{\mu^2} - 1 \right) - \frac{\delta}{32\pi^2} \left(M^2 + \frac{\delta v^2}{2} \right) \left(\log \frac{M^2 + \frac{\delta v^2}{2}}{\mu^2} - 1 \right)$$

Following the same procedure as in the unbroken case one finds

$$m_{\phi'}^2 = \frac{\lambda v^2}{3} + \frac{v^2}{32\pi^2} \left[\lambda^2 \log \frac{m^2 + \frac{\lambda}{2} v^2}{\mu^2} + \delta^2 \log \frac{M^2 + \frac{\delta}{2} v^2}{\mu^2} \right]$$

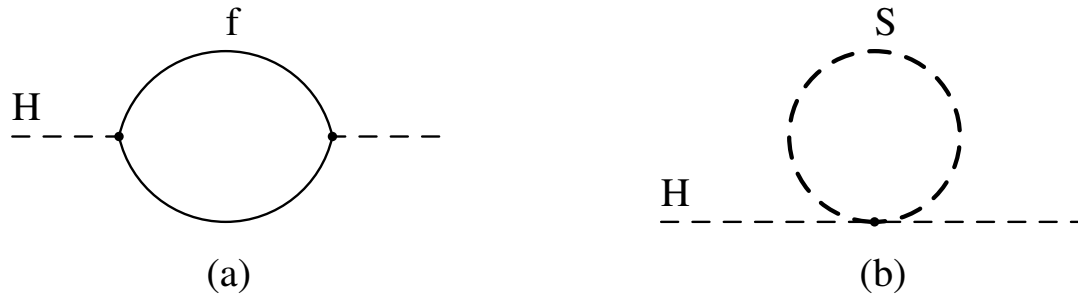
with $v \sim M$ without a suitable tuning of the parameters.

Naturalness: a closer look

The scalar potential in the Standard Model:

$$V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4$$

One-loop corrections to m^2 due to fermionic (a) or bosonic (b) degrees of freedom:



$$(\Delta m^2)_a = \frac{|\lambda_f|^2}{16\pi^2} \left[-2\Lambda^2 + 6m_f^2 \log \frac{\Lambda}{m_f} + \dots \right]$$

$$(\Delta m^2)_b = \frac{\lambda_S}{16\pi^2} \left[\Lambda^2 - 2m_S^2 \log \frac{\Lambda}{m_S} + \dots \right]$$

Here Λ is an ultraviolet cut-off, to be identified with the energy scale at which the SM is no longer reliable, and the dots stand for terms that do not grow with Λ .

In **dimensional regularization** the Λ^2 term would be absent, but contributions proportional to m_f^2, m_S^2 would still be there.

Even if the heavy degrees of freedom are not directly coupled to the SM Higgs, it can be shown that similar contributions arise at higher orders.

In the absence of very special cancellations, the Higgs boson becomes as heavy as the heaviest degrees of freedom.

A **symmetry** that relates fermions to bosons would do the job, at least at one loop. Suppose there are two scalars for each fermion:

$$(\Delta m^2)_{a+b} = \frac{\lambda_S - |\lambda_f|^2}{8\pi^2} \Lambda^2 + \dots$$

For suitable values of the couplings the quadratic divergence disappears.

No surprise: with bosons and fermions in the same multiplet, scalar masses are protected by the same (chiral) symmetry that protects fermion masses from large radiative corrections.

Clearly, more restrictions will be needed in order to guarantee that the cancellation takes place at all orders.

Such a symmetry is called a **supersymmetry**:

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \quad Q|\text{fermion}\rangle = |\text{boson}\rangle$$

The symmetry generator Q (and its hermitian conjugate Q^\dagger) carry spin $1/2$: it is a **space-time symmetry**.

The form of possible supersymmetry algebras is strongly constrained on the basis of very general theorems in field theory. For example, it is impossible with ordinary symmetry generators (elements of a commutator algebra).

There is essentially one possibility:

$$\{Q, Q^\dagger\} = P^\mu$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$$

(more on this later). Further specifications:

- Q, Q^\dagger transform as spinors under the Lorentz group
- Q, Q^\dagger commute with gauge symmetry generators.

In principle, we may have more than one Q : $Q^i, i = 1, \dots, N$
(extended supersymmetry).

Historical remark: symmetries that relate particles with different spin first studied in the context of approximate symmetries of hadrons. Non relativistic quark models have an approximate $SU(6)$ symmetry (3 quark flavors with spin 1/2), which is observed hadron spectrum. It relates hadrons with the same flavor content, but different spin (e.g. $K \leftrightarrow K^*$). Consequence of approximate spin and flavor independence of quark-quark forces.

The **Coleman-Mandula** theorem tells us that this property cannot be possessed by a relativistic theory: with some reasonable assumptions, the most general Lie algebra of symmetry operators that commute with the S matrix consists of **Poincaré generators** P_μ and $J_{\mu\nu}$, **plus ordinary internal symmetry generators** that act on one-particle states with matrices that are diagonal in, and independent of, momentum and spin. Crucial point: the Poincaré group is non compact, it has no non-trivial unitary representations.

Rules out $SU(6)$, but also Lie algebras of supersymmetry generators.

A few basic properties of a supersymmetric theory can already be recognized:

- particles in the same supersymmetric multiplet (which we will call a **supermultiplet**) have equal masses and equal gauge transformation properties (electric charge, weak isospin and color)
- within the same supermultiplet, there is an equal number of bosonic and fermionic degrees of freedom (a proof on the next slide)

A proof (taken from S. Martin):

$$\left. \begin{aligned} (-1)^{2s}|\text{boson}\rangle &= +|\text{boson}\rangle \\ (-1)^{2s}|\text{fermion}\rangle &= -|\text{fermion}\rangle \end{aligned} \right\} \Rightarrow \{(-1)^{2s}, Q\} = \{(-1)^{2s}, Q^\dagger\} = 0$$

Consider the subspace of states $|i\rangle$ within a supermultiplet with the same eigenvalue p^μ of the four-momentum operator P^μ . $\sum_i |i\rangle\langle i| = 1$ within this subspace of states.

$$\begin{aligned} \sum_i \langle i|(-1)^{2s}P^\mu|i\rangle &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \langle i|(-1)^{2s}Q^\dagger Q|i\rangle \\ &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \sum_j \langle i|(-1)^{2s}Q^\dagger|j\rangle\langle j|Q|i\rangle \\ &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_j \langle j|Q(-1)^{2s}Q^\dagger|j\rangle \\ &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle - \sum_j \langle j|(-1)^{2s}QQ^\dagger|j\rangle \\ &= 0. \end{aligned}$$

Fermionic fields: a reminder

We use the Weyl representation of the Dirac matrices:

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix} \quad \sigma^\mu = (\sigma^0, \vec{\sigma}) \quad \bar{\sigma}^\mu = (\sigma^0, -\vec{\sigma}) \quad \gamma_5 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $\vec{\sigma}$ are the three Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The familiar lagrangian for a free, massive Dirac spinor is

$$\mathcal{L} = \bar{\psi} (i\partial\!\!\!/ - m) \psi \quad \rightarrow \quad (i\partial\!\!\!/ - m)\psi = 0$$

Four-component Dirac spinors realize a **reducible** representation of the Lorentz group:

$$\psi = \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix} \quad \psi_L = \begin{pmatrix} \xi_L \\ 0 \end{pmatrix} = \frac{1}{2}(1 - \gamma_5)\psi \quad \psi_R = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix} = \frac{1}{2}(1 + \gamma_5)\psi$$

the two-component spinors ξ_L, ξ_R transform independently under Lorentz transformations.

In terms of ξ_L, ξ_R we have

$$\mathcal{L} = i\xi_L^\dagger \bar{\sigma}^\mu \partial_\mu \xi_L + i\xi_R^\dagger \sigma^\mu \partial_\mu \xi_R - m(\xi_L^\dagger \xi_R + \xi_R^\dagger \xi_L)$$

It can be shown that the transformation law of the two-component spinor ξ_R under Lorentz transformations are the same as those of $\epsilon\xi_L^*$, where ϵ is the antisymmetric matrix

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

It follows that any Dirac spinor may always be written in terms of two left-handed Weyl spinors ξ, χ as

$$\psi = \begin{pmatrix} \xi \\ -\epsilon\chi^* \end{pmatrix}$$

(the minus sign on the second Weyl spinor is conventional).

The free lagrangian for a massive Dirac spinor is

$$\begin{aligned}\mathcal{L} = \bar{\psi} (i\partial\!\!\!/ - m) \psi &= i\chi^T \epsilon^T \sigma^\mu \epsilon \partial_\mu \chi^* + i\xi^\dagger \bar{\sigma}^\mu \partial_\mu \xi - m(\chi^T \epsilon \xi - \xi^\dagger \epsilon \chi^*) \\ &= i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi + i\xi^\dagger \bar{\sigma}^\mu \partial_\mu \xi - m(\chi^T \epsilon \xi - \xi^\dagger \epsilon \chi^*)\end{aligned}$$

Note that

$$\chi^T \epsilon \xi = \xi^T \epsilon \chi$$

because of the anticommutation properties of fermion fields, and that

$$(\chi^T \epsilon \xi)^\dagger = \xi^\dagger \epsilon^T \chi^* = -\xi^\dagger \epsilon \chi^*$$

The shorthand notation

$$\chi\xi = \xi\chi = \chi^T \epsilon \xi \quad \xi^\dagger \chi^\dagger = \chi^\dagger \xi^\dagger = -\xi^\dagger \epsilon \chi^*$$

is often used.

Any theory involving spin-1/2 particles may be written as a collection of left-handed Weyl spinors ψ_i .

Left-handed fermions in the Standard Model:

$$u_i, d_i, e_i, \nu_i$$
$$\bar{u}_i, \bar{d}_i, \bar{e}_i$$

where $i = 1, 2, 3$ is a generation index. For example, the Dirac spinor for the up quark is written in terms of two left-handed Weyl spinors u and \bar{u} as

$$\psi_u \equiv \begin{pmatrix} u_L \\ u_R \end{pmatrix} = \begin{pmatrix} u \\ -\epsilon \bar{u}^* \end{pmatrix}$$

(NB: the bar on \bar{u} here has no special meaning, it is just part of the name)

The Weyl notation is convenient, since left- and right-handed components of Dirac fermions, which behave differently in weak interactions, are treated separately.

The simplest (irreducible) representations of supersymmetry contain Weyl fermions.

Massless supermultiplets of $N = 1$ supersymmetry are formed with a state with helicity λ_{\max} and a state with helicity $\lambda_{\max} - 1/2$.

The simplest supermultiplet candidate: $\lambda_{\max} = 1/2, \lambda_{\min} = 0$

a Weyl fermion ψ and two real (or one complex) scalars ϕ_1, ϕ_2 .

This is called a **chiral or matter supermultiplet**.

