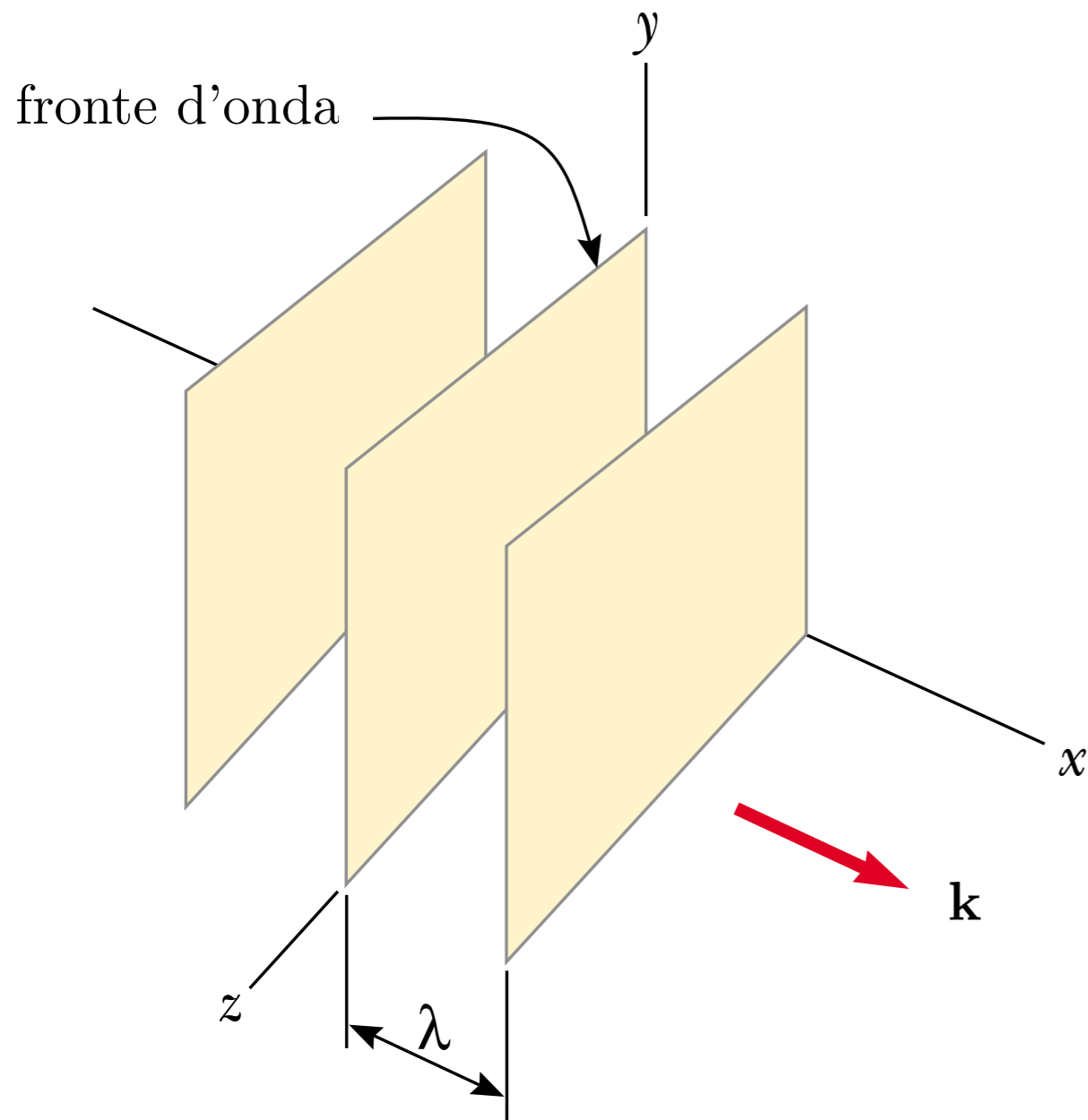


Diffrazione delle onde

Fisica Moderna

Onde piane



$$\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \Delta u$$

$$A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

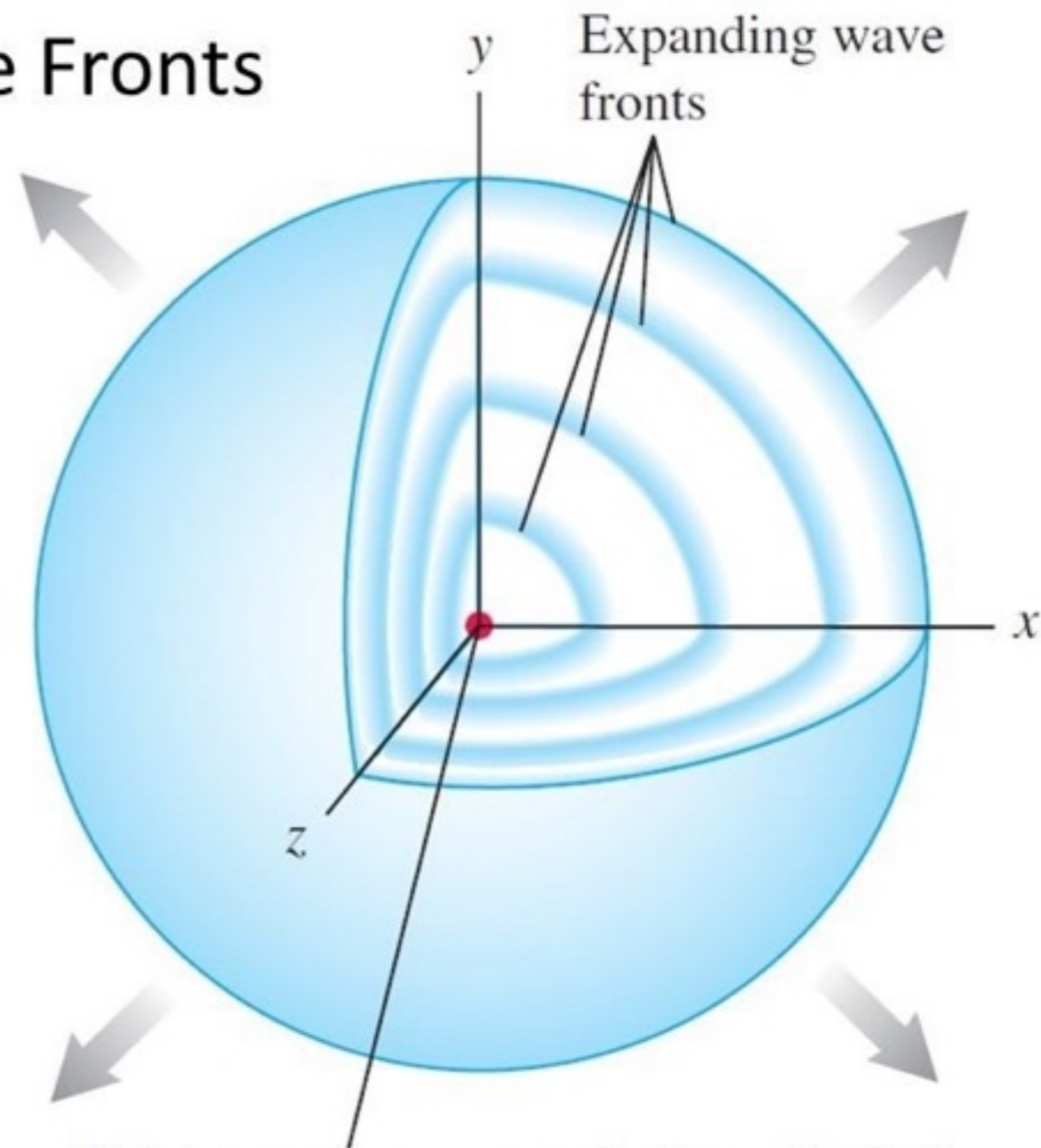
$$k^2 \equiv k_x^2 + k_y^2 + k_z^2$$

$$v_{\text{fase}} = \frac{\omega}{k} = c$$

Onde sferiche

Waves and Wave Fronts

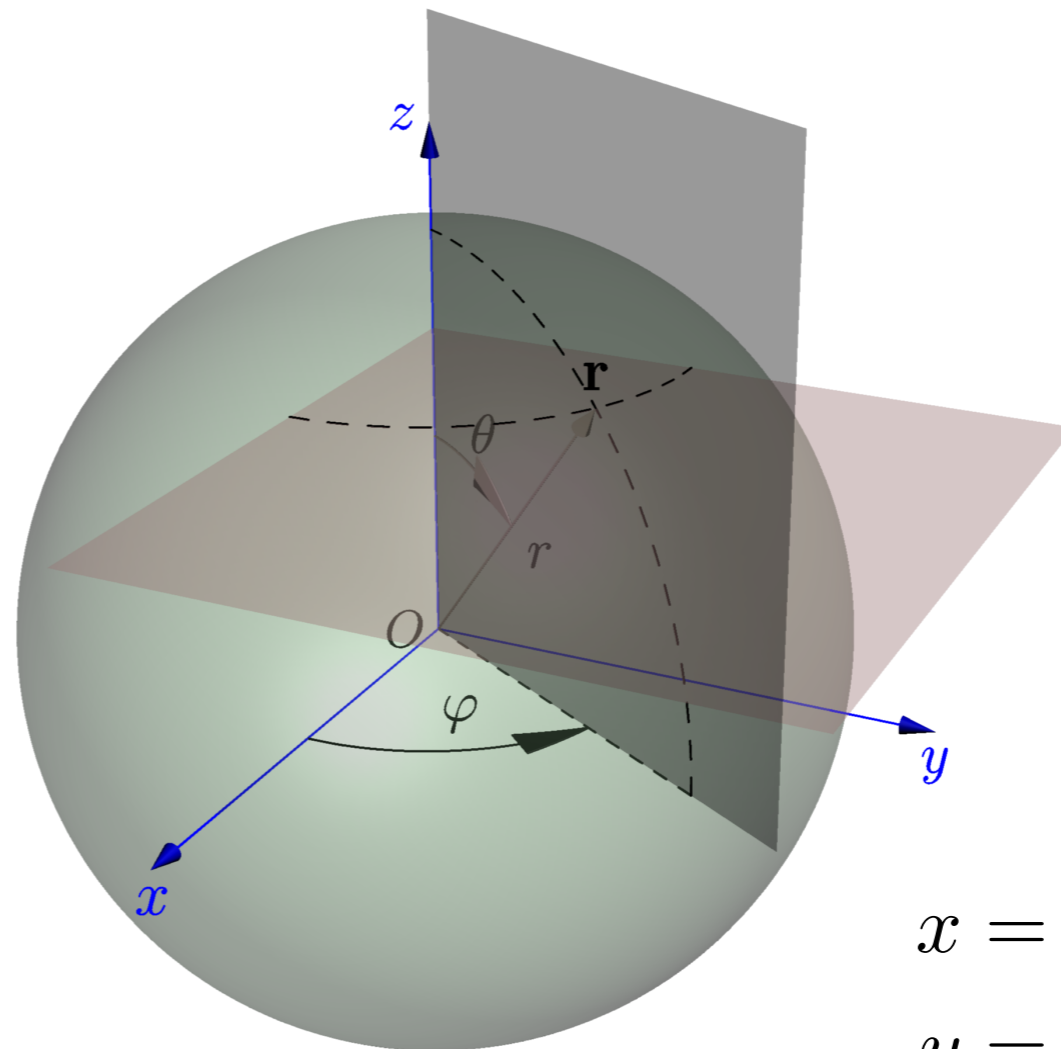
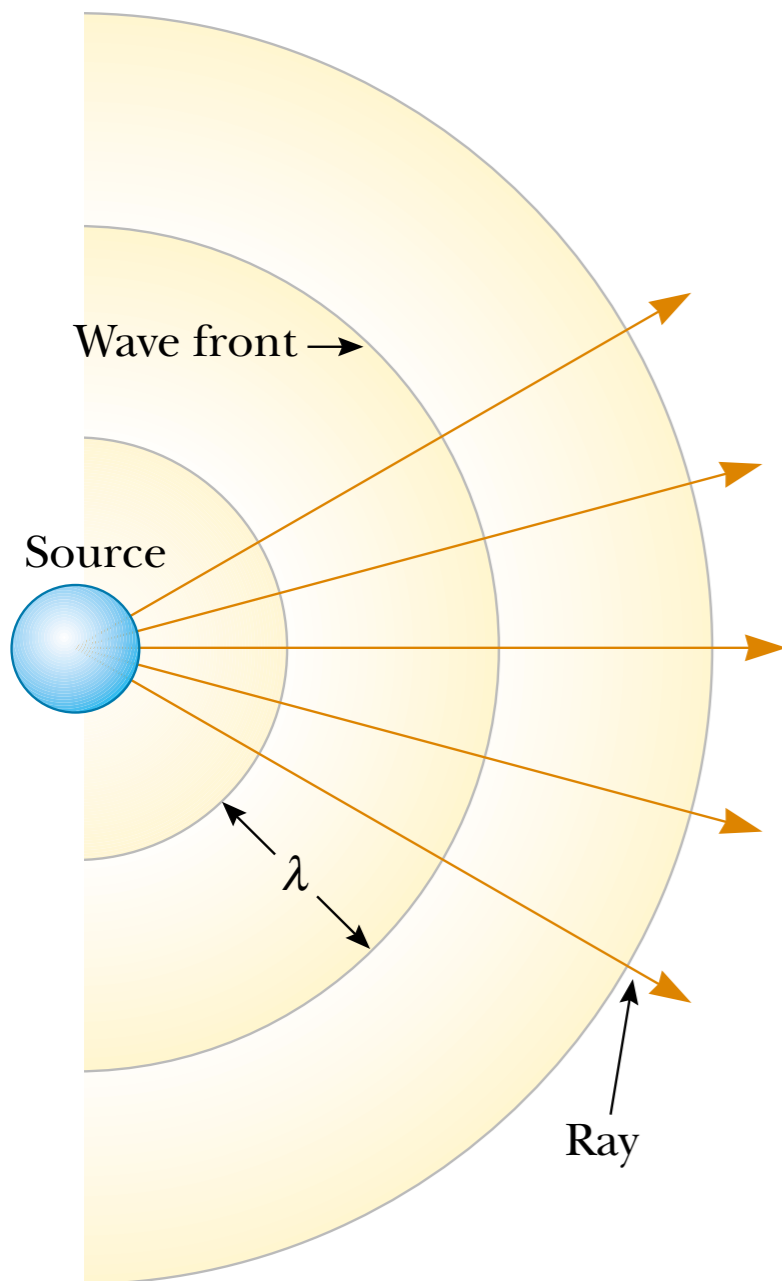
- A **wave front** is the locus of all adjacent points at which the *phase* of a wave is the same.
- Spherical wave fronts of sound spread out uniformly in all directions from a point source.
- Electromagnetic waves in vacuum also spread out as shown here.



Point sound source producing spherical sound waves (alternating compressions and rarefactions of air)

Onde sferiche

