

Some observations about the relationship between theory of relativity and Kantian philosophy (*1946/9-B2)

I

It is a remarkable fact, to which however very little attention is being paid in current philosophical discussions, that at least in one point relativity theory has furnished a very striking confirmation of Kantian doctrines.¹ In fact it is one of the most surprising and counterintuitive tenets of Kant for which this is the case, namely, for that part of his doctrine about time which says that time is neither “something existing in itself” (i.e., a separate entity besides the objects in it), nor “a characteristic or ordering inherent in the objects”,² but only exists in a relative sense. That entity relative to which it exists, *according to Kant*, is the perceiving subject or, more precisely, its “sensibility”;³ *according to relativity theory*, it is certain more general and abstract things, such as material points, world lines, and coordinate systems, which, however, likewise can be conceived most conveniently as characteristics of, or as belonging to, a possible observer.⁴

¹In order to prevent any misunderstanding, I wish to say right in the beginning that I am not an adherent of Kantian philosophy in general. The subsequent considerations only try to show that a surprising similarity subsists in some respects between relativity theory and the Kantian doctrine about time and space and that contradictions between them, as far as they occur, are by far not so fundamental as is widely believed. Above all, however, I wanted to show that the questions arising in such a comparison are interesting and perhaps even fruitful for the future development of physics.

²Cf. B 49, paragraph a).

³“Sensibility” is, according to Kant, the faculty of having sensations under the influence of objects and happenings outside or inside ourselves, and forming images of outer objects and psychic processes out of these sensations. “Faculty of sense perception” would be a more complete, but somewhat cumbersome, expression of what is meant. Sensibility is divided by Kant into outer and inner sense.

⁴Only in [the] case of *special* relativity theory and a straight world line is it unambiguous how a time valid for the whole world is to be defined relative to an observer traveling along this world line. Generalizations to other cases exist, but it is not so clear what the “correct” generalization is, or whether a uniquely determined simplest (or most convenient) generalization at all exists. For the physicist such questions are of little interest, since the measuring results can be predicted also directly from the objective state of affairs. If one takes the viewpoint that time in general relativity theory is the time-like^a coordinate of any Gaussian coordinate system which is so chosen that it has one everywhere time-like and three everywhere space-like coordinates, the dependence of time on the perceiving subject becomes still more manifest, since then anything (within certain limits) which the perceiving subject chooses to consider as time is time.

^aHandwritten in the margin of the manuscript are the words “too general!” and an arrow pointing to the typewritten line beginning with the word “time-like”. Thus the phrase may as well refer to the entire passage as to the specific line or word.

II

There exists, also according to Kant, an objective correlate in the things of our representation of time; e.g., he speaks of “those modifications” (evidently of the things in themselves) “which we represent to ourselves as changes” (B 54). Therefore his thesis of the | “*non-existence of time except* 2
relative to the perceiving subject”, must be understood to refer ONLY to something which is supposed to have the essential characteristics imagined in *our subjective idea* of time⁵ (again in complete agreement with relativity theory).

That furthermore Kant, insofar as he attributes any reality to time *such as we perceive it*, really means^b that temporal properties are certain relations of the things to the perceiving subject appears from many passages of his writings. E.g., in his explanation of the difference between “Schein” and “Erscheinung” he says quite clearly that the properties of the appearances, although they are not properties of the things in themselves,⁶ still are not mere illusions, because they represent relations of the things to the subject. Kant’s own words, in literal translation, are: (1) “the predicates of the appearance can be attributed to the object itself in relation to our sense” (B 69 n.), and: (2) “what is not to be found in the object in itself but always in its relation to the subject ... is appearance” (B 69 n.). Again in the same discussion Kant says that the properties which we attribute to the objects in the appearance (3) “depend ... on the mode of intuition of the subject *in the relation* of the given object to it” (B 69 [emphasis Gödel’s]). In other passages he says that the outer as well as the inner sense (4) “can contain in its representation only the relation of an object to the subject” (B 67), that⁷ time and space are (5) “characteristics not inherent in the things in themselves, but only in their relation to our sensibility”, that⁸ the appearances are something (6) “whose possibility is based on the relation of certain things unknown in themselves ... to our sensibility”. Finally Kant

⁵In particular, e.g., [that it] defines a linear ordering of all events. Only for such an entity is the word “time” without further specification used in the sequel.

⁶The term “things in themselves” reduced to the plain meaning which it has in Kant’s theory of knowledge simply means “the things”, as distinguished from the images they produce in us and the relations they have to our perceptive faculties.

⁷Cf. *Prolegomena*, §11, [p. 284].

⁸See *Prolegomena*, §13, [p. 286]. To this passage, as well as that numbered (2), it is to be noted that for Kant “appearance” does not mean the fact that something appears, but rather an object of thought which has all the properties which are attributed to the object appearing (cf. his definition at B 34). Also the context in which these two passages occur shows clearly that “appearance” is meant in this sense.