Next-to-simplest possibility: $\lambda_{\max} = 1, \lambda_{\min} = 1/2$

a massless gauge vector boson A_{μ}^a and a Weyl fermion, called a *gaugino*, λ^a

This is called a **vector supermultiplet**.

Gauge bosons belong to the adjoint representation of the gauge group, so the same is true for their supersymmetric partners.

Thus,

gauginos are not chiral fermions

because the adjoint representation is equivalent to its conjugate.

The pair A_{μ}^a, λ^a is called a **gauge supermultiplet**.

It turns out that any other representation of the supersymmetry algebra can be reduced to a combination of chiral and gauge supermultiplets, at least if there is only one supersymmetry generator Q ($N = 1$ supersymmetry).

We will not consider extended ($N > 1$) supersymmetries, because they cannot accommodate chiral fermions.

A word on extended supersymmetries

If λ_{\max} is the maximum helicity in a supermultiplet, then $\lambda_{\min} = \lambda_{\max} - N/2$. Hence, in order to exclude states with $|\lambda| > 2$, we cannot allow $N > 8$.

$N = 8$ and $N = 7$ supergravity theories are entirely equivalent (same particle content); $N \leq 6$ are all different from each other.

Global supersymmetry (no spin 2 and 3/2 particles) is possible with $N \leq 4$. $N = 4$ and $N = 3$ global supersymmetries equivalent.

Fermions in extended supersymmetries are either partners of gauge bosons, or have both helicity components in the same supermultiplet; impossible to accommodate chiral fermions.

The particle content of a supersymmetric SM

- No supermultiplet can be formed out of standard particles (e.g. γ, ν).
- Matter fermions must belong to chiral supermultiplets, because left and right fermions transform differently under the weak gauge group.
- Gauge bosons must go into gauge supermultiplets.
- At least two Higgs doublets, with their fermionic partners: cancellation of the axial anomaly.

spin 0	spin 1/2	spin 1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
\tilde{u}_L, \tilde{d}_L	u_L, d_L		3	2	$+\frac{1}{3}$
\tilde{u}_R	u_R		3	1	$+\frac{4}{3}$
\tilde{d}_R	d_R		3	1	$-\frac{2}{3}$
$\tilde{\nu}, \tilde{e}_L$	ν, e_L		1	2	-1
\tilde{e}_R	e_R		1	1	-2
H_u^+, H_u^0	$\tilde{h}_u^+, \tilde{h}_u^0$		1	2	$+1$
H_d^0, H_d^-	$\tilde{h}_d^0, \tilde{h}_d^-$		1	2	-1
	\tilde{g}	g	8	1	0
	$\tilde{w}^\pm, \tilde{w}^0$	W^\pm, W^0	1	3	0
	\tilde{b}^0	B^0	1	1	0

Supersymmetric partners of standard particles (e.g. a scalar electron) with the same masses would have been detected in experiments. Since none of them has been observed so far, we must conclude that

supersymmetry must be broken

in a realistic theory. Recall our formula for scalar mass corrections:

$$(\Delta m^2)_{a+b} = \frac{\lambda_S - |\lambda_f|^2}{8\pi^2} \Lambda^2 + \dots$$

Supersymmetry forces $\lambda_S = |\lambda_f|^2$ to all orders in perturbation theory, so that quadratic divergences are systematically cancelled. This feature must be preserved in the broken theory:

$$\mathcal{L} = \mathcal{L}_{\text{supersymmetric}} + \mathcal{L}_{\text{soft}}$$

where $\mathcal{L}_{\text{soft}}$ only contains mass terms and couplings with positive mass dimension.

Supersymmetric partners of ordinary particles are so heavy that they have escaped detection so far.

Is there a reason for that?

All ordinary particles, including the W, Z bosons and the top quark, would be massless in the absence of spontaneous breaking of the electroweak gauge symmetry.

The contrary is true for their partners: scalar masses are always allowed by gauge symmetries, and gaugino can be massive because they belong to a real representation of the gauge group.

The only exception is the Higgs boson.

A rough estimate of superparticle masses

Call m_{soft} the largest mass scale present in $\mathcal{L}_{\text{soft}}$. Corrections to m^2 arising from $\mathcal{L}_{\text{soft}}$ must vanish as $m_{\text{soft}} \rightarrow 0$, so they cannot grow as Λ^2 (corrections proportional to $m_{\text{soft}}\Lambda$ are also forbidden: UV divergences are either quadratic or logarithmic). Therefore

$$\Delta m^2 \sim \lambda m_{\text{soft}}^2 \log \frac{\Lambda}{m_{\text{soft}}}$$

(λ a generic coupling in $\mathcal{L}_{\text{soft}}$). It follows ($\lambda \sim 1, \Lambda \sim M_P$) that

$$m_{\text{soft}} \lesssim 1 \text{ TeV}$$

in order to get the correct value of the Higgs v.e.v. Furthermore,

$$m_F^2 - m_B^2 \sim m_{\text{soft}}^2$$

within a supermultiplet: superpartners cannot be too heavy.

Building a supersymmetric theory

A simple example: one left-handed Weyl fermion

$$\psi \equiv P_L \psi \quad P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$$

and one complex scalar ϕ , no masses, no interactions:

$$\begin{aligned} \mathcal{L}_{\text{chiral}} &= \mathcal{L}_f + \mathcal{L}_s \\ \mathcal{L}_f &= \bar{\psi} i \not{\partial} \psi \\ \mathcal{L}_s &= \partial^\mu \phi^* \partial_\mu \phi \end{aligned}$$

Is it possible to define a set of transformation rules on the fields, such that scalars are transformed into fermions, under which $\mathcal{L}_{\text{chiral}}$ remains unchanged?

Let us consider the scalar field first, and tentatively define

$$\delta\phi = \bar{\eta}\psi \quad \delta\phi^* = \bar{\psi}\eta$$

where η is a constant, infinitesimal, anticommuting spinor. Then

$$\delta\mathcal{L}_s = (\partial^\mu\bar{\psi})(\partial_\mu\phi)\eta + \bar{\eta}(\partial^\mu\phi^*)(\partial_\mu\psi)$$

How does ψ transform? $\delta\psi$ must be left-handed, linear in η , and must contain one derivative. Only one possibility (up to a proportionality factor):

$$\delta\psi = -i(\partial_\mu\phi)\gamma^\mu\eta \quad \delta\bar{\psi} = i\bar{\eta}\gamma^\mu\partial_\mu\phi^*$$

which tells us that η is right-handed, $P_R\eta = \eta$, and gives

$$\delta\mathcal{L}_f = -\bar{\eta}(\partial_\mu\phi^*)\gamma^\mu\gamma^\nu\partial_\nu\psi + \bar{\psi}\gamma^\nu\gamma^\mu\partial_\nu(\partial_\mu\phi)\eta$$

Using $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and $\partial_\mu \partial_\nu = \partial_\nu \partial_\mu$ we get

$$\begin{aligned} \delta \mathcal{L}_f &= -\bar{\eta} (\partial_\mu \phi^*) (\partial^\mu \psi) - (\partial_\mu \bar{\psi}) (\partial^\mu \phi) \eta \\ &\quad - \bar{\eta} \partial_\nu [(\partial_\mu \phi^*) \gamma^\mu \gamma^\nu \psi] + \bar{\eta} \partial_\mu [(\partial^\mu \phi^*) \psi] + \partial_\mu (\bar{\psi} \partial^\mu \phi) \eta \end{aligned}$$

The first row cancels against $\delta \mathcal{L}_s$, and the rest is a total derivative.

So,

$$\delta \int d^4x \mathcal{L}_{\text{chiral}} = 0$$

Not yet enough to declare that $\mathcal{L}_{\text{chiral}}$ is supersymmetric: we must check that the commutator of two transformations is a symmetry of the theory. For the scalar fields we find

$$(\delta_{\eta_1} \delta_{\eta_2} - \delta_{\eta_2} \delta_{\eta_1}) \phi = -(\bar{\eta}_2 \gamma^\mu \eta_1 - \bar{\eta}_1 \gamma^\mu \eta_2) i \partial_\mu \phi$$

Good news: $i \partial_\mu$ is nothing but the four-momentum operator P_μ , the generator of space-time translations, a symmetry of space-time.

Now consider the fermion fields. After some tedious algebra,* we get

$$\begin{aligned}(\delta_{\eta_1} \delta_{\eta_2} - \delta_{\eta_2} \delta_{\eta_1})\psi &= -(\bar{\eta}_2 \gamma^\mu \eta_1 - \bar{\eta}_1 \gamma^\mu \eta_2) i\partial_\mu \psi \\ &\quad + \frac{1}{2}(\bar{\eta}_2 \gamma^\mu \eta_1 - \bar{\eta}_1 \gamma^\mu \eta_2) i\gamma_\mu \not{\partial} \psi\end{aligned}$$

The first term is similar to what we got for ϕ , while the second one vanishes on the mass shell,

$$\not{\partial} \psi = 0.$$

OK, but it might be useful to close the algebra even off shell.

* For those who wish to try: you need the Fierz identities (and some patience).

This can be done. Let us introduce an **auxiliary scalar field** F , with the lagrangian

$$\mathcal{L}_a = F^* F$$

(note the unusual dimension: m^2 instead of m). It is harmless: the equation of motion is $F = 0$. Now assume

$$\delta F = -i \bar{\eta} \gamma^\mu \partial_\mu \psi \quad \delta F^* = i \partial_\mu \bar{\psi} \gamma^\mu \eta$$

and modify the fermion transformation rules as follows:

$$\delta \psi = P_L [-i \partial_\mu \phi \gamma^\mu \eta + F \eta] \quad \delta \bar{\psi} = [i \bar{\eta} \gamma^\mu \partial_\mu \phi^* + \bar{\eta} F^*] P_R$$

(which amounts to nothing, because the auxiliary field vanishes on shell).

Things start getting complicated:

1. We can no longer assume that $P_R\eta = \eta$, we must promote it to a full four-component spinor. In order not to increase the number of independent parameters, we assume it is a **Majorana** spinor:

$$\eta = \begin{pmatrix} \eta_L \\ \eta_R \end{pmatrix} \quad \eta_L = \epsilon \eta_R^*$$

2. We now have to write explicitly the projector P_L in front of $\delta\psi$, in order to keep ψ left-handed.

Nevertheless, let us press on ...

