

Variance reduction

A primer on simplest techniques

What is variance reduction

- Reduce computer time required to obtain results of sufficient precision
- Random walk sampling modification
 - sampling “important” particles at the expense of the “unimportant”
- Measure:

$$\text{FOM} = 1 / (\sigma_{\text{mr}}^2 T)$$

What it tries to do

$$\sigma_{\text{mr}} = \sigma / (\mu\sqrt{N})$$

To improve it for fixed computing time **t**
must either:

- **decrease s** (by producing tracks)
- **increase N** (by destroying tracks)

‘faster’ than the cost in utilising the technique.

Types of variance reduction

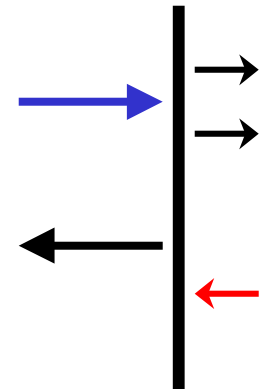
- Energy cutoff

Techniques using weight assigned to a track

- Geometry based
- Energy based
- Geometry/energy “window”
- “Physics” based - biasing

Geometry splitting & Russian Roulette

- Assign each volume an “importance”
- On boundaries compute the ratio $\omega = I_k / I_1$
- If $\omega = 1$ continue
- If $\omega > 1$ **split** the particle
 - into ω particles (if ω integer, else ...)
- If $\omega < 1$ play russian **roulette**
 - kill it with probability $1 - \omega$
 - else increase its weight by ω^{-1}



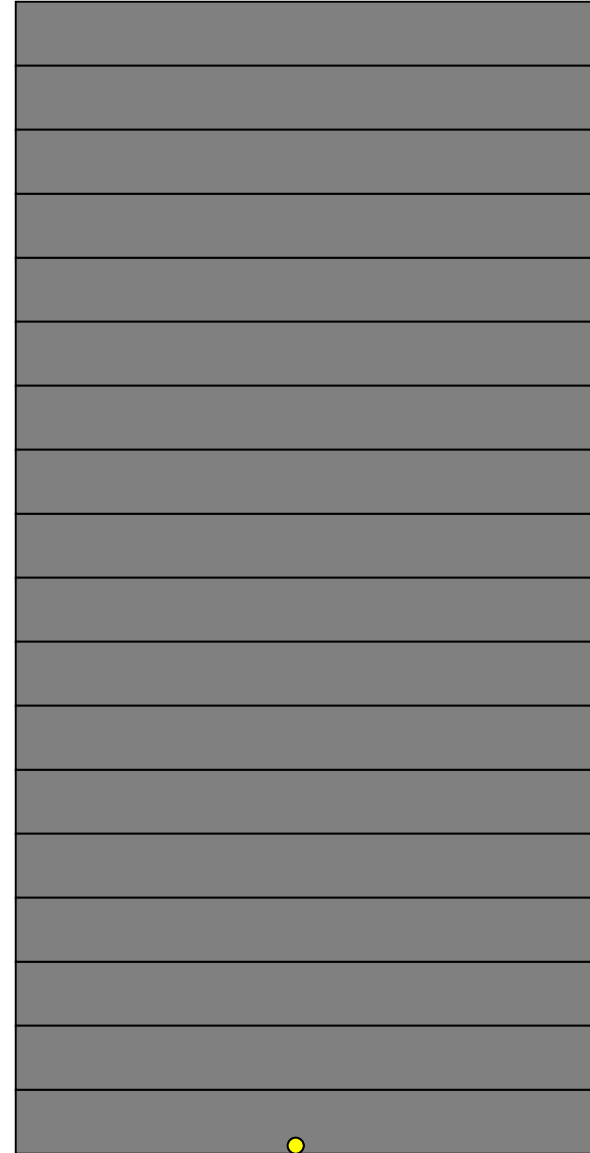
A 'simple' problem

Penetration of thick target

Neutron source (~ 10 MeV)

18 layers of concrete, 10 cm
each

How many neutrons escape
with $E > 0.01$ MeV?