^bThe phrase “by it” has been deleted following “means” by the editors, under the assumption that Gödel had in mind the German “dabei”. As such it was judged to be implied by the remainder of the sentence.

also says⁹ that (7) “our representation of space *is completely adequate to*
 3 the relation | which our sensibility has to the objects” (and this evidently
 is meant to apply to time as well).

But does perhaps the relation to sensibility of which Kant speaks in these passages consist solely in the fact that we represent the things in time and space and as such and such appearances, or that we have the permanent disposition to do so? In that case time would have no more reality than a constant illusion, and every analogy with physics, as to the positive half of Kant’s doctrine (as formulated above) would disappear. Here two questions are to be distinguished:

1. As to the spatial and temporal properties and relations of *objects in time and space*, it is quite clear that, by Kant’s doctrine, they certainly do not mean only that we imagine things to have these properties and relations, but on the contrary to each of them there must correspond some objective relation of the things to us which subsists (independently of our representations) whenever we ascribe that spatial or temporal property or relation to something. This follows from the fact that we determine the spatial and temporal positions and properties of objects by certain well-defined rules¹⁰ on the basis of our sensations and the sensations are caused by actions of things upon us,¹¹ which have nothing to do with our cognitive activity. Hence, generally, to differences in spatial or temporal properties differences in the objective state of affairs must correspond.

4 | That this actually is Kant’s meaning not only follows indirectly from the general considerations just made, but also directly from the wording of some of the passages quoted, in particular the one¹² speaking about “those modifications which we represent to ourselves as changes” and that numbered (3) above, where certain relations of the things to us appear clearly as existing *besides* our “mode of intuition”. Moreover one may allege the passages (2), (4), (6), which (if one takes account of what is said in footnote 8) seem to imply that the relations under consideration are in some sense the object of our representations [and] hence cannot consist solely in the act or disposition of representing.

On the other hand, Kant’s subjectivity of temporal relations cannot be interpreted to refer solely to their qualitative character, so that to each temporal relation an isomorphic objective relation between the things would

⁹See *Prolegomena*, §13, end of note II, [pp. 289–290—emphasis Gödel’s].

¹⁰Almost the whole chapter on the “Second analogy of experience” treats of the rule by which we determine temporal ordering in the appearances. That the formulation of this rule given by Kant can hardly be considered as satisfactory does not change anything in the principle in question.

¹¹Cf. B 34: “The effect of an object on the faculty of representation insofar as we are affected by it is sensation”.

¹²Cf. the passage from B 54 cited above.

correspond. For in that case at least the "ordering" given by time would be something "inherent in the objects", which is explicitly denied by Kant (see the formulation given in the beginning). So the state of affairs as conceived by Kant, as to the point in question, *is very similar* to that subsisting in relativity theory.

2. As to the properties of *space and time themselves*, such as the one-dimensionality of time, the Euclidicity of space, etc., the whole tenor of the Kantian exposition, as well as many particular passages, seem to imply that, by Kant's doctrine, they really exist only insofar as we represent things in (i.e., project our sensations on) these schemes, and that, as far as the reality independent of our representations is concerned, we might quite as well project them on other schemes. E.g., Kant says about time: "So it has subjective reality with regard to my inner experience, i.e., I really have the | representation of time and my modifications in it. Hence it is to be considered to be real not as an object, but only as the mode of intuition of myself as an object." (B 53) The theory of the apriority of time and space in particular seems to require this conception. 4a

On the other hand, however, some of the passages quoted before seem to point in the opposite direction and to attribute some kind of objectivity also to time and space themselves, although, of course, only in the sense of | relations of the things to us (but relations subsisting independently of any *cognitive* facts). Especially the passage (7) can hardly be interpreted to mean that space is adequate to the relation which things have to us owing to our representing them in space. Still less could such an adequacy be used (as Kant does use it) as a defense against the objection that his theory transforms the whole world of sense perception into a mere illusion. For it would be a very poor defense to say that this is not so, because it is an adequate representation of this illusion. 4b

So in this passage a view as to the nature of space and time, slightly different from that usually ascribed to Kant, seems to be implied, which however is not incompatible with the latter insofar as, corresponding to the two parts of sensibility (the faculty of sensation and of representation), both kinds of relations of the things to sensibility may subsist beside each other. The entire neglect of the second kind in the discussion of the epistemological character of geometry seems to me to be the reason why Kant failed to see that geometry in one sense is an empirical science (cf. page 16).

III

5

Having now discussed the historical correctness of the formulation of Kant's thesis given in the beginning, and having thrown some light upon its meaning, let us see in what sense and to what extent this doctrine of Kant has been verified in relativity theory, and let us begin with the negative part of Kant's thesis. Here the agreement is most striking; for the

fundamental temporal relation between two events A and B (“ A before B by t seconds”) is always quantitatively different for suitably chosen different observers and may even be inverse in direction for two different observers. Hence it is, such as it is directly observed, certainly not something inherent in the events.

