

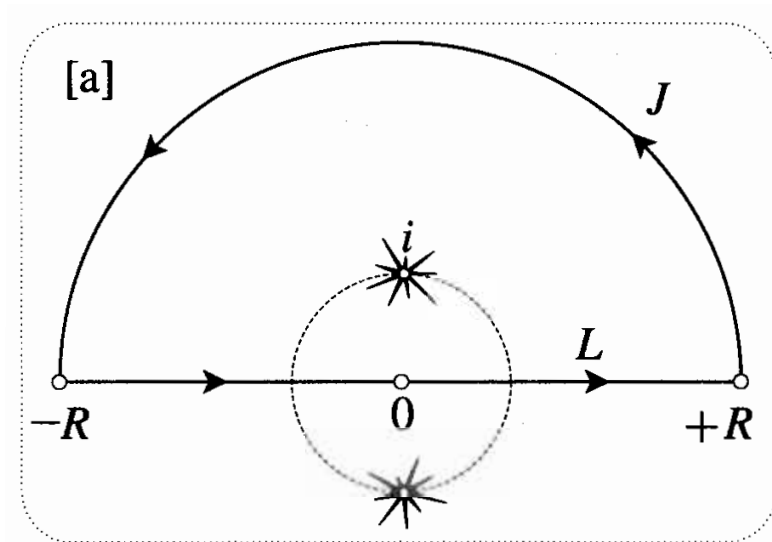
7.2. Calcolo di integrali reali con il metodo dei residui I.

(1)

$$\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^2}$$

Metodo 1

$$f(z) = \frac{1}{(z^2+1)^2} = \frac{1}{(z+i)^2(z-i)^2}$$



$$\int_{L+J} f(z) dz = 2\pi i \text{Res}[f, i] = 2\pi i \frac{d}{dz} \frac{1}{(z+i)^2} \Big|_{z=i} = 2\pi i \frac{-2}{2i^3} = \frac{\pi}{2}$$

$$\int_{L+J} f(z) dz = \int_{-R}^{+R} \frac{dx}{(1+x^2)^2} + \int_J f(z) dz$$

$$\int_J f(z) dz = \int_J \frac{1}{(R^2 e^{i2\theta} + 1)^2} i R e^{i\theta} d\theta \xrightarrow{R \rightarrow \infty} 0$$

$$\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^2} = \lim_{R \rightarrow \infty} \int_{L+J} f(z) dz = \frac{\pi}{2}$$

Metodo 2

È noto che

$$\int_{-\infty}^{+\infty} \frac{dx}{x^2 + a^2} = \left[\frac{1}{a} \tan^{-1} \left(\frac{1}{a} \right) \right] = \frac{\pi}{a}$$

$$\frac{d}{da} \int_{-\infty}^{+\infty} \frac{dx}{x^2 + a^2} = \frac{d}{da} \frac{\pi}{a}$$

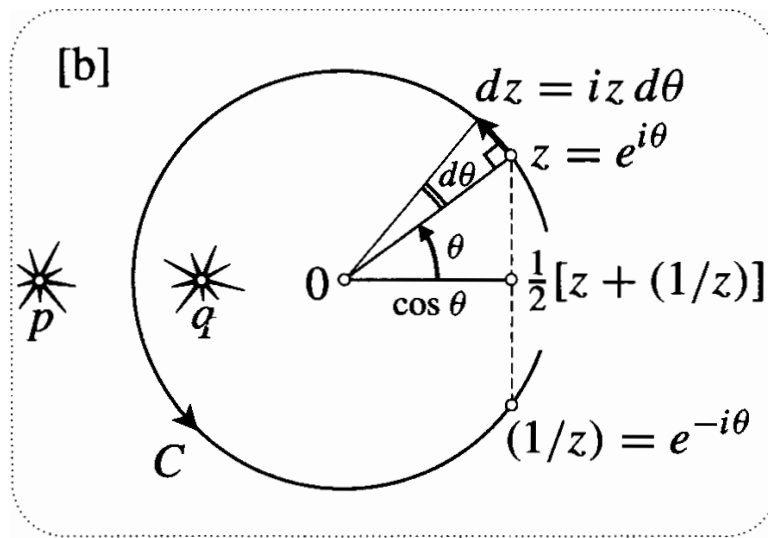
$$\int_{-\infty}^{+\infty} \frac{2adx}{(x^2 + a^2)^2} = \frac{\pi}{a^2}$$

per $a = 1$

$$\int_{-\infty}^{+\infty} \frac{dx}{(x^2 + 1)^2} = \frac{\pi}{2}$$

(2)

$$I = \int_0^{2\pi} \frac{d\theta}{\cos \theta + a}, \quad a > 1$$



$$I = \oint_C \frac{(dz/iz)}{\frac{1}{2}[z + (1/z)] + a} = -2i \oint_C \frac{dz}{z^2 + 2az + 1}.$$

Poiché le singolarità dell'integrando soddisfano $pq = 1$ solo una, q , sarà dentro C e quindi

$$I = 4\pi \text{Res} \left[\frac{1}{(z-p)(z-q)}, q \right] = \frac{2\pi}{\sqrt{a^2 - 1}}$$