FIGURA 2. Coordinate sferiche. Superfici $r = \text{cost.}$, $\theta = \text{cost.}$, $\phi = \text{cost.}$.
L'intersezione di $r = \text{cost.}$ e $\phi = \text{cost.}$ definisce la curva delle θ , di $r = \text{cost.}$
e $\theta = \text{cost.}$, quella delle ϕ , di $\phi = \text{cost.}$ e $\theta = \text{cost.}$, quella delle r .



$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

Onde sferiche

Espressione del laplaciano Δ in coordinate sferiche

$$\Delta = \Delta_r + \frac{1}{r^2} \Delta_S$$

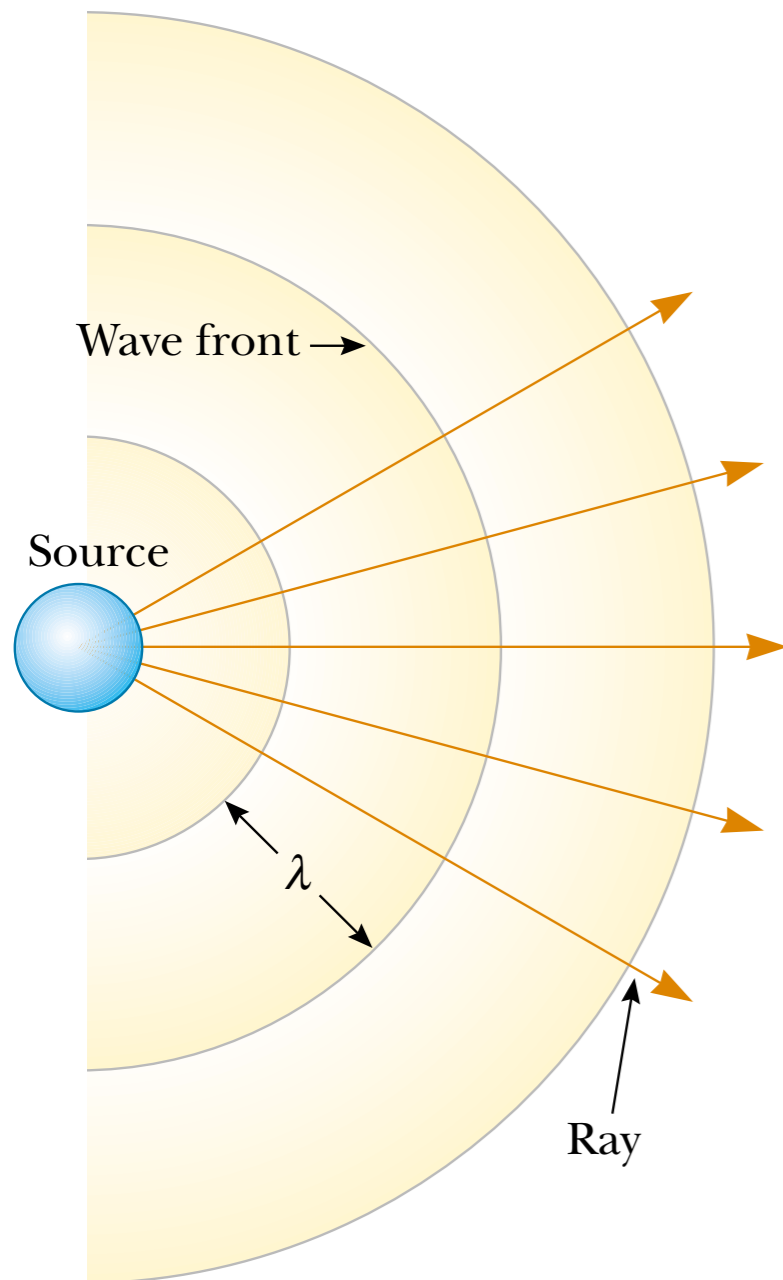
dove Δ_S è un operatore differenziale del secondo ordine che contiene solo derivate rispetto alle variabili angolari:

$$\Delta_S = \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

che è detto *laplaciano sferico*. Mentre il laplaciano radiale

$$\Delta_r = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} = \frac{1}{r} \frac{\partial^2}{\partial r^2} r$$

dipende solo da r .