- 6 | Of course there might nonetheless exist some objective reality independent of the observer and the frame of reference, which has the character of time such as we imagine it, but which does not adequately express itself in the observations. In relativity theory however this is not the case either, since what remains of time as an objective characteristic inherent in the events themselves, namely, the relation “ A is before B for all observers” and its quantitative specification,¹³ is quite different from what we imagine by temporal sequence.

First of all it determines only a partial ordering, i.e., some of the pairs of events have no absolute temporal relation whatsoever to each other, insofar^e as the relation “neither before nor after” cannot be interpreted as “simultaneousness”. (If it were, two events simultaneous with a third one would in general not be simultaneous with each other.) Still more

¹³I.e., the relation “ A before B in general by at least t seconds (but not more than t seconds) for every observer”.^c This magnitude is exactly the main invariant of the space-time scheme of special relativity theory. This is all that remains in relativity theory of time as an objective characteristic inherent in the events. [Or, to be more exact, every generally applicable definition of an absolute “before” leads to a relation which in some possible worlds has the above-explained properties distinguishing it from the intuitive “before”. (See the paragraph about the definition of an absolute world time below.^d)]

^cThere is an obvious slip here, for “by at least t seconds (but not more than t seconds) for every observer” just means *by exactly t seconds for every observer*—which is an assertion that time is “absolute”! Gödel’s original note, in A and $B1$, does not contain the parenthetical phrase; this is inserted, in longhand, in $B2$. But without that phrase the statement is still wrong. For two events, A and B , in Minkowski space-time (“the space-time scheme of special relativity theory”), with A earlier than B , the proper time-lapse from A to B along any time-like path is *less than or equal to* the Minkowski “interval” t between them (the maximum, t , is attained along the “straight” path); the greatest *lower* bound of this time-lapse is always zero.

What seems to have happened is this: First, Gödel overlooked the point just made: he thought of the straight path as the *geodesic* from A to B (which is quite right) and overlooked for the nonce that in Minkowski space-time a *time-like* geodesic does not minimize, but rather *maximizes*, length between its extremities. His intent, therefore, was to characterize the Minkowski interval as the greatest lower bound of the proper time-lapses. Gödel then noticed this logical slip; and (somewhat hastily) thought to correct it by the addition of the parenthetical phrase—which fails to accomplish its purpose. To say what he *meant*, he should have written: “the relation ‘ t is the largest number such that A is before B by at least t seconds for every observer’”. To say what he *should* have meant, he should have written: “the relation ‘ t is the *smallest* number such that A is before B by at *most* t seconds for every observer’”. A comedy of errors!

^dGödel evidently meant to refer to the full paragraph on p. 10 below.

^eSurely the word intended here is “inasmuch”.

different from our idea of temporal sequence is the absolute “before” in its quantitative properties. For it is not additive (i.e., except for special cases, $AB + BC \neq AC$ for the temporal distances of three events A, B, C) and cannot be represented by a line, but only by a four-dimensional space; in short, objective “time” is not one-, but four-dimensional. Furthermore for some of the pairs of events no absolute temporal distance at all exists (for such events any temporal distance whatsoever is valid for some possible observer).

But not only as to the structure, but also as to the specific content of our idea of time—namely, the character of the “flowing” of time and of “change” of things in time—relativity theory gives the unambiguous answer that the objective correlate of our representation of time does *not* have this character. For that time elapses and change exists means, or at least implies, that at any moment of our existence only a certain portion of the totality of facts of which the world | is composed exists (and different 7 portions at different moments). Moreover, if there is to be real change in the things, not only apparent change for the observer, the word “exists” in the preceding sentence must mean existence of the facts in themselves (not just for me), i.e., the facts existing now must be the same no matter which observer, at the present moment, judges about the question. But for the “now” of relativity theory this is not the case; and a generally applicable objective concept of “now”, which would have this property, cannot be introduced at all within the framework of relativity theory (see the paragraph below about the definition of an absolute world time^f). Hence, if relativity theory gives a correct description of reality, the assumption that at any moment of time only a certain portion of the facts composing the world exists objectively is wrong, i.e., there exists no objective change and no objective lapse of time.¹⁴

This state of affairs matches surprisingly well with a certain passage of the *Critique of pure reason*, where Kant says that for beings with other forms of cognition “those modifications which we represent to ourselves as changes would give rise to a perception in which the idea of time, and therefore also of change, would not occur at all” (B 54) (such as, e.g., the perception of an inclination of the world-lines in a certain place of Minkowski’s four-dimensional world, one is almost tempted to add).

