



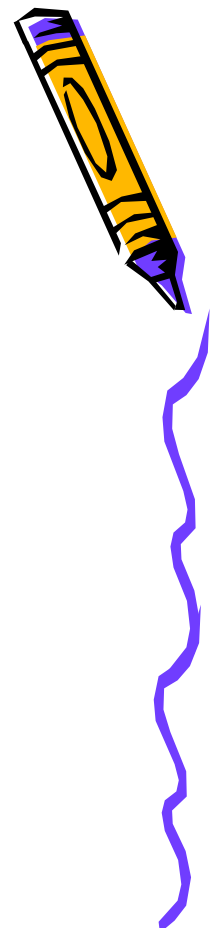
A general assistant tool for the checking results from Monte Carlo simulations

Koi, Tatsumi
SLAC/SCCS



Contents

- Motivation
- Precision and Accuracy
- Central Limit Theorem
- Testing Method
- Current Status of Development
- Summary



Motivation

- After a Monte Carlo simulation, we get an answer. However how to estimate quality of the answer.

What we must remember is

- Large number of history does not valid result of simulation.
- Small Relative Error does not valid result of simulation



Motivation (Cont.)

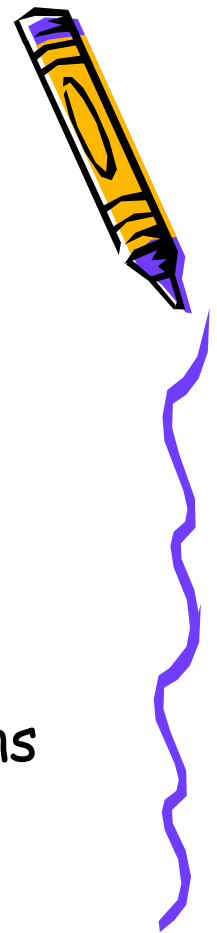
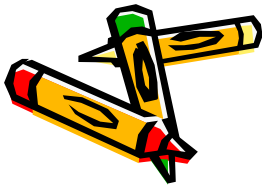
- To provide “statistical information to assist establishing valid confidence intervals for Monte Carlo results” for users, something like MCNPs did.



Subject of this study

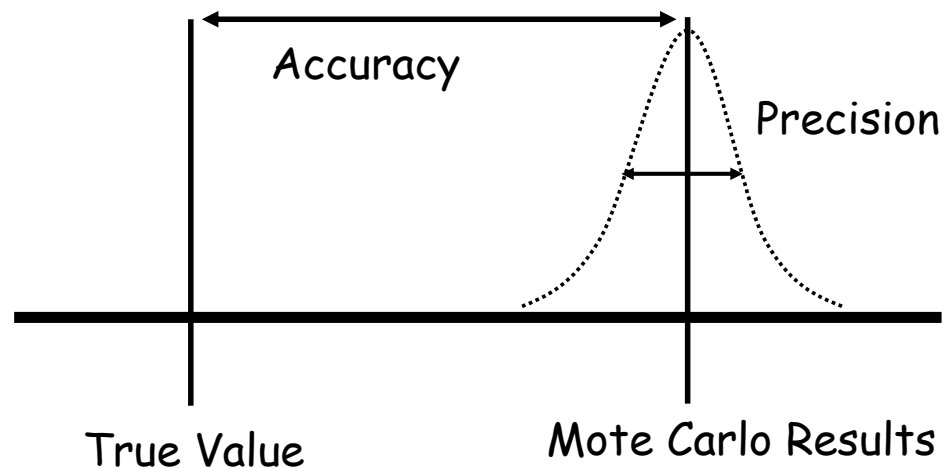
- Precision of the Monte Carlo simulation
- Accuracy of the result is NOT a subject of this study

At first we have to define
Precision and Accuracy of simulations



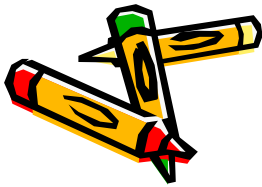
Precision and Accuracy

- Precision: Uncertainty caused by statistical fluctuation
- Accuracy: Difference between expected value and true physical quantity.