Onde sferiche

L'equazione delle onde per $u(r, t)$ allora diventa

$$\frac{1}{r} \frac{\partial^2 r u}{\partial r^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

vale a dire

$$\frac{\partial^2 r u}{\partial r^2} = \frac{1}{c^2} \frac{\partial^2 r u}{\partial t^2}$$

Quindi ru è soluzione dell'equazione delle onde in una dimensione ed è quindi della forma

$$ru = f(r - ct) + g(r + ct)$$

In definitiva,

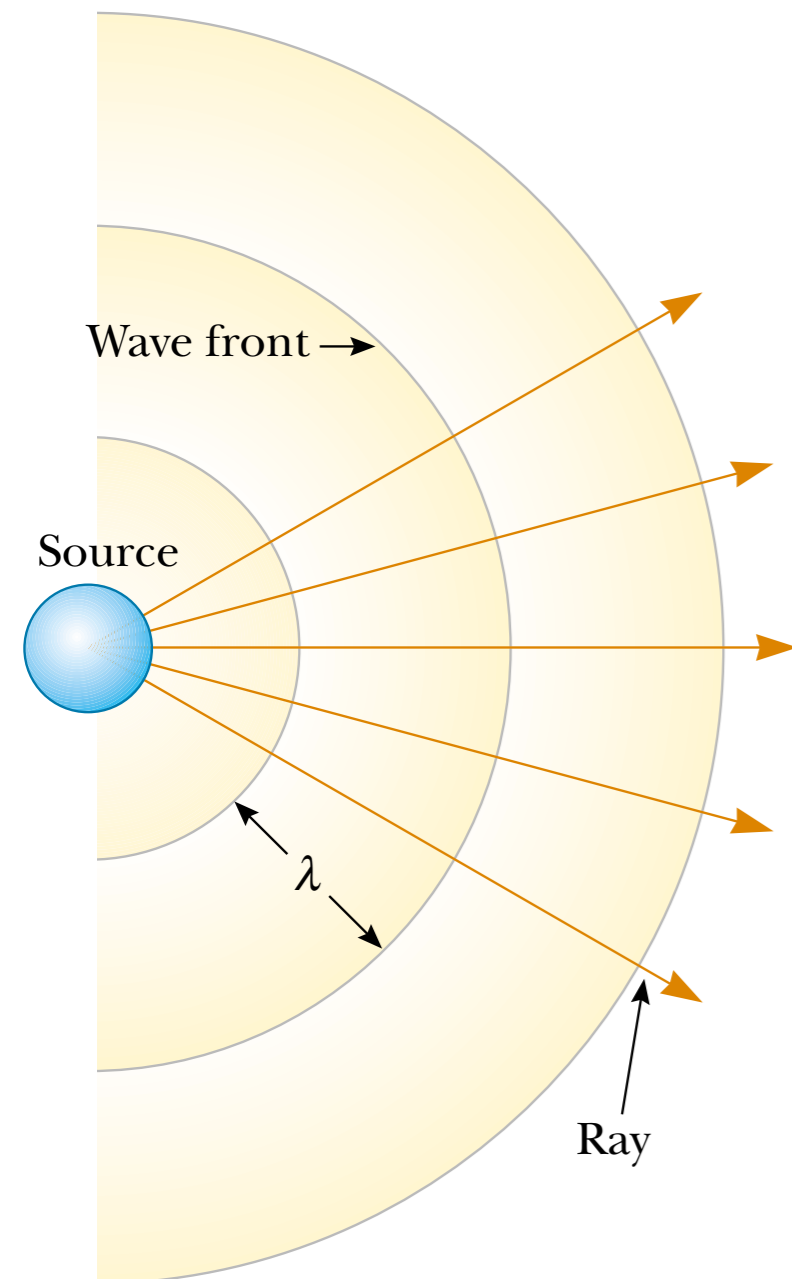
$$u(r, t) = \frac{f(r - ct)}{r} + \frac{g(r + ct)}{r}$$

Un'onda progressiva armonica di frequenza ω è quindi della forma

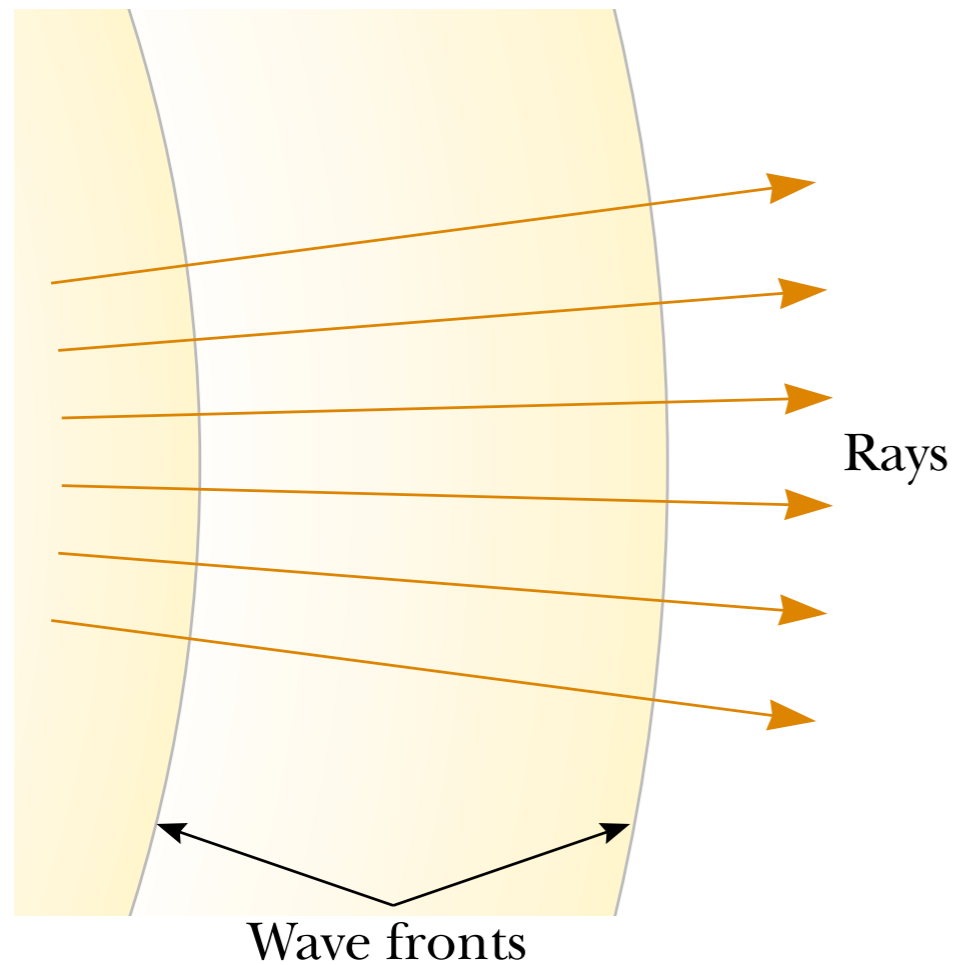
$$u_k(r, t) = u_0 \frac{e^{i(kr - \omega t)}}{r}$$

