

# MAID partial wave analysis using the Pietarinen expansion method

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in collaboration with

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Baryon Resonances, PWA7, Camogli (Italy), September 23 - September 27, 2013







## our general motivation



precise knowledge of meson photoproduction amplitude  
is important for:

- designing of proposals, setting up experiments and data analysis
- comparison with EFT, near threshold and near resonances
- dispersion theoretical applications, Compton scattering, sum rules
- baryon resonance analysis, besides  $\pi N \rightarrow \pi N$ ,  
 $\gamma N \rightarrow \pi N$  is the most important source
- comparisons with quark models and lattice QCD,  
especially for  $N^*$  physics

## our specific motivation



MAID is a unitary isobar model build with

- simple background (Born terms + vector meson t-ch exchange)
- and a set of Breit-Wigner resonances (currently 13 \*\*\*\*)
- and unitarization constraints

it cannot provide the underlying fundamental pole structure  
with pole positions and residues

and it cannot easily be extended with further degrees of freedom  
in order to fit any new polarization data with high statistics

we believe, a solution to these problems can be the application of  
the L+P expansion method to energy dependent (ED) and independent (SE) PWA

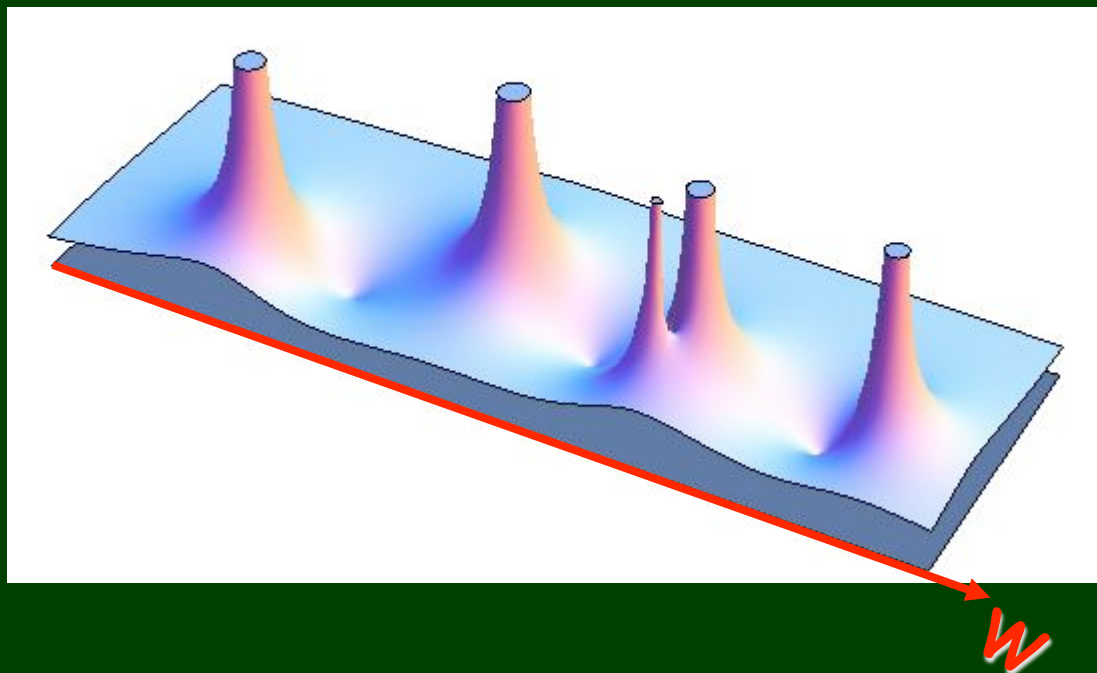
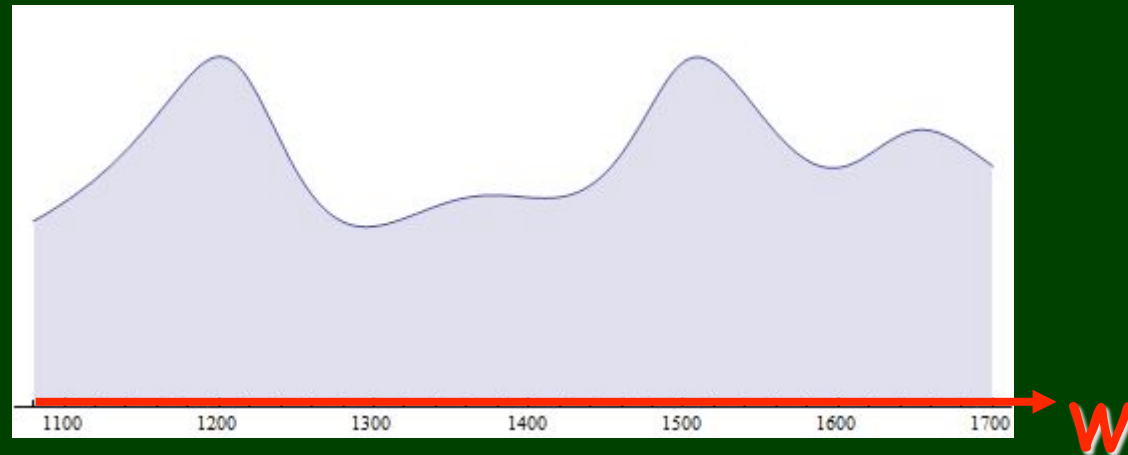


# experimental bumps and theoretical poles



bumps on the  
physical axis

typically described with  
Breit-Wigner functions



poles in the  
complex plane

typically described with  
Laurent Series

# PWA groups in Baryon Spectroscopy



- SAID** model indep. single ch. PWA  $\pi N \rightarrow \pi N$ ,  $\pi N \rightarrow \eta N$ ,  $\gamma N \rightarrow \pi N$ ,  $\gamma N \rightarrow \eta N$   
<http://gwdac.phys.gwu.edu/>
- BnGa** multichannel partial wave analysis,  $\pi N$ ,  $\eta N$ ,  $\pi\pi N$ ,  $K\Lambda$ ,  $K\Sigma$ ,  $\gamma N$   
<http://pwa.hiskp.uni-bonn.de/>
- MAID** unitary isobar model, single ch.  $\gamma N \rightarrow \pi N$ ,  $\gamma N \rightarrow \eta N$ ,  $\gamma N \rightarrow K\Lambda(\Sigma)$   
<http://www.kph.uni-mainz.de/MAID/>
- DMT** dynamical model with few coupled channels,  $\pi N$ ,  $\eta N$ ,  $\pi\pi N$ ,  $\gamma N$   
<http://www.kph.uni-mainz.de/MAID/>
- Jülich** dynamical model with coupled ch.,  $\pi N$ ,  $\eta N$ ,  $\pi\pi N$ ,  $K\Lambda$ ,  $K\Sigma$ , ...,  $\gamma N$
- Gießen** coupled ch. unitary Lagrangian model,  $\pi N$ ,  $\eta N$ ,  $\pi\pi N$ ,  $K\Lambda$ ,  $K\Sigma$ ,  $\gamma N$
- Kent State** K matrix coupled channels,  $\pi N$ ,  $\eta N$ ,  $\pi\pi N$ ,  $K\Lambda$ ,  $K\Sigma$ ,  $\gamma N$
- ANL-Osaka** dynamical model with coupled ch.,  $\pi N$ ,  $\eta N$ ,  $\pi\pi N$ ,  $K\Lambda$ ,  $K\Sigma$ , ...,  $\gamma N$



# comparison of pole parameters from PWA groups



## ■ SAID 2006:

Arndt, Briscoe, Strakovsky, Workman, Phys. Rev. C 74, 045205 (2006)

## ● DMT 2007:

Chen, Kamalov, Yang, Drechsel, Tiator, Phys. Rev. C 76, 035206 (2007)

## ● BNGA 2012:

Anisovich, Beck, Klempt, Nikonov, Sarantsev, Thoma, Eur. Phys. J. A 48, 15 (2012)

## ■ Jülich 2013:

Rönchen, Döring, Huang, Haberzettl, Haidenbauer, Hanhart, Krewald, Meißner, Nakayama, Eur. Phys. J. A 49, 44 (2013)

## ◆ ANL-Osaka 2013:

Kamano, Nakamura, Lee, Sato, arXiv:1305.4351 (2013)

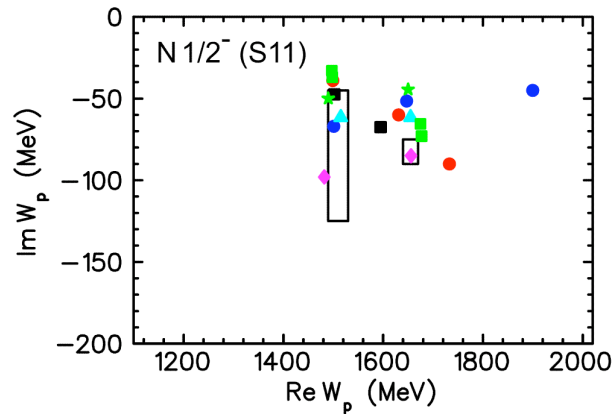
## ▲ KSU 2012:

Shrestha, Manley, Phys. Rev. C 86, 055203 (2012)

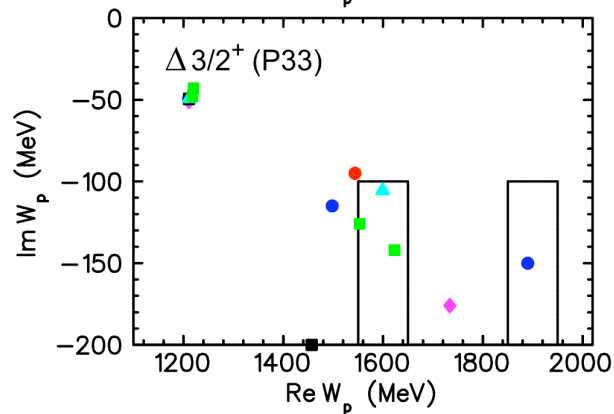
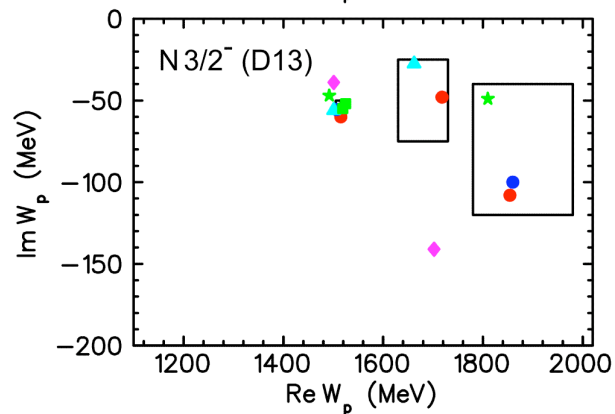
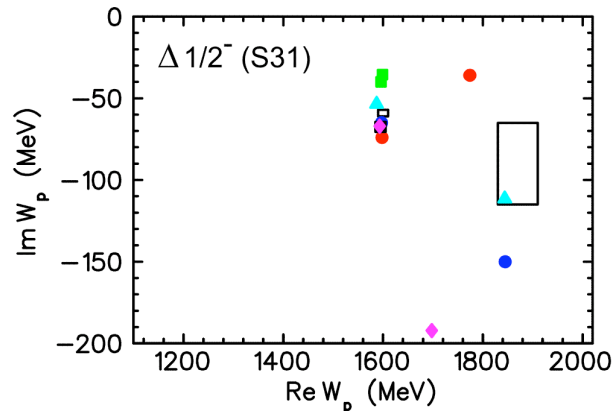
## ★ Gießen 2013:

Shklyar, Lenske, Mosel, Phys. Rev. C 87, 015201 (2013)

# pole positions for selected resonances



- PDG ranges
- SAID 2006
- DMT 2007
- BnGa 2012
- Jülich 2013
- ◆ ANL-Osaka 2013
- ▲ KSU 2012
- ★ Giessen 2013



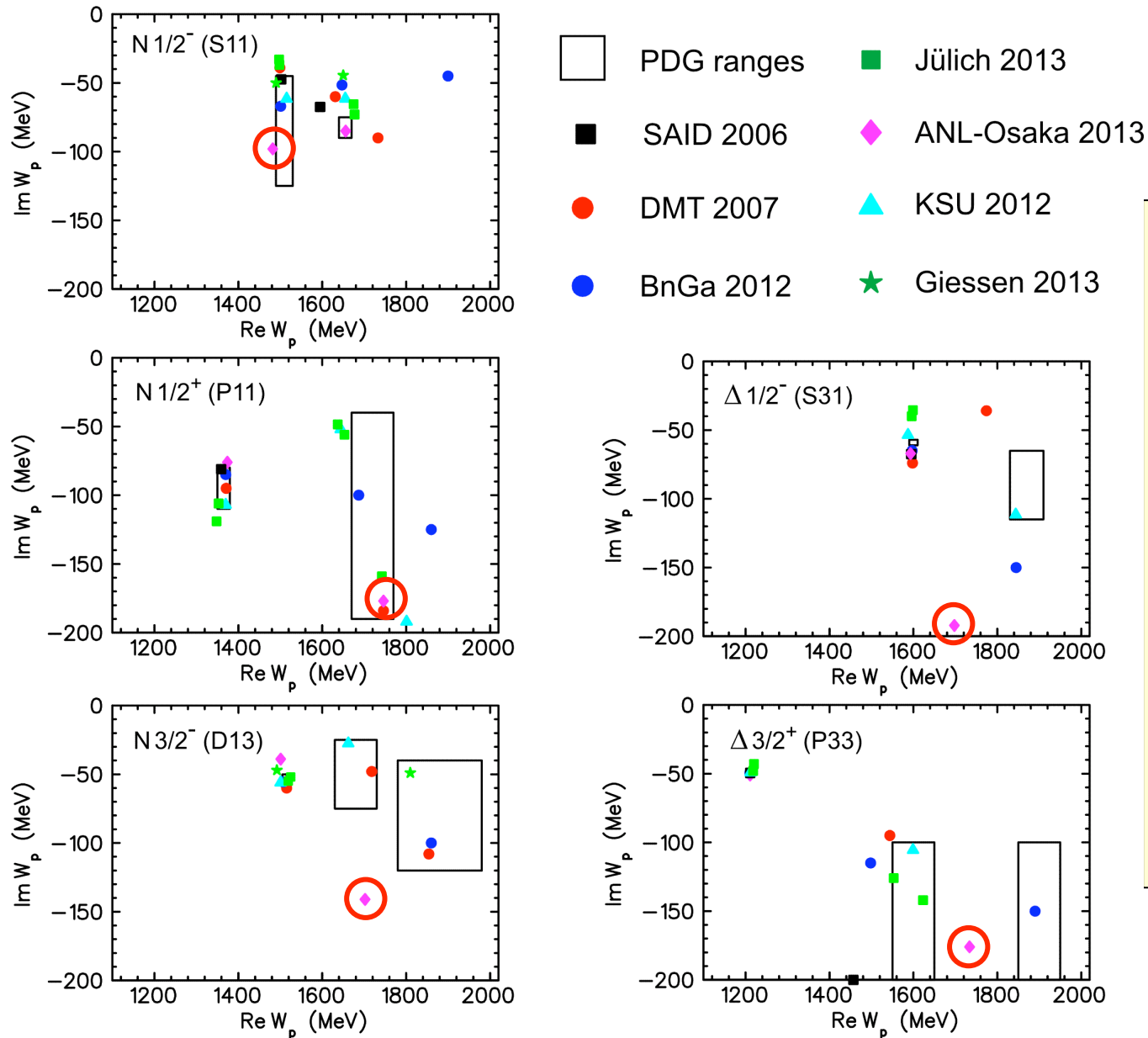
consistent analysis  
only for  
4-star resonances

3\*, 2\*, 1\* res.  
show a large spread  
in their pole positions

a more sophisticated  
error analysis  
in PWA is demanded



# pole positions for selected resonances



consistent analysis  
only for  
4-star resonances

some larger deviations  
observed in  
ANL-Osaka analysis

the S11, S31, D13 poles  
are on hidden sheets

# baryon photo-decay amplitudes at the pole



R.L. Workman, L. Tiator, and A. Sarantsev, Phys. Rev. C **87**, 068201 (2013)

photo-decay amplitudes:  $A_{1/2}, A_{3/2}$

e.g. for  $\Delta(1232)$  PDG2012:  $A_{1/2} = -0.135 \pm 0.006 \text{ GeV}^{-1/2}$ ,  $A_{3/2} = -0.250 \pm 0.008 \text{ GeV}^{-1/2}$

all these numbers have traditionally be given as real amplitudes at the Breit-Wigner position

**standard BW definition:**

$$A_h^{BW} = \sqrt{\frac{\pi(2J+1)M_r\Gamma_r^2}{2k_r^2 m_N \Gamma_{\pi,r}}} \text{Im } T_{\gamma_h N \rightarrow \pi N} \Big|_{W=M_r}$$

**new definition at the pole:**

$$\tilde{A}_h^{pole} = \sqrt{\frac{\pi(2J+1)W_{pole}}{k_{pole}^2 m_N \text{Res } T_{\pi N \rightarrow \pi N}}} \text{Res } T_{\gamma_h N \rightarrow \pi N} \Big|_{W=W_{pole}}$$

relation between e.m. multipoles and T-matrix for photoproduction:

$$T_{\gamma_h N \rightarrow \pi N} = \sqrt{2kq} C \mathcal{A}_\alpha^h$$



# photon decay amplitudes for selected resonances

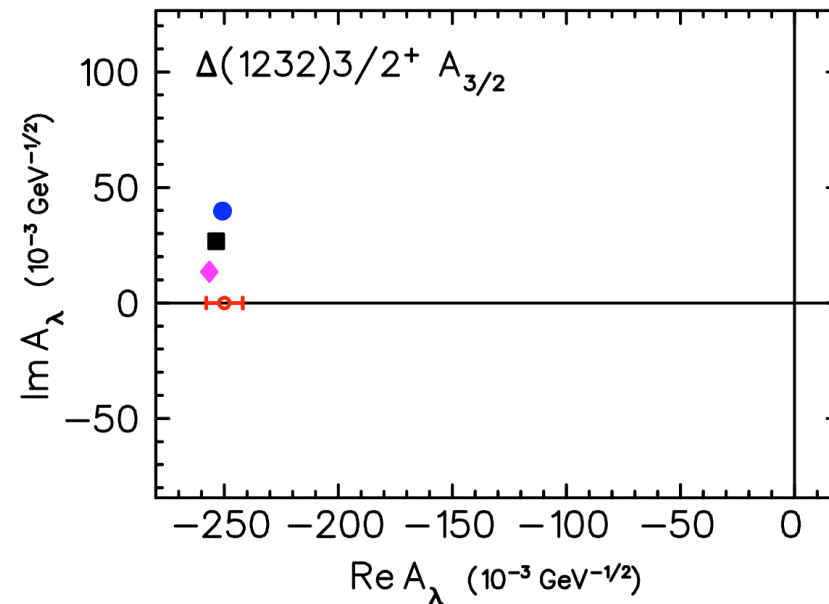
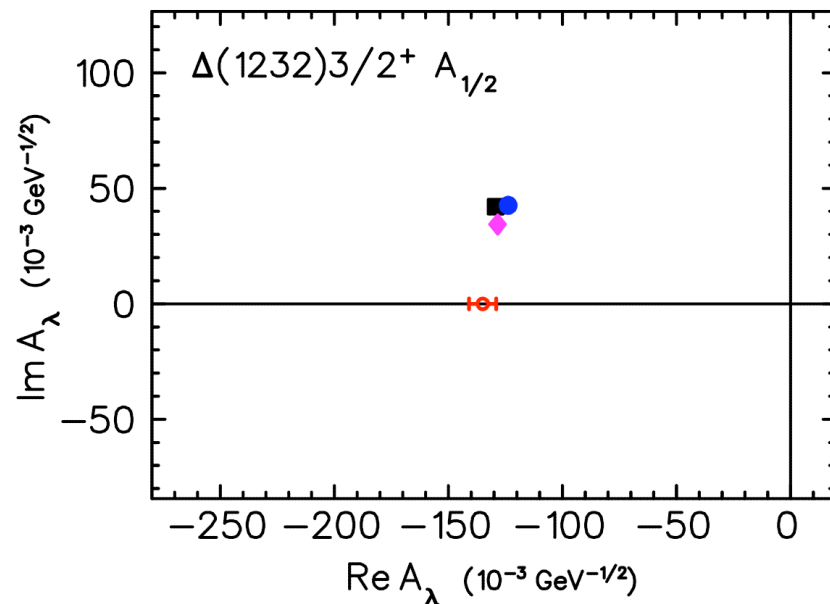


amplitudes at the pole :

- SAID 2006
- BnGa 2012
- ◆ ANL-Osaka 2013

Breit-Wigner amplitudes :

- ⊖ BW value, PDG 2012



# photon decay amplitudes for selected resonances



amplitudes at the pole :

Breit-Wigner amplitudes :