Subject of this study (Cont.)

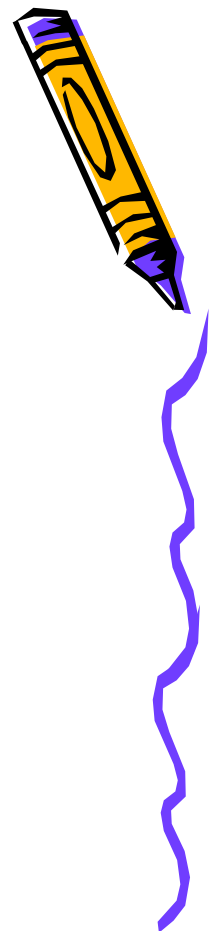
- Precision of the Monte Carlo simulation is subject of this study.
- To state accuracy of simulations, we should consider details of simulation, i.e., uncertainties of physical data, modeling of physical processes, variance reduction techniques and so on.
- To make a generalized tool, we have to limit subjects only for precision.



Accuracy is a subject for most of presentations in this workshop.

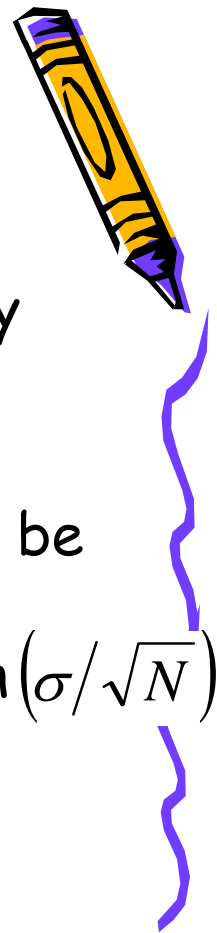


Principal of this study is
Central Limit Theorem



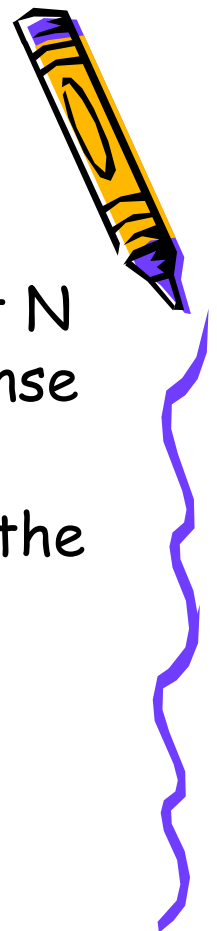
Central Limit Theorem

- Every data which are influenced by many small and unrelated random effects has normally distribution.
- The estimated mean will appear to be sampled from normal distribution with a KNOWN standard deviation (σ/\sqrt{N}) when N approaches infinity.

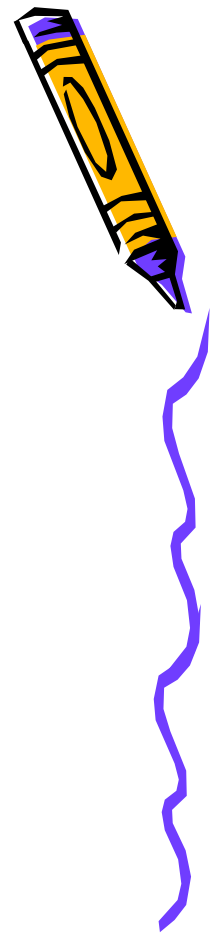
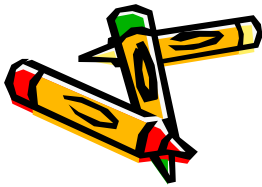


Central Limit Theorem (Cont.)

- Therefore, We have to check that N have approached infinity in the sense of the CLT, or not.
- This corresponds to the checking the *complete* sampling of interested phase space has occurred, or not.