dove

$$k^2 \equiv k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

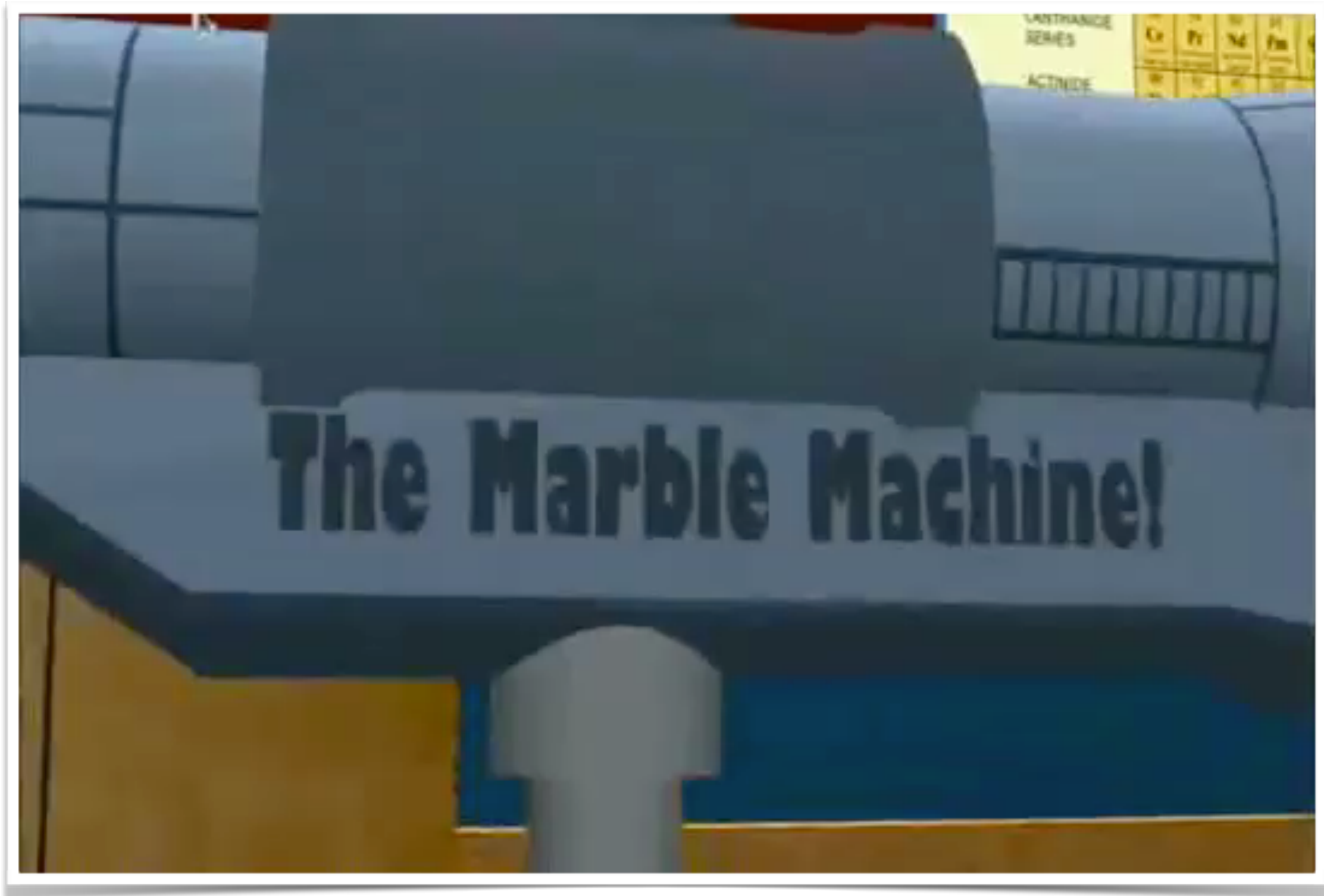


Onde sferiche

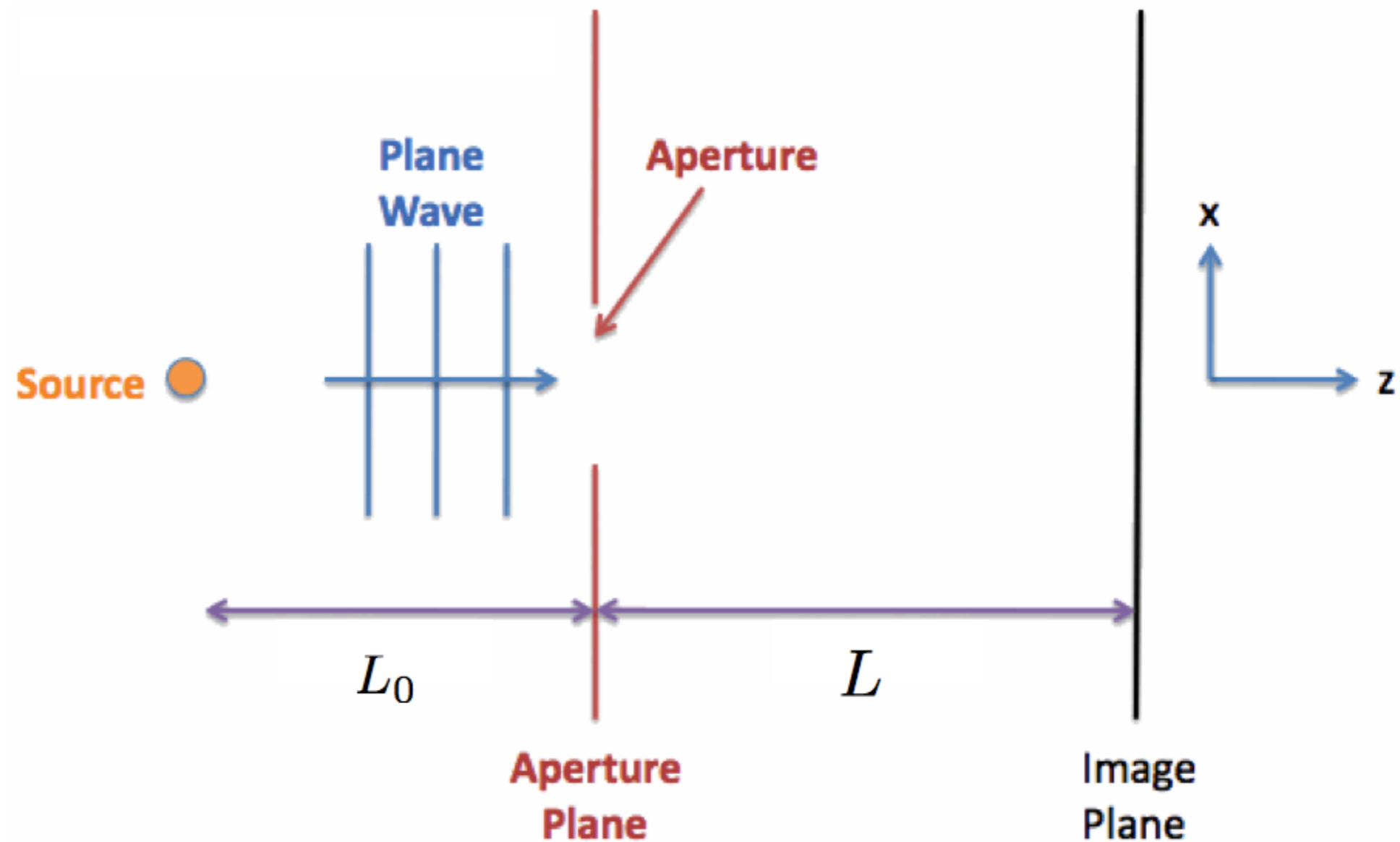


Far away from a point source, the wave fronts are nearly parallel planes, and the rays are nearly parallel lines perpendicular to the planes. Hence, a small segment of a spherical wave front is approximately a plane wave.

Diffrazione

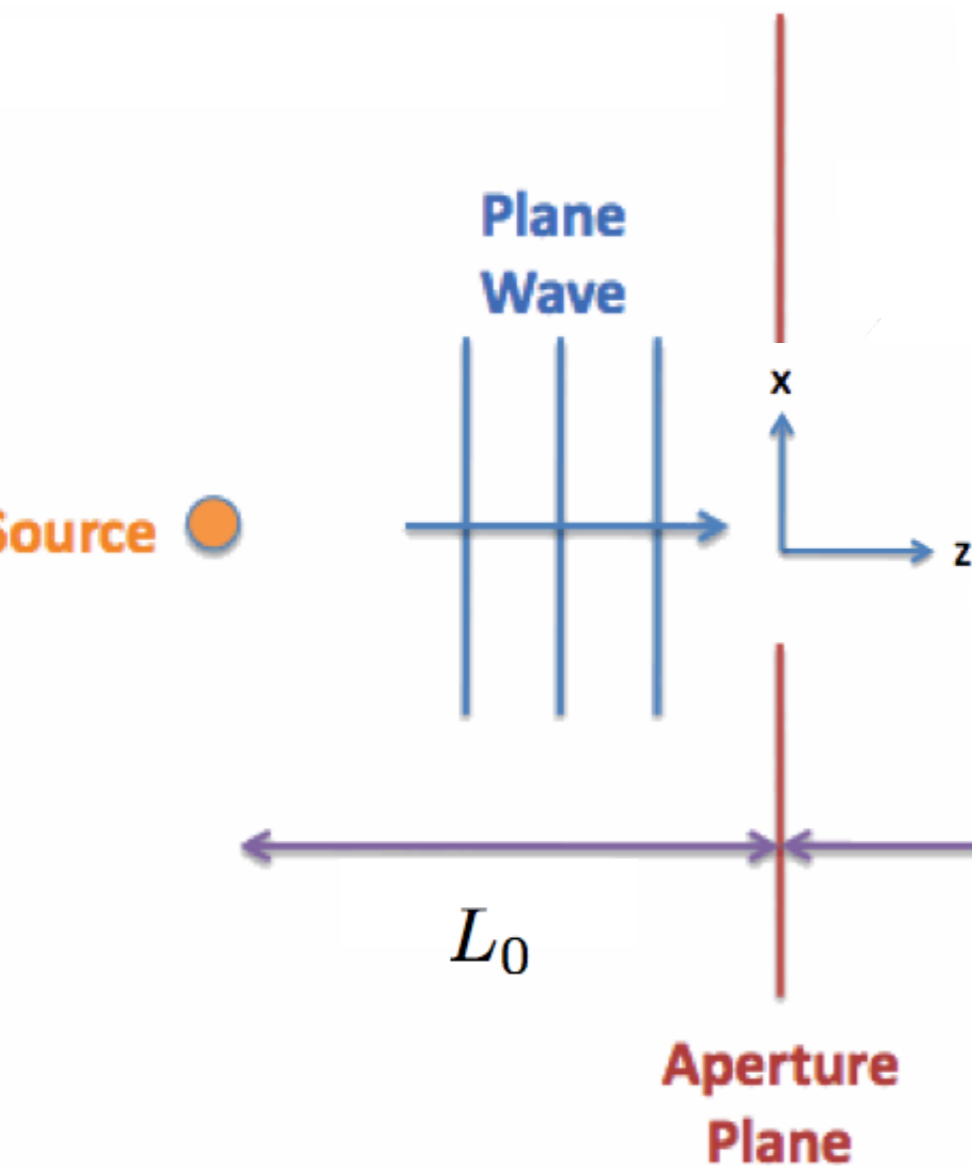


Diffrazione



Qual è il campo u sul piano dell'immagine, che si trova ad una distanza L più in giù lungo l'asse z ?