... It is immediate to check that

$$\mathcal{L}_{\text{chiral}} = \mathcal{L}_s + \mathcal{L}_f + \mathcal{L}_a$$

is **our first supersymmetric lagrangian** (not very exciting: it's a non-interacting theory), invariant under the transformation

$$\delta\phi = \bar{\eta}\psi$$

$$\delta\phi^* = \bar{\psi}\eta$$

$$\delta\psi = P_L [-i\partial_\mu\phi\gamma^\mu\eta + F\eta]$$

$$\delta\bar{\psi} = [i\bar{\eta}\gamma^\mu\partial_\mu\phi^* + \bar{\eta}F^*] P_R$$

$$\delta F = -i\bar{\eta}\gamma^\mu\partial_\mu\psi$$

$$\delta F^* = i\partial_\mu\bar{\psi}\gamma^\mu\eta$$

with

$$(\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1})X = -(\bar{\eta}_2\gamma^\mu P_R\eta_1 - \bar{\eta}_1\gamma^\mu P_R\eta_2)i\partial_\mu X$$

for $X = \phi, \psi, F$, both on and off the mass shell. This structure can be replicated for an arbitrary number of chiral supermultiplets.

The same results take a much simpler form when recast in Weyl language. Define Weyl spinors through

$$\psi \rightarrow \begin{pmatrix} \psi \\ 0 \end{pmatrix} \quad \eta \rightarrow \begin{pmatrix} \eta \\ -\epsilon\eta^* \end{pmatrix}$$

Recalling the definitions of γ matrices, we get

$$\delta\phi = \eta\psi$$

$$\delta\psi = i(\sigma^\mu\epsilon\eta^*)\partial_\mu\phi + F\eta$$

$$\delta F = -i\eta^\dagger\bar{\sigma}^\mu\partial_\mu\psi$$

and

$$(\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1})X = -(\eta_1^\dagger\bar{\sigma}^\mu\eta_2 - \eta_2^\dagger\bar{\sigma}^\mu\eta_1)i\partial_\mu X$$

Why do we need auxiliary fields? Count **fermionic** and **bosonic** degrees of freedom:

- on shell: ψ has two helicity states (it is a Weyl fermion): $n_f = 2$
 ϕ is a complex scalar: $n_b = 2$
- off shell: ψ is a complex two-component spinor: $n_f = 4$
 ϕ is a complex scalar: $n_b = 2$
 F is a complex scalar: $n_b = 2$

Without the auxiliary fields, the counting rule $n_f = n_b$ would be violated off the mass shell.

Ordinary symmetries: a reminder

Consider a lagrangian density

$$\mathcal{L}(\phi, \partial\phi)$$

functions of a set of fields $\phi_i, i = 1, \dots, n$. The transformation

$$\delta\phi_i = i\eta^A T_{ij}^A \phi_j; \quad (T^A)^\dagger = T^A; \quad [T^A, T^B] = if^{ABC} T^C$$

is a symmetry transformation if

$$\delta\mathcal{L} = \eta^A \partial^\mu K_\mu^A \Rightarrow \delta S[\phi] = \delta \int d^4x \mathcal{L} = 0$$

As a consequence, a set of *conserved currents* exists:

$$\partial^\mu J_\mu^A = 0; \quad J_\mu^A = T_{ij}^A \phi_j \frac{\partial\mathcal{L}}{\partial\partial^\mu\phi_i} - K_\mu^A$$

$$Q^A(t) = \int d^3x J_0^A(t, \mathbf{x}); \quad \frac{d}{dt} Q^A(t) = 0; \quad [Q^A, Q^B] = if^{ABC} Q^C$$

The same procedure can be applied to our supersymmetry: a conserved (spinor) current J_μ and a conserved charge

$$Q = \sqrt{2} \int d^3x J_0(x)$$

can be built by tedious but straightforward calculations

The supersymmetry charge Q (a Weyl spinor!) obey the anticommutator algebra

$$\{Q, Q^\dagger\} = 2\sigma^\mu P_\mu; \quad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

as anticipated.

Summary:

$$\mathcal{L}_{\text{chiral}} = \psi^\dagger i \bar{\sigma}^\mu \partial_\mu \psi + \partial^\mu \phi^* \partial_\mu \phi + F^* F$$

(ψ left-handed fermion, ϕ and F complex scalars) is invariant under the supersymmetric transformation

$$\delta\phi = \eta\psi$$

$$\delta\psi = i(\sigma^\mu \epsilon \eta^*) \partial_\mu \phi + F\eta$$

$$\delta F = -i \eta^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

The field F is auxiliary: $F = 0$ on shell. For all fields $X = \psi, \phi, F$

$$(\delta_{\eta_1} \delta_{\eta_2} - \delta_{\eta_2} \delta_{\eta_1})X = -(\eta_1^\dagger \bar{\sigma}^\mu \eta_2 - \eta_2^\dagger \bar{\sigma}^\mu \eta_1) i \partial_\mu X$$

Masses and non-gauge interactions

The most general renormalizable supersymmetric term has the form (in Weyl notation)

$$\mathcal{L}_{\text{ng}} = -\frac{1}{2} W_{ij} \psi_i \psi_j + W_i F_i + \text{h.c.}$$

where

$$W_i = \frac{\partial W}{\partial \phi_i} \quad W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

and W is the **superpotential**:

$$W = \frac{1}{2} M_{ij} \phi_i \phi_j + \frac{1}{6} y_{ijk} \phi_i \phi_j \phi_k,$$

The constants M_{ij} and y_{ijk} must be totally symmetric in their indices.

A proof

Let us write in general

$$\mathcal{L}_{\text{ng}} = -\frac{1}{2}W_{ij}\psi_i\psi_j + W_i F_i + \text{h.c.}$$

where W_{ij} and W_i are some functions of the bosonic fields with dimensions of (mass) and (mass)² respectively. W_{ij} is symmetric under $i \leftrightarrow j$.

No term which is a function of the scalar fields ϕ_i, ϕ_i^* only can be included in \mathcal{L}_{ng} : under a supersymmetry transformation it would go into another function of the scalar fields only, multiplied by $\eta\psi_i$ or $\eta^\dagger\psi^{\dagger i}$, and with no spacetime derivatives or F_i, F_i^* fields: nothing of this form can possibly be cancelled by the supersymmetry transformation of any other term in the lagrangian.

Renormalizability: each term has field content with mass dimension ≤ 4 . Hence

- W_{ij} and W_i cannot be functions of the fermionic or auxiliary fields
- W_i is a quadratic polynomial, and W_{ij} is linear, in the fields ϕ_i and ϕ_i^* .

Now require supersymmetry invariance of \mathcal{L}_{ng} . We do it in steps:

1) Part which contains four spinors:

$$[\delta\mathcal{L}_{\text{ng}}]_{4\text{-spinor}} = -\frac{1}{2} \frac{\partial W_{ij}}{\partial\phi_k} (\eta\psi_k)(\psi_i\psi_j) - \frac{1}{2} \frac{\partial W_{ij}}{\partial\phi_k^*} (\eta^\dagger\psi_k^\dagger)(\psi_i\psi_j) + \text{h.c.}$$

The first term cannot cancel against any other. It vanishes if and only if $\partial W_{ij}/\partial\phi_k$ totally symmetric in i, j, k , because of the Fierz identity:

$$(\eta\psi_i)(\psi_j\psi_k) + (\eta\psi_j)(\psi_k\psi_i) + (\eta\psi_k)(\psi_i\psi_j) = 0.$$

No such identity for the second term. Hence, supersymmetry requires

$$\frac{\partial W_{ij}}{\partial\phi_k^*} = 0$$

An important feature: the superpotential W must be an **analytic function** of the scalar fields. It is a function of ϕ , not of ϕ and ϕ^* .

A complex function $f(z, z^*) = u(x, y) + iv(x, y)$ of a complex variable $z = x + iy$ is analytic if

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

This implies

$$\begin{aligned} \frac{\partial f}{\partial z^*} &= \frac{\partial(u + iv)}{\partial x} \frac{\partial x}{\partial z^*} + \frac{\partial(u + iv)}{\partial y} \frac{\partial y}{\partial z^*} \\ &= \frac{\partial(u + iv)}{\partial x} + i \frac{\partial(u + iv)}{\partial y} \\ &= \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) + i \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) = 0 \end{aligned}$$

It follows that

$$W_{ij} = M_{ij} + y_{ijk}\phi_k$$

with M_{ij} and y_{ijk} totally symmetric. We define the **superpotential**

$$W = \frac{1}{2}M_{ij}\phi_i\phi_j + \frac{1}{6}y_{ijk}\phi_i\phi_j\phi_k$$

so that

$$W_{ij} = \frac{\partial}{\partial\phi_i} \frac{\partial}{\partial\phi_j} W$$

M_{ij} fermion mass matrix, y_{ijk} Yukawa couplings.

2) Parts of $\delta\mathcal{L}_{\text{ng}}$ which contain a spacetime derivative:

$$\begin{aligned} [\delta\mathcal{L}_{\text{ng}}]_{\partial} &= -iW_{ij} \psi_i \partial_{\mu} \phi_j \sigma^{\mu} \epsilon \eta^* - iW_i \eta^{\dagger} \bar{\sigma}^{\mu} \partial_{\mu} \psi_i + \text{h.c.} \\ &= -i (W_{ij} \psi_i \partial_{\mu} \phi_j + W_i \partial_{\mu} \psi_i) \sigma^{\mu} \epsilon \eta^* + \text{h.c.} \end{aligned}$$

Using

$$W_{ij} \partial_{\mu} \phi_j = \partial_{\mu} \frac{\partial W}{\partial \phi_i}$$

this becomes

$$[\delta\mathcal{L}_{\text{ng}}]_{\partial} = -i \left(\psi_i \partial_{\mu} \frac{\delta W}{\delta \phi_i} + W_i \partial_{\mu} \psi_i \right) \sigma^{\mu} \epsilon \eta^* + \text{h.c.}$$

which is a total derivative provided

$$W_i = \frac{\partial W}{\partial \phi_i} = M_{ij} \phi_j + \frac{1}{2} y_{ijk} \phi_j \phi_k$$

The remaining terms in $\delta\mathcal{L}_{\text{ng}}$ cancel, provided W_i and W_{ij} are the first and second derivatives of the same function W .

The field equations for the auxiliary fields F_i are now

$$F_i^* = -W_i = -M_{ij} \phi_j - \frac{1}{2} y_{ijk} \phi_j \phi_k$$

and therefore

$$\begin{aligned} \mathcal{L}_{\text{ng}} + \mathcal{L}_a &= -\frac{1}{2} W_{ij} \psi_i \psi_j - \frac{1}{2} W_{ij}^* \psi_i^\dagger \psi_j^\dagger + W_i F_i + W_i^* F_i^* + F_i F_i^* \\ &= -\frac{1}{2} W_{ij} \psi_i \psi_j - \frac{1}{2} W_{ij}^* \psi_i^\dagger \psi_j^\dagger - W_i W_i^* \end{aligned}$$

The scalar potential is entirely determined by the superpotential:

$$\begin{aligned} V(\phi) &= W_i W_i^* = F_i F_i^* \\ &= M_{ij} M_{il}^* \phi_j \phi_l^* \quad \text{scalar masses} \\ &\quad + \frac{1}{2} M_{ij} y_{ilm}^* \phi_j \phi_l^* \phi_m^* + \frac{1}{2} M_{il}^* y_{ijk} \phi_j \phi_k \phi_l^* \quad \text{trilinear couplings} \\ &\quad + \frac{1}{4} y_{ijk} y_{ilm}^* \phi_j \phi_k \phi_l^* \phi_m^* \quad \text{quartic couplings} \end{aligned}$$

The remaining terms contain mass terms for the fermions and Yukawa couplings:

$$\begin{aligned} -\frac{1}{2} W_{ij} \psi_i \psi_j - \frac{1}{2} W_{ij}^* \psi_i^\dagger \psi_j^\dagger &= -\frac{1}{2} M_{ij} \psi_i \psi_j - \frac{1}{2} M_{ij}^* \psi_i^\dagger \psi_j^\dagger \\ &\quad - \frac{1}{2} y_{ijk} \phi_i \psi_j \psi_k - \frac{1}{2} y_{ijk}^* \phi_i^* \psi_j^\dagger \psi_k^\dagger \end{aligned}$$

Many interaction terms, but couplings related by supersymmetry.

