

A new theoretical tool for extracting resonance parameters

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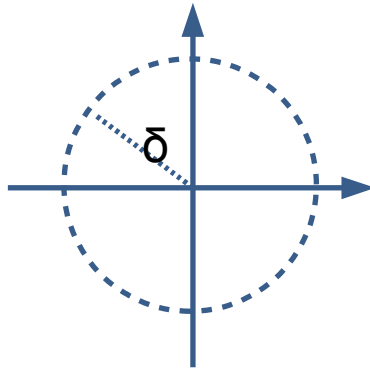
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Work in collaboration with JJ Sanz-Cillero
arXiv:1306.6308 [hep-ph] (accepted at EPJC)

Application to PWA together with Lothar Tiator

Analytical properties

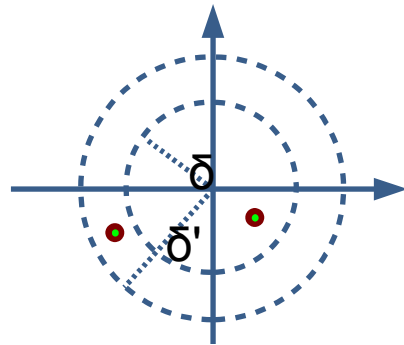
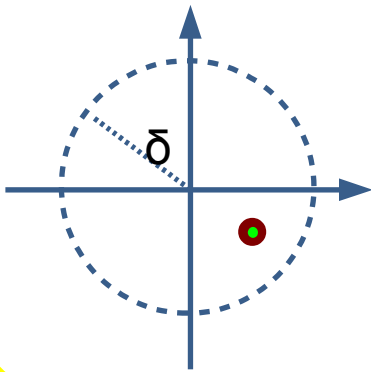


Analytical function $f(x)$ in a disk $B_\delta(0)$

$$f(x) = \sum_{n=0}^N a_n x^n \text{ converges to } f(x) \text{ for } N \rightarrow \infty$$

Experimentally, $f(0)$, $f'(0)$, ...
can be extracted from successive polynomial fits

Analytical function $f(x)$ but for a n -number of poles:



$$f(x) = \sum_{n=0}^N a_n x^n$$

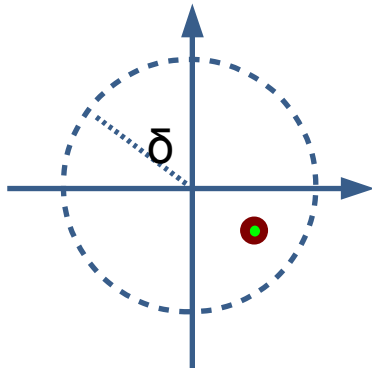
Doesn't converge to $f(x)$ for $N \rightarrow \infty$

No extraction of $f(x)$ experimentally
using polynomials

The Montessus' theorem

[R. Montessus de Ballore 1902]

Analytical function $f(x)$ but for a n -number of poles (for ex. one pole):



A series of single-pole Padé Approximants:

$$P_1^N(s, s_0) = \sum_{k=0}^{N-1} a_k (s - s_0)^k + \frac{a_N (s - s_0)^N}{1 - \frac{a_{N+1}}{a_N} (s - s_0)}$$

Converge unif. to $f(x)$ when $N \rightarrow \infty$

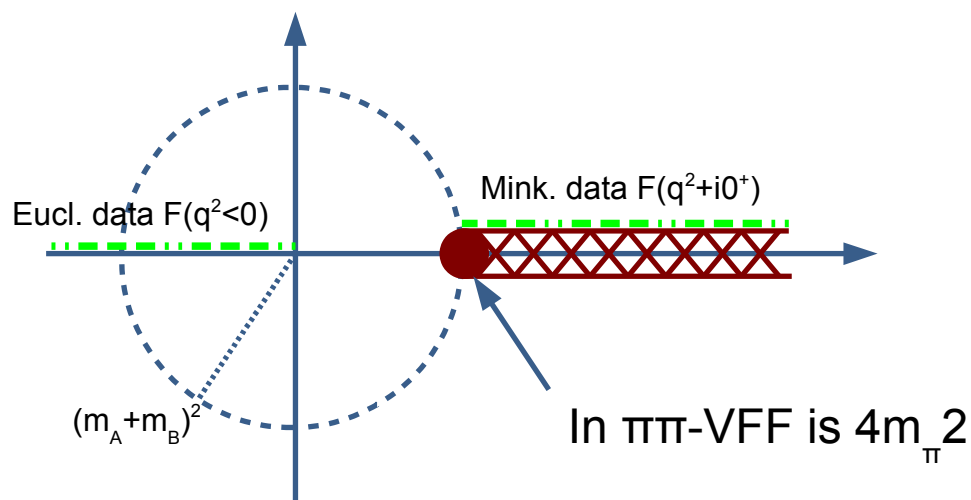
Hence, also the PA pole $x_p = s_0 + \frac{a_L}{a_{L+1}} \rightarrow s_p$

Montessus' theorem \rightarrow extract resonance poles

Application of Montessus' theorem

Usually Padé Approximants constructed at $q^2=0$ [Baker, Graves-Morris'96]

For physical amplitude $F(q^2) \rightarrow$ only analytical up to threshold

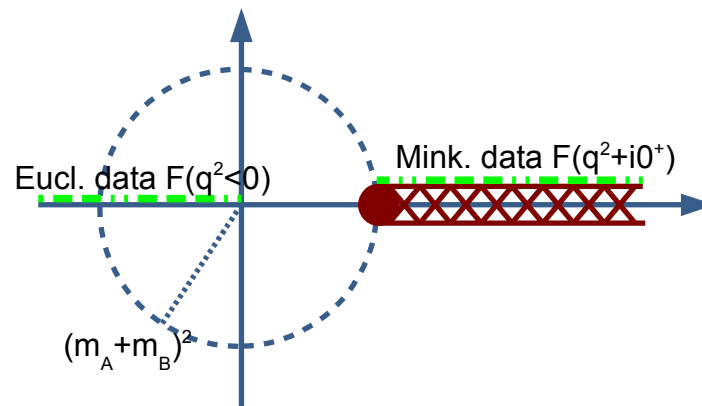


Experimental data:

- Space-like: $F(q^2 < 0) \rightarrow$ [PM, Peris, Sanz-Cillero'08] & [PM '12]
- Time-like: $F(q^2 + i0^+) \rightarrow$ NO \rightarrow what can we do?
[PM, Sanz-Cillero, '13]

Outline

- What is a Padé Approximant?
- Applications of the Montessus Theorem
 - Genuine PA
 - Fitted PA
- PWA analyses
 - MAID2007
 - ANL-Osaka
- Conclusions



What is this approximation?

What is a Padé Approximant?

[Baker, Gave-Morris '05]

Let $f(z)$ be a function of complex z with an expansion

$$f(z) = \sum_{n=0}^{\infty} f_n z^n, \quad z \rightarrow 0.$$

Define a rational function $P_N^M(z)$ such that:

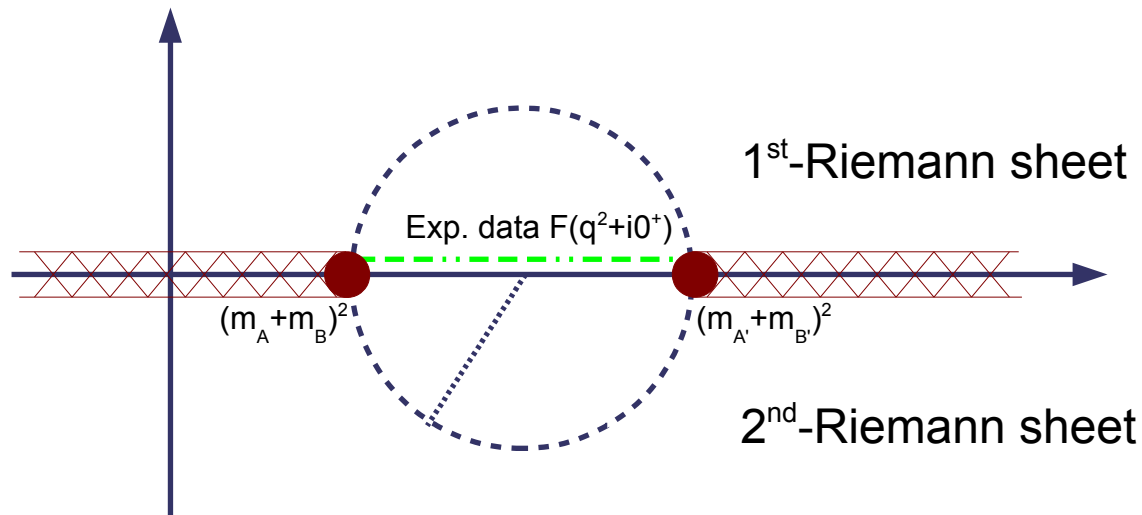
$$P_N^M(z) \equiv \frac{Q_M(z)}{R_N(z)} \approx f_0 + f_1 z + f_2 z^2 + \cdots + f_{M+N} z^{M+N} + O(z^{M+N+1})$$

$P_N^M(z)$ is a **Padé** Approximant \rightarrow Montessus' theorem!