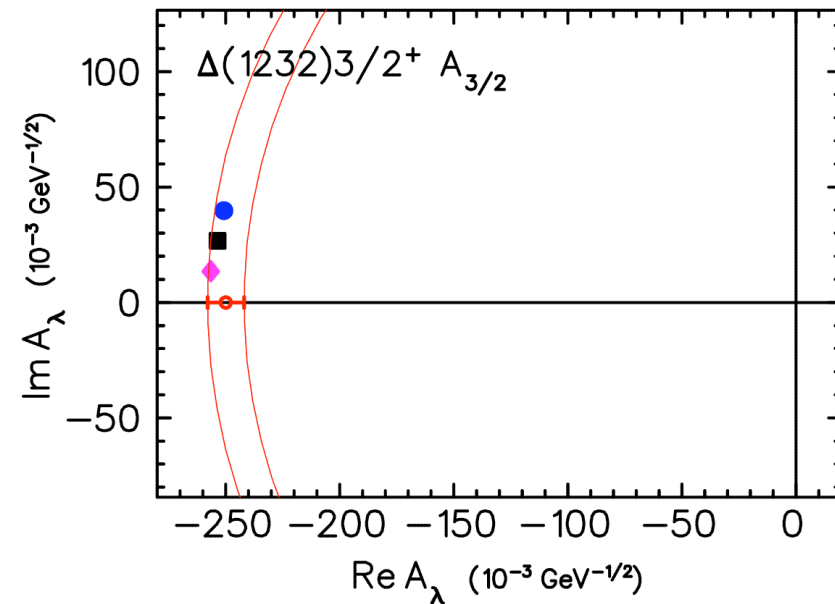
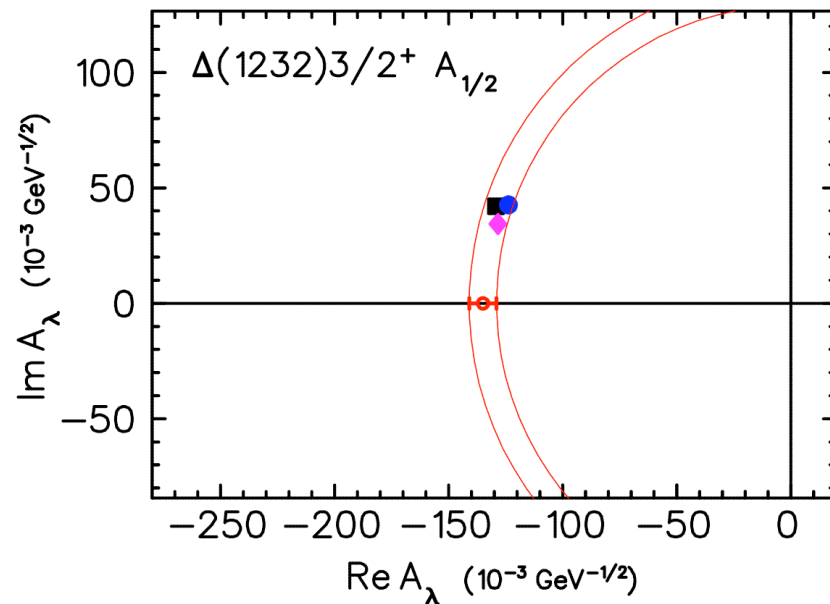
■ SAID 2006

● BnGa 2012

◆ ANL-Osaka 2013

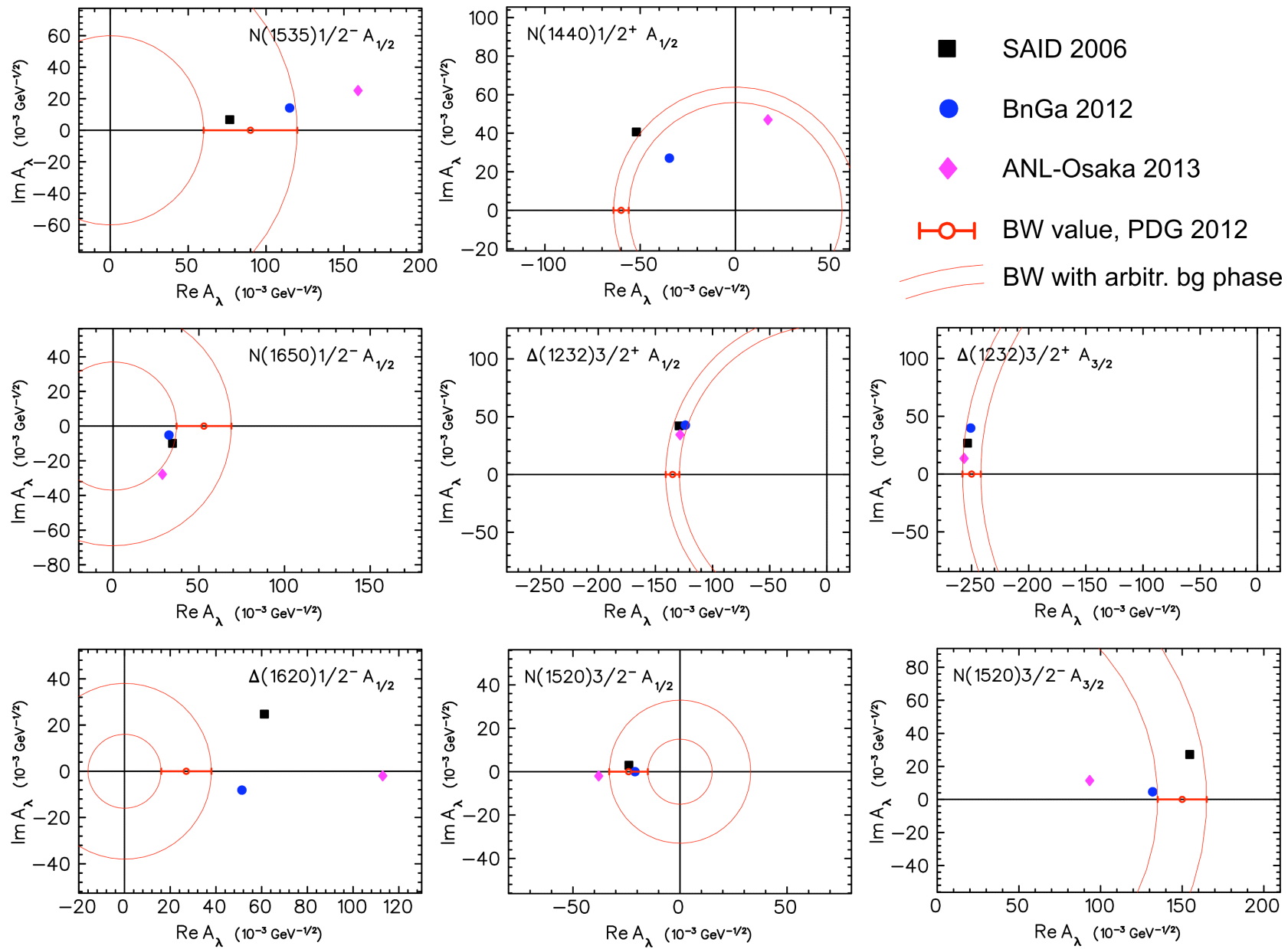
○ BW value, PDG 2012

— BW with arbitr. bg phase





# photon decay amplitudes for selected resonances

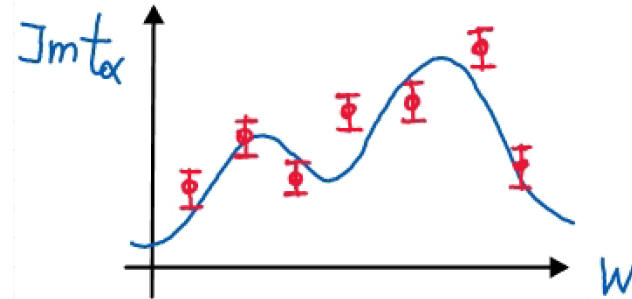


# SE and ED partial wave analysis $t_a(w)$



SE : single-energy analysis

ED : energy-dependent analysis



intelligent parametrization using symmetries, thresholds, branch points, poles, unitarity, dispersion relations, ...

closer to the exp. data, no constraints in ideal case  
problem: multiple solutions very likely

in practise: often loosely bound to ED solutions,

$$\text{e.g. } \chi^2 = \sum_i^{N_{data}} \left( \frac{\theta_i - \theta_i^{exp}}{\delta\theta_i} \right)^2 + \sum_j^{N_{mult}} \left( \frac{X_j - 1}{\Delta} \right)^2$$

usual chisquared      penalty term

and do not have the same statistics as the underlying real data !!

# result of single-energy and energy-dependent fitting



results of latest version MAID2007 (Drechsel, Kamalov, Tiator, EPJ A34, 69 (2007) :

reduced  $\chi^2$  from Maid07 fits to  $\gamma, \pi$  data in different energy regions

	proton			
$E_\gamma$ [MeV]	$N_{\text{mult}}$	$N_{\text{data}}$	$\chi_{\text{se}}^2/N$	$\chi_{\text{gl}}^2/N$
140—200	10	990	0.79	2.37
200—450	10	5622	0.97	2.53
450—850	24	6403	1.09	3.55
850—1210	24	2965	1.39	8.43
1210—1610	24	1454	3.26	8.67
total		17434	1.27	4.41
	neutron			
$E_\gamma$ [MeV]	$N_{\text{mult}}$	$N_{\text{data}}$	$\chi_{\text{se}}^2/N$	$\chi_{\text{gl}}^2/N$
140—200	5	51	2.21	2.96
200—450	5	872	1.85	2.53
450—850	12	902	1.94	3.78
850—1210	12	334	1.95	6.92
1210—1610	12	83	1.29	7.02
total		2242	1.88	4.13

single-energy (SE) fits

energy-dependent (ED) fits

but with higher energies the analysis becomes more difficult and less accurate

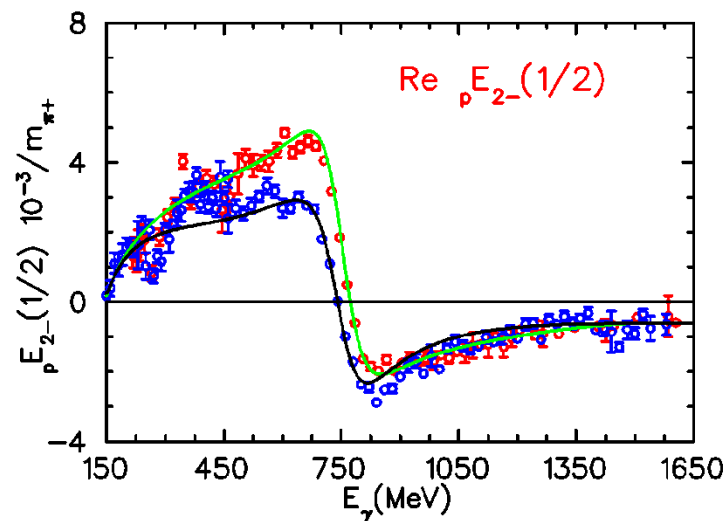
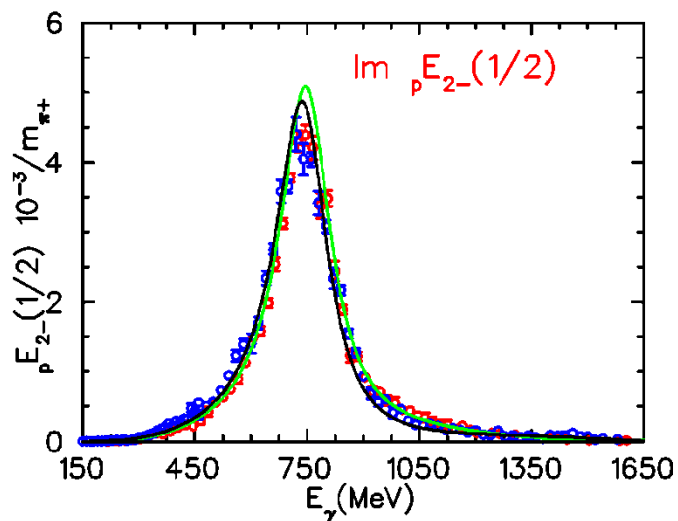
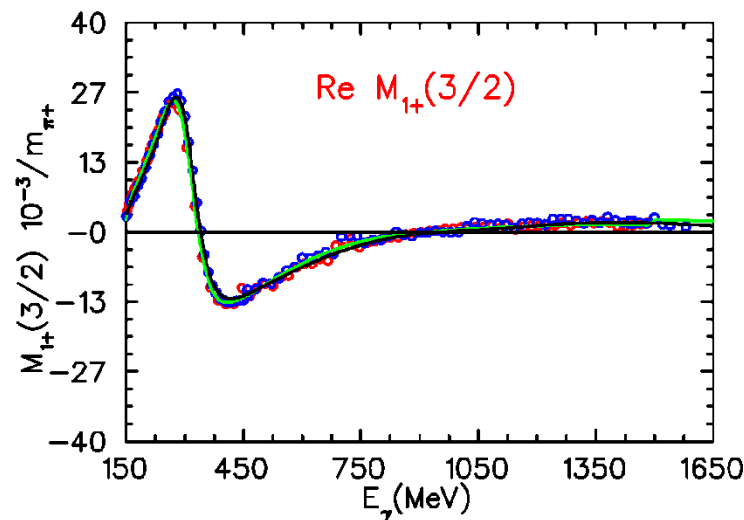
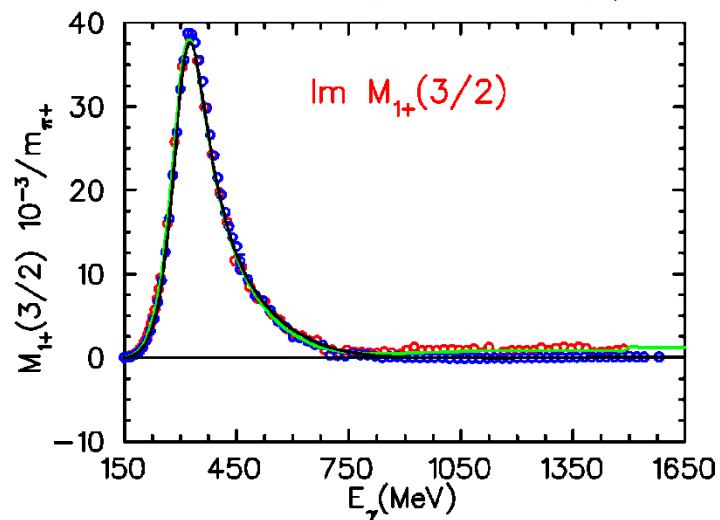
# comparison between MAID and SAID