This is not
a simple static test
but
check of results from
nature of Monte Carlo
simulations



Checking Values

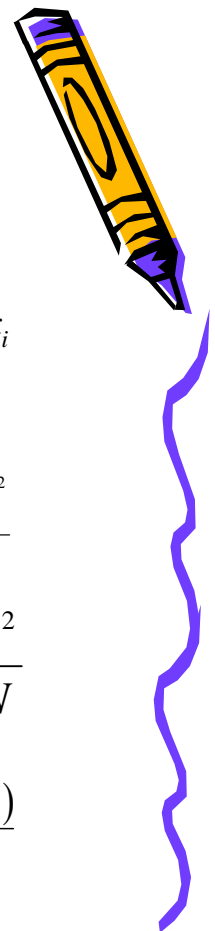
- Mean
- Variance and Standard Deviation
- Relative error
- Variance of Variance

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$S^2 = \frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N-1}$$

$$R = \frac{S^2}{\bar{x}}, \quad \text{where } S^2 = \frac{S^2}{N}$$

$$VOV = \frac{S^2(S_{\bar{x}})}{S_{\bar{x}}^4}$$



Checking Values (Cont.)

- Figure of Merit

$$FOM = \frac{1}{R^2 T}$$

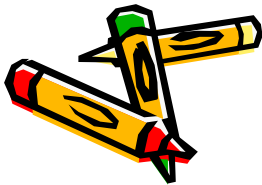
- Scoring Efficiency
 $R_{\text{intrinsic}}$ and $R_{\text{efficiency}}$ $q = \frac{\text{number of NON ZERO histories}}{N}$

- Shift

$$SHIFT = \sum (x_i - \bar{x})^3 / (2S^2 N)$$

- SLOPE

Fit to the Largest history scores



What we check?

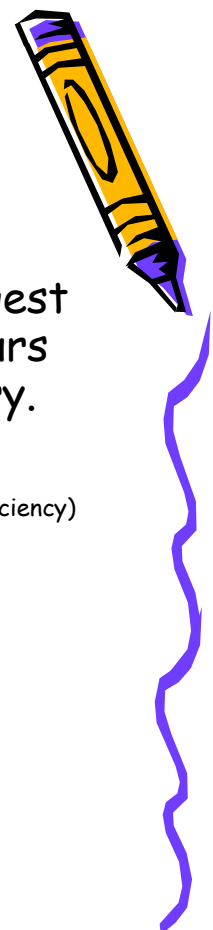
- Behavior of MEAN
- Values of R
- Time profile of R
- Values of VOV
- Time profile of VOV
- Time profile of FOM
- Behavior of FOM
- Value of SLOPE
- Value of SHIFT

- Effect of the largest history score occurs on the next history.
 - MEAN
 - R ($R_{\text{intrinsic}}$ and $R_{\text{efficiency}}$)
 - VOV
 - FOM
 - SHIFT



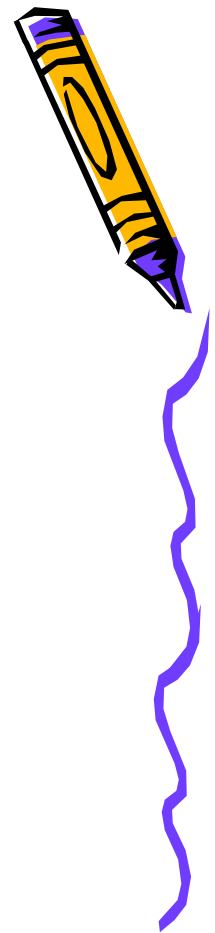
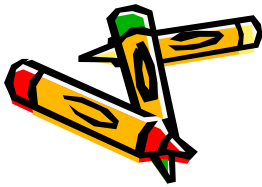
Boolean
Answer