Application of Montessus theorem

To unfold the 2nd Riemann sheet:

→ PA centered over the branch cut $q^2 > (m_A + m_B)^2$

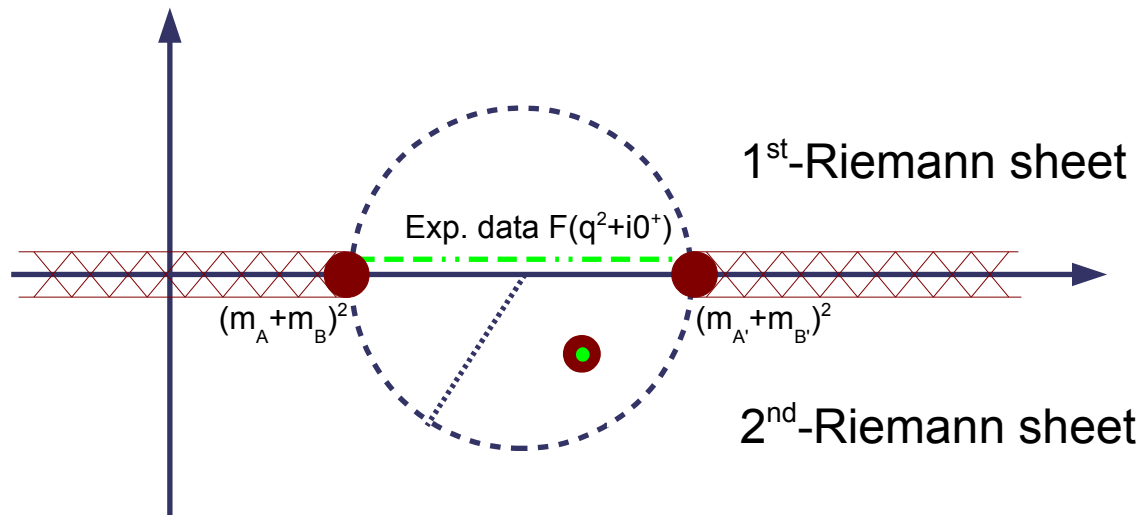


For example, in $\pi\pi$ -VFF, the effective range $4m_\pi^2 \leftrightarrow 4m_K^2$

Application of Montessus theorem

In the case of resonant amplitudes:

→ a 2nd Riemann sheet pole appears related to hadronic state



We can apply the Montessus' theorem
and locate the resonant pole in a model-independent way!

A test with a model

A preliminary analysis of the method (ρ -like model):

$$F(s) = \frac{M^2}{M^2 - s + \frac{1}{\pi} M G \xi(s)}$$

with a pole at $s_p = (M_p - i\Gamma_p/2)^2$
+
Log singular at $s=0$

with M and Γ chosen to produce the pole at the physical $s_p \approx (0.77 - i0.15/2)^2 \text{GeV}^2$

Model A): $\xi(s) = \ln \frac{-s}{M^2}$, (1 threshold)

Model B): $\xi(s) = \frac{s}{M} \ln \frac{-s}{M^2}$,

Model C): $\xi(s) = \frac{s}{M^2} \ln \frac{-s}{M^2} + \frac{1}{2} \frac{s \rho_K^2(s)}{M^2} \left[2 - \rho_K(s) \ln \left(\frac{\rho_K(s) + 1}{\rho_K(s) - 1} \right) \right]$

(2 thresholds)

A test with a model

A preliminary analysis of the method (ρ -like model):

$$s_{PA}^{(N)} = s_0 + \frac{a_N}{a_{N+1}},$$

$$Z_{PA}^{(N)} = -\frac{(a_N)^{N+2}}{(a_{N+1})^{N+1}},$$

$$a_N = \frac{1}{N!} \left. \frac{d^N F}{ds^N} \right|_{s=s_0}$$

(a_N are complex numbers)

Double application:

- first using $f(x) = \sum_{n=0}^N a_n x^n$ (genuine PA)

- second using simulated data (fitted PA)

Genuine PA: ρ -like VFF

The pole of the model located at:

$$s_p \approx (0.77 - i0.15/2)^2 \text{GeV}^2$$

We expand $F(s)$:

$$F(s) = a_0(s - s_0)^0 + a_1(s - s_0)^1 + a_2(s - s_0)^2 + \mathcal{O}(a_0(s - s_0)^3)$$

We construct the simplest PA around s_0 :

$$P_1^0(s) = \frac{a_0}{1 - \frac{a_1}{a_0}(s - s_0)} \longrightarrow s_{P_1^0} = s_0 + \frac{a_0}{a_1}$$

$$P_1^N(s) = P_1^1(s), P_1^2(s), P_1^3(s), \dots$$

where $s_0 \approx \text{Re}[s_p] + i\epsilon$

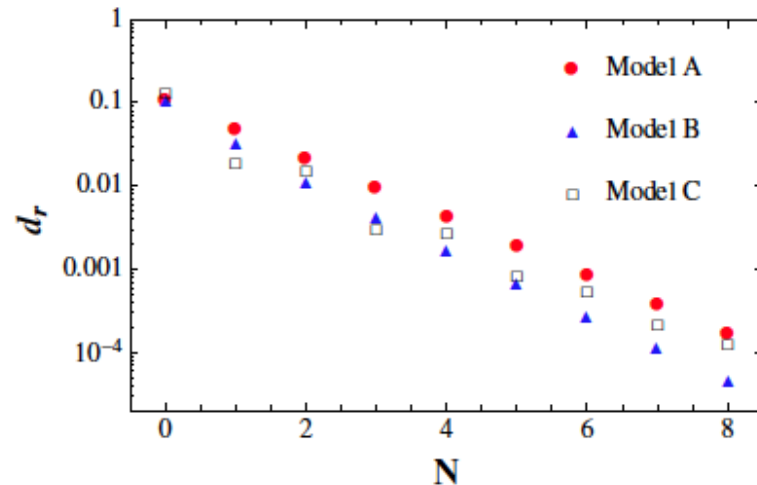
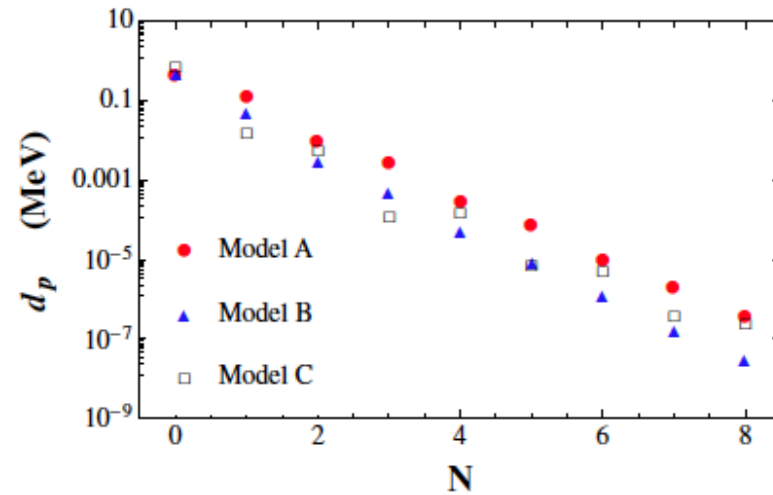
Genuine PA: ρ -like VFF

The pole of the model located at:

$$s_p \approx (0.77 - i0.15/2)^2 \text{GeV}^2$$

$$d_p = \sqrt{(M_{PA} - M_{pole})^2 + (\Gamma_{PA} - \Gamma_{pole})^2}, \quad d_p \text{ (MeV)}$$

$$d_r = \left| \frac{Z_{PA}}{Z} - 1 \right|$$



Genuine PA: ρ -like VFF

- ρ -like resonance:

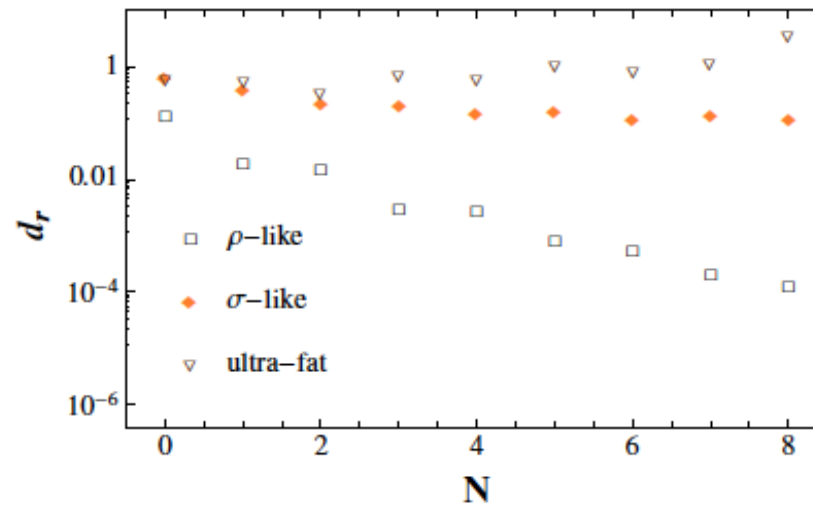
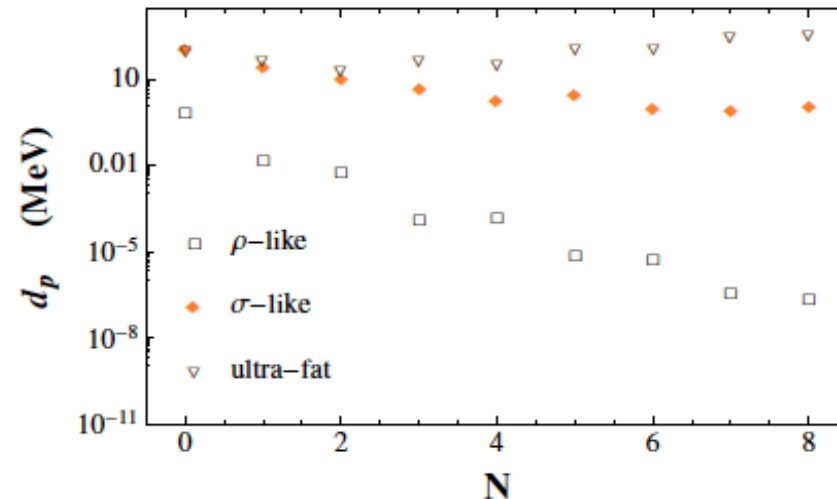
$$s_p = (0.77 - i 0.15/2)^2 \text{ GeV}^2$$