¹⁴The unreality of change has been asserted frequently, also quite independently of the theory of relativity (already in antiquity), simply on the ground that the idea of change is self-contradictory (cf., e.g., *McTaggart 1908*). Indeed if change is real, “the present” seems to be the same thing as “the existing” (or at least “the existing” in the proper sense of the word, whereas past and future have only some weakened, shadowy form of existence). But this entails that we can, at different times, rightly assert about the same thing that it exists and that it does not exist.

^fSee the paragraph cited in fn. 13.

Summarizing the foregoing considerations we can say that what remains of time in relativity theory as an objective reality inherent in the things neither has the structure of a linear ordering nor the character of flowing or allowing of change. Something of this kind, however, can hardly be called time. So Kant's view about time *has* been confirmed by relativity theory at least | as to its negative part.

As to its positive part the similarity to relativity theory is less [pronounced]⁸ in the details, but still there exists a close affinity in principle. Time in the ordinary sense of the word (i.e., the time obtained from that immediately experienced by a simple process of idealization and extrapolation outside ourselves) is in both theories a relation to the observer, but in relativity theory it is a very simple relation to the sense organs of the observer (to be more precise it depends only on the place and state of motion of his sense organs or other observational apparatus); for Kant it is a relation to his "sensibility" (which presumably means that it depends on the special structure of his organs of sense and representation). Considering what has been said above on pages 3-4 (cf. in particular (7)) this difference between Kant and relativity theory is not so fundamental as it might at first seem because also for Kant the relation in question is to a large extent factual (e.g., physical), not merely cognitive. But still it doubtless means that the relativity of time is asserted in a much stronger sense by Kant than in relativity theory. A similar relationship exists between the two theories also in other respects (see what is said about space on page 13) so that it may be said in general that (as far as the nature of space and time is concerned) Kant and relativity theory go in exactly the same direction, but relativity theory goes only one step in the direction indicated by Kant.

But also as to the positive part of Kant's doctrine there really exists a deep-rooted affinity (not only a connection made up artificially). The linear ordering of all events by the time immediately experienced, e.g., really expresses a "relation of the things to our sensibility". For it comes about only by projecting the events on the world line of our body, i.e., on the line of immediate sensorial contact we have with the reality outside ourselves.

9

IV

The agreement described in the preceding pages between certain consequences of modern physics and a doctrine which Kant set up 150 years ago, in contradiction both to common sense and to the physicists and philosophers of his times, is greatly surprising, and it is hard to understand why so

⁸See the textual notes.

little attention is being paid to it in philosophical discussions of relativity theory. Quite on the contrary, as far as any relationship is at all noted, Kant is usually stamped (together with Newton) as an arch-champion of absolute time and space, and then either his doctrine or relativity theory is rejected, or some "reconciliation" is attempted. If however someone sees in relativity theory a confirmation of Kant, he nearly always means it in some entirely different sense, or he passes with a few words over the questions considered above.

Moreover, disregarding the relations to Kant, there is, with very few exceptions, a tendency to minimize the subjectivistic consequences of relativity theory. Much store is laid by the existence of the absolute relation of "before" mentioned above, but it is overlooked that this relation has a structure entirely different from that of the intuitive "before" and in particular does not permit of any objective lapse of time or objective change.

With much more justification, it is objected¹⁵ that the impossibility of defining any absolute time (among or besides the various relative ones) in the empty space-time scheme of special relativity theory (upon which the foregoing considerations have been based) does not exclude that matter and the curvature of space-time produced by it, | if the structure of the world as a whole is taken into account, may enable us to determine some objectively distinguished ordering of all events to which the properties contained in our intuitive idea of time could consistently be attributed, and compared to which the various observed times would appear as something like systematic errors due to motion of the observers. 10

This view is supported by the fact that in all known cosmological solutions (i.e., relativistically possible structures in the large of non-empty worlds) such an "absolute world time" really can be defined.¹⁶ But nevertheless the conclusions drawn above can be maintained because there exist other cosmological solutions¹⁷ for which a definition in terms of physical magnitudes of an absolute world time is demonstrably impossible. If, however, such a world time were to be introduced in these worlds as a new entity, independent of all observable magnitudes, it would violate the principle of sufficient reason, insofar as one would have to make an arbitrary choice between infinitely many physically completely indistinguishable possibilities, and introduce a perfectly unfounded asymmetry. Therefrom it

¹⁵See *Jeans 1936*, pp. 22–23.

¹⁶Prima facie only for a world in which matter is strictly homogeneously distributed, but this difficulty could very likely be overcome, e.g., by considering the homogeneous solution with the least total deviation from the given one.

¹⁷Not yet published result.^h

^hPublished in *Gödel 1949*, with supplementary results in 1952; cf. also *1949b in this volume.

follows that the possibility of a determination of an absolute world time, where it exists at all, is certainly not due to the laws of nature (which are satisfied in all cosmological solutions), but only to the special distribution and motion which matter has in those instances. A lapse of time, however, would have to be founded, one should think, in the laws of nature, i.e., it could hardly be maintained that whether or not an objective lapse of time exists depends on the special manner in which matter and its impulse are distributed in the world.