Diffrazione



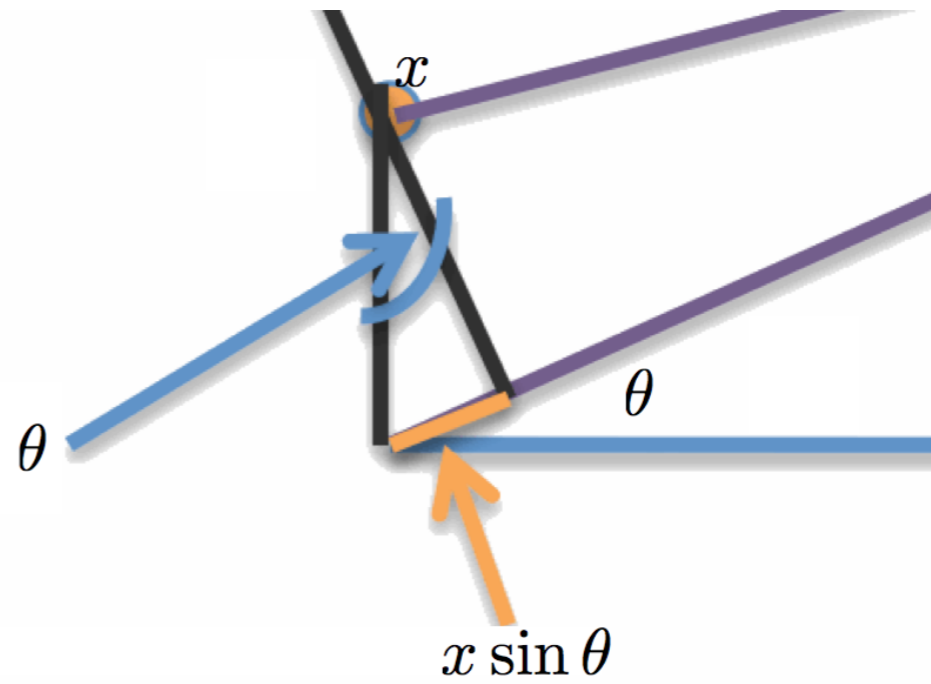
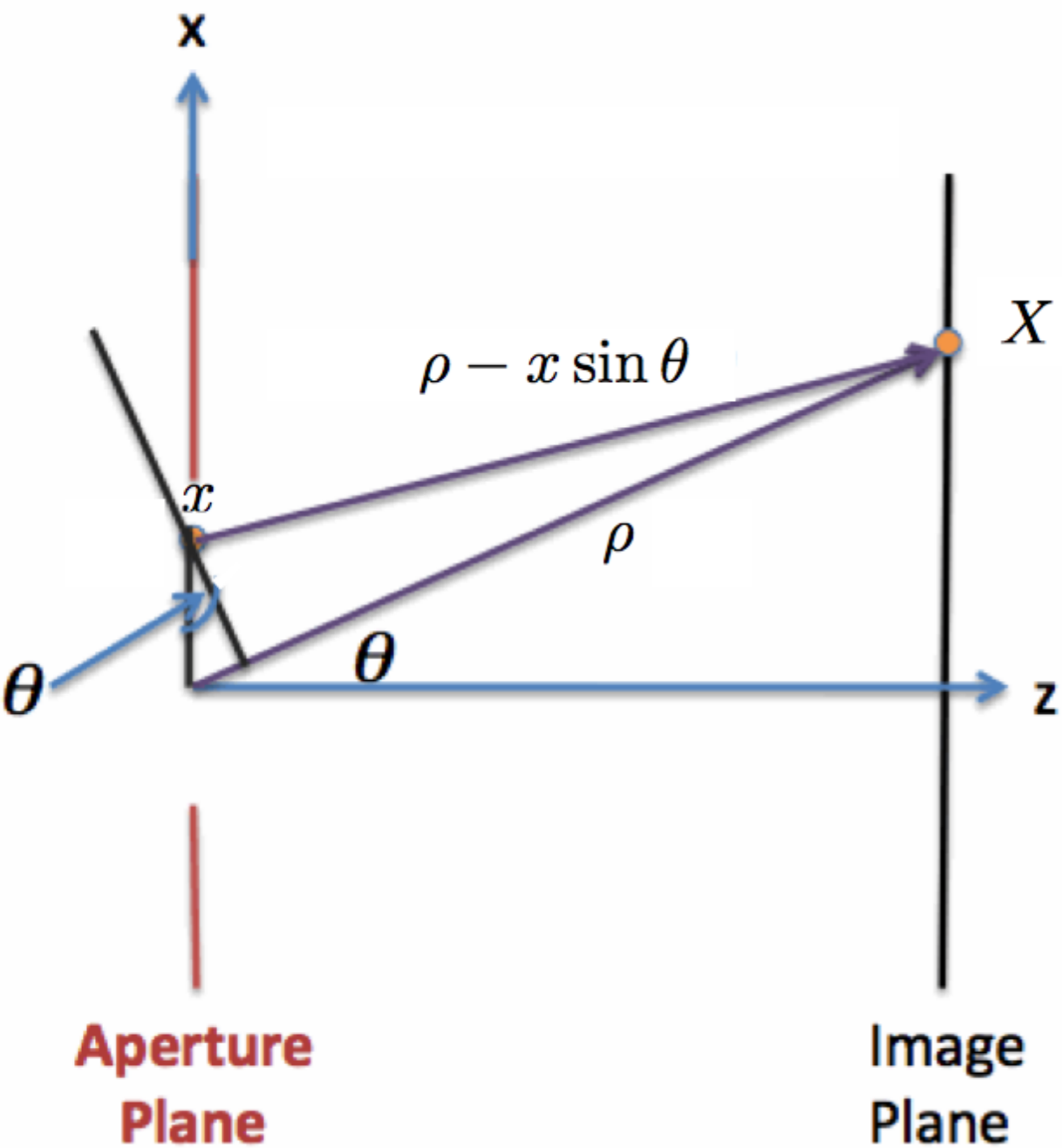
$$u = u_0 e^{ikz} e^{-i\omega t}$$

origine delle coordinate nel piano dell'apertura (p.e., al centro)