- σ -like resonance:

$$s_p = (0.5 - i 0.5/2)^2 \text{ GeV}^2$$

- "Ultra-fat" resonance:

$$s_p = (1 - i/2)^2 \text{ GeV}^2$$



Genuine PA: ρ -like VFF

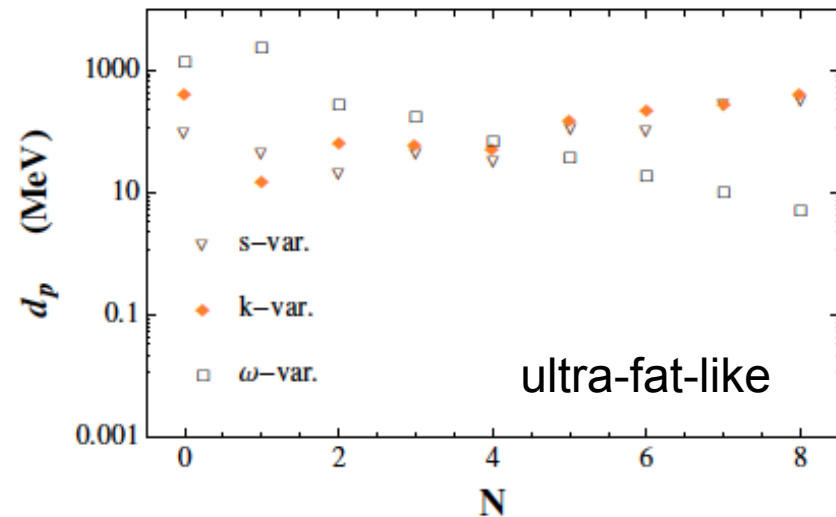
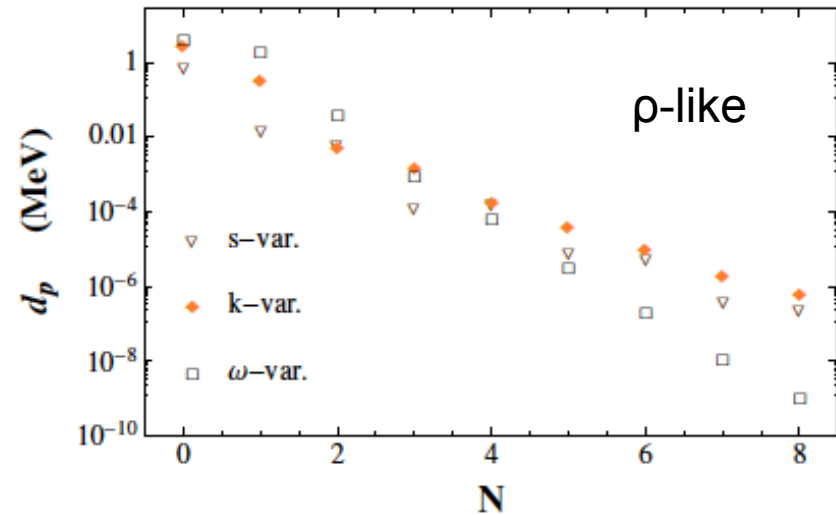
- s-variable

- k-variable

$$k = \frac{\lambda(s, m_1^2, m_2^2)^{\frac{1}{2}}}{2\sqrt{s}}$$

- w-variable

$$w = \frac{\sqrt{s - s_1^{th}} - \sqrt{s_2^{th} - s}}{\sqrt{s - s_1^{th}} + \sqrt{s_2^{th} - s}}$$

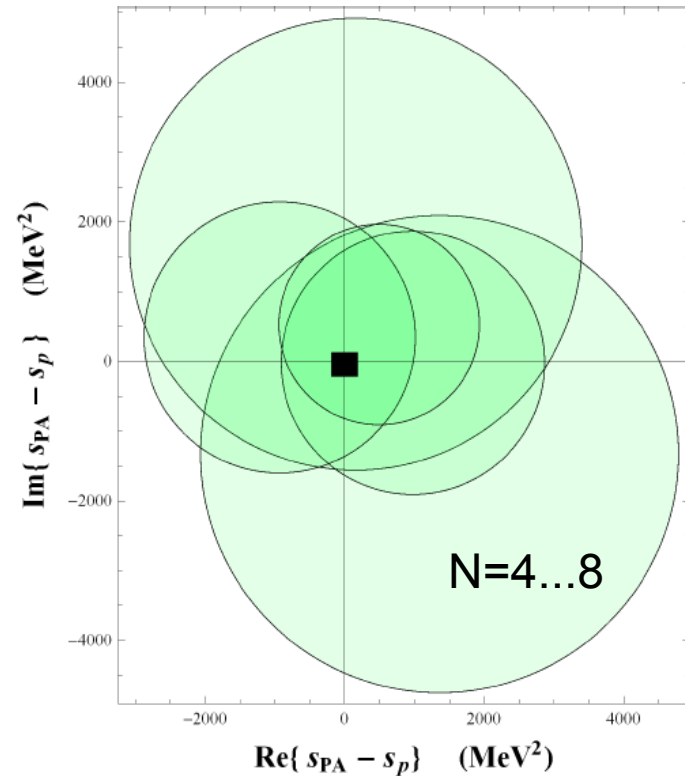
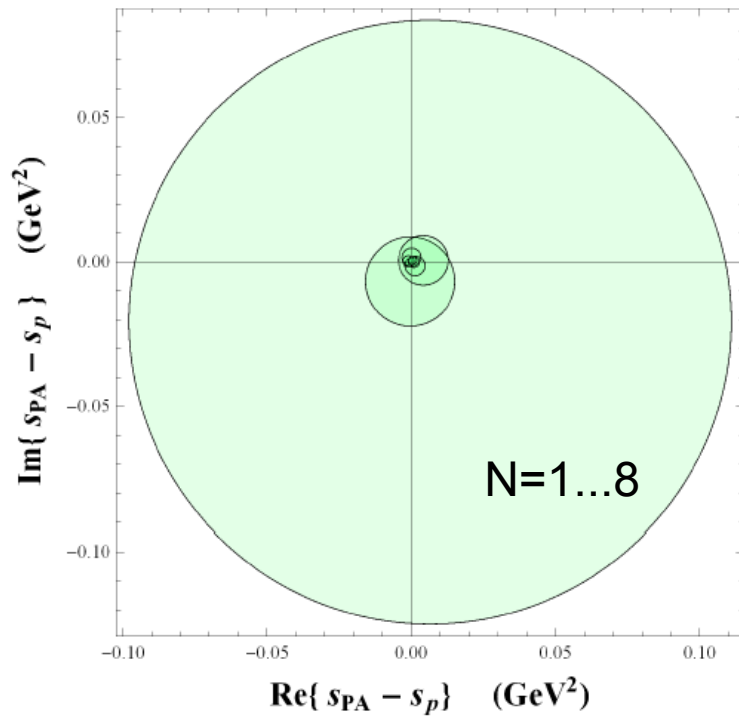


Error criterium

Systematic error estimate:

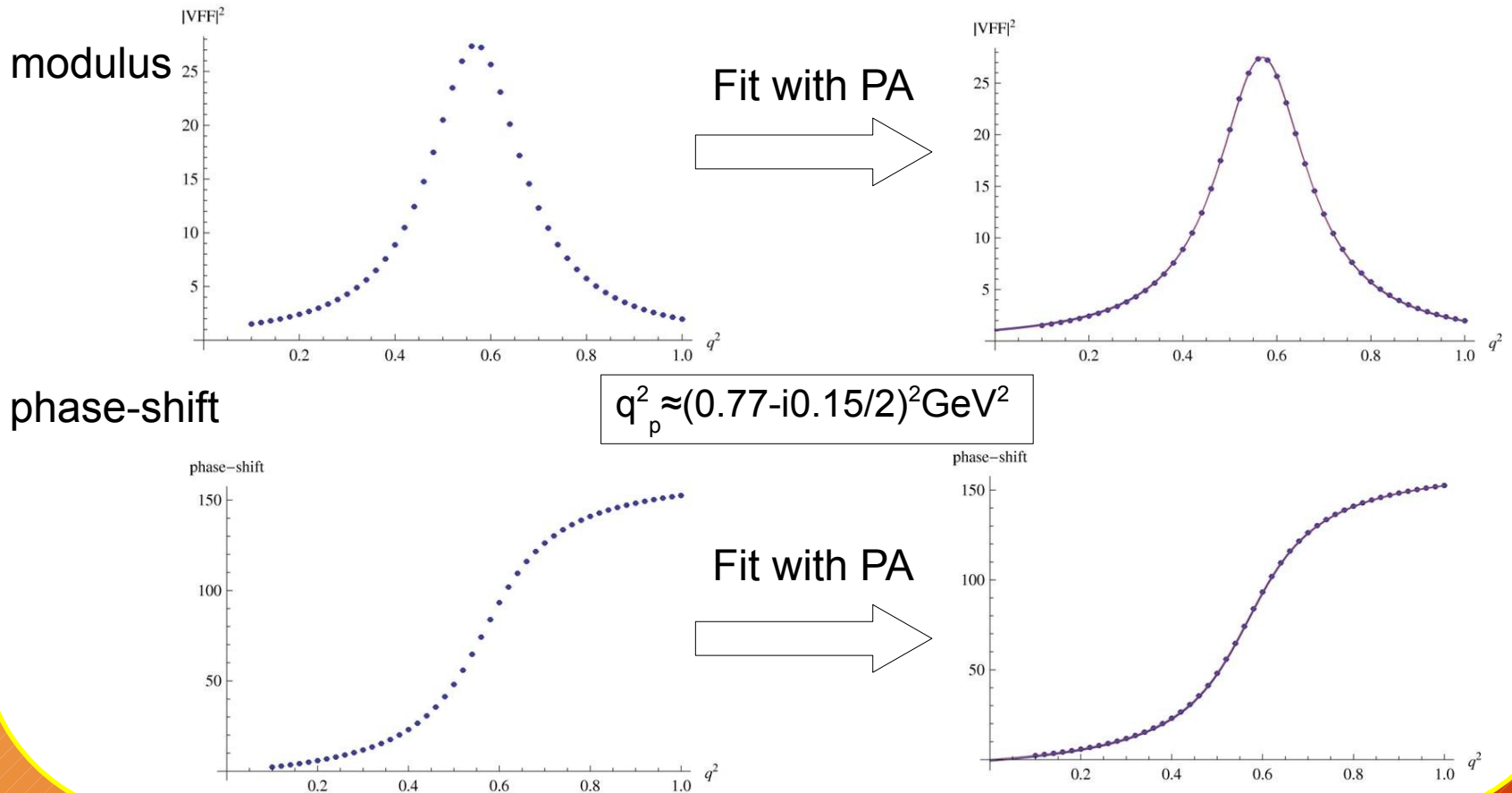
$$\Delta s_N \equiv |s_{PA}^{(N)} - s_{PA}^{(N-1)}|$$