Numeric
Answer



Of cause, Boolean check is
carried out mathematically
(statistically)

value
behavior
time profile



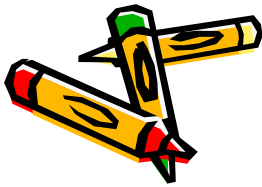
For behaviors and
time profiles check

- Derive Pearson's r from data (results and theoretical values)
 - $r=1(-1)$, perfect positive (negative) correlation
 - $r=0$, uncorrelated
- null hypothesis is set to uncorrelated
- Then, $t = r\sqrt{N-2/1-r^2}$ follows student t distribution of degree of freedom $\nu = N - 2$
- Checking significance of r with null hypothesis.
- Rejection level of null hypothesis is 68.28% (1σ)

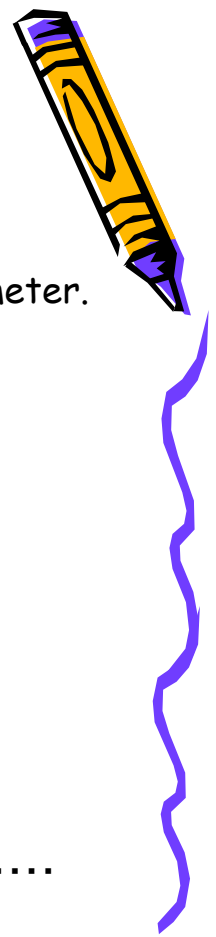
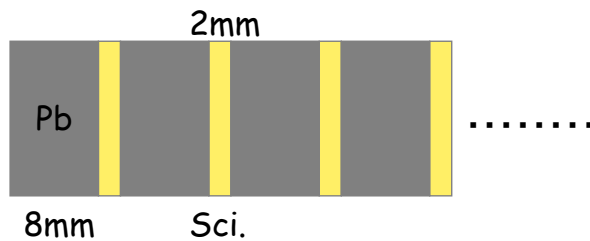


Example

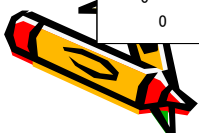
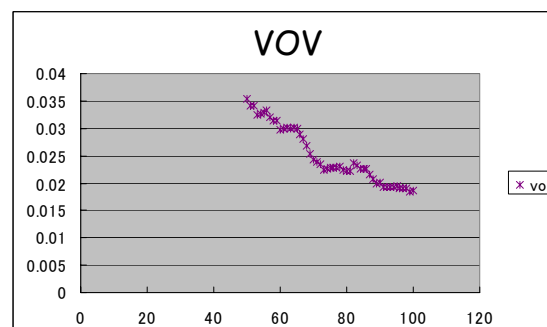
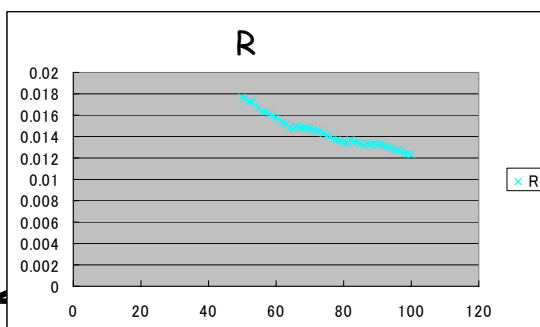
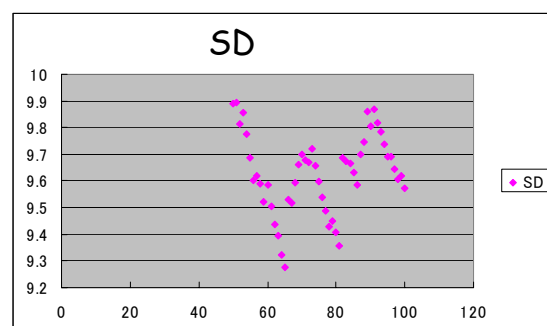
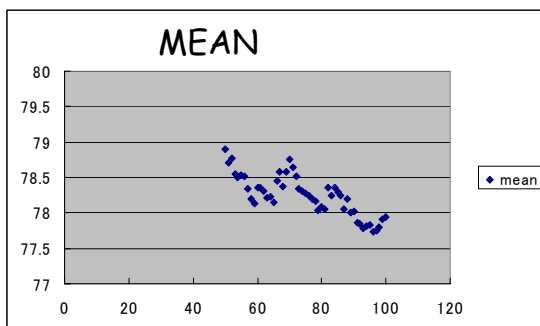
- Checking value: Observable Energy of Sampling Calorimeter.
- Material
 - Pb (Lead)-Scintillator
- Thicknesses
 - Pb: 8.0 mm/layer, Sci: 2.0 mm/layer
- Layers
 - 120 layers
 - 1 m x 1 m – interaction surface
- Beam
 - Electron 4 GeV
- Range Cuts
 - 1 mm



