Cramèr-Von Mises test in chi-squared approximation

Tiku (1965) developed a method to approximate Cramèr-Von Mises distance in terms of a central chi-square. This method permits to bypass the problem of the not obvious integration of a weighting function. The statistic

$$\omega^2 = n \cdot \int_{-\infty}^{\infty} [F(x) - F_n(x)]^2 dF$$

is used to test an hypothetical cumulative distribution function $F(x)$ given the sample cumulative distribution function $F_n(x)$ for a sample of size n. The exact sampling distribution under the null hypothesis of ω^2 for any sample size n is not known, Anderson et Darling (1952) have given the exact distribution in the limiting case when $N \to \infty$, while Marshall (1958) has given solution for $N = 1, 2$ and 3. Tiku obtained the approximation for the distribution:

$$\omega^2 = a + b\chi^2$$

where χ^2 is distribuited as central chi-squares with degrees of freedom f, and the parameters a,b and f are determined as function of n:

$$a = \frac{336n^2 - 959n + 609}{210(32n^2 - 61n + 30)}$$

$$b = \frac{32n^2 - 61n + 30}{84n(4n - 3)}$$

$$f = \frac{98}{5} \frac{n(4n - 3)^3}{(32n^2 - 61n + 30)^2}$$

On the same principle, we shall obtain an approximation for the distribution of index t in the Fisz-Cramèr-von Mises test for the comparison between two sample cumulative distribution function $F_n(x), G_m(x)$.

As illustrative example consider two binned distributions, the distance calculated by the Fisz-Cramer vonMises test is $t = 0.2980$. From the formulas, replacing the number of bins $n = 20$ to calculated the central chi-squared:

$$f = 1.658, \; b = 0.089, \; a = 0.048$$

so that

$$\chi^2 = \frac{0.2980 - 0.048}{0.089} = 2.806$$

$$P(t \geq 0.2980) \sim P(\chi^2 \geq 2.806 \,|f = 1.658) = 0.094$$

Tiku's approximation test can be performed on continuous variables or on binned data, the simplicity of calculation makes this test very useful.