Example with a σ -like profile:



Fitting data with PA

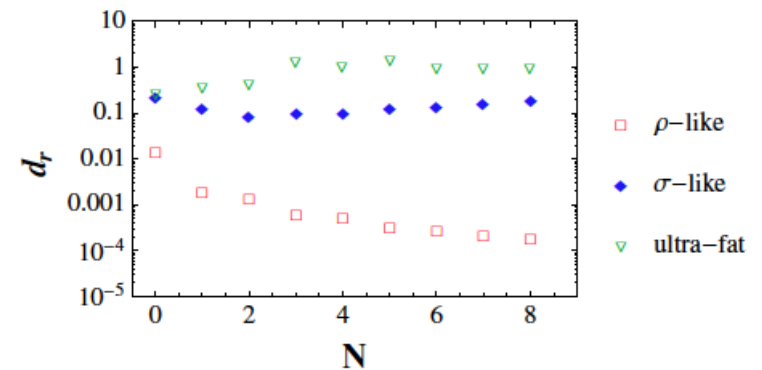
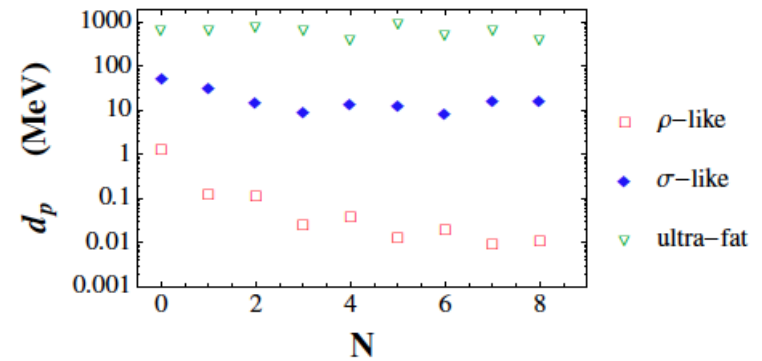
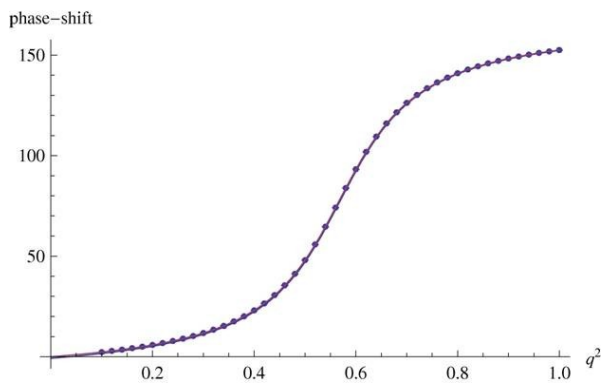
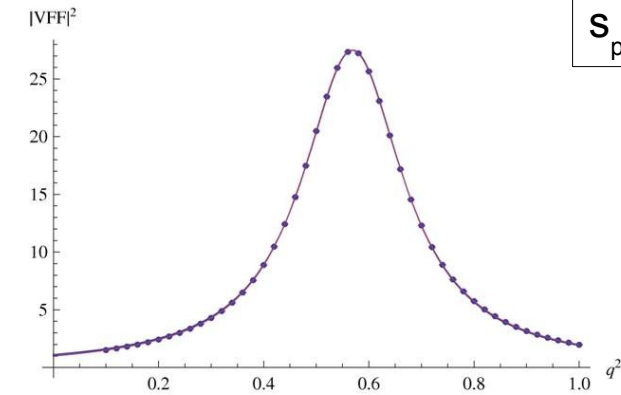
Using the model we generate a series of data points with 0 error



Fitting data with PA

A convergence pattern

$$s_p \approx (0.77 - i0.15/2)^2 \text{GeV}^2$$



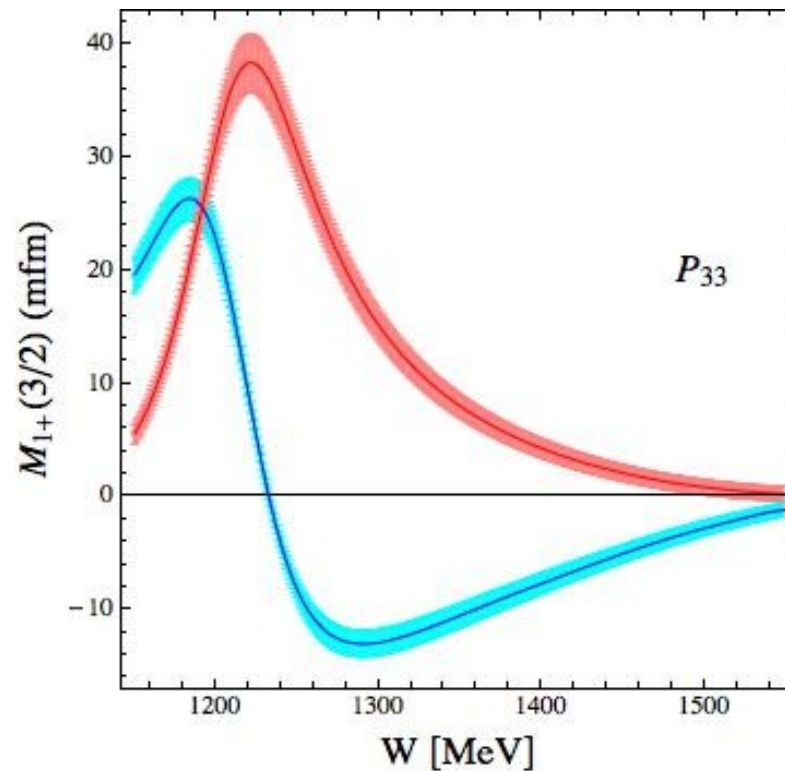
Fitting MAID2007 data

PRELIMINARY RESULTS

Fitting MAID2007 data: P33

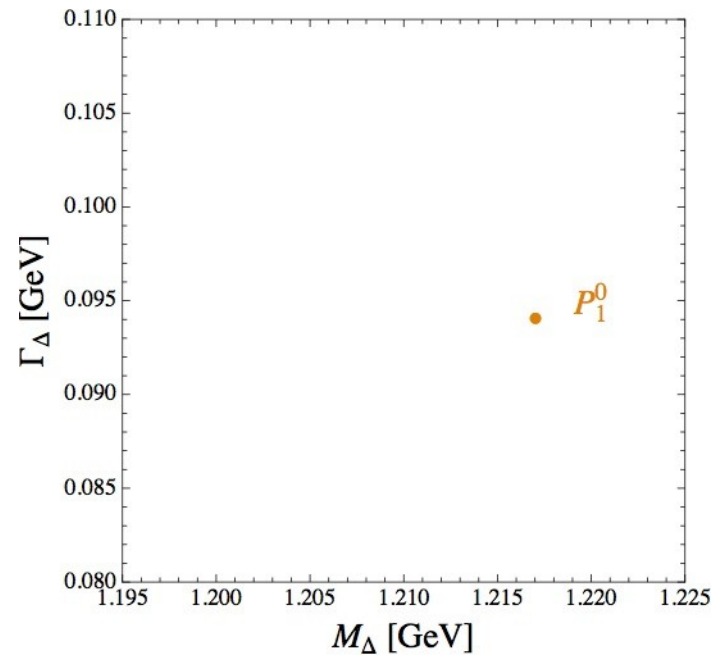
PRELIMINARY RESULTS

Consider the global solutions of MAID2007 for the multipole $M_{1+}(3/2)$



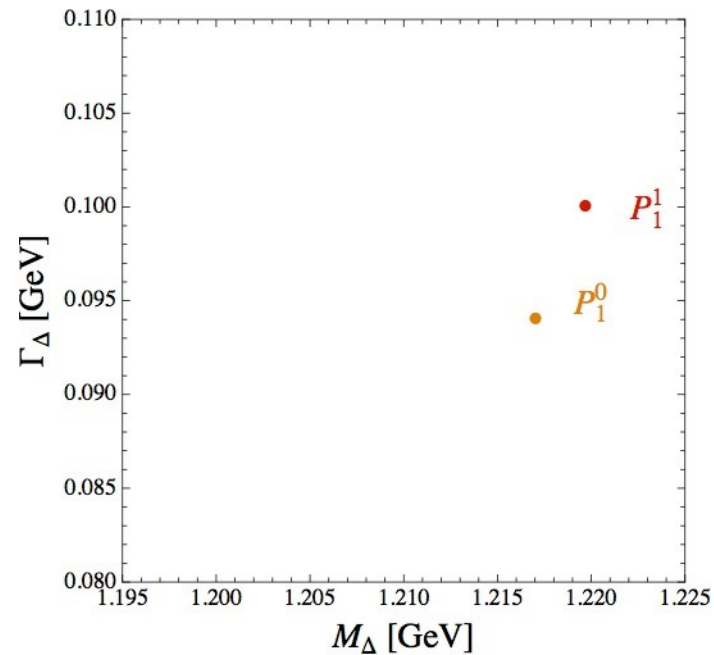
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



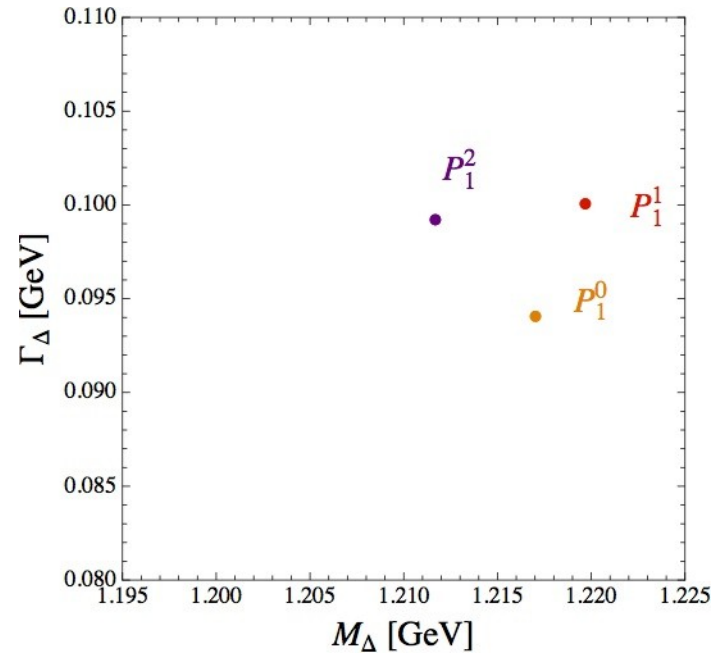
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