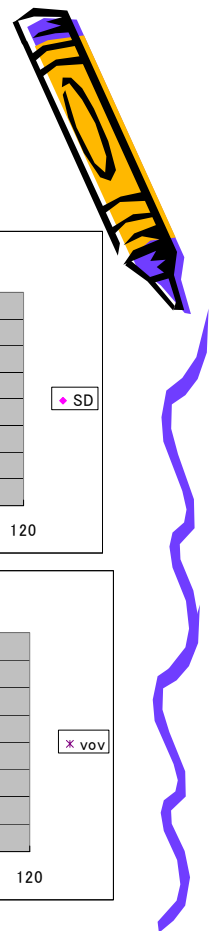
e^- →



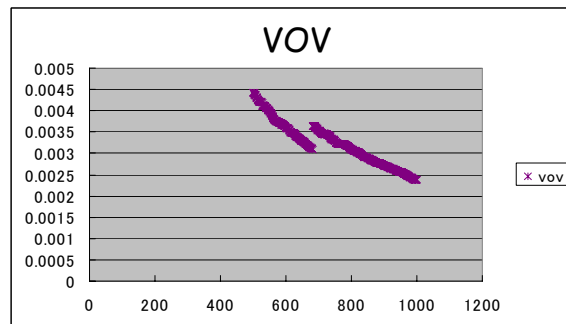
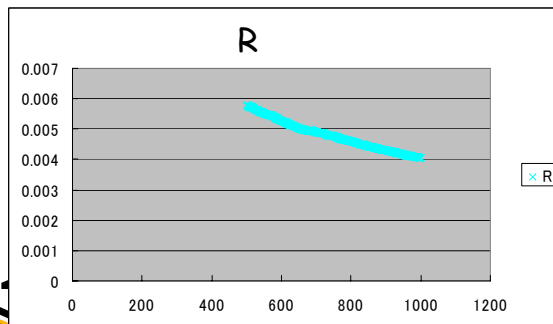
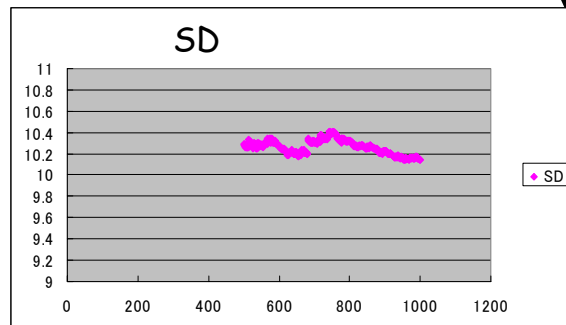
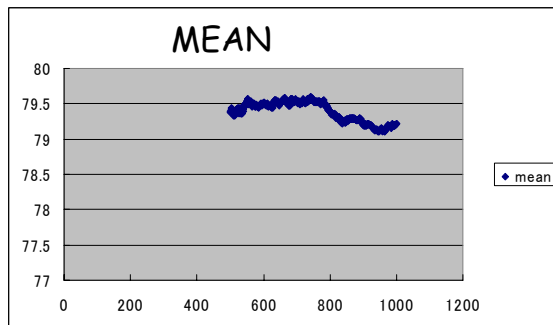
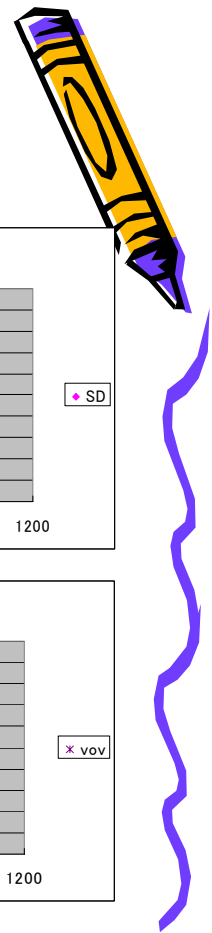
Example 100 histories