A simple example: just one supermultiplet ϕ, ψ .

$$\begin{aligned}\mathcal{L}_{\text{ng}} + \mathcal{L}_a &= |M|^2 |\phi|^2 + \frac{1}{2} M y^* \phi \phi^* \phi^* + \frac{1}{2} M^* y \phi \phi \phi^* + \frac{1}{4} |y|^2 |\phi|^4 \\ &\quad - \frac{1}{2} M \psi \psi - \frac{1}{2} M^* \psi^\dagger \psi^\dagger \\ &\quad - \frac{1}{2} y \phi \psi \psi - \frac{1}{2} y^* \phi^* \psi^\dagger \psi^\dagger\end{aligned}$$

Some comments:

- The scalar potential is **positive semi-definite**: (almost) no problems with the stability of the ground state.
- The quartic scalar self-coupling and the fermion-fermion-scalar Yukawa coupling are related: $\lambda_S = |\lambda_f|^2$.
- It is easy to check that scalars and fermions have the same squared mass matrix

$$(M^2)_{ij} = M_{ik}M_{kj}^*$$

which is symmetric and can be diagonalized.

- The analyticity of W requires the introduction of two Higgs doublets in the supersymmetric version of the Standard Model.

In summary, we have the following

Recipe

for non-gauge interaction terms of a set of chiral supermultiplets

ϕ_i, ψ_i, F_i :

1. Give a superpotential

$$W = \frac{1}{2} M_{ij} \phi_i \phi_j + \frac{1}{6} y_{ijk} \phi_i \phi_j \phi_k,$$

with M_{ij} and y_{ijk} completely symmetric.

2. Build the interaction lagrangian as

$$\mathcal{L}_{\text{ng}} = -\frac{1}{2} W_{ij} \psi_i \psi_j + W_i F_i + \text{h.c.}$$

where

$$W_i = \frac{\partial W}{\partial \phi_i} \quad W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

Gauge interactions

A gauge supermultiplet contains a vector field A_μ^a and a Weyl fermion λ^a . An infinitesimal gauge symmetry transformation with parameter α^a acts on A_μ^a and λ^a as

$$\delta_{\text{gauge}} A_\mu^a = g f^{abc} A_\mu^b \alpha^c - \partial_\mu \alpha^a$$

$$\delta_{\text{gauge}} \lambda^a = g f^{abc} \lambda^b \alpha^c$$

color $SU(3)$	$a = 1, \dots, 8$	$f^{abc} \rightarrow f_{SU(3)}^{abc}$	$g \rightarrow g_s$
weak isospin $SU(2)$	$a = 1, \dots, 3$	$f^{abc} \rightarrow \epsilon^{abc}$	$g \rightarrow g$
weak hypercharge $U(1)$	$a = 1$	$f^{abc} \rightarrow 0$	$g \rightarrow g'$

Counting fermionic and bosonic degrees of freedom, we see that we should add one real scalar auxiliary field, D^a (off shell, $n_b = 3$ from A_μ and $n_f = 4$ from λ). Then

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i\lambda^{a\dagger} \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a$$

where

$$D_\mu \lambda^a = \partial_\mu \lambda^a - gf^{abc} A_\mu^b \lambda^c$$

The corresponding supersymmetry transformation laws are easily worked out, requiring that δA_μ^a be real, and δD^a proportional to the equation of motion of λ^a :

$$\begin{aligned}\delta A_\mu^a &= -\frac{1}{\sqrt{2}} \left(\eta^\dagger \bar{\sigma}_\mu \lambda^a + \lambda^{a\dagger} \bar{\sigma}_\mu \eta \right) \\ \delta \lambda^a &= -\frac{i}{2\sqrt{2}} \sigma^\mu \bar{\sigma}^\nu \eta F_{\mu\nu}^a + \frac{1}{\sqrt{2}} D^a \eta \\ \delta D^a &= -\frac{i}{\sqrt{2}} \left[\eta^\dagger \bar{\sigma}^\mu D_\mu \lambda^a - (D_\mu \lambda^a)^\dagger \bar{\sigma}^\mu \eta \right]\end{aligned}$$

With these definitions,

$$(\delta_{\eta_1} \delta_{\eta_2} - \delta_{\eta_2} \delta_{\eta_1}) X = -(\eta_1^\dagger \bar{\sigma}^\mu \eta_2 - \eta_2^\dagger \bar{\sigma}^\mu \eta_1) i D_\mu X$$

on $X = F_{\mu\nu}^a, \lambda^a, D^a$.

Back to the chiral supermultiplet $X = \phi, \psi, F$. All of them must be in the same representation of the gauge group:

$$\delta_{\text{gauge}} X_i = i g \alpha^a (T^a)_{ij} X_j$$

where T^a are the generators of the gauge group in the representation of X . Ordinary derivatives must be replaced by covariant derivatives everywhere:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i g A_\mu^a T^a$$

Is this enough to turn our simple model into a gauge-invariant supersymmetric theory?

Not quite: there are other possible gauge invariant and renormalizable terms that can be formed out of the fields in the gauge and chiral supermultiplets:

$$\phi^* T^a \psi \lambda^a \quad (\phi^* T^a \phi) D^a$$

They are not taken into account by the covariant derivatives, because they involve the superpartners of the gauge fields.

They can be added to our supersymmetric lagrangian, with appropriate coefficients, provided the transformation law of the auxiliary field F is slightly modified:

$$\delta\phi = \eta\psi$$

$$\delta\psi = i(\sigma^\mu\epsilon\eta^*) D_\mu\phi + F\eta$$

$$\delta F = -i\eta^\dagger\bar{\sigma}^\mu D_\mu\psi + g\sqrt{2}(T^a\phi)\lambda^{a\dagger}\eta^\dagger$$

Then

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{chiral}} + g\sqrt{2} \left(\phi^* T^a \psi \lambda^a + \phi T^a \lambda^{a\dagger} \psi^\dagger \right) + g(\phi^* T^a \phi) D^a$$

is supersymmetric, provided the superpotential is gauge-invariant:

$$\delta_{\text{gauge}} W = W_i (T^a)_{ij} \phi_j = 0$$

The equations of motion for D^a are now

$$D^a = -g(\phi^* T^a \phi)$$

and the scalar potential becomes

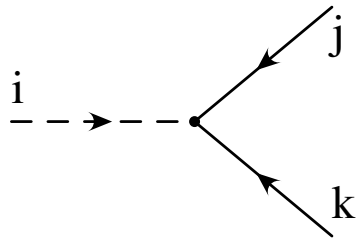
$$V(\phi) = F_i^* F_i + \frac{1}{2} D^a D^a = W_i^* W_i + \frac{1}{2} g^2 (\phi^* T^a \phi) (\phi^* T^a \phi)$$

completely determined by the auxiliary fields. This fact is relevant for the discussion of spontaneous supersymmetry breaking.

- The scalar potential is positive semidefinite: a sum of squares.
- A unique feature of supersymmetric theories: **the scalar potential is entirely determined by other interactions.** F - terms are fixed by Yukawa couplings, D - terms by gauge couplings

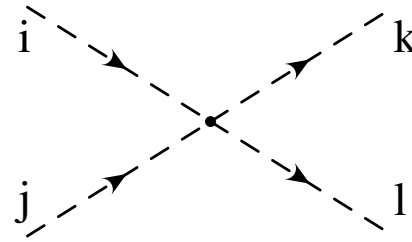
In particular, the quartic Higgs couplings are no longer free parameters.

Superpotential couplings



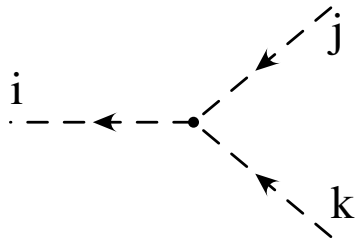
(a)

$$y_{ijk}$$



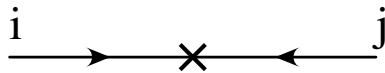
(b)

$$y_{ijm}y_{klm}^*$$



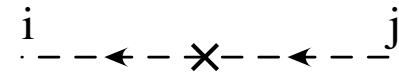
(a)

$$M_{im}^* y_{jkm}$$



(b)

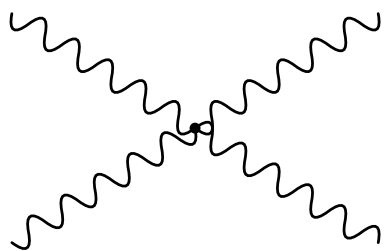
$$M_{ij}$$



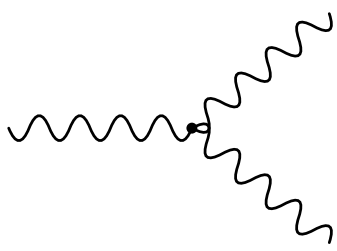
(c)

$$M_{ik}^* M_{kj}$$

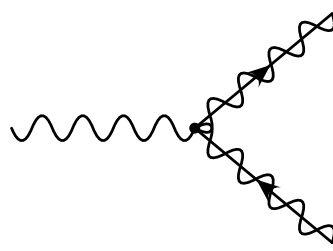
Gauge couplings



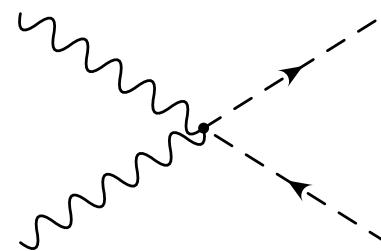
(a)



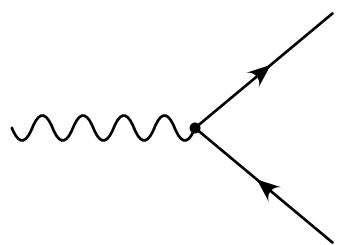
(b)