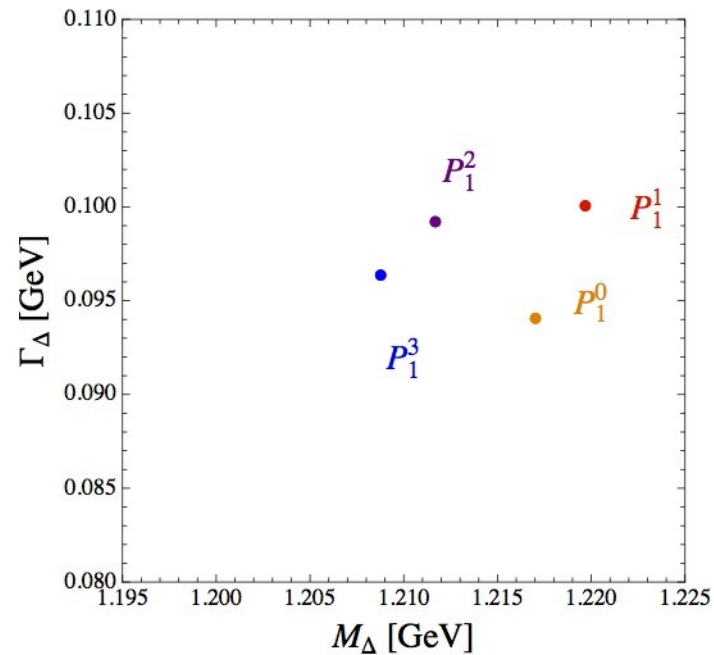
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



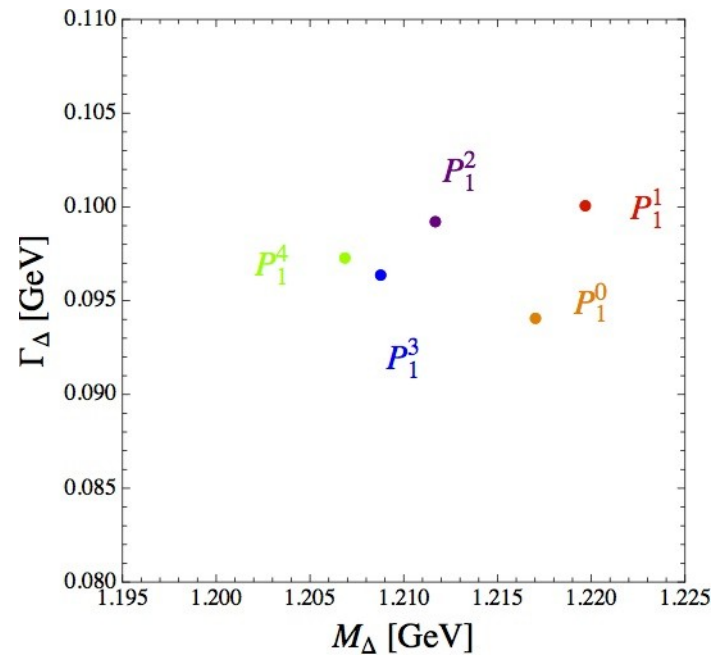
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



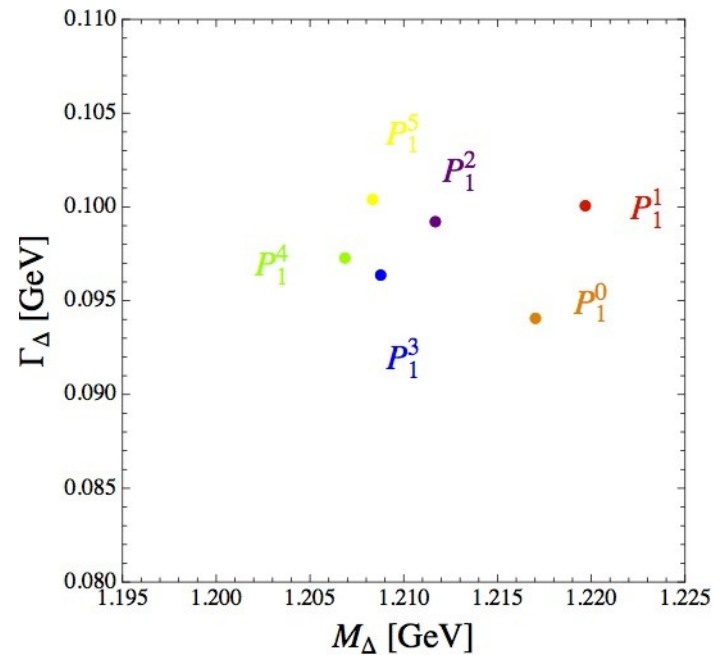
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



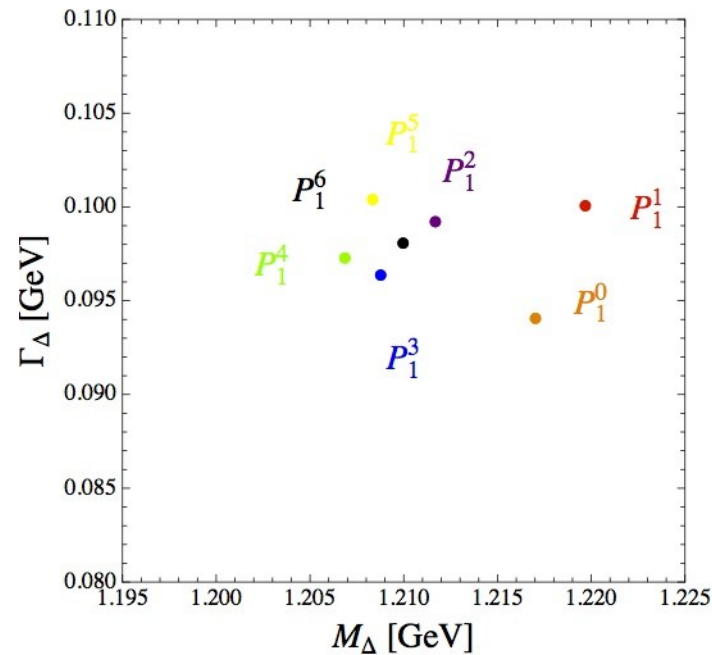
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



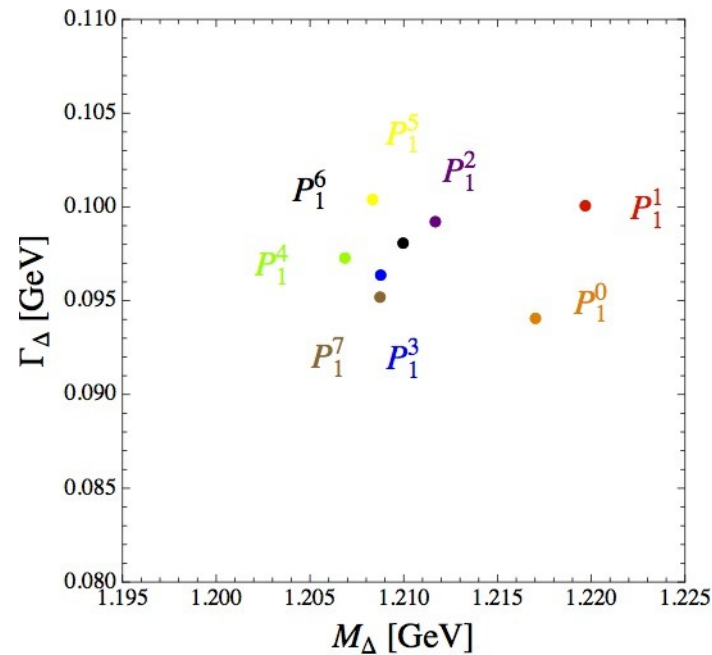
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



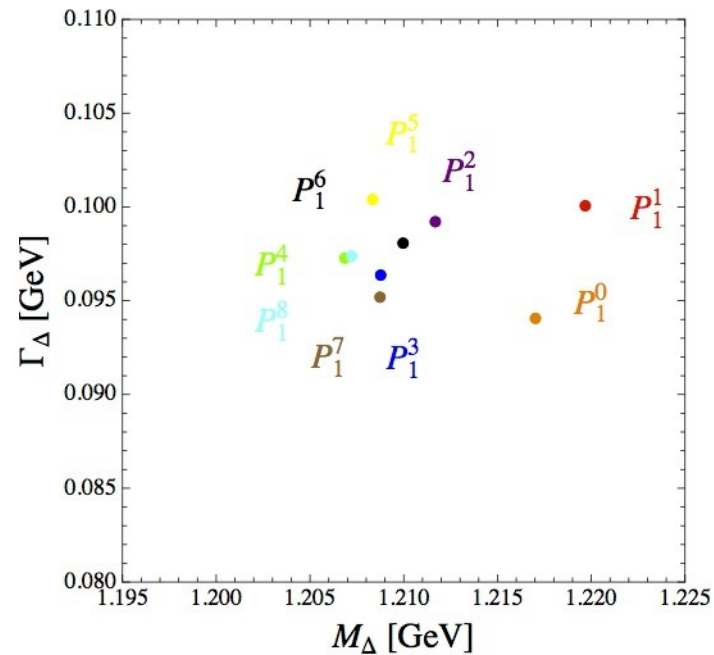
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