Brute force - “analog” calculation

| Volume | Tracks entering |
|--------|-----------------|
| 1 | 4783 |
| 2 | 2176 |
| 3 | 1563 |
| 4 | 939 |
| 5 | 511 |
| 6 | 287 |
| 7 | 170 |
| 8 | 87 |
| 9 | 44 |
| 10 | 31 |
| 11 | 31 |
| 12 | 18 |
| 13 | 4 |
| 14 | 0 |
| 15 | 0 |
| 16 | 0 |
| 17 | 0 |
| 18 | 0 |
| 19 | 0 |
| 20 | 0 |

| Events | Hits (tally) | relative error | FOM |
|--------|--------------|----------------|-----|
| 3920 | 0 | 0 | 0 |

Energy cutoff calculation

| Volume | Tracks entering |
|--------|-----------------|
| 1 | 15416 |
| 2 | 4445 |
| 3 | 2197 |
| 4 | 973 |
| 5 | 467 |
| 6 | 233 |
| 7 | 110 |
| 8 | 56 |
| 9 | 40 |
| 10 | 20 |
| 11 | 8 |
| 12 | 3 |
| 13 | 0 |
| 14 | 0 |
| 15 | 0 |
| 16 | 0 |
| 17 | 0 |
| 18 | 0 |
| 19 | 0 |
| 20 | 0 |

| Events | Hits (tally) | relative error | FOM |
|--------|--------------|----------------|-----|
| 10000 | 0 | 0 | 0 |
| 13970 | 0 | 0 | 0 |

Imposing energy cutoff of 0.010 MeV

The problem with geometry splitting & russian roulette

Set importance of bottom region to 1.

At each boundary double the importance.

128
64
32
16
8
4
2
1



Results with geometry splitting, RR

| Volume | Tracks entering |
|--------|-----------------|
| 1 | 2329 |
| 2 | 1278 |
| 3 | 1323 |
| 4 | 1321 |
| 5 | 1326 |
| 6 | 1353 |
| 7 | 1358 |
| 8 | 1261 |
| 9 | 1182 |
| 10 | 1089 |
| 11 | 998 |
| 12 | 823 |
| 13 | 792 |
| 14 | 734 |
| 15 | 664 |
| 16 | 525 |
| 17 | 514 |
| 18 | 406 |
| 19 | 163 |

| Events | Hits (tally) | relative error | FOM |
|--------|--------------|----------------|-----|
| 2220 | 5.87 e-07 | 0.244 | 27 |

Fewer tracks simulated (2200 vs 13000)
Yet a 'tally' was created, estimating
roughly the number of neutrons
escaping with $E > 0.01$ MeV

Rule of thumb: flat distribution of tracks
gives best result (but broad optimum)
for 1-d problems

Other techniques

- Biasing the source
 - direction, energy
- Energy roulette
 - roulette at energy ‘cutoffs’
- Forced collisions
 - split into collision (weight ‘ w ’), non ($1-w$)
- More advanced techniques
 - Weight Window techniques

Caveats

- Application of variance reduction methods require care and knowledge to choose the appropriate technique(s)
- Several simple techniques can be combined
- Advanced techniques require expert knowledge

Geant4 considerations

- Energy cutoffs and parameterisations there
- Can already implement most VR schemes as user actions ('unfriendly')
- Simple measures will allow generic implementation of simple VR schemes (geometry/energy splitting)
 - adding 'importance' to physical volumes
 - creating process(es) for splitting/roulette
- Sophisticated schemes can follow later ...

Some reading

Primary reference for this (excellent introduction)

- A Sample Problem for Variance Reduction in MCNP, LA-10363-MS, T. Booth, Oct 1985

Good modern book, with coverage of VR:

- Monte Carlo Transport Methods: Neutron and Photon Calculations, I. Lux and L. Koblinger, CRC Press, 1991