11 |

V

One reason for the neglect in contemporary philosophical literature of the relations set forth between Kant and relativity theory may lie in the fact that space and time occur on the same footing in Kant, whereas the foregoing considerations about time can be transferred to space only in a limited measure.

It is true that something like the space of Newtonian physics, which is supposed to exist beside and in complete independence of matter, likewise has no reality in relativity theory (its non-existence follows even from prerelativistic physics and the principle of the objective equality of states which cannot be distinguished by observations¹⁸). Kant however denied the objective existence not only of space and time, but also of spatial and temporal relations (B 42 a), and therein exactly consists the novelty of Kant's view. That time and space have no existence independent of and beside the things was asserted already by Leibniz.

Now as to time this view of Kant's has been verified to a large extent by relativity theory insofar as, in general, for some of the pairs of events no absolute temporal relation whatsoever exists in relativity theory, and for the remaining ones [[there exist]] relations of an entirely different nature and structure. But for space the situation is different. The spatial relations directly observed and measured, it is true, also have no absolute meaning, since they are different for different observers. But, in contradistinction to
12 | time, there also exist for any two material | objects (i.e., things persisting in time, not events) absolute spatial relations¹⁹ and they lead to a structure

¹⁸If there existed an absolute space, a state in which the whole world moves in a certain direction with respect to this space would be objectively different but experimentally indistinguishable from a state in which it is at rest.

¹⁹Of course every property or relation changing in time is, strictly speaking, not "inherent in the things", because it subsists only relative to a certain moment of time. But nevertheless the spatial relations under consideration are absolute as far as anything changing in time can be absolute, i.e., they are the same for different observers at moments of their respective times which correspond to each other in a certain way.¹

¹Written beside this footnote in shorthand was the word "falsch" (false).

(see below) not so very different from that of intuitive spatial relations. Nor do the latter include anything in their qualitative character which would contradict relativity theory (as does the idea of the flow of time).

In particular a "true spatial distance" of two points of the same or different material objects at a given moment of time (as opposed to the various apparent distances relative to various observers) can be defined (e.g., in case of not too large distances, by their distance in a coordinate system in which the centre of the line segment connecting them is at rest).²⁰ Moreover, in addition to the system of absolute spatial distances of the actually existing physical objects, there also exists (independent of the observer and the frame of reference) the schema of these distances for all *possible* movements of material points or (which is essentially the same) the schema of all possible movements of rigid bodies.²¹ These movements can be described by the world lines of all points of these bodies, and these world lines in their turn are described by their coincidences with or relative positions to the material points of the actually existing bodies. This schema is what may be called the objective physical geometry of the world, of which the various geometries existing for various observers are only different aspects.²²

²⁰In order to obtain a structure not too different from intuitive space, one may confine this definition to infinitely small distances. This leads to a Riemannian space changing in time, possibly with discontinuous g_{ik} , if the velocity distribution is discontinuous. This space is different for two observers A, B only insofar as the phases of development of its several parts (in consequence of the different meanings of simultaneousness for A and B) are collected together in different ways into one phase of the whole space. The asymmetry between time and space which shows up in the situation described is not due to the space-time scheme as such, but only to its content, namely, the fact that there exist world lines of material points and that they always go in time-like directions, whereas in general there do not exist any physically distinguished space-like three-dimensional hypersurfaces. It is worth mentioning that the extent to which it is possible to speak of objective spatial relations in relativity theory (namely, always for objects persisting in time, but in general not for events) is essentially the same as in classical physics supplemented by the principle mentioned above. The difference is only that in classical physics the "true distance" and the apparent distance relative to an observer always agree with each other. Whether perhaps even "absolute" points of space persisting in time and identifiable at different times, can always be introduced by reference to the world as a whole and the matter in it is not known. This question (which corresponds exactly to that about time discussed on pp. 9–10) is complicated by the circumstance that not even its meaning is quite clear. For, evidently, in order to obtain something not too far apart from the intuitive idea of "points of space", some requirement about an invariability (at least in some respect) of this absolute space is necessary. Otherwise points of space may simply be identified with material points.

²¹Strictly speaking only infinitely small bodies (i.e., in practice, bodies small with respect to the stars).

²²When, e.g., a man is included in a rotating box, the geometrical behaviour of rigid bodies changes for him only insofar as bodies at rest relative to him behave like bodies moving in a certain way in the former state and vice versa, and in general moving bodies behave like bodies moving in another way in the former state.