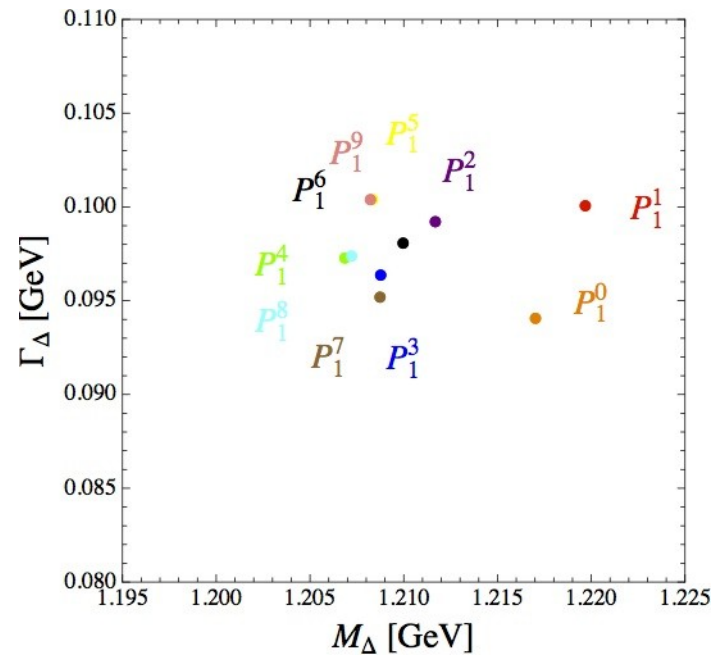
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



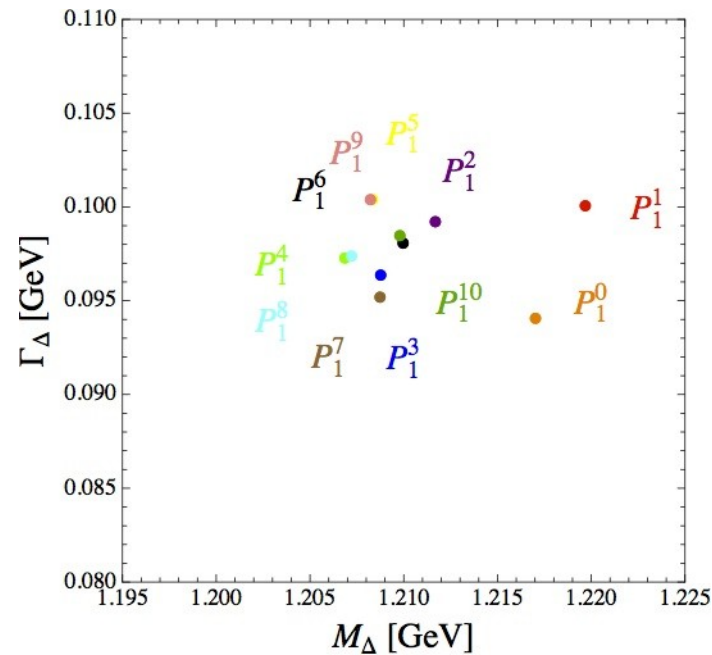
Fitting MAID2007 data: P33

PRELIMINARY RESULTS



Fitting MAID2007 data: P33

PRELIMINARY RESULTS

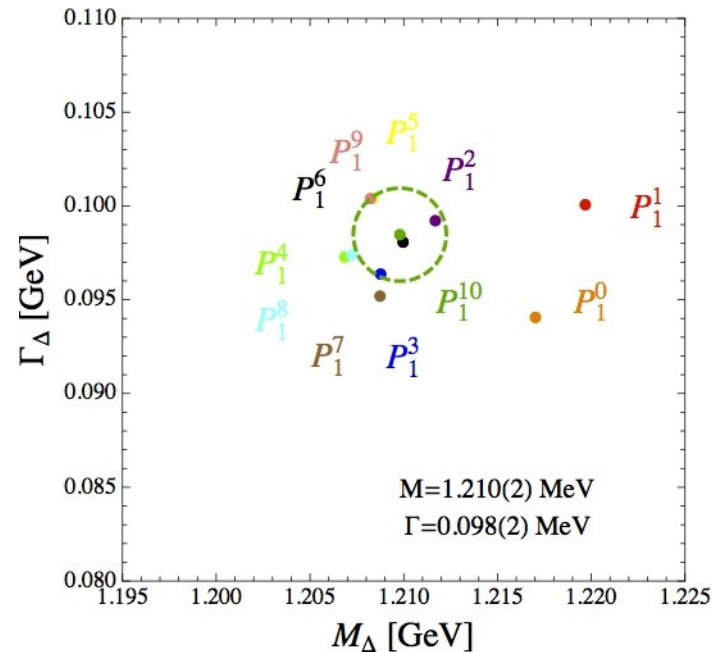


Fitting MAID2007 data: P33

PRELIMINARY RESULTS

We finally take $N=10$ as the best estimate:

- For $N \geq 4$ the theoretical errors are found $0.1 \leftrightarrow 2$ MeV



$$M_{\text{fit}} = 1210 \pm 2 \text{ MeV}$$

$$\Gamma_{\text{fit}} = 98 \pm 2 \text{ MeV}$$

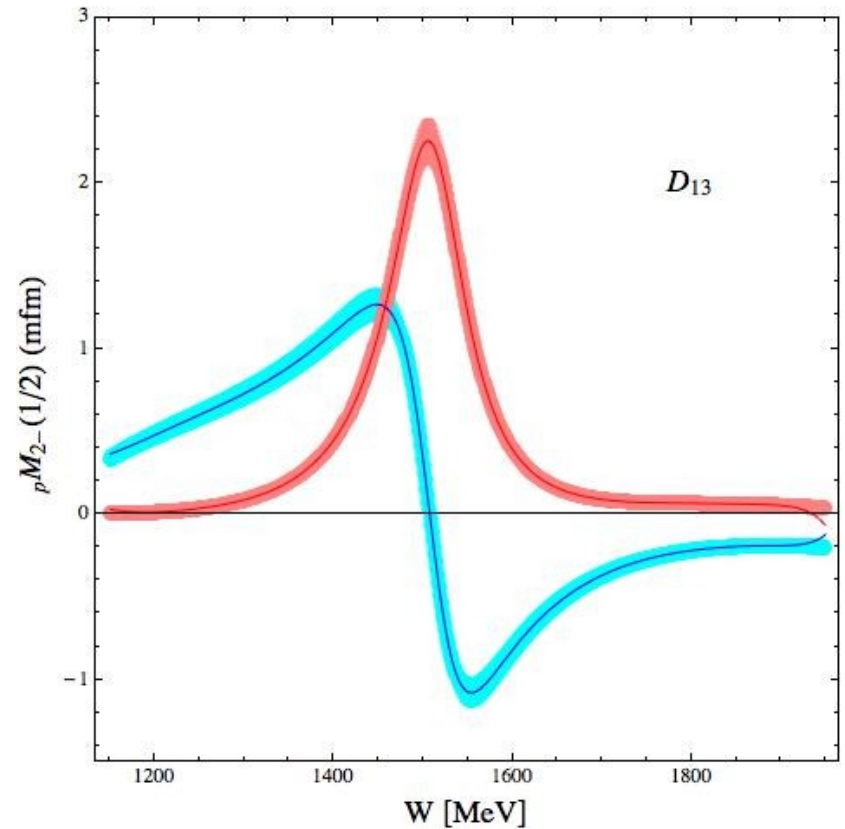
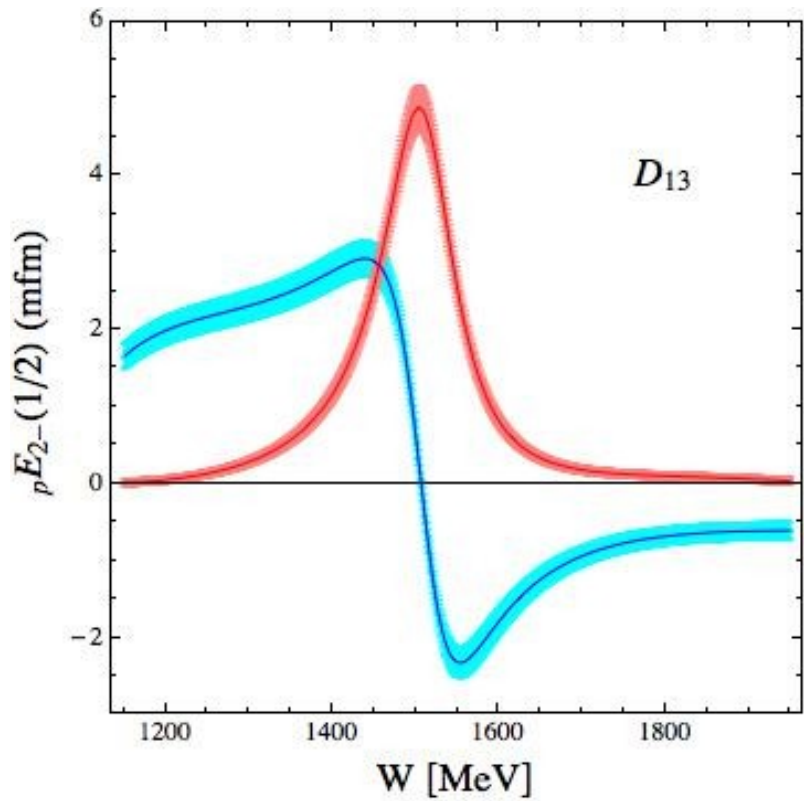
$$M_{\text{MAID}} = 1212 \pm 1 \text{ MeV}$$

$$\Gamma_{\text{MAID}} = 99 \pm 1 \text{ MeV}$$

[Tiator, Drechsel, Kamalov, and Vanderhaeghen, '11]