Does not pass most of Boolean tests

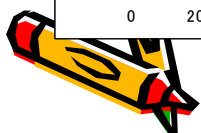
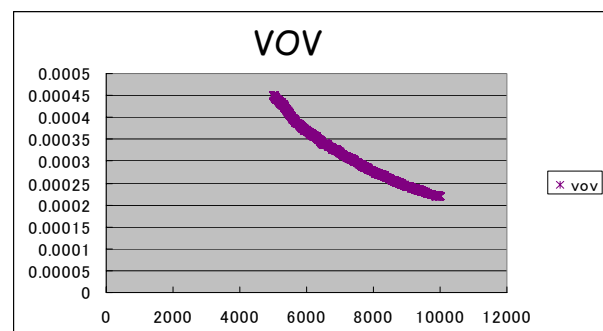
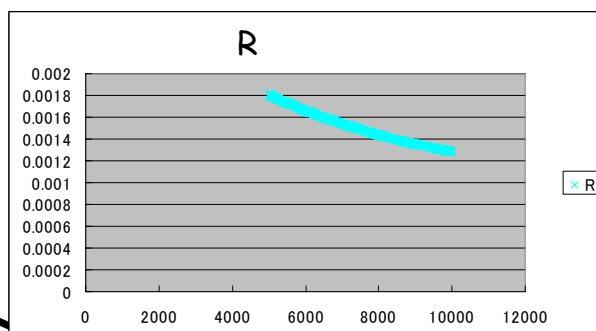
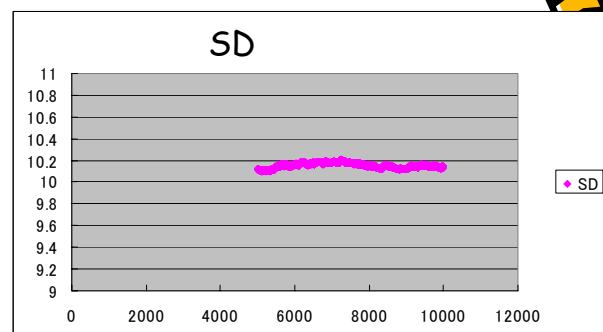
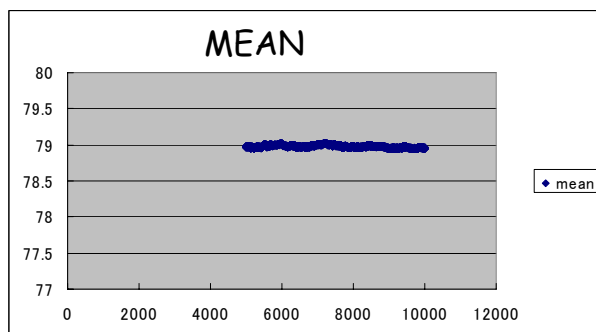
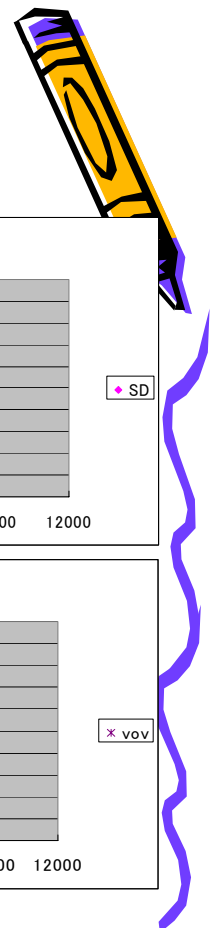