Quindi l'onda piana incidente sull'apertura è

$$u = u_0 e^{-i\omega t}$$

Diffrazione

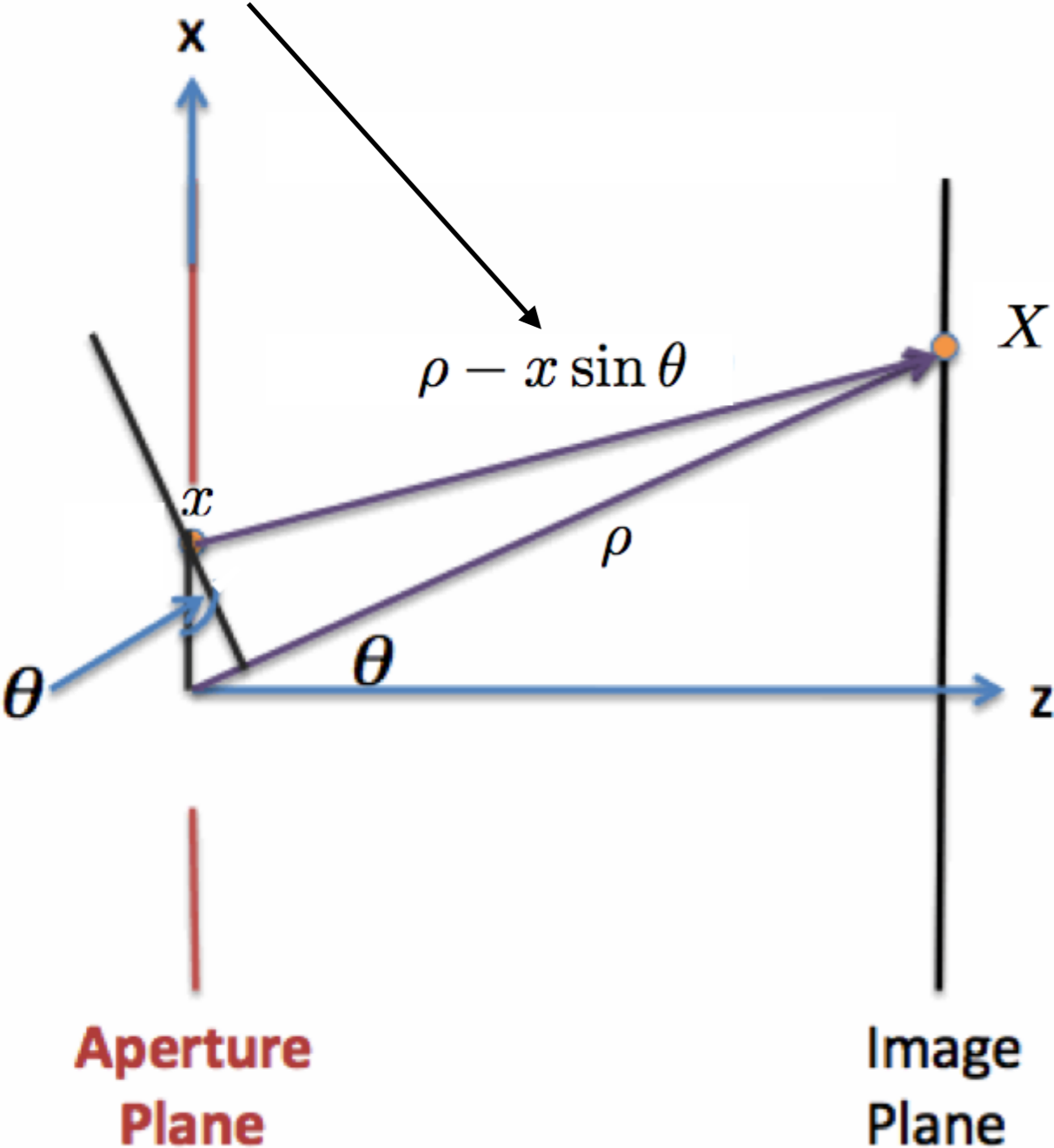


r : distanza tra x e X

$$r \approx \rho - x \sin \theta$$

Diffrazione

r : distanza tra x e X



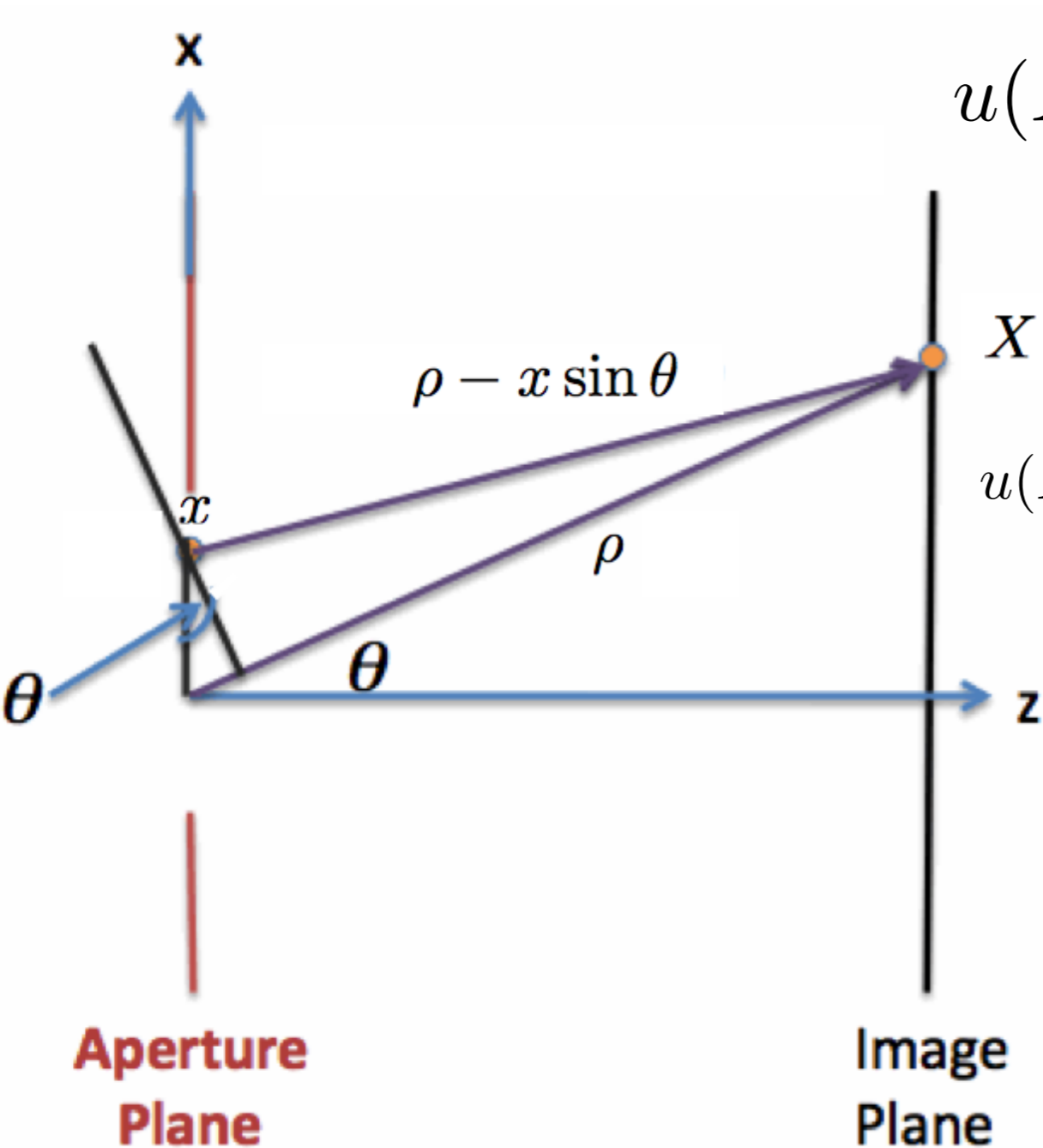
campo in X generato da sorgente in x

$$u = u_0 \frac{e^{i(kr - \omega t)}}{r} = u_0 e^{-i\omega t} \frac{e^{ikr}}{r}$$

campo in X generato da tutti i punti dell'apertura

$$u(X) = \int_{\text{apertura}} u \, dx$$

Diffrazione



$$u(X) = \int_{\text{apertura}} u \, dx$$

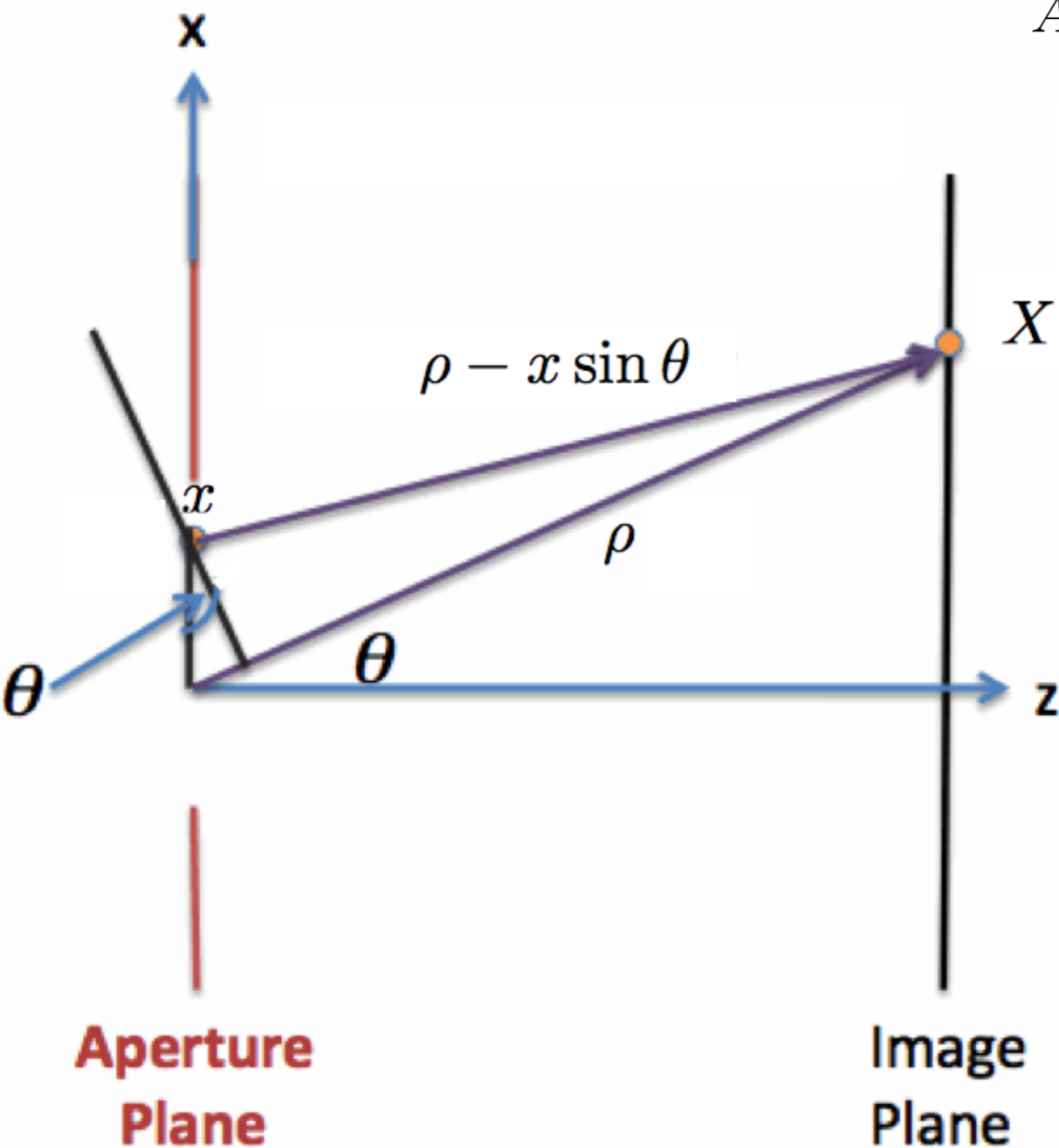
$$u(X) = u_0 e^{-i\omega t} \int_{\text{apertura}} \frac{1}{\rho - x \sin \theta} e^{ik(\rho - x \sin \theta)} dx$$

$$= u_0 e^{-i\omega t} \int_{-\infty}^{\infty} \frac{A(x)}{\rho - x \sin \theta} e^{ik(\rho - x \sin \theta)} dx$$

$$A(x) = \begin{cases} 1 & \text{per } -a \leq x \leq a \\ 0 & \text{altrimenti} \end{cases}$$

Diffrazione

$$A(x) = \begin{cases} 1 & \text{per } -a \leq x \leq a \\ 0 & \text{altrimenti} \end{cases}$$