13 These absolute spatial relations, it is true, are not strictly isomorphic with the corresponding ones of intuitive space (insofar as, e.g., they are non-Euclidean), but these differences are insubstantial except for one point: namely, the finiteness of Riemannian space and of the straight lines in it disagrees *completely* with our spatial intuition. | [To speak of an ideality (i.e., subjectivity) of space, however, is hardly justified, unless the conditions prevailing among the things are fundamentally, not only slightly, different.] Note that even for temporal relations there exist in relativity theory certain marked similarities between the objective state of affairs and that subsisting relative to an observer, so that a still higher degree of subjectivity is conceivable. This applies in particular to the fact that for the series of events happening to one material point the “before” has always an objective meaning.²³

In the present imperfect state of physics, however, it cannot be maintained with any reasonable degree of certainty that the space-time scheme of relativity theory really describes the objective structure of the material world. Perhaps it is to be considered as only one step beyond the appearances and towards the things (i.e., as one “level of objectivation”, to be followed by others²⁴). Quantum physics in particular seems to indicate that physical reality is something still more different from the appearances than even the four-dimensional Einstein-Minkowski world.²⁵ T. Kaluza’s fifth dimension points in the same direction.²⁶

²³I.e., it subsists relative to all observers. Kant, incidentally, would not acknowledge this as a sufficient criterion for objectivity. Cf. B 72.

²⁴Cf. in this connection *Bollert 1921*, where one may find a description in more detail of these steps or “levels of objectivation”, each of which is obtained from the preceding one by the elimination of certain subjective elements. The “natural” world picture, i.e., Kant’s world of appearances itself, also must of course be considered as one such level, in which a great many subjective elements of the “world of sensations” are already eliminated. Unfortunately, whenever this fruitful viewpoint of a *distinction* between subjective and objective elements in our knowledge (which is so impressively suggested by Kant’s comparison with the Copernican system; see below p. 20) appears in epistemology, there is at once a tendency to exaggerate it into a boundless subjectivism, whereby its effect is annulled. Kant’s thesis of the unknowability of the things in themselves is one example, and another one is the prejudice that the positivistic interpretation of quantum mechanics, the only one known at present, must necessarily be the final stage of the theory.

²⁵Quantum physics has not yet succeeded in giving a satisfactory description of a physical reality which would make the success of its rules of computation understandable. Cf. *Einstein, Podolsky and Rosen 1935*, where it is proved that the wave function cannot represent the whole reality, unless an action at distance not diminishing with the distance is assumed. Unfortunately, this condition, which is necessary for the proof given, is not stated explicitly in the result.

²⁶See *Kaluza 1921*.

VI

14

There exists, in addition to the situation just discussed, a second circumstance which tends to conceal the close relationship which exists in some respects between relativity theory and Kant, namely, the fact that Kant, in addition to the subjectivity of time and space, also asserted the apriority of all our knowledge concerning them, and that he even based his main proof for their subjectivity on their apriority. Their apriority, however, seems flagrantly to contradict relativity theory, since the newly discovered properties of time and space (such as the relativity of simultaneousness, the non-Euclidicity of space, the Lorentz contraction, etc.), far from being a priori recognizable, even flatly contradict our supposed a priori intuition.

Closer examination however shows that the Kantian a priori is incompatible with relativity theory only in one minor point (made by Kant in only one rather obscure passage) and that Kant's thesis of the apriority of Newtonian time and Euclidean space as he meant it does not contradict relativity theory at all.

In the case of geometry, e.g., the fact that the physical bodies surrounding us move by the laws of a non-Euclidean geometry does not exclude in the least that we should have a Euclidean "form of sense perception", i.e., that we should possess an a priori representation of Euclidean space and be able to form images of outer objects only by projecting our sensations on this representation of space, so that, even if we were born in some strongly non-Euclidean world, we would nevertheless invariably imagine space to be Euclidean, but material objects to change their size and shape in a certain regular manner, when they move with respect to us or we with respect to them. | Nor does the non-Euclidicity of physical geometry (defined by the behaviour of rigid bodies) mean that this Euclidean pure intuition, if it exists, is simply wrong. For geometrical concepts satisfying Euclid's axioms can be defined also in a non-Euclidean world, although (*in surprising conformity with Kant's views*) in general only as *relations* to our sense organs (or some other arbitrarily chosen objects). 15

Such definitions can be obtained, e.g., in the following way: We select some physical object (e.g., the earth or our body), a point O on it, and three (preferably orthogonal) directions α , β , γ at this point. Then we define the "coordinates" a , b , c of any material point P by the number of times a measuring rod must be applied along the directions α , β , γ (respectively, the directions obtained by suitable parallel displacements) in order to reach P from O .²⁷ If then straight lines are defined by linear

²⁷In a closed (e.g., a Riemannian) world this definition of course would not give unique values for the coordinates, i.e., it would have the consequence that either every object is in infinitely many different places simultaneously, or that for every object there exist infinitely many exactly equal ones. But this undesirable consequence could be

equations in these coordinates, the Euclidean axioms for them are true irrespective of the physical properties of the world. The same, of course, is true for other concepts and axioms of Euclidean geometry.