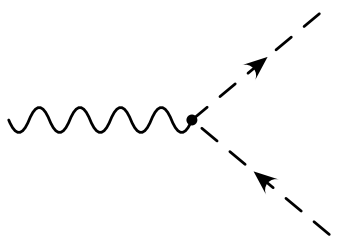
(c)



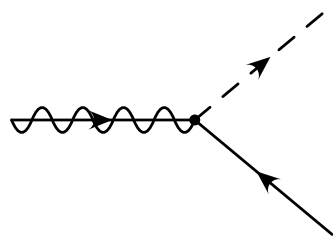
(d)



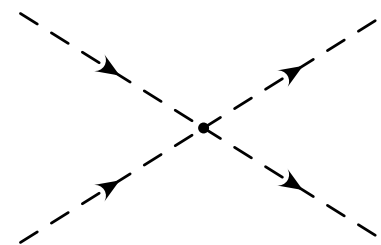
(e)



(f)



(g)



(h)

Comments

- M_{ij} and y_{ijk} in general restricted by gauge invariance
- Pattern of Yukawa couplings required for the cancellation of quadratic divergences
- Fermion mass terms flip chirality, scalar masses do not

Soft supersymmetry breaking

The most general soft supersymmetry breaking lagrangian is given by

$$\mathcal{L}_{\text{soft}} = -m_{ij}^2 \phi_i \phi_j^* - \left(\frac{1}{2} m_\lambda \lambda^a \lambda^a + \frac{1}{2} b_{ij} \phi_i \phi_j + \frac{1}{6} a_{ijk} \phi_i \phi_j \phi_k + \text{h.c.} \right) - \frac{1}{2} c_{ijk} \phi_i^* \phi_j \phi_k + \text{h.c.}$$

It can be shown that a theory with exact supersymmetry, plus $\mathcal{L}_{\text{soft}}$, is free of quadratic divergences.

The last term is usually negligible in most supersymmetry breaking scenarios.

A word on gaugino masses. A mass term for a left-handed Weyl spinor

$$\xi^T \epsilon \xi + \text{h.c.}$$

is called a **Majorana mass term**. It is allowed by Lorentz invariance, but generally forbidden if ξ belongs to a complex representation of a gauge group. A simple example: a phase transformation $\delta\xi = i\alpha\xi$. Matter fermions in the Standard Model (with the possible exception of a **right-handed neutrino**) cannot have a Majorana mass term.

The case of gauginos λ^a is different: they transform according to the adjoint representation,

$$\delta_{\text{gauge}}\lambda_a = g f_{abc}\lambda_b\alpha_c$$

which is self-conjugate. This gives

$$\delta_{\text{gauge}}(\lambda_a^T \epsilon \lambda_a) = g f_{abc}\alpha_c(\lambda_b^T \epsilon \lambda_a + \lambda_a^T \epsilon \lambda_b) = 0$$

because f_{abc} is completely antisymmetric.

The minimal supersymmetric Standard Model

With some specifications, there is a unique superpotential which is allowed by the Standard Model gauge invariance and by renormalizability:

$$W_{\text{MSSM}} = \tilde{u}_R^* y_u \tilde{q}_L H_u - \tilde{d}_R^* y_d \tilde{q}_L H_d - \tilde{e}_R^* y_l \tilde{\ell}_L H_d + \mu H_u H_d$$

(call it W from now on). The first term in detail:

$$\tilde{u}_R^* y_u \tilde{q}_L H_u = (\tilde{u}_R^*)_j{}^f (y_u)_{fg} (\tilde{q}_L)_\alpha{}^{gj} (H_u)_\beta \epsilon^{\alpha\beta}$$

j is an $SU(3)_{\text{color}}$ index, $j = 1, 2, 3$

α, β are $SU(2)_L$ indices, $\alpha = 1, 2, \beta = 1, 2$

f, g are generation indices, $f = 1, 2, 3, g = 1, 2, 3$

If ϕ_1 and ϕ_2 are $SU(2)$ doublets,

$$\delta\phi_1 = i\alpha^a \sigma_a \phi_1 \quad \delta\phi_2 = i\alpha^a \sigma_a \phi_2$$

then

$$\phi_1^T \epsilon \phi_2, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \equiv i\sigma_2$$

is an $SU(2)$ scalar:

$$\delta(\phi_1^T \epsilon \phi_2) = i\alpha^a \phi_1^T (\sigma_a^T \epsilon + \epsilon \sigma_a) \phi_2 = 0$$

because σ_1, σ_3 are symmetric, σ_2 antisymmetric, and $\{\sigma_a, \sigma_b\} = 2\delta_{ab}$. The quantities

$$\phi_1^\dagger \phi_1 \quad \phi_1^\dagger \phi_2 \quad \phi_2^\dagger \phi_2$$

are also, obviously, $SU(2)$ invariants, by they cannot enter the superpotential, which is an analytic function of all scalar fields.

Some comments:

- It is now clear why we need two Higgs doublets: in the Standard Model, H gives mass to up-type quarks, while $H^c = \epsilon H^*$ gives mass to down quarks and leptons. Here, H and H^* cannot appear simultaneously in the superpotential.
- y_u, y_d, y_l are 3×3 matrices in family space of dimensionless Yukawa couplings. They contain fermion masses and generation mixing. A good approximation:
 $y_u^{33} = y_t, y_b^{33} = y_b, y_l^{33} = y_\tau$, all other entries equal to 0.
- No bilinear term other than $\mu H_u H_d$ is allowed by gauge symmetry. μ is the only dimensionful parameter in the superpotential; in particular, mass terms for fermions are forbidden.

The superpotential originates a large variety of interaction vertices, with coupling constants related by supersymmetry.

Recall our recipe,

$$\mathcal{L}_{\text{ng}} = -W_i W_i^* - \frac{1}{2} W_{ij} \psi_i \psi_j - \frac{1}{2} W_{ij}^* \psi_i^\dagger \psi_j^\dagger$$

where

$$W_i = \frac{\partial W}{\partial \phi_i}, W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

We find e.g.

- Yukawa couplings, such as

$$\frac{\partial W}{\partial \tilde{u}_R^* \partial \tilde{u}_L} \bar{u} u = \bar{u} y_u u H_u^0$$

as in the Standard Model.

- quark-squark-higgsino couplings,

$$\frac{\partial W}{\partial \tilde{u}_R^* \partial H_u^0} \bar{u} \tilde{h}_u^0 = \bar{u} y_u \tilde{q}_L h_u^0$$

- Quartic scalar couplings:

$$\left| \frac{\partial W}{\partial \tilde{u}_R^*} \right|^2 = |y_u \tilde{q}_L H_u|^2$$

The term $\mu H_u H_d$ provides a higgsino mass term

$$-\mu (\tilde{h}_u^+ \tilde{h}_d^- - \tilde{h}_u^0 \tilde{h}_d^0 + \text{h.c.})$$

and a scalar Higgs mass term

$$-|\mu|^2 \left(|H_u^+|^2 + |H_u^0|^2 + |H_d^0|^2 + |H_d^-|^2 \right)$$

This term cannot induce spontaneous breaking of the electroweak gauge symmetry. We must rely upon soft breaking terms.

The value of μ is a problem: we expect it to be of the same order (~ 100 GeV) as the soft breaking terms, in order to provide the correct value of the W and Z masses, but it has a completely different origin.

Accidental symmetries and R-parity

Baryon and lepton number conservation in the Standard Model follow from accidental symmetries: a consequence of renormalizability.

This is no longer true in supersymmetry:

$$W_{\Delta L=1} = \frac{1}{2} \lambda_{ijk} \tilde{\ell}_L^i \tilde{\ell}_L^j (\tilde{e}_R^k)^* + \frac{1}{2} \lambda'_{ijk} \tilde{\ell}_L^i \tilde{q}_L^j (\tilde{d}_R^k)^* + \mu'_i \tilde{\ell}_L^i H_u$$

$$W_{\Delta B=1} = \frac{1}{2} \lambda''_{ijk} (\tilde{u}_R^i)^* (\tilde{d}_R^j)^* (\tilde{d}_R^k)^*$$

are allowed by renormalizability and gauge symmetry.

In order to implement B and L conservation we must impose some extra symmetry (not B and L themselves: they are violated by nonperturbative effects!).

Consider the so-called **matter parity**:

$$P_M = (-1)^{3(B-L)}$$

where $B = 1/3$ for quarks and squarks and 0 otherwise, $L = 1$ for leptons and sleptons, 0 otherwise.

P_M commutes with supersymmetry: all particles in the same supermultiplet have the same P_M (-1 for quarks and leptons, $+1$ for Higgs and gauge).

Requiring $P_M = +1$ for all terms in the lagrangian (or in the superpotential) eliminates B and L violating terms.

A different, but equivalent language: define R -parity as

$$P_R = (-1)^{3(B-L)+2s}$$

Conserved if P_M is conserved, but **does not** commute with supersymmetry:

$$P_R = +1 \quad \text{for standard particles}$$

$$P_R = -1 \quad \text{for superpartners}$$

R -parity conserved \Leftrightarrow matter parity conserved.

Useful for phenomenology: each vertex must contain an even number of fields with $P_R = -1$.

Consequences of R-parity conservation

- The lightest particle with $P_R = -1$ is stable. It is called the **LSP**. A candidate for dark matter.
- Supersymmetric particles decay into states with an odd number of $P_R = -1$ particles.
- Supersymmetric particles are always produced in even numbers (usually 2).

Soft supersymmetry breaking in the MSSM

The most general soft supersymmetry breaking term for the MSSM is

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -\frac{1}{2} \left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{w}\tilde{w} + M_1 \tilde{b}\tilde{b} + \text{h.c.} \right) \\ & - \left(\tilde{u}_R^* A_u \tilde{q}_L H_u - \tilde{d}_R^* A_d \tilde{q}_L H_d - \tilde{e}_R^* A_e \tilde{\ell}_L H_d \right) + \text{h.c.} \\ & - \tilde{q}_L^\dagger m_Q^2 \tilde{q}_L - \tilde{\ell}_L^\dagger m_L^2 \tilde{\ell}_L - \tilde{u}_R^\dagger m_U^2 \tilde{u}_R - \tilde{d}_R^\dagger m_D^2 \tilde{d}_R - \tilde{e}_R^\dagger m_E^2 \tilde{e}_R \\ & - m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - (B H_u H_d + \text{h.c.})\end{aligned}$$

An enormous number of new independent parameters (105).

All of them are expected to be of order m_{soft} (to the appropriate power), something between 100 GeV and 1 TeV.

Do we have any guiding principle that can help us reducing the number of arbitrary parameters?

$m_Q^2, m_U^2, m_D^2, m_E^2$ are arbitrary 3×3 matrices in flavour space; they are subject from strong constraints from

- flavour changing neutral current phenomena (e.g. $K_0 \bar{K}_0$ mixing)
- individual leptonic number conservations (e.g. $\mu \rightarrow e\gamma$)
- CP violation

The (unknown) mechanism that generates soft breaking terms must respect these constraints.