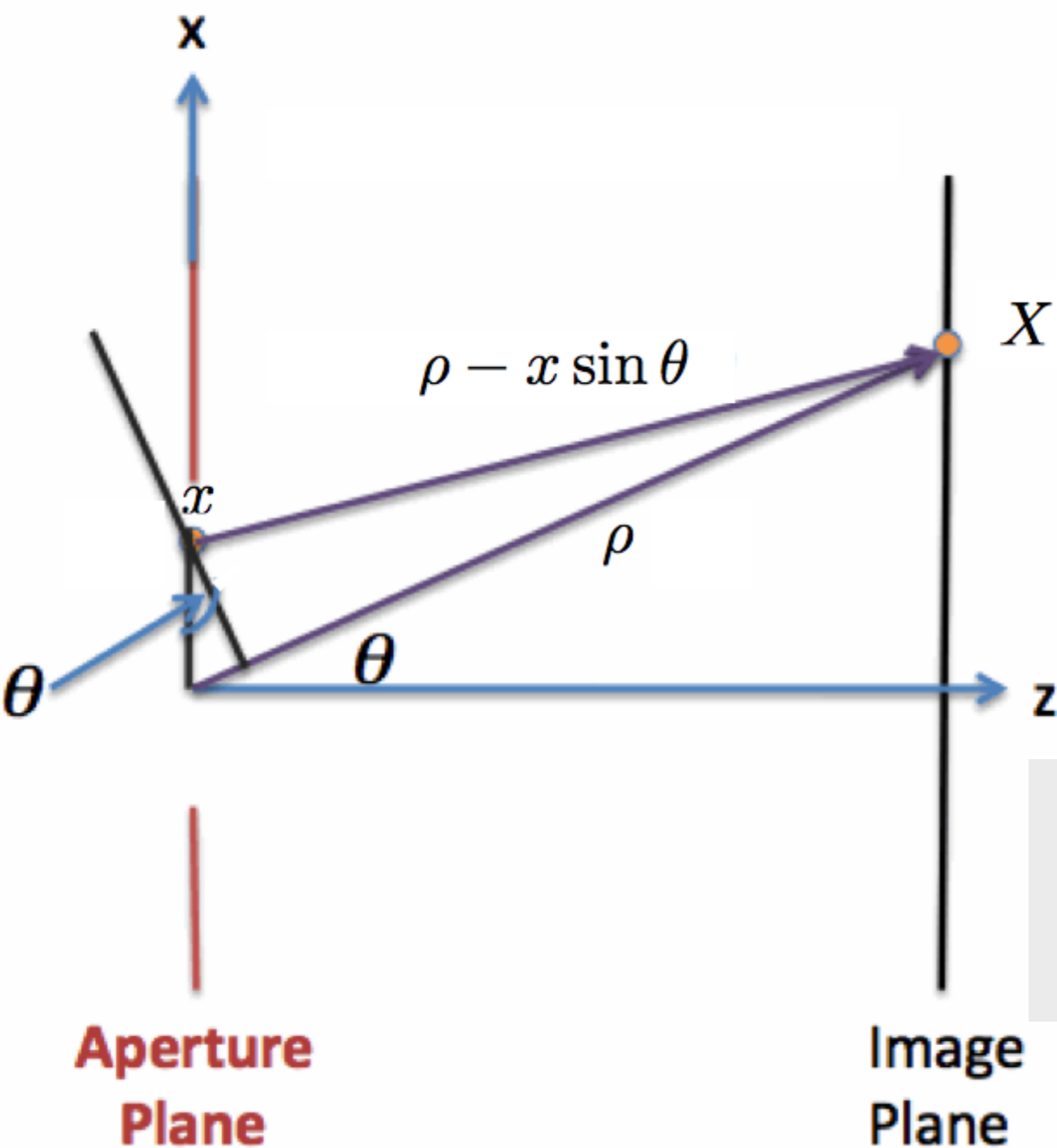


$$u(X) = u_0 e^{-i\omega t} \int_{-\infty}^{\infty} \frac{A(x)}{\rho - x \sin \theta} e^{ik(\rho - x \sin \theta)} dx$$

$$\frac{1}{\rho - x \sin \theta} \approx \frac{1}{\rho}$$

$$u(X) = u_0 \frac{e^{-i\omega t} e^{ik\rho}}{\rho} \int_{-\infty}^{\infty} A(x) e^{-ikx \sin \theta} dx$$

Diffrazione



$$u(X) = u_0 \frac{e^{-i\omega t} e^{ik\rho}}{\rho} \int_{-\infty}^{\infty} A(x) e^{-ikx \sin \theta} dx$$

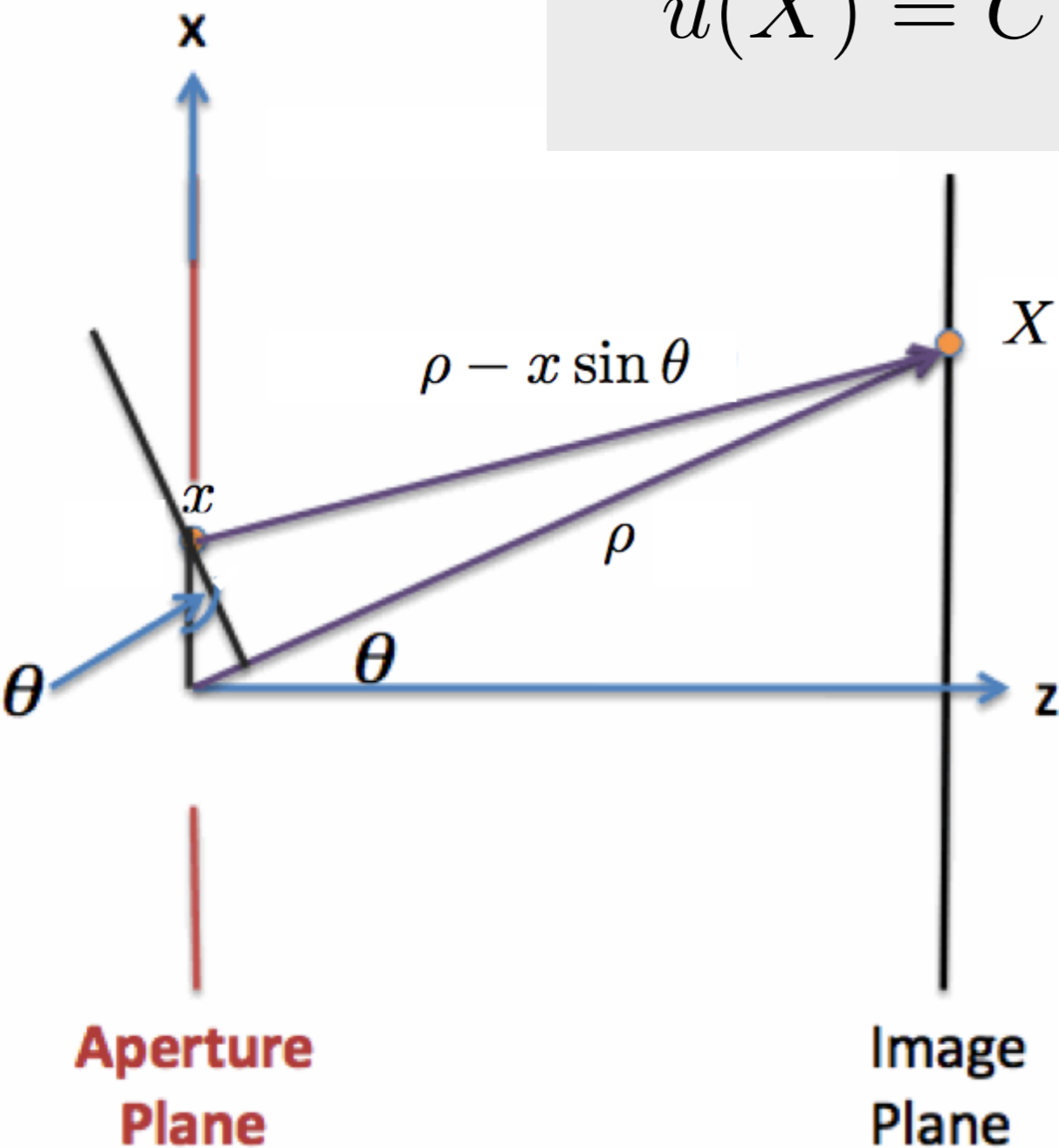
$$q = k \sin \theta$$

$$C = u_0 \frac{e^{-i\omega t} e^{ik\rho}}{\rho}$$

$$u(X) = C \int_{-\infty}^{\infty} A(x) e^{-iqx} dx$$

Diffrazione

$$u(X) = C \int_{-\infty}^{\infty} A(x) e^{-iqx} dx$$



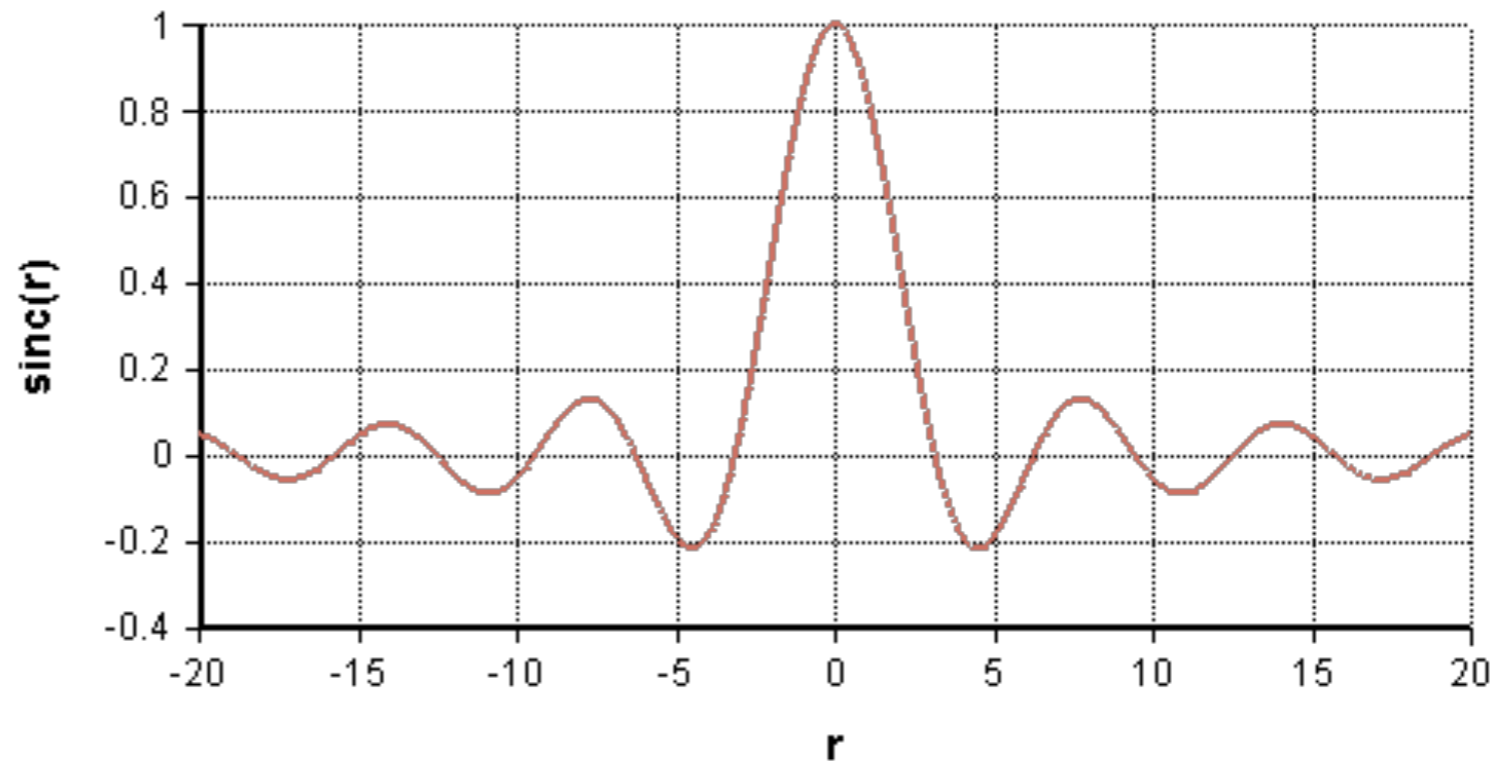
Abbiamo stabilito che i campi sul piano dell'immagine sono la **trasformata di Fourier** della funzione di apertura $A(x)$!