Hence, if we should possess an innate intuition representing geometrical entities of this kind,²⁸ the axioms of this intuitive geometry, although referring to reality (namely, to measuring processes) would be a priori true.²⁹

16 Nevertheless, | they would not be analytic from Kant's standpoint, since arithmetic, to which geometry is reduced in this way, was not considered as analytic by Kant.³⁰ What, however, Kant did not take into account is that (irrespective of whether we have such an a priori intuition) space and its properties express themselves also in the sensations, which we know only a posteriori; namely, in the fact that by projecting the sensations in a certain way into a three-dimensional Euclidean space the laws connecting them can be stated much more easily. That this is not a necessary property of our sensations follows from the possibility of a physical non-Euclidean geometry and other considerations along these lines [which show that our sensations might be such that to project them into a three-dimensional Euclidean space, though possible, would be of no use for stating the laws governing them].

This insight does not necessitate any weakening of Kant's thesis of the subjectivity of space, since *the sensations also* are something subjective (based on the actions of the things on us) and therefore any of their properties may as well be due to subjective as to objective conditions, i.e., to properties of the senses as well as to properties of the objects. Nor does it

avoided by choosing other definitions. Only if we were to project our world, by the procedure described above, on a space of a higher dimensionality (e.g., four) would consequences of this kind become inevitable.

²⁸Of course I do not want to say that pure intuition according to Kant refers exactly to those geometrical entities which are defined above. Rather I only wanted to show that, surprisingly, an innate Euclidean geometrical intuition which is a priori valid and whose concepts have a well-determined physical meaning is possible and compatible with the existence and physical reality of non-Euclidean geometry and with relativity theory.

²⁹This seems to contradict Einstein's well-known aperçu about the relation between geometry and experience (cf. *Einstein 1921*), but the contradiction is rather one of words than of meaning, since the phrase "sich auf die Wirklichkeit beziehen" is evidently meant by Einstein in the sense of "say something about reality", whereas the geometrical axioms as interpreted above, although they refer to physical reality, do not say anything about it. Also, according to Kant, the geometrical propositions are synthetic, not because they say anything about that which is given (i.e., the sensations), or about the things in themselves, but because they say something about that which we, or rather our intellectual faculties, imagine (assume, construct) in the process of mental assimilation of the sensations.

³⁰Whether or not Kant was justified in this view depends on what is meant by analyticity which, on the basis of Kant's own definition, is by no means so unambiguous as it looks at first sight.

necessitate a weakening of his thesis of the apriority of space, since Kant did not assert that the “adequacy of our representation of space to the relation of the objects to our sensibility” can be known a priori.^j [That some particular relations of the things to us are adequately represented by Euclidean geometry in every world (cf. the definitions given above) certainly does not mean yet “a complete adequacy | to *the* relation of the things to our sensibility”.]

In point of fact the Kantian assertion about adequacy is factually (although not a priori) true in this sense, that the deviations from Euclidean geometry in the small are far below everything directly observable. [It *might* turn out to be true also in the more interesting sense that some Euclidean geometry defined relative to an observer (i.e., a world line) is in some way physically distinguished—in contradistinction to the Euclidean geometry defined above which is of no physical interest³¹ (very unlikely).]

One might moreover say that our supposed a priori intuition (irrespective of the question of its adequacy) would certainly be wrong in a strongly non-Euclidean world, at least insofar as it would represent Euclidean space, not as *our* scheme of arraying things (on the basis of relations of the things to *us*), but as a physical reality entirely independent of our existence. But it may be replied that this objectivation of space is a misrepresentation also by Kant’s own theory, no matter whether physical geometry is Euclidean or not; that Kant however avoided this consequence [that pure intuition is in this respect wrong] by laying the blame solely on our interpretation of the data of pure intuition.³²

I do not think the question whether, in accordance with Kant’s view, we have an *innate* (and therefore a priori) intuition of Euclidean space (i.e., whether we would develop the same intuition also in a strongly non-Euclidean world) has yet been decided; nor the related question whether we are able (in our world) to learn to imagine³³ a non-Euclidean space. [For

³¹This question is closely connected with that discussed in fn. 4. It is worth noting that for time the answer to the corresponding question is trivially in the affirmative, since the reasonably defined time relative to an observer is of course isomorphic with Newtonian time.

³²See B 69 and *Prolegomena*, §13, note III.

³³Usually it is not realized how greatly different a non-Euclidean spatial intuition would be from ours in case of a sufficiently large curvature of space. E.g., in a certain Lobachevskian space (which stretches into infinity like ours) the circumference of a circle of radius 1 inch would be 1 yard.