All these constraints are satisfied with the simple assumption that **flavour mixing and CP violation are only originated by Yukawa couplings**. This is achieved by assuming that

- $(m_Q^2)_{fg} = m_Q^2 \delta_{fg}$ $(m_U^2)_{fg} = m_U^2 \delta_{fg}$ $(m_D^2)_{fg} = m_D^2 \delta_{fg}$
 $(m_E^2)_{fg} = m_E^2 \delta_{fg}$
- $(A_u)_{fg} = A_u (y_u)^{fg}$ $(A_d)_{fg} = A_d (y_d)^{fg}$ $(A_e)_{fg} = A_e (y_e)^{fg}$
- all soft breaking parameters are real.

This corresponds to assuming that the sector that generates soft-breaking terms is flavor-blind.

This (or any other) structure of soft breaking terms results from some underlying mechanism that breaks supersymmetry at a scale Q_0 , presumably very large (e.g. the grand-unification scale).

Predictions at energy scales relevant for our experiments, m_{exp} , typically of order 100 to 1000 GeV, will therefore contain large logarithms

$$\log \frac{Q_0}{m_{\text{exp}}}$$

induced by radiative corrections. These must be resummed by standard renormalization group techniques.

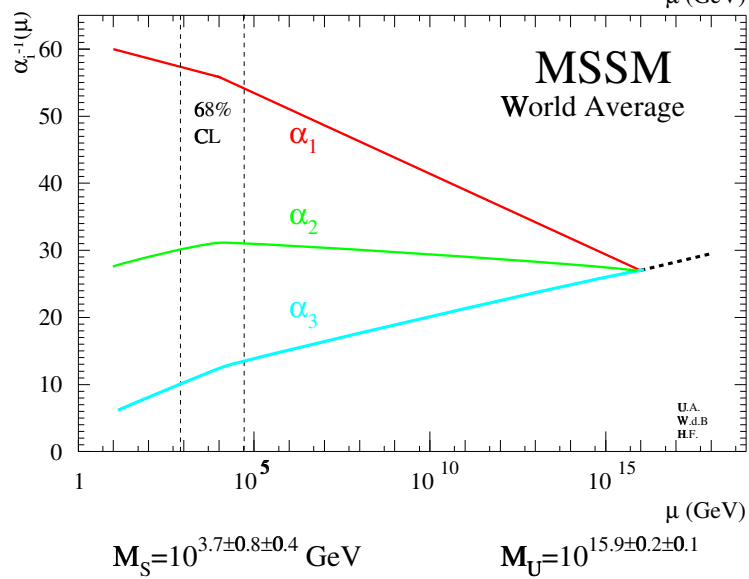
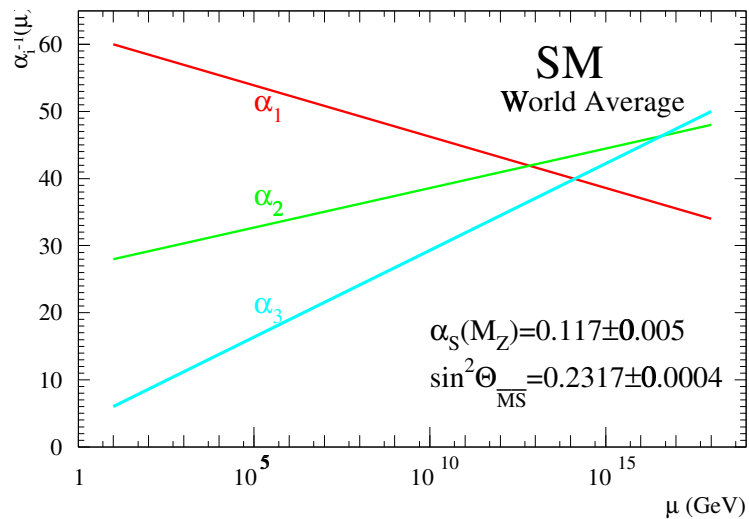
An encouraging fact: coupling constant unification works much better in the MSSM than in the Standard Model. Define

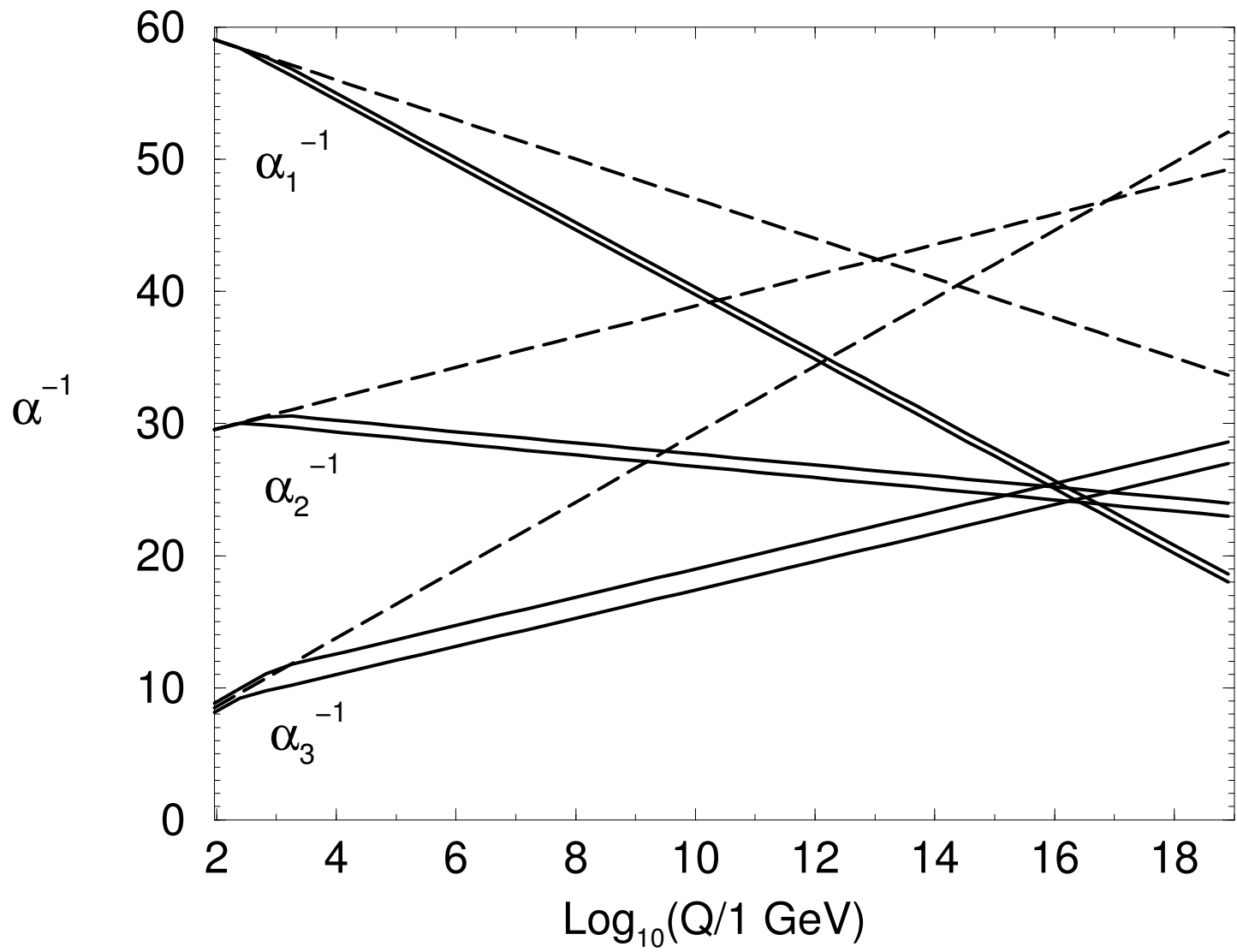
$$\begin{aligned}g_3 &= g_s \\g_2 &= g = \frac{e}{\sin \theta_W} \\g_1 &= \sqrt{\frac{5}{3}} g' = \sqrt{\frac{5}{3}} \frac{e}{\cos \theta_W}\end{aligned}$$

(in order to have the same normalization for symmetry generators). The corresponding RGEs at one loop are

$$\mu^2 \frac{dg_i}{d\mu^2} = \frac{b_i}{8\pi} g_i^3 \quad \Rightarrow \quad \mu^2 \frac{d}{d\mu^2} \frac{1}{\alpha_i} = -b_i$$

with $\alpha_i = g_i^2/(4\pi)$.





The mass spectrum

1. The Higgs sector

The Higgs potential of the MSSM is

$$\begin{aligned} V_{\text{Higgs}} = & (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) + (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) \\ & + B (H_u^+ H_d^- - H_u^0 H_d^0) + \text{h.c.} \\ & + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\ & + \frac{1}{2}g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 \end{aligned}$$

No arbitrary quartic couplings: they are fixed by the D terms.

Minimization: We can take $H_u^+ = H_d^- = 0$ (good news: EM unbroken) and H_u^0, H_d^0 real and positive at the minimum without loss of generality. Then we restrict to the neutral sector:

$$V_{\text{Higgs}} = (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 + (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 - (BH_u^0H_d^0 + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

B may also be taken to be real and positive: no CP violating phases in the Higgs sector.

Perform an $SU(2)_L$ gauge rotation that makes $\langle H_u^+ \rangle = 0$; then, the condition $\partial V / \partial H_u^+ = 0$ is only fulfilled if also $\langle H_d^- \rangle = 0$.

The only place where the phases of H_u^0, H_d^0 appear is the term

$$-BH_u^0 H_d^0 + \text{h.c.}$$

We can make B real and positive by a suitable phase choice for H_u^0, H_d^0 .

Clearly, the potential has a minimum if also $H_u^0 H_d^0$ is real and positive, so $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ must have opposite phases, which can be rotated away by a $U(1)_Y$ gauge transformation.

Conditions for electroweak spontaneous symmetry breaking:

1. The origin $H_u^0 = H_d^0 = 0$ is not a minimum of the potential
2. The potential is bounded from below

Condition 1. leads to the constraint

$$(|\mu|^2 + m_{H_d}^2)(|\mu|^2 + m_{H_u}^2) < B^2$$

In some cases, guaranteed by large negative radiative corrections to $m_{H_u}^2$.

Condition 2. is almost automatic: the quartic couplings are positive definite. There are however **D-flat** directions $|H_u^0| = |H_d^0|$. We must require that the potential in these directions is stabilized by quadratic terms:

$$2B < 2|\mu|^2 + m_{H_d}^2 + m_{H_u}^2$$

Define $v_u = \langle H_u^0 \rangle$ **and** $v_d = \langle H_d^0 \rangle$. **We find**

$$m_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_u^2 + v_d^2) \Rightarrow v^2 \equiv v_u^2 + v_d^2 \simeq (174 \text{ GeV})^2$$

Following tradition, we also define an angle β through

$$\tan \beta = \frac{v_u}{v_d}$$

Clearly $0 \leq \beta \leq \pi/2$. The minimization conditions are

$$|\mu|^2 + m_{H_d}^2 = B \tan \beta - \frac{m_Z^2}{2} \cos 2\beta$$

$$|\mu|^2 + m_{H_u}^2 = B \cot \beta + \frac{m_Z^2}{2} \cos 2\beta$$

The μ problem is now evident.