status 2007

— MAID energy dep. sol.  
○ MAID single-energy sol.

— SAID energy dep. sol.  
○ SAID single-energy sol.



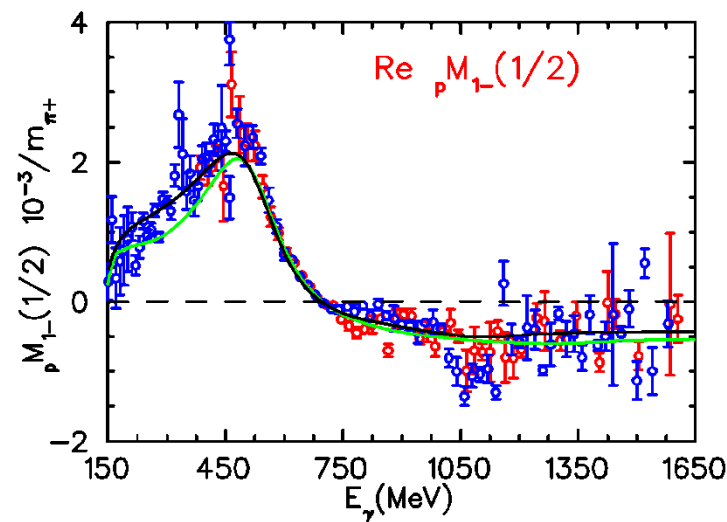
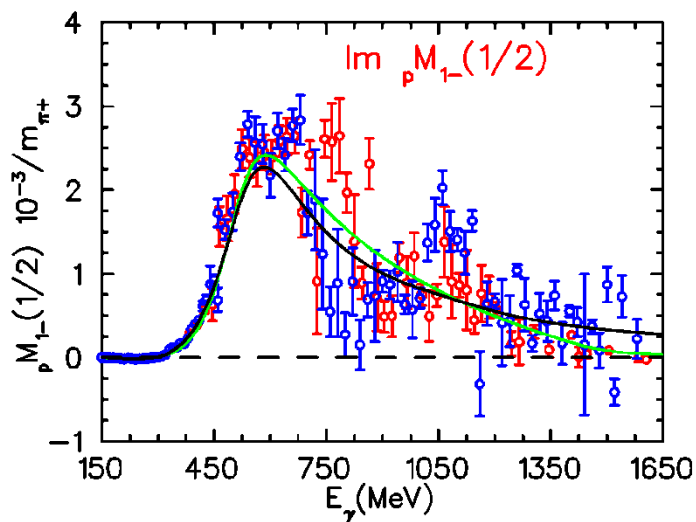
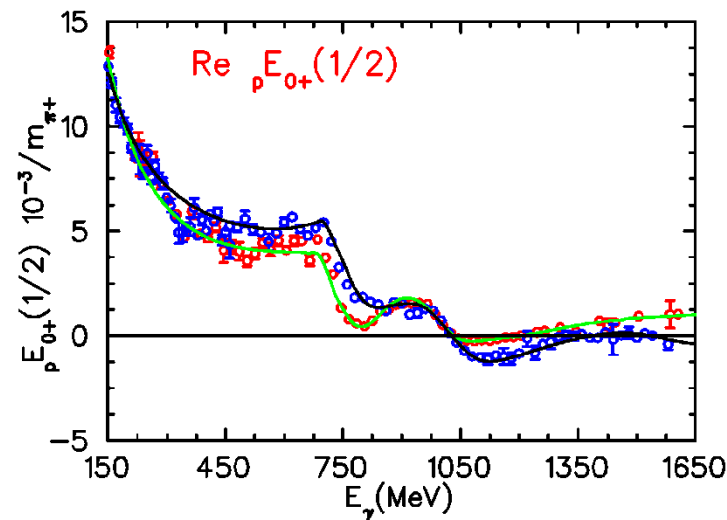
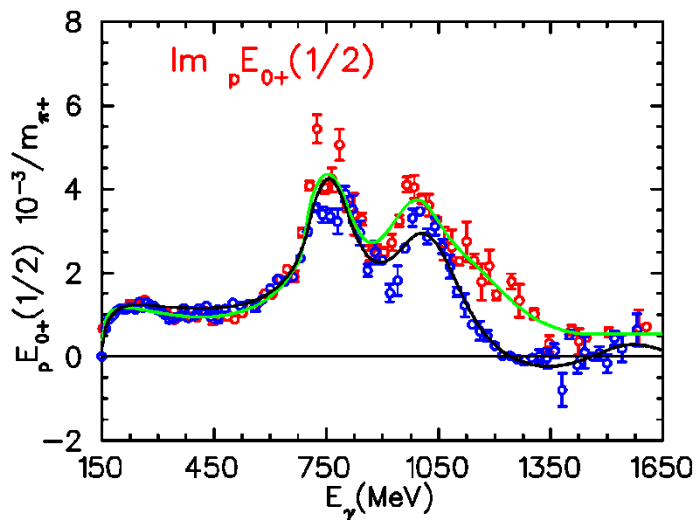
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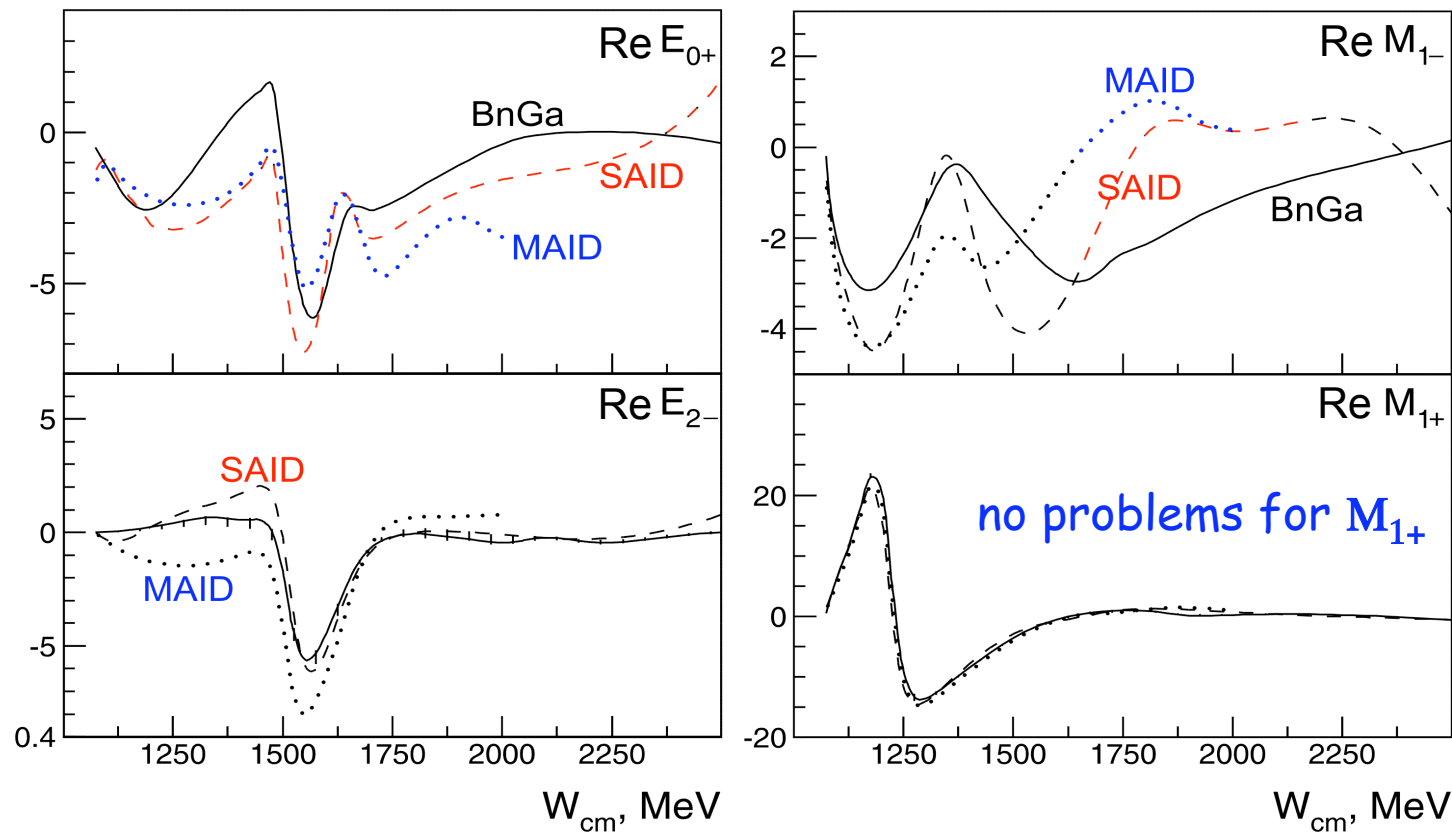


# comparison of multipoles: MAID - SAID - BnGa



from Anisovich et al., Eur. Phys. J. A. **44**, 203-220 (2010)

real parts of  $\gamma, \pi^0$  multipoles



even though the world data is equally well described, **surprisingly large differences!**  
due to an incomplete data base

# unique solutions only in a Complete Experiment



## a) complete amplitude analysis:

(Keaton Workman, 1996; Chiang, Tabakin, 1997)

