
Fisz-Cramér-von Mises test

Cramér-von Mises statistics ω^2 measures the quadratic deviations between the empirical distribution function of a sample and the empirical distribution function of a theoretical function, multiplied by a weighting function $\psi(x)$:

$$\omega^2 = \int_{-\infty}^{\infty} [F(x) - F_n(x)]^2 \psi(x) dF$$

In the case of Cramér-von Mises test statistics the weighing function is:

$$\psi(x) = 1$$

Fisz generalised Cramér-von Mises test statistics to the specific case of the comparison of two samples.

The aim of this test is to compare two distributions, f and g with respectively n and m observations, in order to test whether the two independent samples involved have been drawn from the same population.

Deriving from Cramér-von Mises test statistics, the test statistics t computation involves the two empirical distribution functions F and G . If the two samples have been drawn from the same population distribution, the cumulative distributions of both samples may be expected to be close to each other; if the two sample cumulative distributions differ, this suggests that the samples come from different populations.

$$t = \frac{n \cdot m}{(n + m)^2} \left\{ \sum_{i=1}^{n+m} [F_n(x_i) - G_m(x_i)]^2 \right\}$$

This test can be applied in the case of binned distributions; it is satisfactory in the case of symmetric distributions and of right-skewed distributions.

In the case of unbinned distributions, this test is not independent on the distribution parameters.

A table of the asymptotic distribution is available in the paper by Anderson et Darling¹ and Anderson².

¹T.W.Anderson, D.A.Darling, *A test of goodness-of-fit* American Statistical Association Journal, 1954.

²T.W.Anderson, *On the distribution of the two-sample Cramér-von Mises criterion*, Annals of Mathematical Statistics, 1962.