Mass eigenstates:

1. Two CP-odd neutral scalars

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \sin \beta & -\cos \beta \\ \cos \beta & \sin \beta \end{pmatrix} \begin{pmatrix} \text{Im } H_u^0 \\ \text{Im } H_d^0 \end{pmatrix}$$

2. Two charged scalars

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \sin \beta & -\cos \beta \\ \cos \beta & \sin \beta \end{pmatrix} \begin{pmatrix} H_u^\pm \\ H_d^\pm \end{pmatrix}$$

G^0, G^\pm are **pseudo-Goldstone bosons**: they can be removed from the spectrum by an appropriate gauge transformation.

A^0, H^\pm are **physical** degrees of freedom.

3. Two CP-even neutral scalars

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \text{Re } H_u^0 - v_u \\ \text{Re } H_d^0 - v_d \end{pmatrix}$$

Finding the mass eigenvalues is a matter of algebra:

$$m_A^2 = \frac{2B}{\sin 2\beta}$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2$$

$$m_{h,H}^2 = \frac{1}{2} \left(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta} \right)$$

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_A^2 + m_Z^2}{m_H^2 - m_h^2} \quad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}$$

It is easy to show that

$$m_{H^\pm}^2 \geq m_A^2$$

$$m_h \leq m_A \leq m_H$$

$$m_h < |\cos 2\beta| m_Z$$

Radiative corrections weaken this bound. At one loop, and neglecting squark mixing, one finds

$$\Delta m_h^2 \simeq \frac{3g^2}{8\pi^2} \frac{m_t^4}{m_W^2} \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}$$

An extra term appears if mixing is taken into account:

$$(\Delta m_h^2)_{\text{mix}} \simeq \frac{3g^2}{8\pi^2} \frac{m_t^4}{m_W^2} \frac{(A_t - \mu \cot \beta)^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \left[\log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + (A_t - \mu \cot \beta)^2 f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right]$$

$$f(a, b) = \frac{1}{a - b} \left[1 - \frac{1}{2} \frac{a + b}{a - b} \log \frac{a}{b} \right]$$

Including all corrections, one obtains approximately

$$m_h \lesssim 130 \text{ GeV}$$

still very interesting in view of LHC.

Yukawa couplings are related to fermion masses in the third generation by

$$y_t = \frac{g m_t}{\sqrt{2} m_W \sin \beta} \quad y_b = \frac{g m_b}{\sqrt{2} m_W \cos \beta} \quad y_\tau = \frac{g m_\tau}{\sqrt{2} m_W \cos \beta}$$

A difference with respect to the Standard Model: y_b and y_τ may be as large as y_t , if $\tan \beta$ is large enough ($\tan \beta < 1$ disfavoured: y_t becomes nonperturbative for $\tan \beta > 1.2$).

The couplings of h, H, A to standard particles are the same as in the Standard Model, rescaled by α - and β -dependent factors:

	$d\bar{d}, s\bar{s}, b\bar{b}$ $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$	$u\bar{u}, c\bar{c}, t\bar{t}$	W^+W^-, ZZ
h	$-\sin \alpha / \cos \beta$	$\cos \alpha / \sin \beta$	$\sin(\beta - \alpha)$
H	$\cos \alpha / \cos \beta$	$\sin \alpha / \sin \beta$	$\cos(\beta - \alpha)$
A	$-i\gamma_5 \tan \beta$	$-i\gamma_5 \cot \beta$	0

At tree level, the Higgs sector is described by two parameters (a common choice: m_A^2 and $\tan \beta$).

In addition, there are $h(H)AA$ and $h(H)AZ$ couplings.

2. Neutralinos and charginos

The supersymmetric partner of neutral gauge bosons, \tilde{w}^0 and \tilde{b} , and of Higgs scalars, $\tilde{h}_u^0, \tilde{h}_d^0$, are not mass eigenstates. We find

$$\mathcal{L}_n = -\frac{1}{2} \tilde{\psi}_0^T M_n \tilde{\psi}_0 + \text{h.c.}$$

where the matrix M_n in the basis $\tilde{b}, \tilde{w}^0, \tilde{h}_d^0, \tilde{h}_u^0$ is given by

$$\begin{bmatrix} M_1 & 0 & -m_Z \cos \beta \sin \theta_W & m_Z \sin \beta \sin \theta_W \\ 0 & M_2 & m_Z \cos \beta \cos \theta_W & -m_Z \sin \beta \cos \theta_W \\ -m_Z \cos \beta \sin \theta_W & m_Z \cos \beta \cos \theta_W & 0 & -\mu \\ m_Z \sin \beta \sin \theta_W & -m_Z \sin \beta \cos \theta_W & -\mu & 0 \end{bmatrix}$$

The mass matrix M_n is symmetric, and can be diagonalized by a unitary transformation N . The four mass eigenstates

$$\tilde{\chi}_i^0 = N_{ij} \tilde{\psi}_j^0; \quad M_n = N^T \hat{M}_n N$$

are called **neutralinos**. Their coupling are entirely determined by the matrix N , and thus by the four parameters

$$M_1, M_2, \mu, \tan \beta$$

The lightest neutralino is in most models (= for most of the allowed parameter choices) the **LSP**; neutralinos play a crucial role in phenomenology.

Charged gauginos and higgsinos

$$\tilde{w}^+, \tilde{h}_u^+, \tilde{w}^-, \tilde{h}_d^-$$

mix in a similar way:

$$\mathcal{L}_c = -\frac{1}{2} \tilde{\psi}_c^T M_c \tilde{\psi}_c + \text{h.c.}$$

$$M_c = \begin{bmatrix} 0 & X^T \\ X & 0 \end{bmatrix} \quad X = \begin{bmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & \mu \end{bmatrix}$$

The mass eigenstates (called **charginos**) are found by diagonalizing X with a bi-unitary transformation:

$$\begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_2^+ \end{pmatrix} = V \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_u^+ \end{pmatrix} \quad \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix} = U \begin{pmatrix} \tilde{w}^- \\ \tilde{h}_d^- \end{pmatrix}$$

$$X = U^T \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix} V$$

$$m_{\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm}^2 = \frac{1}{2} \left[\left(|M_2|^2 + |\mu|^2 + 2m_W^2 \right) \mp \sqrt{\left(|M_2|^2 + |\mu|^2 + 2m_W^2 \right)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2} \right]$$

As in the case of neutralinos, the matrices U, V contain all the relevant information about chargino couplings.

It is quite natural to assume that gaugino masses have a common value $m_{1/2}$ at the reference scale Q_0 , where the (properly normalized) gauge couplings take approximately the same value $\alpha_U \sim 0.04$.

It turns out that gaugino running masses obey the same renormalization group equations. Therefore, with the above assumption,

$$M_i(Q) = \frac{\alpha_i(Q)}{\alpha_U} m_{1/2}$$

This assumption reduces (from 4 to 3) the number of arbitrary parameters in the gaugino sector.

3. Squarks and sleptons

The scalar partners of matter fermions (squarks and sleptons) have mass matrices with contributions from various terms:

- quartic Higgs-Higgs-squark-squark vertices
- trilinear Higgs-squark-squark couplings
- soft-breaking masses
- D -terms

In general, there is mixing among different families, and mixing between scalar partners of left- and right-handed fermions. This can be sizeable in the third generation.

Mass matrices for squarks and sleptons in the third generation, in the simplified situation outlined above:

$$\mathbf{s - top :} \quad (\tilde{t}_L^* \quad \tilde{t}_R^*) \begin{pmatrix} m_Q^2 + m_t^2 + \Delta_Q & m_t(A_t - \mu \cot \beta) \\ m_t(A_t - \mu \cot \beta) & m_U^2 + m_t^2 + \Delta_U \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

$$\mathbf{s - bottom :} \quad (\tilde{b}_L^* \quad \tilde{b}_R^*) \begin{pmatrix} m_Q^2 + m_b^2 + \Delta_Q & m_b(A_b - \mu \tan \beta) \\ m_b(A_b - \mu \tan \beta) & m_D^2 + m_b^2 + \Delta_D \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix}$$

$$\mathbf{s - tau :} \quad (\tilde{\tau}_L^* \quad \tilde{\tau}_R^*) \begin{pmatrix} m_L^2 + m_\tau^2 + \Delta_L & m_\tau(A_\tau - \mu \tan \beta) \\ m_\tau(A_\tau - \mu \tan \beta) & m_E^2 + m_\tau^2 + \Delta_E \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

where

$$\Delta = m_Z^2 (T_3 - Q \sin^2 \theta_W) \cos 2\beta$$

Spontaneous supersymmetry breaking

How are soft supersymmetry-breaking terms generated? We need a mechanism of spontaneous supersymmetry breaking (explicit breaking not acceptable).

A symmetry is said to be spontaneously broken when the vacuum state is not invariant under that symmetry:

$$Q|0\rangle \neq 0$$

Supersymmetry is no exception. This can happen only if some of the auxiliary fields F_i or D^a acquires a non-zero vacuum expectation value.

Proof: the supersymmetry algebra implies

$$H = P^0 = \frac{1}{4} \left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2 \right)$$

which implies

$$Q_i |0\rangle \neq 0 \Leftrightarrow \langle 0|H|0\rangle > 0 \Leftrightarrow \langle 0|V|0\rangle > 0$$

for a translation-invariant vacuum state. But

$$V = F_i F_i^* + \frac{1}{2} D^a D^a$$

so supersymmetry is spontaneously broken if and only if the system

$$F_i = 0 \quad D^a = 0$$

has no solution.

Spontaneous breaking based on $\langle D^a \rangle \neq 0$ for some D^a is called **à la Fayet-Iliopoulos**. If there is a $U(1)$ factor in the gauge group, a term

$$\mathcal{L}_{D\text{-breaking}} = kD$$

can be added to the lagrangian. Then

$$V = \frac{1}{2}D^2 - kD + gD \sum_i q_i \phi_i \phi_i^*$$

$$D = k - g \sum_i q_i \phi_i \phi_i^*$$

D is different from zero if for some reason all scalar fields are zero at the minimum. This is *not* the case in the MSSM.

Not very promising, although not completely ruled out.