4 complex amplitudes depending on energy and angle:

$$H_i(E, \theta), \quad i = 1, \dots, 4$$

for that we need 8 observables for any given energy and angle  $E, \theta$

e.g.:  $\sigma_0, \Sigma, T, P, E, F, C_{x'}, O_{x'}$

## b) complete truncated partial wave analysis:

(Omelaenko, 1981; Wunderlich, Beck, Tiator, 2013)

$4 * L_{max}$  complex multipoles depending only on energy:

$$E_\ell \pm (E), M_\ell \pm (E), \quad \ell = 1, \dots, L_{max}$$

for that we need only 5 observables over the whole angular range  $\theta$

e.g.:  $\sigma_0, \Sigma, T, P,$  and  $F$

$\pi N$  elastic

2

4

$2 * L_{max} + 1$

3

for data analysis with experimental uncertainties and statistical fluctuations,  
at least 2-3 more observables are needed for a unique solution

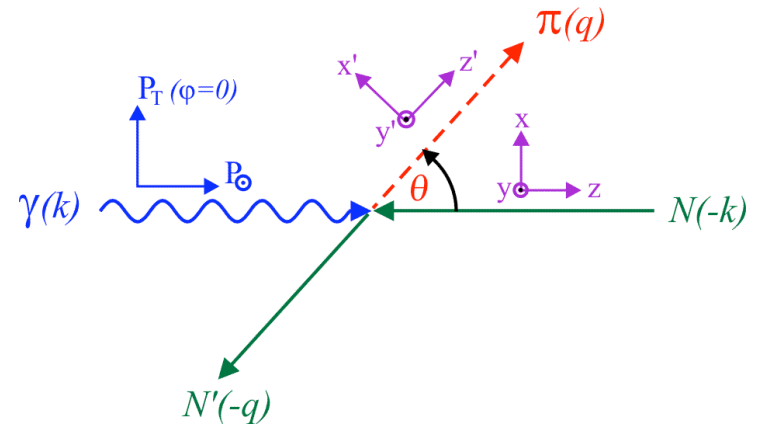
# 16 spin observables in photoproduction



linear and circular polarized beams

longitudinal and transverse polarized targets

recoil polarization, in particular for  $K\Lambda$  and  $K\Sigma$



Photon		Beam - Target			Beam - Recoil			Target - Recoil					
	-	-	-	-	$x'$	$y'$	$z'$	$x'$	$x'$	$z'$	$z'$		
	-	$x$	$y$	$z$	-	-	-	$x$	$z$	$x$	$z$		
unpolarized	$\sigma$	0	$T$	0	0	$P$	0	$T_{x'}$	$L_{x'}$	$T_{z'}$	$L_{z'}$		
linear polariz.	$\Sigma$	$H$	$P$	$G$	$O_{x'}$	$T$	$O_{z'}$	$L_{z'}$	$T_{z'}$	$L_{x'}$	$T_{x'}$		
circular polariz.	0	$F$	0	$E$	$C_{x'}$	0	$C_{z'}$	0	0	0	0		

8 observ.      12 observ.

# new prel. Mainz data with transversely polarized target

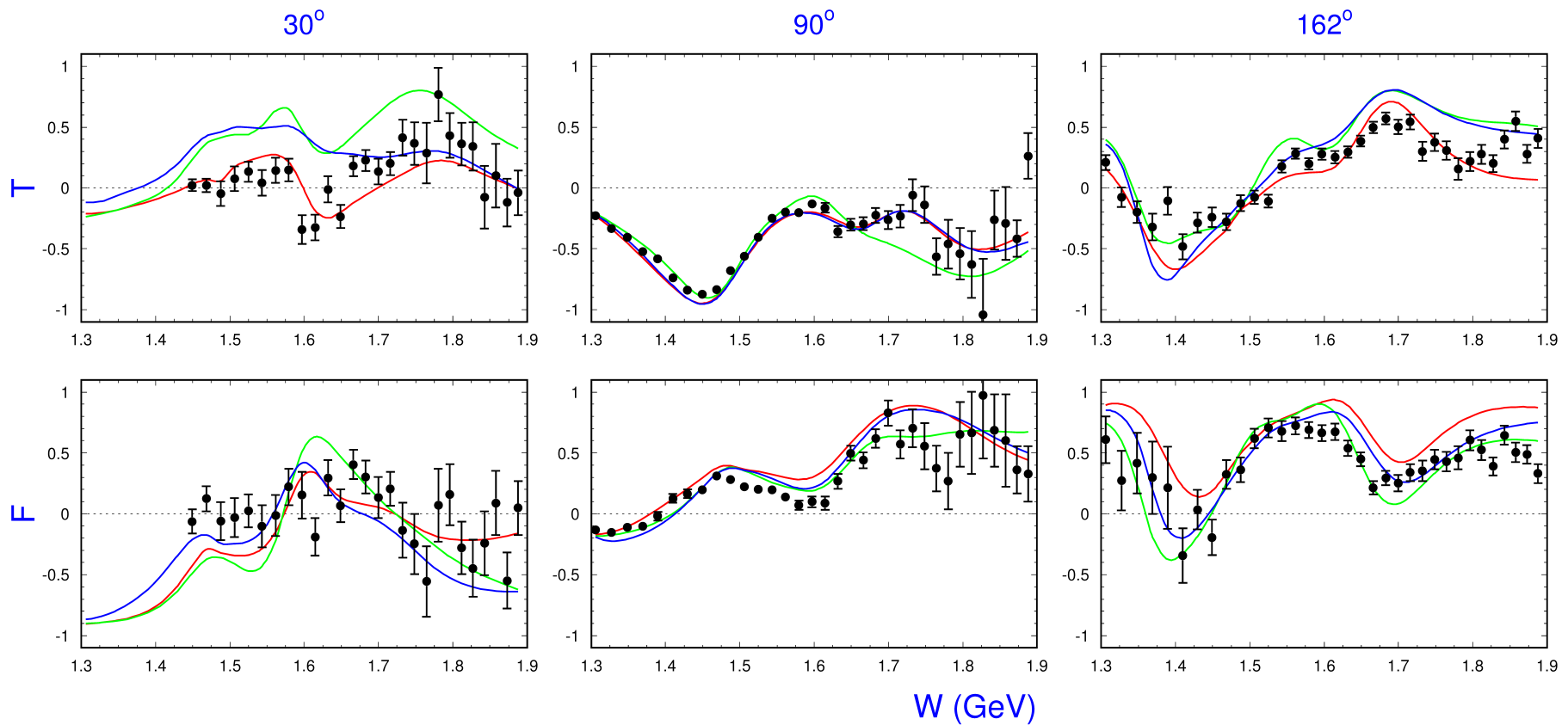


— MAID  
— SAID  
— BnGa

preliminary MAMI data:

$T$  : target asymmetry

$F$  : lin. pol. photon beam - transv. target pol.

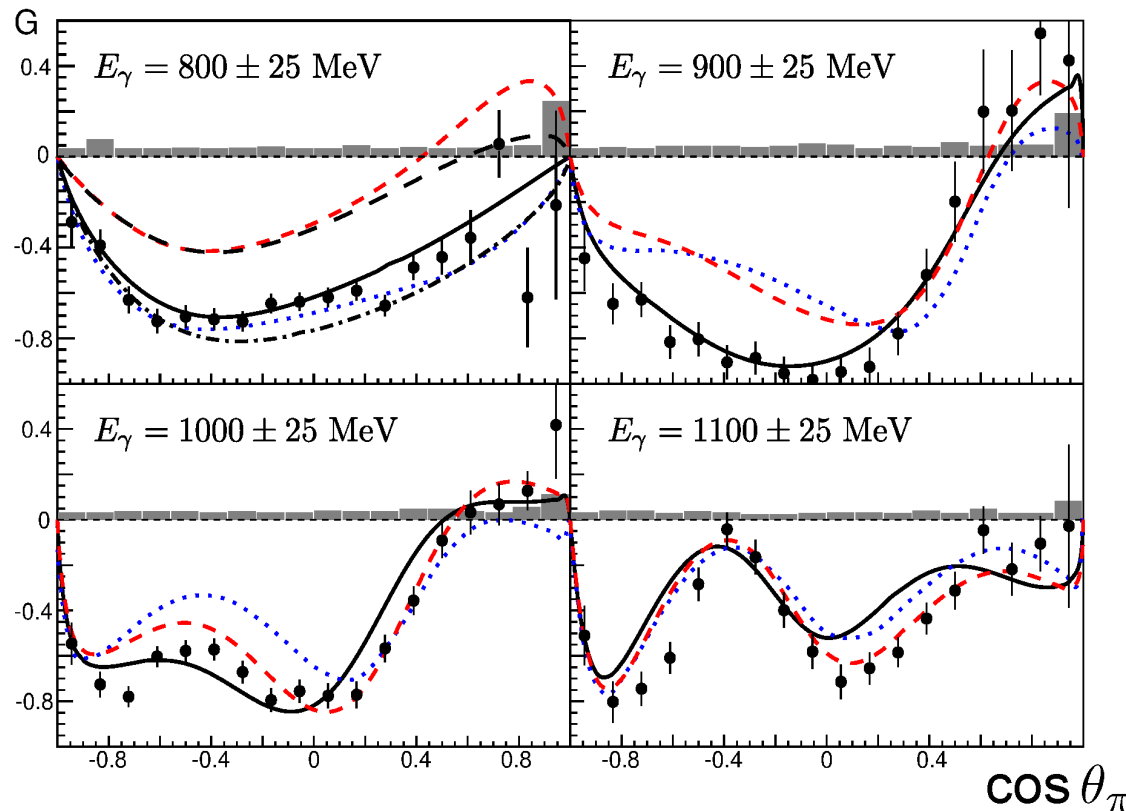




## Asymmetry $G$ in $\vec{\gamma} \vec{p} \rightarrow p \pi^0$ (CBELSA/TAPS)

$$\frac{d\sigma}{d\Omega} = \sigma_0 \{ 1 - \delta_I \Sigma \cos 2\phi$$

$$+ \Lambda_x (-\delta_I H \sin 2\phi + \delta_{\odot} F) \\ - \Lambda_y (-T + \delta_I P \cos 2\phi) \\ - \Lambda_z (-\delta_I G \sin 2\phi + \delta_{\odot} E) \}$$



Surprisingly,  $\pi$  production also not well understood at lower energies:

- BoGa
- - SAID
- ⋯ MAID



A. Thiel *et al.* [CBELSA/TAPS], PRL **109**, 102001 (2012)



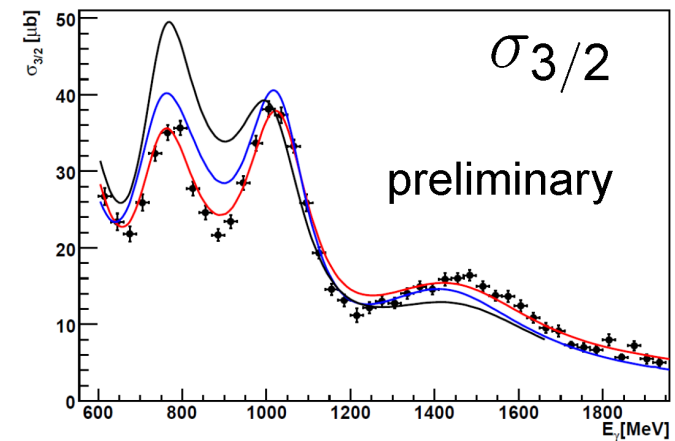
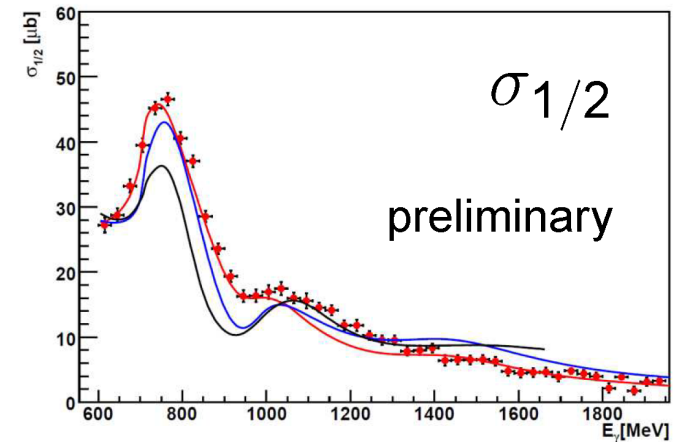
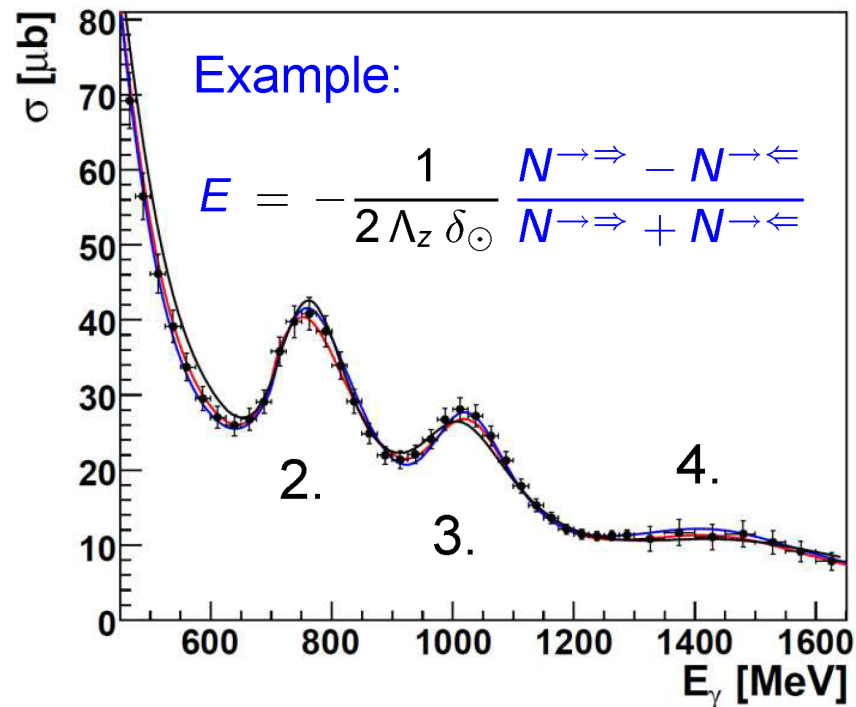


# new Bonn data with longitudinally polarized target



## Example: Ambiguities in $\gamma p \rightarrow p \pi^0$

- Bonn-Gatchina (2011\_2)
- SAID (SN11, CM12)
- MAID



# how can we improve MAID ?



main question: are the discrepancies due to background or resonance contributions?

for background: we could add polynominal functions

for resonance: we could add more Breit-Wigner terms  
PDG lists 50 resonances, MAID uses only 13 \*\*\*\* resonances

our new strategy: obtain fits of partial waves to SE analysis  
then go back to observables  
perform a new SE-fit starting from new solution  
obtain a new fit of partial waves to new SE-fit  
continue this iteration until it converges

for the fitting we use the Pietarinen expansion, discussed this morning  
it is very suitable to describe partial wave amplitudes,  
especially in the presence of branch-points  
and it is able to find the underlying  $N^*$  and  $\Delta$  resonances  
and extract precise pole positions and residues

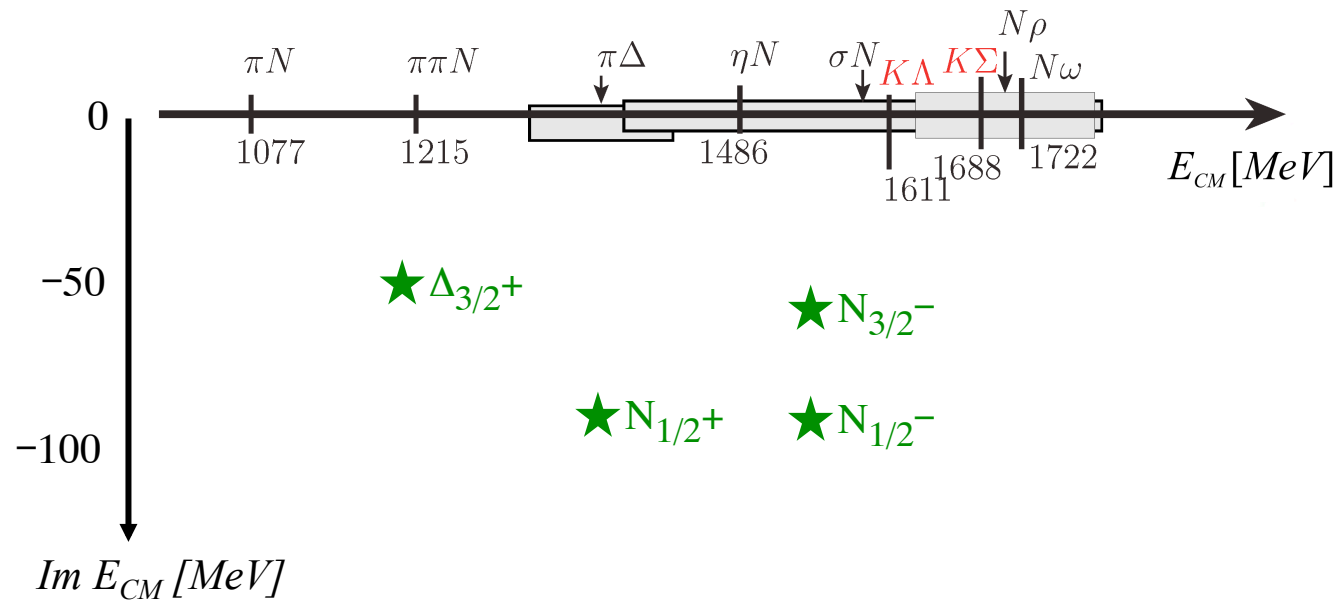
# Nucleon Resonance Analysis with Pietarinen expansion



in collaboration with Svarc (Zagreb), Hadzimehmedovic, Osmanovic, Stahov (Tuzla), Workman (GWU), Phys. Rev. C88, 035206 (2013)

The singularities that strongly influence the partial wave amplitudes in the physical region are the thresholds (branch-points) on the real axis and the poles in the closest (2nd) Riemann sheet:

poles and branch points (regions) in the Jülich coupled channels model:



# Nucleon Resonance Analysis with Pietarinen expansion



in collaboration with Svarc (Zagreb), Hadzimehmedovic, Osmanovic, Stahov (Tuzla), Workman (GWU), Phys. Rev. C88, 035206 (2013)

The L+P (Laurent+Pietarinen) expansion method is defined as:

$$T(\omega) = \sum_{i=1}^k \frac{a_{-1}^{(i)}}{\omega_i - \omega} + B^L(\omega)$$
$$B^L(\omega) = \sum_{n=0}^M c_n Z(\omega)^n + \sum_{n=0}^N d_n W(\omega)^n + \dots$$
$$Z(\omega) = \frac{\alpha - \sqrt{x_P - \omega}}{\alpha + \sqrt{x_P - \omega}}; \quad W(\omega) = \frac{\beta - \sqrt{x_Q - \omega}}{\beta + \sqrt{x_Q - \omega}} + \dots$$

$a_{-1}^{(i)}, \omega_i, \omega \in \mathbb{C}$   
 $c_n, x_P, d_n, x_Q, \alpha, \beta \dots \in \mathbb{R}$   
and  $k, M, N \dots \in \mathbb{N}$ .

1 Pietarinen series  
for each branch point

we have typically 3 Pietarinen  
1 in unphysical region  $E < \text{thresh}$   
2 in physical region,  
e.g.  $\pi N, \pi\pi N, \eta N$  thresholds

→ the Pietarinen expansion is a conformal mapping of the  $\omega$ -plane onto the interior of the unit circle of the  $Z$ -plane

E. Pietarinen, Nuovo Cim. Soc. Ital. Fis. 12A, 522 (1972)  
(successfully applied in the Karlsruhe  $\pi N$  partial wave analysis)

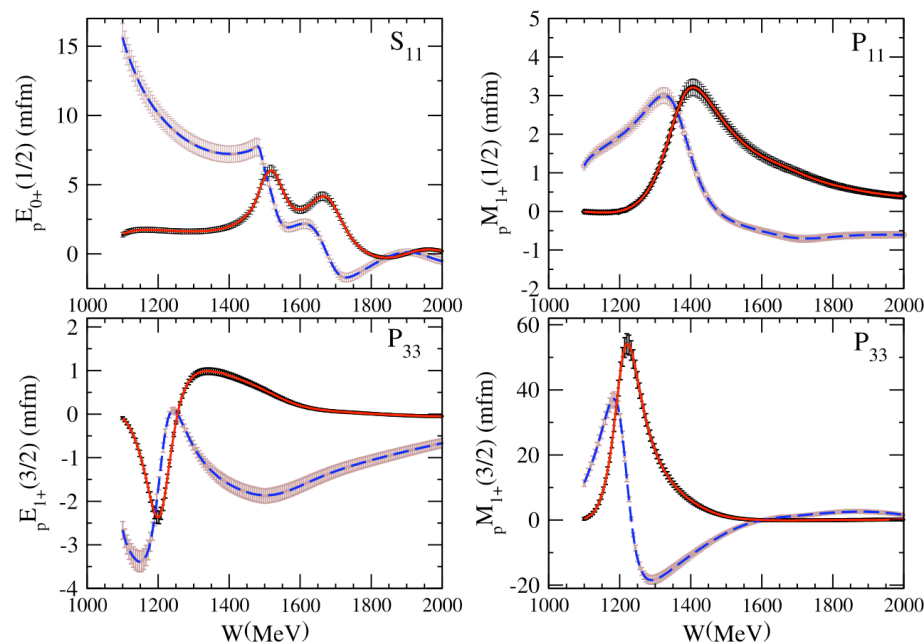
# Pietarinen expansion for the MAID $\gamma, \pi$ PWA



in collaboration with Svarc (Zagreb), Hadzimehmedovic, Osmanovic, Stahov (Tuzla), Workman (GWU), paper in preparation.