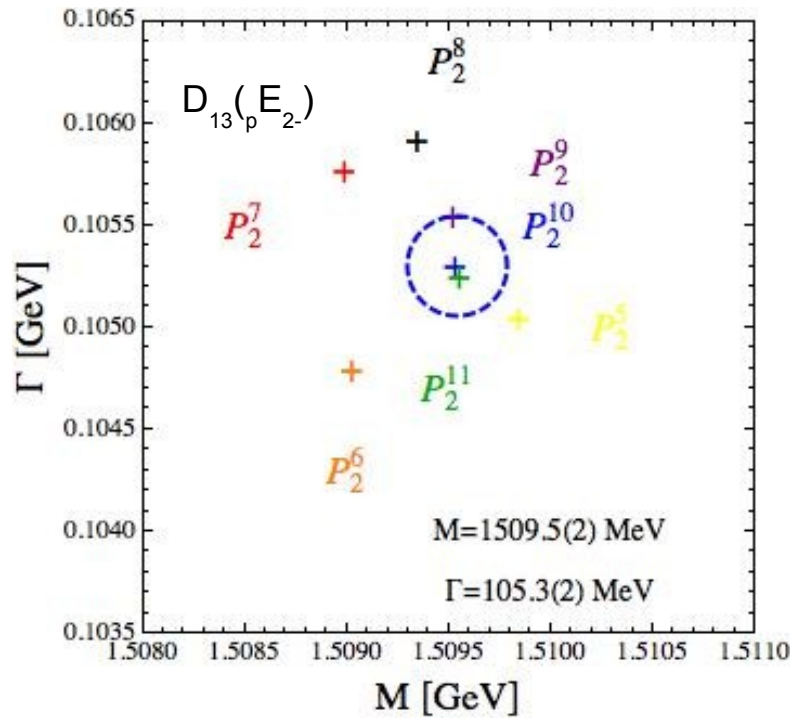
Fitting MAID2007 data: D13

PRELIMINARY RESULTS

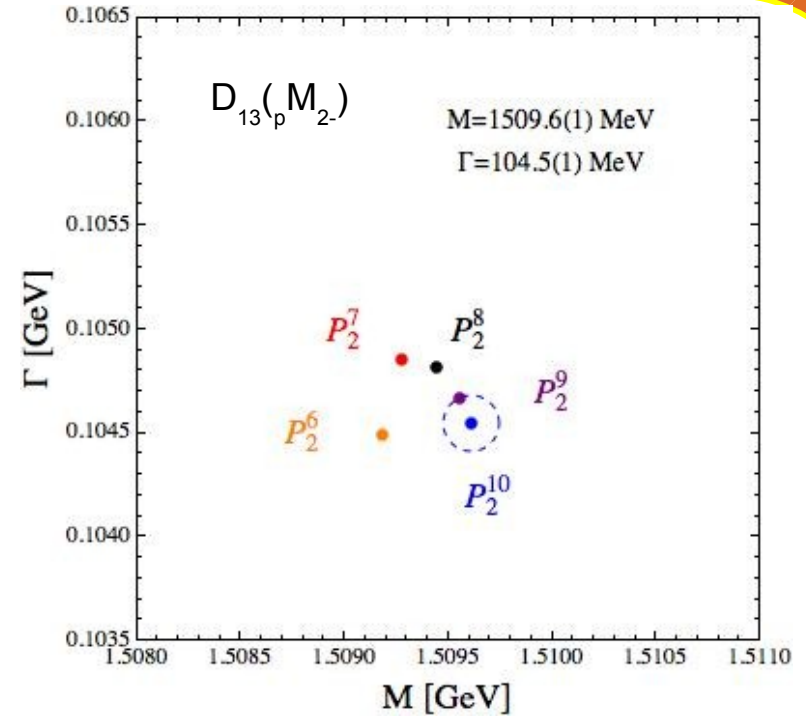


Fitting MAID2007 data: D13

PRELIMINARY RESULTS



$M_{\text{fit}} = 1509.5 \pm 0.2 \text{ MeV}$
 $\Gamma_{\text{fit}} = 104.9 \pm 0.4 \text{ MeV}$

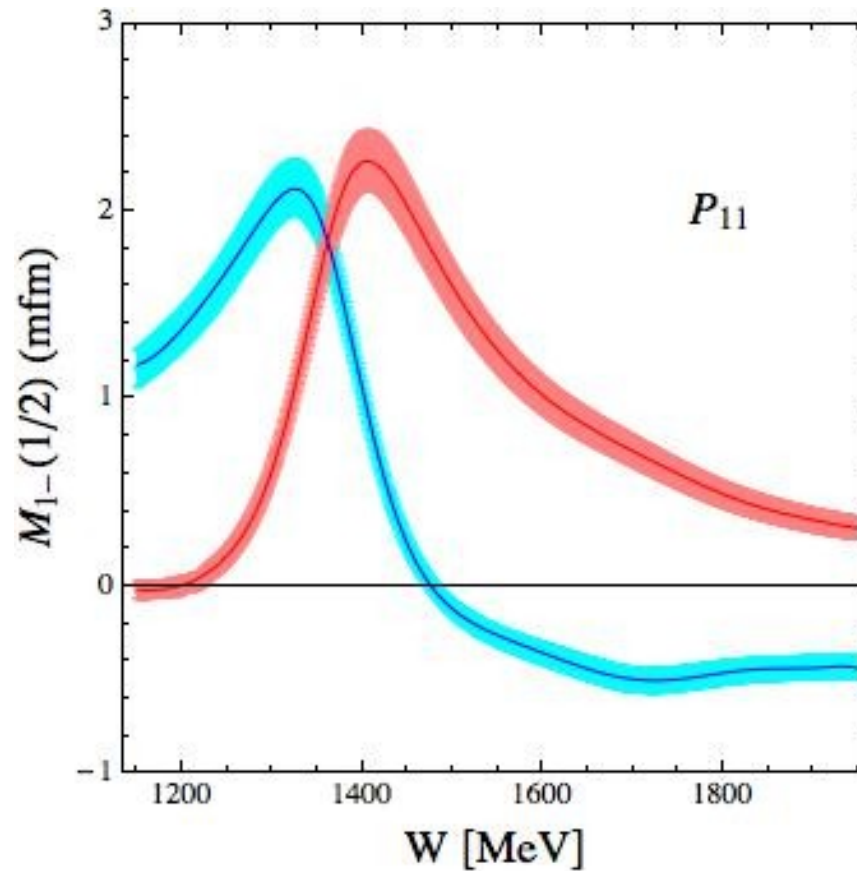


$M_{\text{MAID}} = 1509 \pm 1 \text{ MeV}$
 $\Gamma_{\text{MAID}} = 106 \pm 1 \text{ MeV}$

[Tiator, Drechsel, Kamalov, and Vanderhaeghen'11]

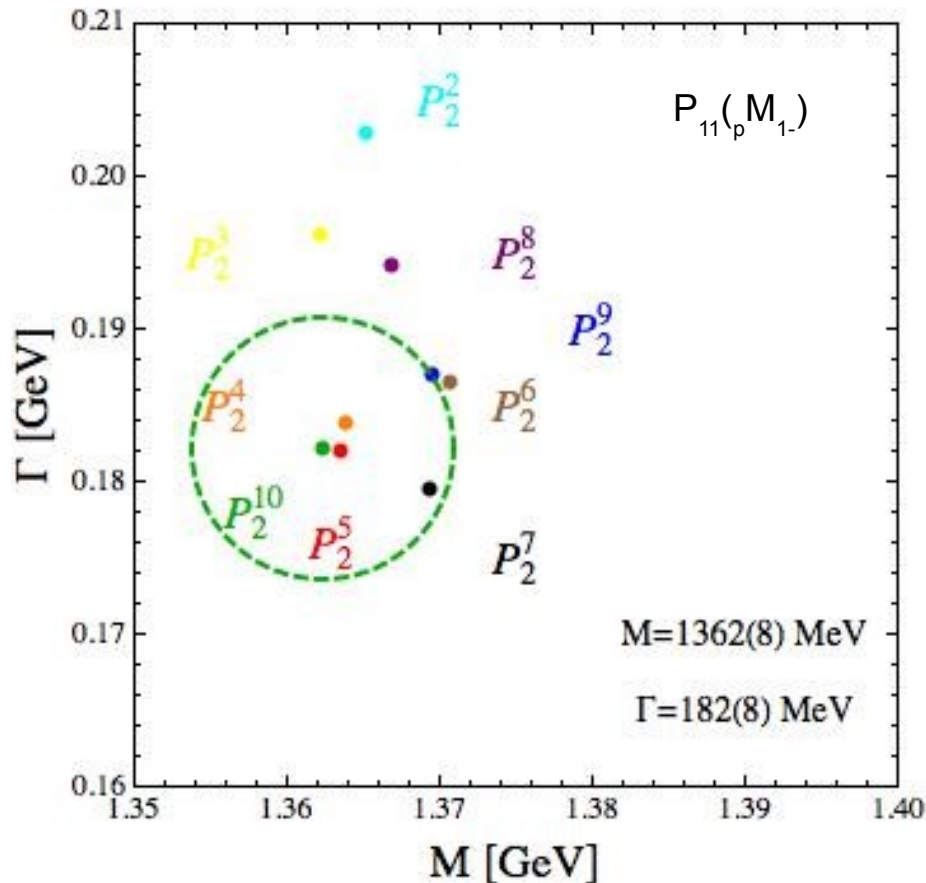
Fitting MAID2007 data: P11

PRELIMINARY RESULTS



Fitting MAID2007 data: P11

PRELIMINARY RESULTS



$$M_{\text{MAID}} = 1379 \pm 5 \text{ MeV}$$

$$\Gamma_{\text{MAID}} = 183 \pm 6 \text{ MeV}$$

[Tiator, Drechsel,
Kamalov, and
Vanderhaeghen'11]