Models in which supersymmetry is spontaneously broken by $\langle F_i \rangle \neq 0$ for some F_i are called **O’Raifeartaigh** models. An explicit example: three chiral superfields, and

$$W = -k\phi_1 + m\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

where ϕ_1 must be a **gauge singlet**. Then

$$V = |F_1|^2 + |F_2|^2 + |F_3|^2$$

$$F_1 = k - \frac{y}{2}\phi_3^{*2} \quad F_2 = -m\phi_3^* \quad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*$$

$F_1 = F_2 = F_3 = 0$ has no solution. For $m^2 > yk$, the minimum of V is for $\phi_2 = \phi_3 = 0$, ϕ_1 undetermined (flat direction).

The solution is $F_1 = k$, $V = k^2$ at the minimum. The flat direction is no longer flat after radiative corrections, and the minimum is at $\phi_1 = 0$. The spectrum: six scalars with masses

$$0 \quad 0 \quad m^2 \quad m^2 \quad m^2 - yk \quad m^2 + yk$$

and three fermions with masses

$$0 \quad m \quad m$$

The zero-mass fermion is the analog of a Goldstone boson in broken ordinary symmetry: we call it a **goldstino**. It is the partner of the auxiliary field F_1 , responsible for supersymmetry breaking.

A general conclusion: **spontaneous breaking of supersymmetry cannot take place within the MSSM**. There is no gauge-singlet supermultiplet whose auxiliary field can take a non-vanishing vacuum expectation value.

Hard to do by simply extending the MSSM with new fields and new renormalizable interactions:

- gaugino masses cannot arise from renormalizable couplings
- the spectrum must include light scalars:

$$\text{Tr}M_{\text{real scalars}}^2 = 2 \text{Tr}M_{\text{chiral fermions}}^2$$

even after supersymmetry breaking.

Mass terms are generated after spontaneous symmetry breaking as, for example, quark masses in the standard model:

$$y \phi \bar{\psi} \psi \rightarrow y \langle \phi \rangle \bar{\psi} \psi = m \bar{\psi} \psi$$

In supersymmetry, the same mechanism is at work. A gaugino mass could be generated either by a coupling

$$\phi \lambda \lambda \rightarrow \langle \phi \rangle \lambda \lambda$$

but such a term is absent in a supersymmetric theory, or by

$$F \lambda \lambda \rightarrow \langle F \rangle \lambda \lambda$$

which is not renormalizable.

Way out: assume that supersymmetry is broken in a **hidden sector** of non-observable degrees of freedom, that communicate somehow with the observable sector. Historically, two proposals: supersymmetry breaking communicated to the observable sector by

1. gravitational interactions

2. gauge interactions

The differences between the two cases are due to a different size of the supersymmetry breaking scale

$$\langle 0|F|0\rangle$$

In **gravity mediated** supersymmetry breaking, soft breaking terms arise from (in general nonrenormalizable) interaction terms (of gravitational strength) of observable fields with combinations of fields in the hidden sector that acquire a non-zero vev. We expect therefore

$$m_{\text{soft}} \sim \frac{\langle F \rangle}{M_P}$$

on dimensional grounds: m_{soft} must vanish as $\langle F \rangle$ goes to zero (no spontaneous breaking) and as $M_P \rightarrow \infty$ (no interaction between hidden and observable sectors).

For $m_{\text{soft}} \sim 1 \text{ TeV}$ we get

$$\sqrt{\langle F \rangle} \sim 10^{11} \text{ GeV}$$

In the **gauge mediated** scenario, supersymmetry breaking is communicated to the observable sector by the ordinary gauge interactions via loop effects due to particles called **messengers**.

A rough estimate of the relevant supersymmetry breaking scale can be obtained. We have in this case

$$m_{\text{soft}} \sim \frac{\alpha}{4\pi} \frac{\langle F \rangle}{M}$$

where M is the typical messenger mass. This corresponds to a much lower value for the supersymmetry order parameter:

$$\sqrt{\langle F \rangle} \sim 10^4 - 10^5 \text{ GeV}$$

if $\langle F \rangle$ and M^2 are assumed to be roughly of the same size.

A closer look at the goldstino

The goldstino field can be easily identified. The full fermion mass matrix of a generic supersymmetric theory is

$$M_f = \begin{pmatrix} 0 & \sqrt{2}g_\alpha(T^a\langle\phi\rangle)_i \\ \sqrt{2}g_\alpha(\langle\phi^*\rangle T^a)_j & \langle W_{ij}\rangle \end{pmatrix}$$

There is always one combination of fermion fields that corresponds to zero mass:

$$\tilde{G} = \begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle F_i\rangle \end{pmatrix}$$

is an eigenvector of M_f with zero eigenvalue. This is the goldstino.

The first component of $M_f \tilde{G}$ is

$$\sqrt{2} \langle g_a (T^a \phi)_i F_i \rangle = -\sqrt{2} \langle g_a (T^a \phi)_i W_i^* \rangle = -\sqrt{2} \left\langle \frac{\partial W^*}{\partial \phi_i^*} \delta_{\text{gauge}} \phi_i^* \right\rangle = 0$$

by gauge-invariance of the superpotential.

The second component is given by

$$\langle g_a (\phi^* T^a)_j D^a + W_{ij} F_i \rangle$$

The scalar potential is

$$V = F_i F_i^* + \frac{1}{2} D^a D^a = W_i W_i^* + \frac{1}{2} g_a^2 (\phi^* T^a \phi)^2$$

It follows that

$$\begin{aligned} \left\langle \frac{\partial V}{\partial \phi_k} \right\rangle &= \langle W_i^* W_{ik} + g_a^2 (\phi^* T^a \phi) (\phi^* T^a)_k \rangle \\ &= -\langle F_i W_{ik} + g_a D^a (\phi^* T^a)_k \rangle = 0 \end{aligned}$$

Let us consider the case of **local** supersymmetry: $\eta = \eta(x)$. Then

$$(\delta_{\eta_1} \delta_{\eta_2} - \delta_{\eta_2} \delta_{\eta_1})X = -(\eta_1^\dagger \bar{\sigma}^\mu \eta_2 - \eta_2^\dagger \bar{\sigma}^\mu \eta_1) i\partial_\mu X$$

is a **local** translation, i.e. a general coordinate transformation.

Gravitation is included in a natural way.

Local supersymmetry is called **supergravity**.

In this case, the so-called **super-Higgs** effect takes place: the spin-3/2 partner of the graviton, the **gravitino**, acquires a mass.

The missing spin degrees of freedom are provided by the goldstino.

By dimensional analysis,

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \quad \left(m_{3/2} = \frac{\langle F \rangle}{\sqrt{3}M_P} \right)$$

Clearly, the gravitino mass has very different values in gravity-mediated and gauge mediated scenarios:

- gravity-mediated: the gravitino mass

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \sim m_{\text{soft}}$$

and its interactions are gravitational. Almost no interest for phenomenology at colliders.

- gauge-mediated:

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \sim \frac{10^5}{10^{19}} \text{ GeV}$$

if $M \sim \langle F \rangle \ll M_P$. Its goldstino components may have non-negligible couplings, and may therefore play a role in collider physics ($\tilde{X} \rightarrow X\tilde{G}$).

Gravity-mediated models: some details

The supergravity lagrangian is non-renormalizable (as expected). Assume there is a hidden chiral superfield X whose auxiliary component F_X gets a non-zero vev. The lagrangian includes

$$\begin{aligned}\mathcal{L} = & -\frac{1}{M_P} F_X \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + \text{h.c.} \\ & + \frac{1}{M_P^2} F_X F_X^* k_{ij} \phi_i^* \phi_j \\ & - \frac{1}{M_P} F_X \left(\frac{1}{6} y'_{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'_{ij} \phi_i \phi_j + \text{h.c.} \right)\end{aligned}$$

When $F_X \rightarrow \langle F_X \rangle$, this gives precisely the soft terms we need.

In a *minimal* version of supergravity, there are considerable simplifications:

$$f_a = f \quad k_{ij} = k\delta_{ij} \quad y' = \alpha y \quad \mu' = \beta\mu$$

This leads to

$$M_1 = M_2 = M_3 = m_{1/2}$$

$$m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = m_{H_u}^2 = m_{H_d}^2 = m_0^2$$

$$A_u = A_0 y_u \quad A_d = A_0 y_d \quad A_e = A_0 y_e$$

$$B = B_0 \mu$$

Neutral flavour changing effects automatically suppressed.

Gauge-mediated models: some details

In the simplest version, the model contains four messenger supermultiplets:

$$q \sim \left(\mathbf{3}, \mathbf{1}, -\frac{2}{3} \right) \quad \bar{q} \sim \left(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3} \right) \quad \ell \sim (\mathbf{1}, \mathbf{2}, +1) \quad \bar{\ell} \sim (\mathbf{1}, \mathbf{2}, -1)$$

and a chiral multiplet S , whose scalar and auxiliary components take a vev (e.g. by the O’Raifeartaigh mechanism). They interact through a superpotential

$$W_{\text{messengers}} = y_2 S \ell \bar{\ell} + y_3 S q \bar{q}$$

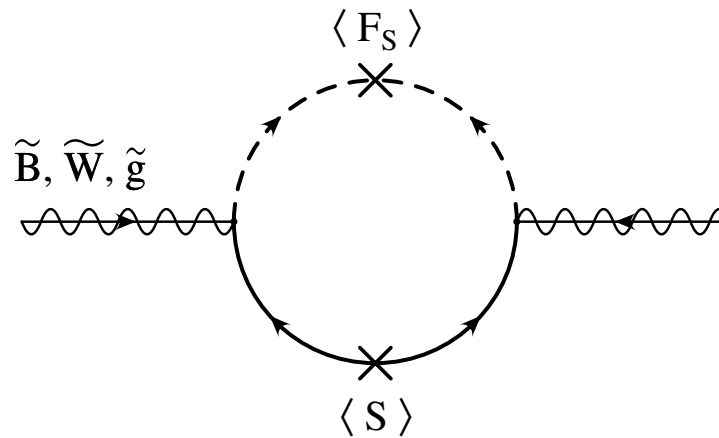
which becomes effectively

$$W_{\text{messengers}} = y_2 \langle S \rangle \ell \bar{\ell} + y_3 \langle S \rangle q \bar{q}$$

Gauginos get masses

$$M_a = \frac{\alpha_a}{4\pi} \frac{\langle F_S \rangle}{\langle S \rangle}$$

through one-loop diagrams:



Soft scalar masses and trilinear couplings arise at two-loops. One gets

$$A_u = A_d = A_e = 0$$

at the messenger scale.

General features of soft masses in gauge-mediated models:

- larger for strongly-interacting particles
- determined by gauge properties only

The second property leads to a degeneracy in squark and slepton masses, that suppresses FCNC effects.

Summary

- Supersymmetry provides a solution to the naturalness problem without spoiling the good features of the Standard Model.
- It has a number of welcome side-effects:
 - a better context for grand unification
 - a natural candidate for cold dark matter.
- A large portion of the parameter space for supersymmetric theories has already been explored.
- Supersymmetry breaking in realistic models is still an open problem.