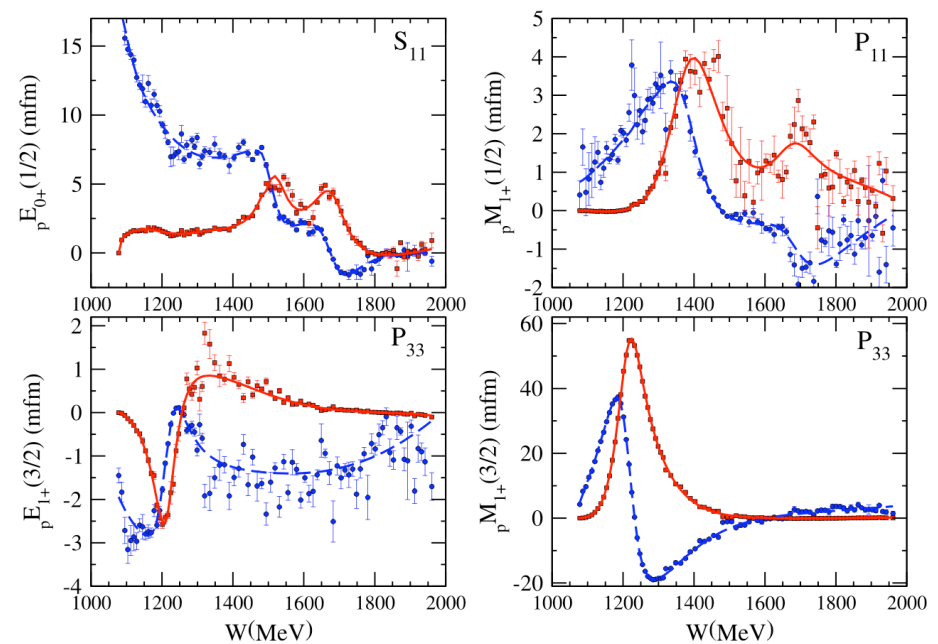
## MAID energy-dependent solution (ED)

for ED solutions, L+P expansion  
gives a numerical approximation  $\sim 10^{-3}$



## MAID single-energy solution (SE)

for SE solutions, L+P expansion  
gives the best-fit with a statistically significant  $\chi^2 \sim 1$



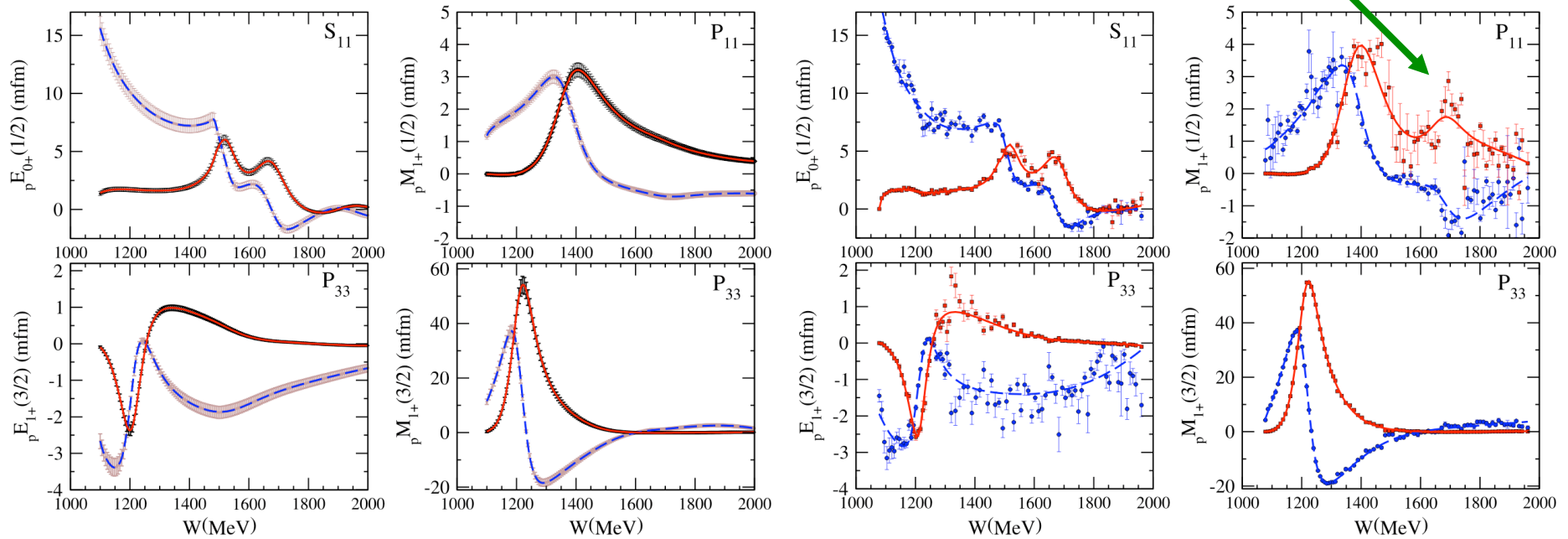


# Pietarinen expansion for the MAID $\gamma, \pi$ PWA



MAID  
energy-dependent solution (ED)

$P_{11}(1710)$   
is not included in MAID  
but it is found in the L+P expansion of  
the MAID single-energy analysis

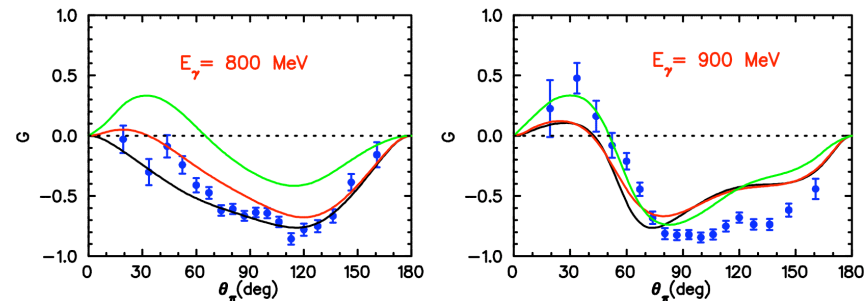
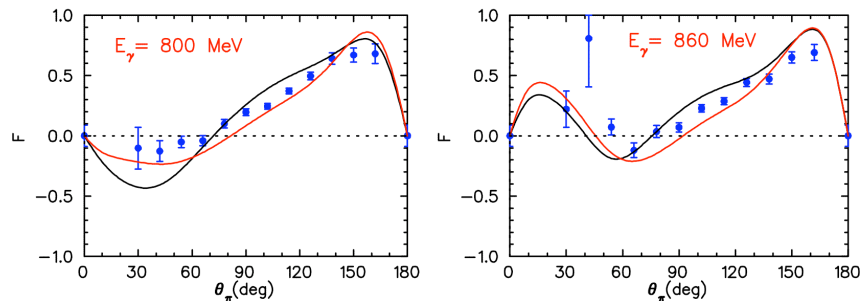
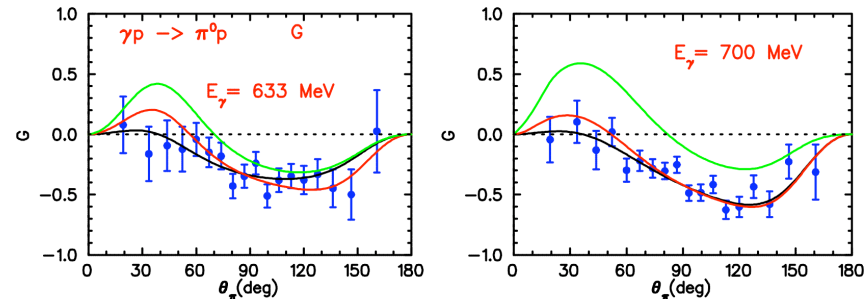
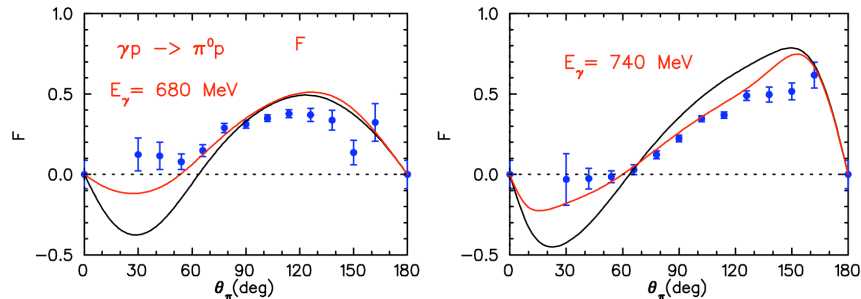
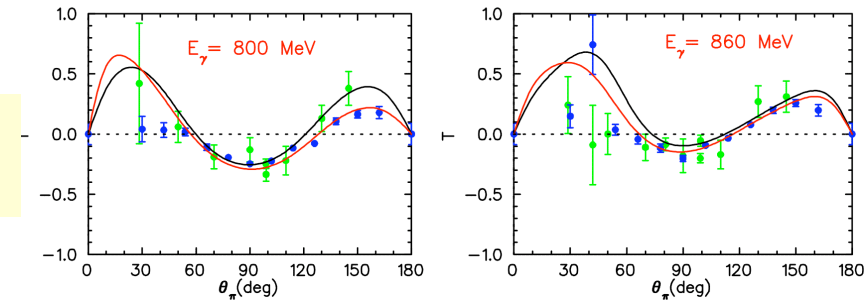
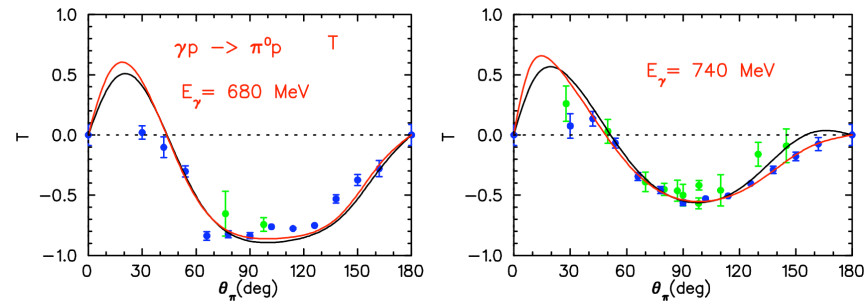


# new L+P fit to new polarization data from Mainz and Bonn



— new L+P fit  
— MAID2007  
— SAID-SN11

for  $E < 900$  MeV the fit looks reasonable, with  $G$  observable we are not yet satisfied