^jWhether this passage, as here rendered, genuinely reflects Gödel’s considered opinion is very doubtful, in the light of the details of his manuscript text at this point and of its history. For a discussion, see the introductory note above, §7, especially pp. 224–226.

we can imagine this also in terms of regular changes of size and shape due to motion in a Euclidean space.³⁴] [For we would certainly not identify (in our intuitive world picture) the behaviour of | physically rigid bodies with geometry in every possible world, e.g., not in a world which is Euclidean for the most part, but has certain non-Euclidean spots.] But whatever the answers to these questions may be, there can certainly never result from them any incompatibility between Kant and relativity theory, but at most between Kant and psychology (or phenomenology) of sense perception.

VII

A real contradiction between relativity theory and Kantian philosophy seems to me to exist only in one point, namely, as to Kant's opinion that natural science in the description it gives of the world must necessarily retain the forms of our sense perception and can do nothing else but set up relations between appearances within this frame.

This view of Kant has doubtless its source in his conviction of the unknowability (at least by theoretical reason) of the things in themselves,³⁵ and at this point, it seems to me, Kant should be modified, if one wants to establish agreement between his doctrines and modern physics; i.e., it should be assumed that it is possible for scientific knowledge, at least partially and step by step, to go beyond the appearances and approach the world of things.³⁶

The abandoning of that "natural" picture of the world which Kant calls the world of "appearance" is exactly the main characteristic distinguishing modern physics from Newtonian physics. Newtonian physics, except for the elimination of secondary qualities (which in principle was known already to Democritus), is only a refinement, but not a correction, of this picture of the world; modern physics however has an entirely different character. This is seen most clearly from the distinction which has developed between "laboratory | language" and the theory, whereas Newtonian physics can be completely expressed in a refined laboratory language.

³⁴Cf., e.g., *Einstein 1921*.

³⁵There is of course no strict equivalence between these two tenets. There is, however, a certain affinity between them, insofar as the abandoning of our ordinary forms of thinking is at least a first step towards a knowledge of the things in themselves.

³⁶Cf. fn. 24.

VIII

20

So the trend of modern physics is in one respect opposed to Kantian philosophy. On the other hand, however, it should not be overlooked that the very refutation of Kant's assertion concerning the impossibility for theoretical science of stepping outside the limits of our natural conception of the world has furnished in several points a most striking confirmation of his main doctrine concerning this natural world picture, namely, its largely subjectivistic character, even as to those concepts which seem to constitute the very backbone of reality.

Moreover, it is to be noted that the possibility of a knowledge of things beyond the appearances is by no means so strictly opposed to the views of Kant himself as it is to those of many of his followers. For (1) Kant held the concept of things in themselves to be meaningful and emphasized repeatedly that their existence must be assumed, (2) the impossibility of a knowledge concerning them, in Kant's view, is by no means a necessary consequence of the nature of all knowledge, and perhaps does not subsist even for human knowledge in *every* respect. One may compare [with respect] to this point, e.g., what he says in the preface to the second edition of the *Critique of pure reason* (B xxi, Bxxii n. and Bxxvi n.) about the positive use of the *Critique*, not only for religious belief, but possibly also for knowledge (B xxvi n.). In the second of the passages cited above Kant even goes so far as to compare his subjectivistic theory of knowledge, as to its possible positive use, with Copernicus's explanation of the apparent motions of the planets by the motion of the observer, pointing out that this new viewpoint of Copernicus led to the discovery of the "hidden power connecting the structure of the universe, the Newtonian attraction".

Kant, it is true, wanted to base such knowledge on ethics, but, even as far as theoretical reason is concerned, he evidently did not want to say that nothing whatsoever can be asserted about the things in themselves. For Kant himself asserted, | e.g., that they exist, affect our sensibility, and do not exist in time and space, but that the ideas of time and space are completely adequate to their relationship to our sensibility. 21

Doubtless Kant would least of all have held an overstepping of the world of appearances to be possible, on the basis of conclusions drawn from experiments (already his thesis of the empirical reality of time and space seems to exclude this). But [with respect] to this point it is to be noted that, in perfect conformity with Kant, the experiments by themselves really do not force us to abandon Newtonian time and space as objective realities,³⁷ but

³⁷In special relativity theory, e.g., some arbitrarily chosen inertial system can be considered as representing absolute space and time (with all properties Newton attributed to them) and the varying observational results of observers moving with different velocities can be explained by the effect which motion relative to this absolute space has on the

only the experiments together with certain general principles, in particular the principle that two states of affairs which cannot be distinguished by observations are also objectively equal. Generally speaking, it can be said that relativity theory (especially general relativity theory) owes its origin, perhaps more than any other physical theory, to the consistent application of certain very general principles, and was only subsequently verified in its consequences by experience.

physical bodies and processes, in particular on the measuring instruments (effects which incidentally follow without any ad hoc hypothesis from the empirically verifiable electromagnetic equations of Maxwell and the very natural assumption that the constitution of physical bodies is based on electromagnetic forces or forces of a similar nature). It remains however entirely arbitrary which coordinate system is in this way distinguished as the absolute space-time scheme and this, by the principle formulated above, entails that absolute time and space do not exist.