Example 1,000 histories



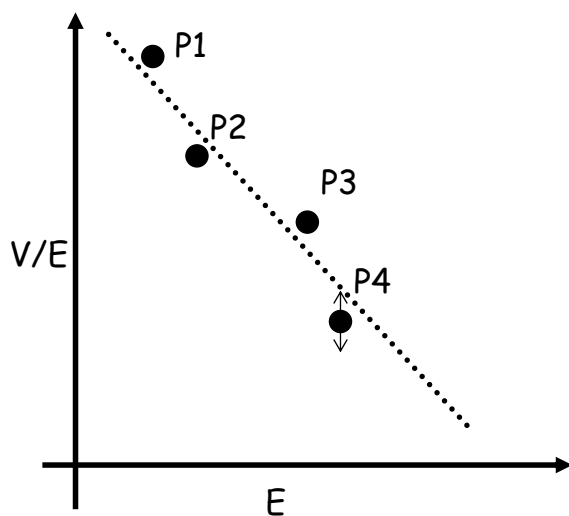
Does not pass some of Boolean tests

Example 10,000 histories



Does not pass one of Boolean tests (SLOPE check)

How to apply Energy Spectrum estimation etc.



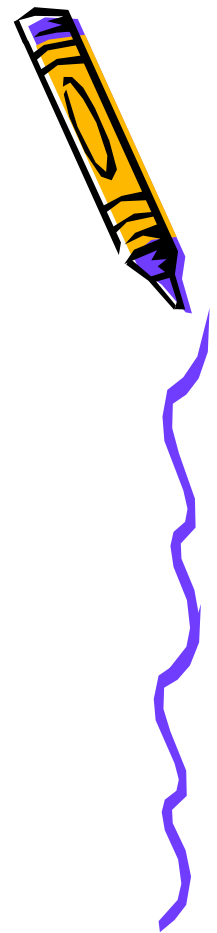
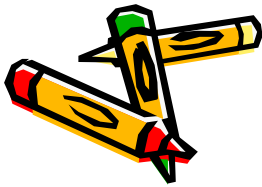
- Checking each confidence level of $P1, P2, P3, P4, \dots$
- Of course, scoring efficiency becomes low.



Unfortunately, this tool does not work well with some deterministic variance reduction techniques. This is come from limitation of CLT (means some variance are required for distribution), so that we can not over come.



And some simulations
becomes deterministic
without awaking of users.
Please check your
simulation carefully.



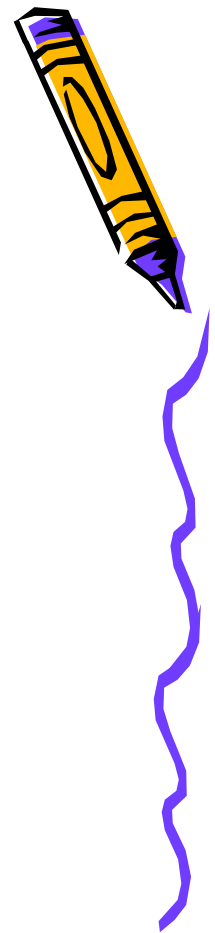
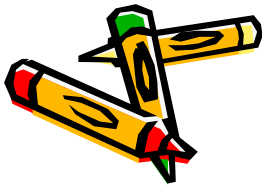
Current Status of Development

- Most part of developments has been done.
- Following items are remained under development.
 - Output of testing result
 - Class or function for minimization of multi dimensional functions



We want to include this tool in
Geant4
but
what category is suite for this
tool?

Run, SD, Hits and its collections,
Tally??



Summary

- We have successfully developed a general assistant tool for the checking the results from Monte Carlo simulations like MCNPs.
- Through this tool, users easily know the confidence intervals for Monte Carlo results.