Quindi, la luce che passa attraverso una fessura, nelle giuste condizioni (monocromatica, campo lontano) produrrà la trasformata di Fourier della fenditura

Diffrazione

$$A(x) = \begin{cases} 1 & \text{per } -a \leq x \leq a \\ 0 & \text{altrimenti} \end{cases} \quad \hat{A}(q) = \int_{-a}^a e^{-iqx} dx = \frac{2}{q} \sin qa$$

$$\text{sinc } x \equiv \frac{\sin x}{x}$$



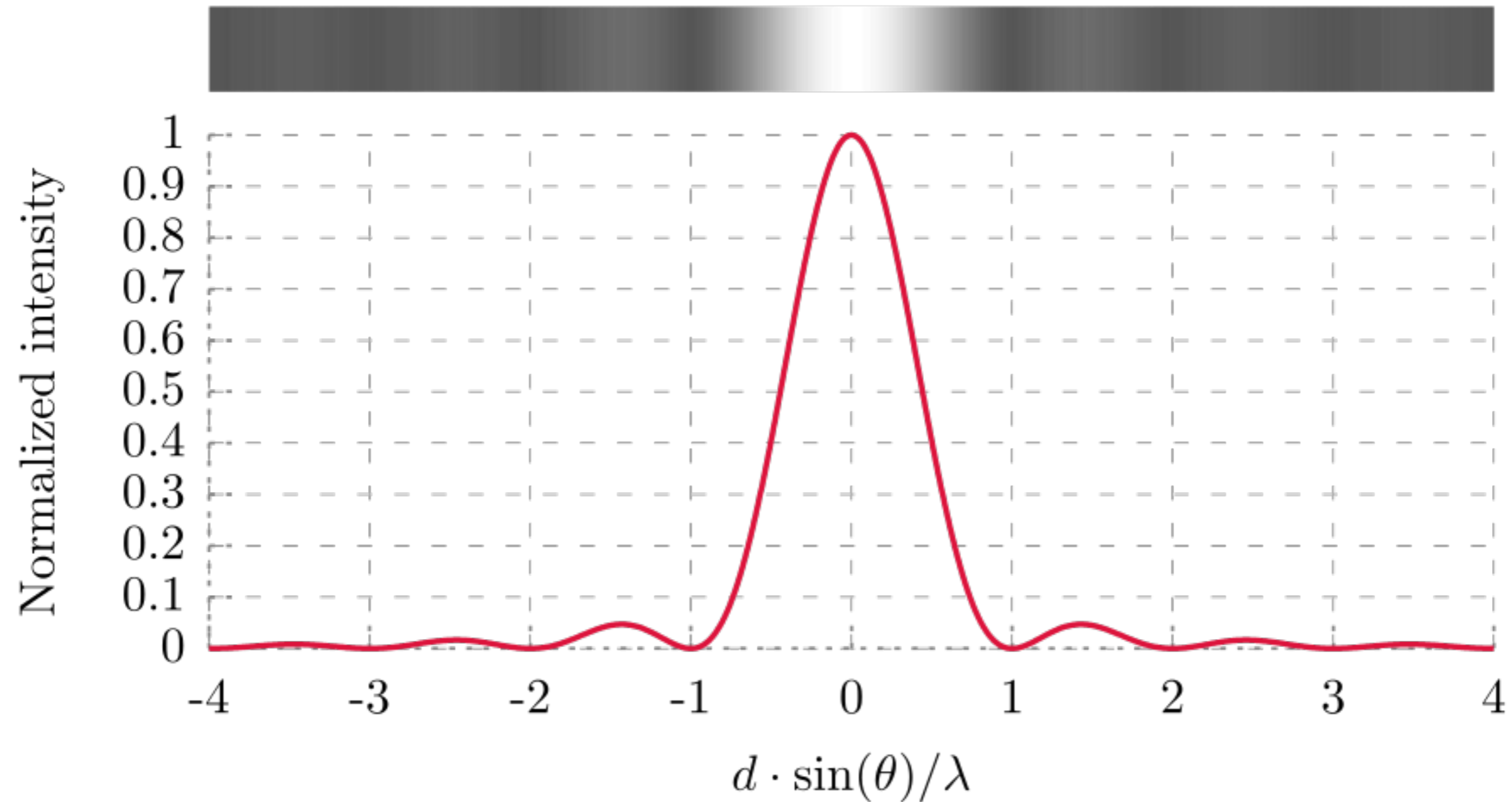
$$\hat{A}(q) = 2a \text{ sinc } qa$$

Diffrazione

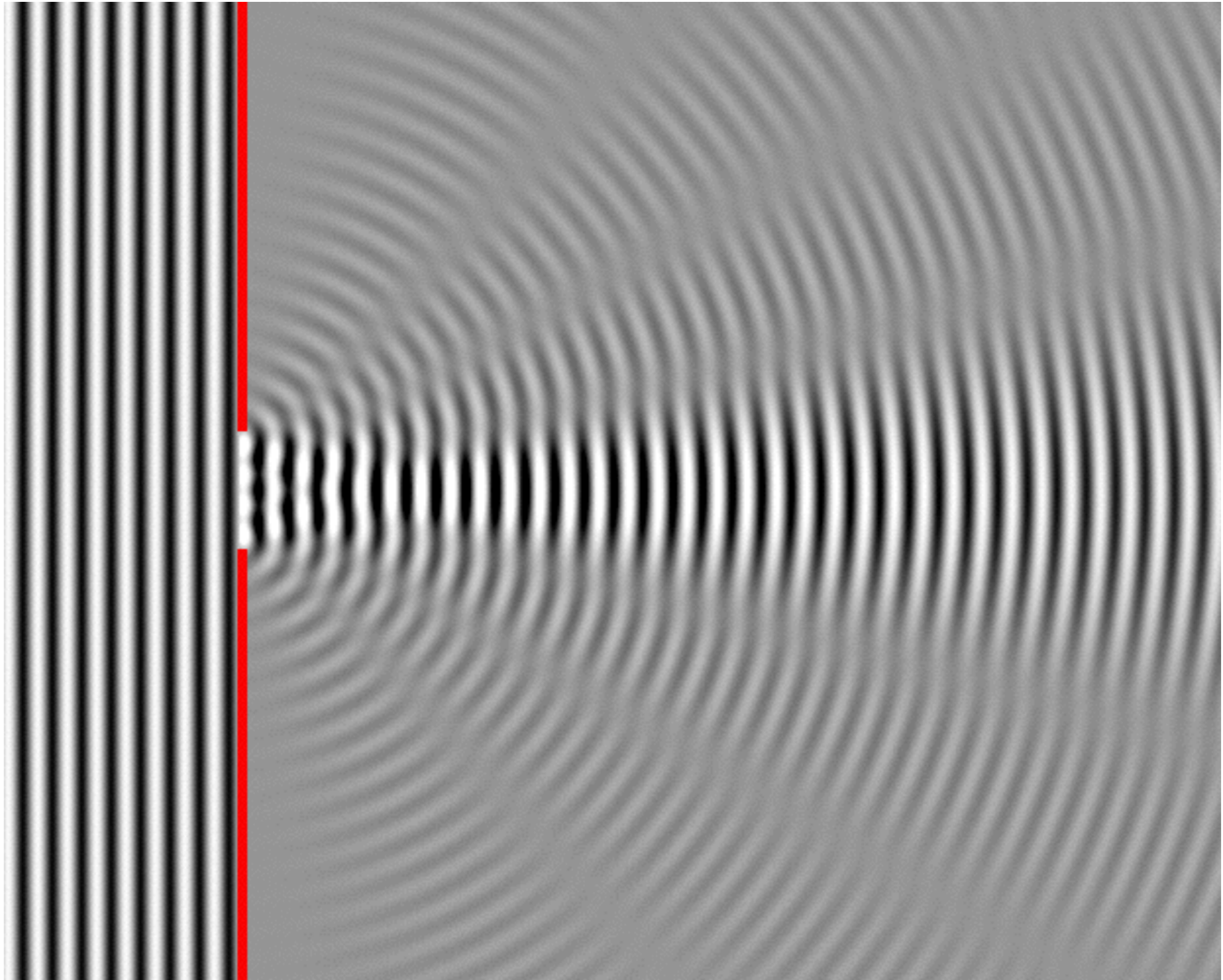
$a = d/2$, $d =$ apertura della fenditura

$$I(\theta) = I_0 \operatorname{sinc}^2 \left[\frac{d\pi}{\lambda} \sin \theta \right]$$

Diffrazione



Diffrazione



Esperimento delle due fenditure

