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## Kuiper test

Kolmogorov test<sup>1</sup> measures the maximum absolute deviation between the empirical distribution function of a sample and the empirical distribution function of a theoretical function:

$$D_n(x) = \sup_x |F(x) - F_n(x)|$$

Kolmogorov defined also the directional deviations:

$$D_n^+(x) = \sup_x [F(x) - F_n(x)] \quad D_n^-(x) = \sup_x [F_n(x) - F(x)]$$

Kuiper test statistics  $V_n$  involves both  $D_n^+$  and  $D_n^-$ , *i.e.* the maximum deviation above and below of the two empirical distribution functions:

$$V_n(x) = D_n^+(x) + D_n^-(x) = \sup_x [F_n(x) - F(x)] + \sup_x [F(x) - F_n(x)]$$

Its asymptotic distribution is:

$$P(V > V_{obs}) = Q_{KP} \cdot \left( [\sqrt{n} + 0.155 + \frac{0.24}{\sqrt{n}}] \cdot V \right)$$

where:

$$Q_{KP} = 2 \sum_{i=1}^{\infty} (4i^2 \lambda^2 - 1) \cdot e^{-2i^2 \lambda^2}$$

Kuiper statistics is valid also when the comparison involves two samples,  $f$  and  $g$  with respectively  $n$  and  $m$  observations. In this case Kuiper test statistics  $V_{n,m}$  involves the signed deviations between the two empirical distribution functions  $F$  and  $G^2$ :

$$V_{n,m}(x) = \sup_x [F_n(x) - G_m(x)] + \sup_x [G_m(x) - F_n(x)]$$

Its asymptotic distribution is:

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<sup>1</sup>A.Kolmogorov, *On the empirical determination of a distribution function*, Giornale dell'Istituto Italiano degli Attuari, 1933.

<sup>2</sup>N.H.Kuiper, *Tests concerning random points on a circle*, Proc. Koninkl. Neder. Akad. van Wetenschappen, 1960.

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$$P(V > V_{obs}) = Q_{KP} \cdot \left( \left[ \sqrt{\frac{nm}{(n+m)}} + 0.155 + \frac{0.24}{\sqrt{\frac{nm}{(n+m)}}} \right] \cdot V \right)$$

where:

$$Q_{KP} = 2 \sum_{i=1}^{\infty} (4i^2 \lambda^2 - 1) \cdot e^{-2i^2 \lambda^2}$$

In both cases, this test can be applied if the observations are cyclic, on a circle, because the test statistics is independent on the choice of the origin. For this reason, this test is invariant for cyclic transformations of the independent variable. Thanks to  $V_n$  and  $V_{n,m}$  definitions, the test as sensitive to the tails as to the median<sup>3</sup>.

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<sup>3</sup><http://encyclopedia.thefreedictionary.com/Kuiper's%20test>