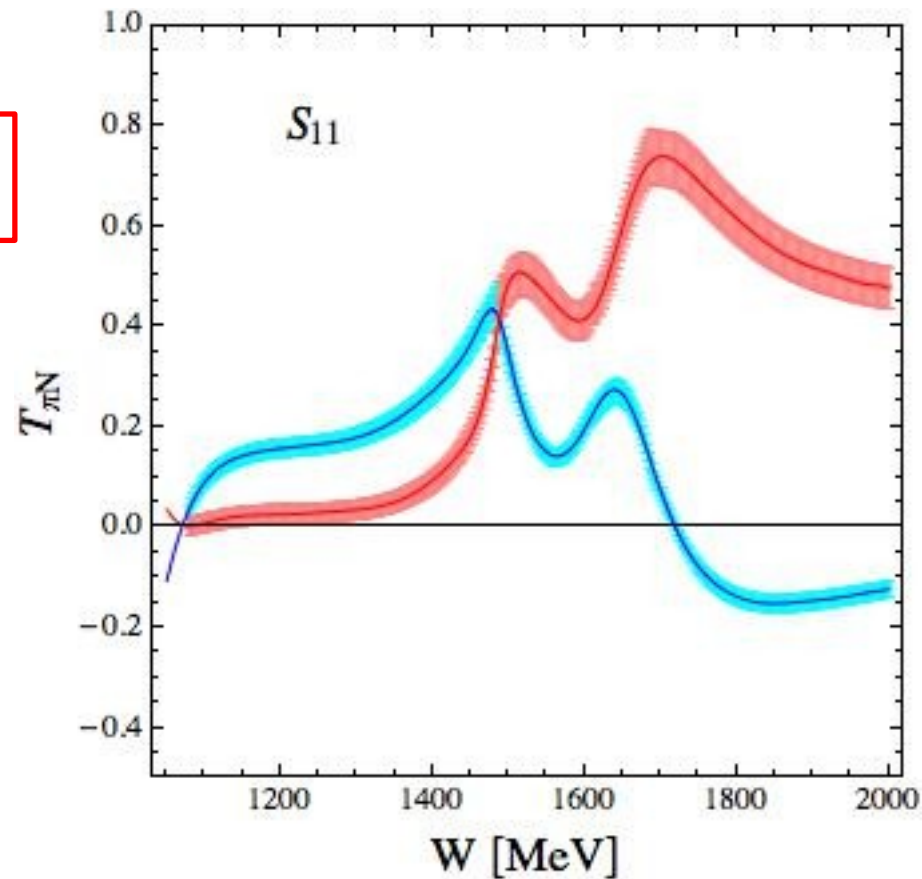
Fitting ANL-Osaka data: S11

PRELIMINARY RESULTS

Fitting ANL-Osaka data: S11

PRELIMINARY RESULTS

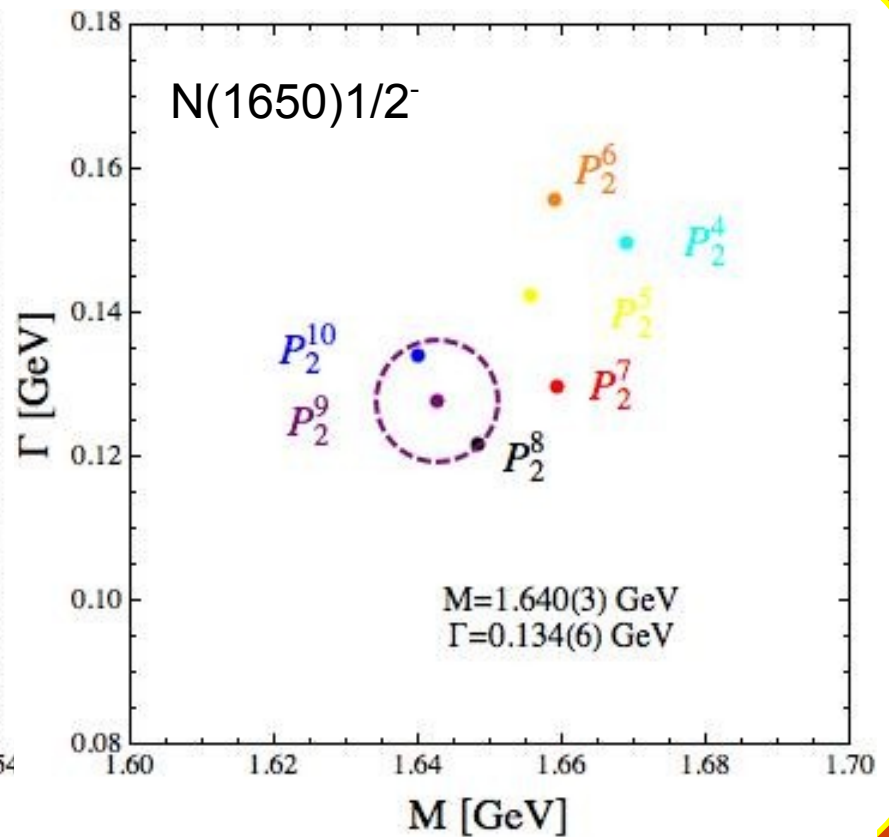
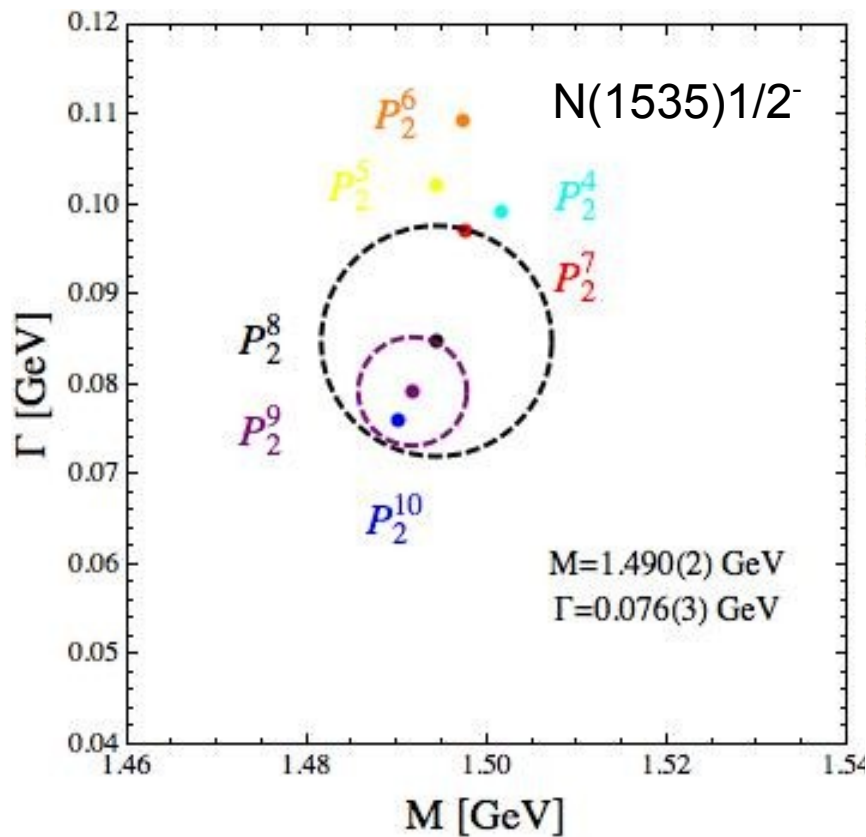
Padé Approximants
method



[Courtesy of
T. Sato's group]

Fitting ANL-Osaka data: S11

PRELIMINARY RESULTS

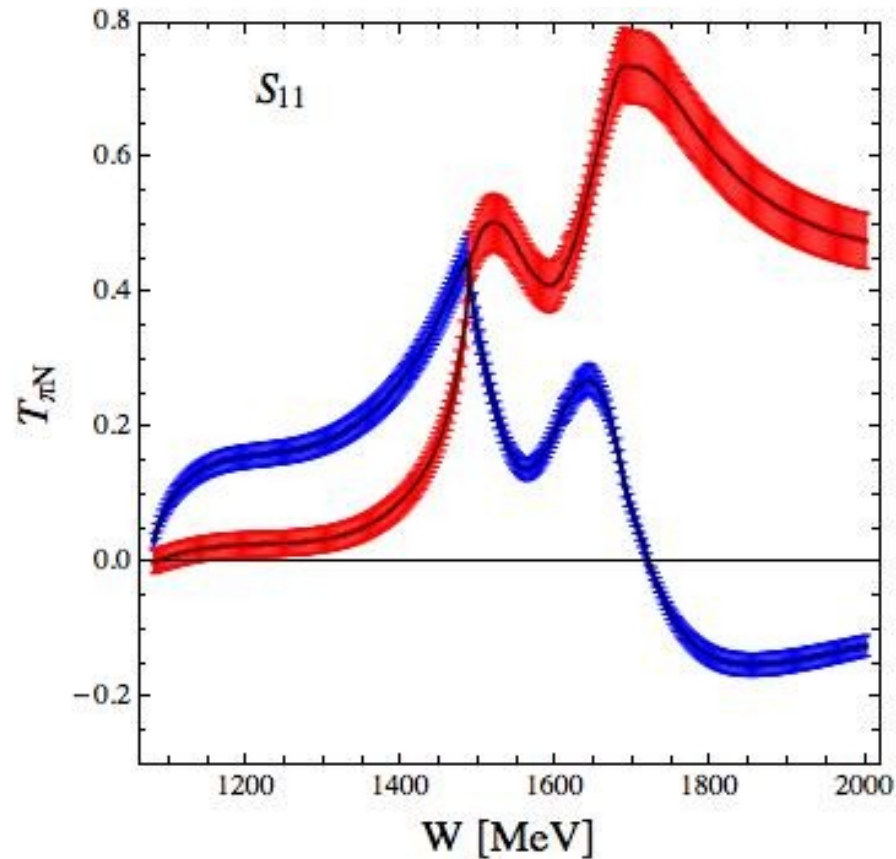


Fitting ANL-Osaka data: S11

PRELIMINARY RESULTS

L+P method

[Courtesy of
A. Svarc and
H. Osmanovich]



Fitting ANL-Osaka data: S11

PRELIMINARY RESULTS

N(1535)1/2-	$Re[W]$ MeV	$-2Im[W]$ MeV	Residue R	ϕ [Degree]
Kamano <i>et al</i>	1482	196	63	-44
L+P	1491(1)	82(2)	11	-59
Padé	1490(2)	76(3)	11	-58

N(1650)1/2-	$Re[W]$ MeV	$-2Im[W]$ MeV	Residue R	ϕ [Degree]
Kamano <i>et al</i>	1656	170	53	-70
L+P	1643(1)	144(1)	33	-97
Padé	1640(3)	134(6)	27	-109

Conclusions

- We use sequences of rational approximants, called Padé Appr. as a tool for extracting resonance parameters.
- PA not always work. We rely on Montessus theorem.
- We explore the *genuine* PA and the *fitted* PA.
- We study MAID2007 and ANL-Osaka.
- The method is simple, systematic and provides a systematic error estimation.

Thank you!