Anderson-Darling two samples test

Anderson-Darling statistics A^2 measures the quadratic deviations between the empirical distribution function F_n of a sample and the empirical distribution function of a theoretical function F, multiplied by a weighting function $\psi(x)$:

$$A^2 = \int_{-\infty}^{\infty} [F(x) - F_n(x)]^2 \psi(x) \mathrm{dF}$$

In the case of Anderson-Darling test statistics the weigthing function is[1]:

$$\psi(x) = \frac{1}{F_n(x)(1 - F_n(x))}$$

When the comparison involves two samples, f and g with respectively n and m observations, the test statistics A^2 performes the comparison between the two empirical distribution function F and G, involving also the empirical distribution function combined sample H[2]:

$$A^2 = \frac{n+m-1}{(n+m)^2}[\frac{1}{n}\sum_{j=1}^{L} h_j \frac{((n+m)F_j - nH_j)^2}{H_j(n+m-H_j) - \frac{(n+m)h_j}{4}} + \frac{1}{m}\sum_{j=1}^{L} h_j \frac{((n+m)G_j - mH_j)^2}{H_j(n+m-H_j) - \frac{(n+m)h_j}{4}}]$$

where:

- h_j is the number of observations of the combined sample equal to z_j;
- H_j is the number of observations of the combined sample lower or equal to z_j plus one half of the values in the combined sample equal to z_j;
- F_j is the number of observations of the distribution f that are lower than z_j plus one half of observations in this group equal to z_j;
- G_j is the number of observations of the distribution g that are lower than z_j plus one half of observations in this group equal to z_j;

[1] T.W.Anderson, D.A.Darling, *Asymptotic theory of certain goodness-of-fit criteria based on stochastic processes*, Annals of Mathematical Statistics, 1952.

[2] `http://www.itl.nist.gov/div898/software/dataplot/refman1/auxillar/andeksam.htm`

If the two samples have been drawn from the same population distribution, the cumulative distributions of both samples may be expected to be close to each other; if the two sample cumulative distributions differ, this suggests that the samples come from different populations.
The critical level for Anderson-Darling two sample statistics is:

$$A^2_{0.05} = 1.992743$$

Anderson-Darling test statistics can be applied for both binned and unbinned data. For its mathematical formulation of the weighting function $\psi(x)$, this test is sensitive on the distributions' tails.