Kuiper two-samples test

Kolmogorov test[1] measures the maximum absolute deviation between the empirical distribution function F_n of a sample and the empirical distribution function of a theoretical function F:

$$D_n(x) = sup_x|F(x) - F_n(x)|$$

Kolmogorov defined also the directional deviations:

$$D_n^+(x) = sup_x[F(x) - F_n(x)]$$
$$D_n^-(x) = sup_x[F_n(x) - F(x)]$$

Kuiper test statistics V_n involves both D_n^+ and $D - n^-$, *i.e.* the maximum deviation above and below of the two empirical distribution functions:

$$V_n(x) = D - n^+(x) + D_n^-(x) = sup_x[F_n(x) - F(x)] + sup_x[F(x) - F_n(x)]$$

Its asymptotic distribution is:

$$P(V > V_{obs}) = Q_{KP}([\sqrt{n} + 0.155 + \frac{0.24}{\sqrt{n}}]V)$$

where:

$$Q_{KP} = 2\sum_{i=1}^{\infty}(4i^2\lambda^2 - 1)e^{-2i^2\lambda^2}$$

Kuiper statistics is valid also when the comparison involves two samples, f and g with respectively n and m observations. In this case Kuiper test statistics $V_{n,m}$ involves the signed deviations between the two empirical distribution functions F and G[2]:

$$V_{n,m}(x) = sup_x[F_n(x) - G_m(x)] + sup_x[G_m(x) - F_n(x)]$$

Its asymptotic distribution is:

[1] A.Kolmogorov, *On the empirical determination of a distribution function*, Giornale dell'Istituto Italiano degli Attuari, 1933.

[2] N.H.Kuiper, *Tests concerning random points on a circle*, Proc. Koninkl. Neder. Akad. van Wetenschappen, 1960.

$$P(V > V_{obs}) = Q_{KP}([\sqrt{\frac{nm}{(n+m)}} + 0.155 + \frac{0.24}{\sqrt{\frac{nm}{(n+m)}}}]V)$$

where:

$$Q_{KP} = 2\sum_{i=1}^{\infty}(4i^2\lambda^2 - 1)e^{-2i^2\lambda^2}$$

In both cases, this test can be applied if the observations are cyclic, on a circle, because the test statistics is independent on the choice of the origin. For this reason, this test is invariant for cyclic transformations of the independent variable. Thanks to V_n and $V_{n,m}$ definitions, the test as sensitive to the tails as to the median[3].
Deriving from Kolmogorov statistics, Kuiper test is based on the same assumptions and for this reason this test can be applied only on unbinned data.

[3]`http://encyclopedia.thefreedictionary.com/Kuiper's%20test`