# new L+P fit to new polarization data from Mainz and Bonn

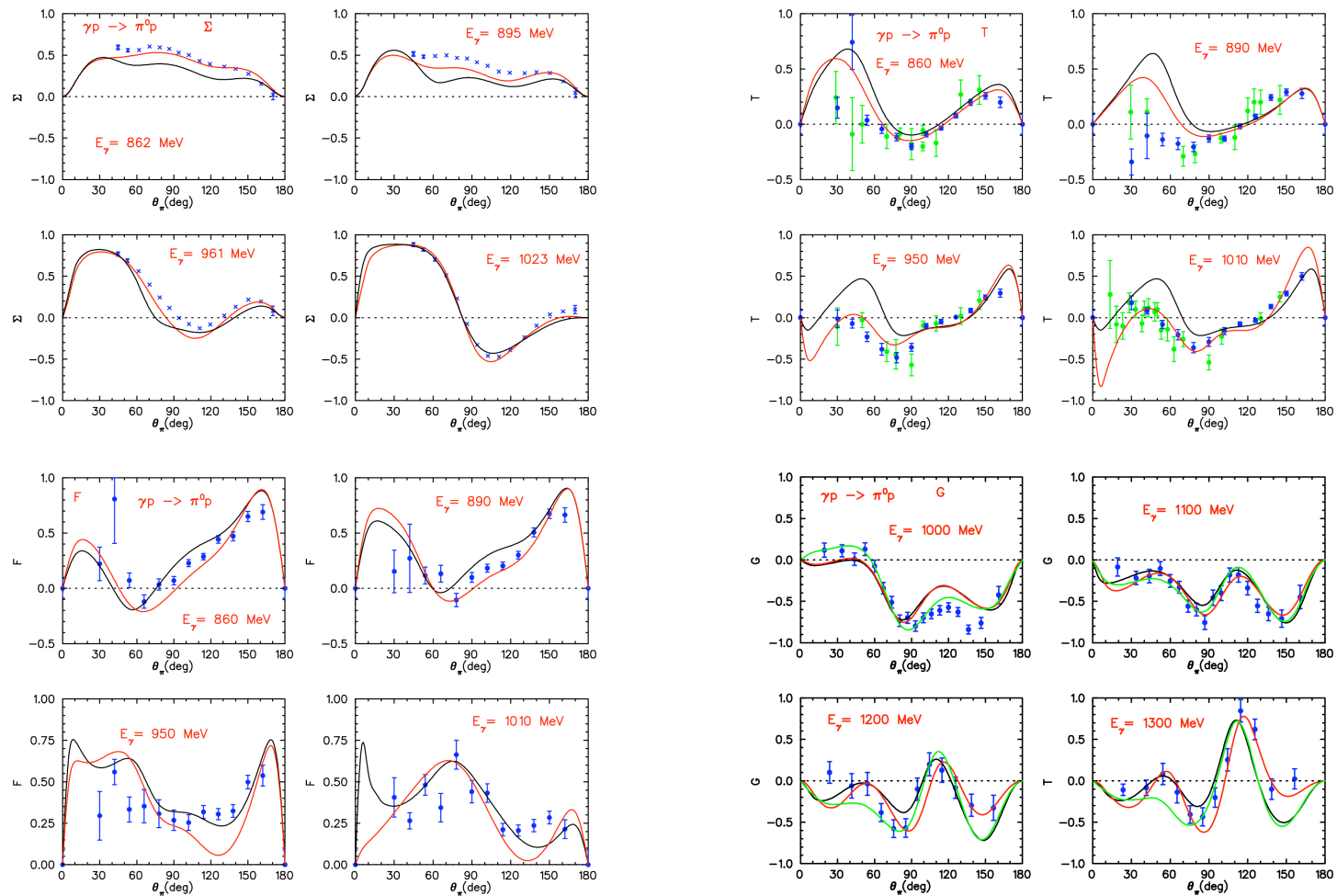


— new L+P fit

— MAID2007

— SAID-SN11

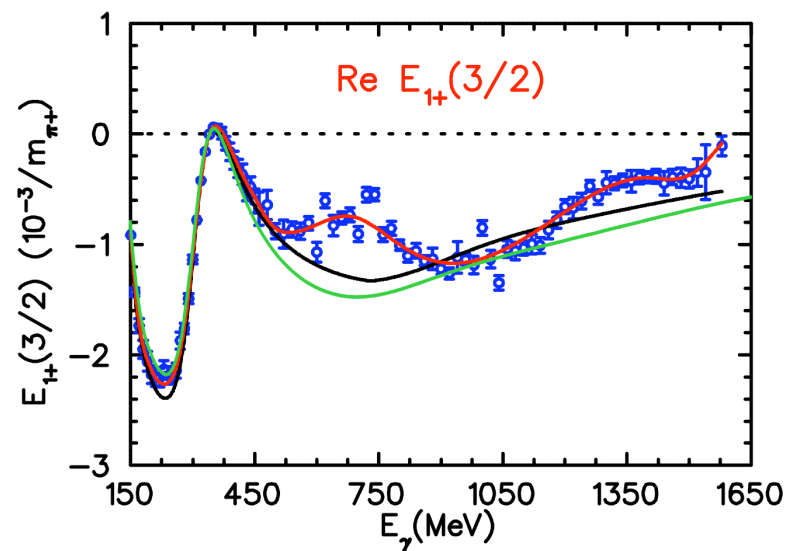
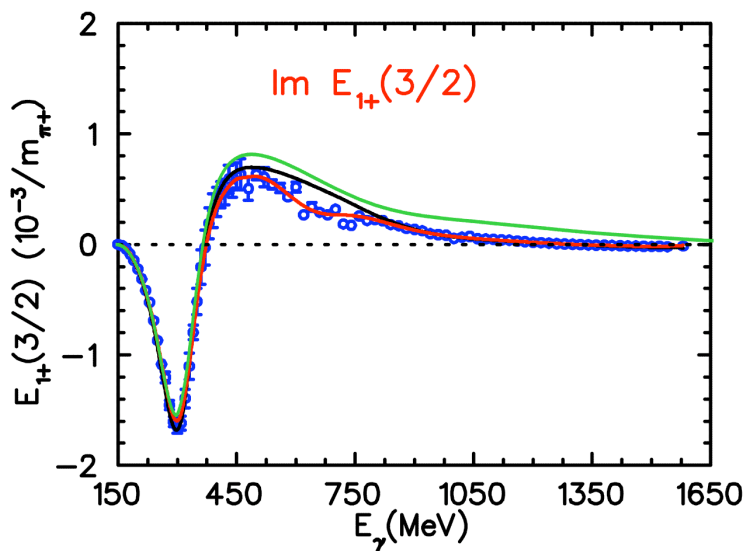
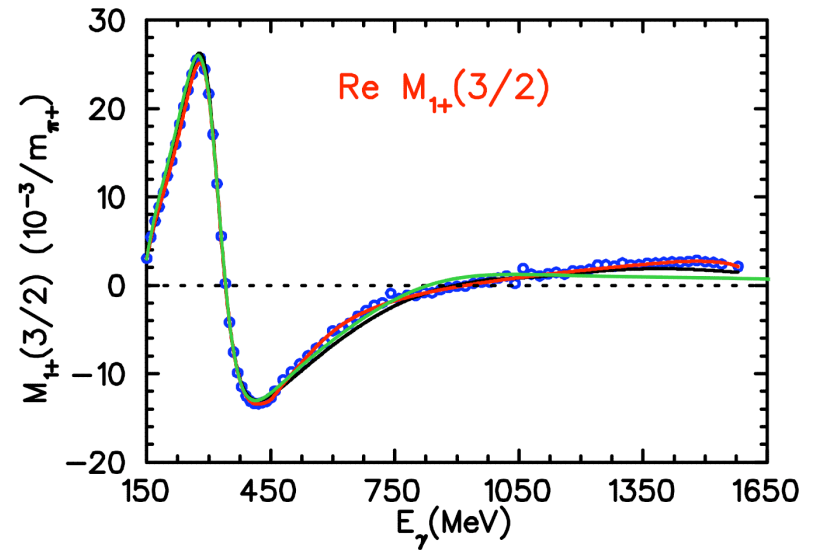
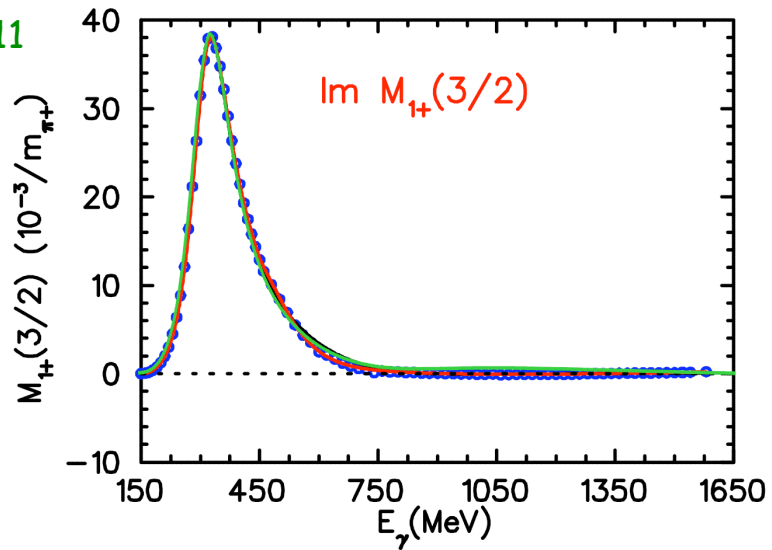
for higher energies,  $E > 900$  MeV the fits are not so good



# new L+P fit to new polarization data from Mainz and Bonn



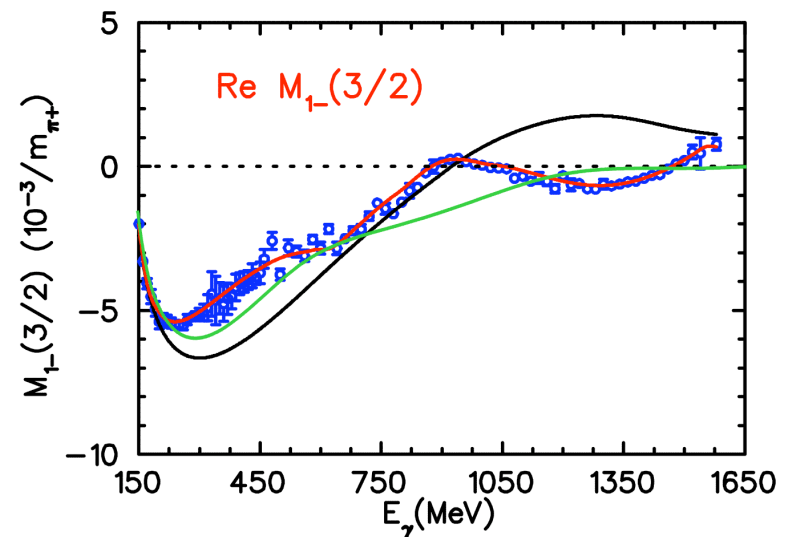
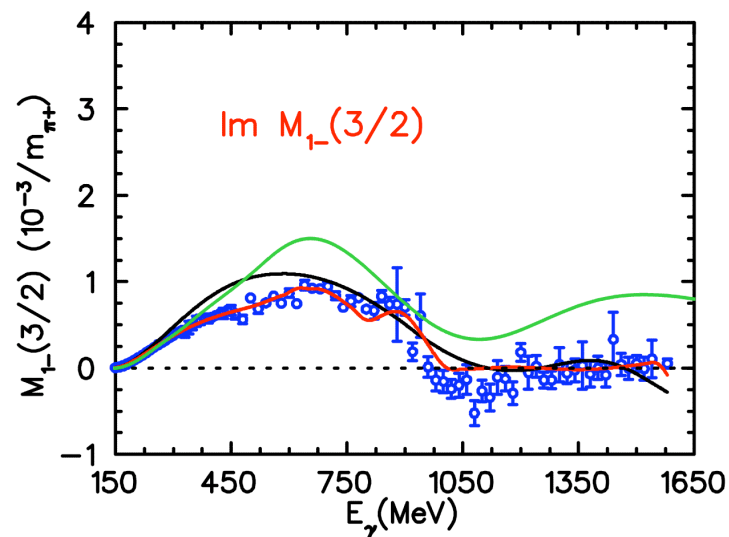
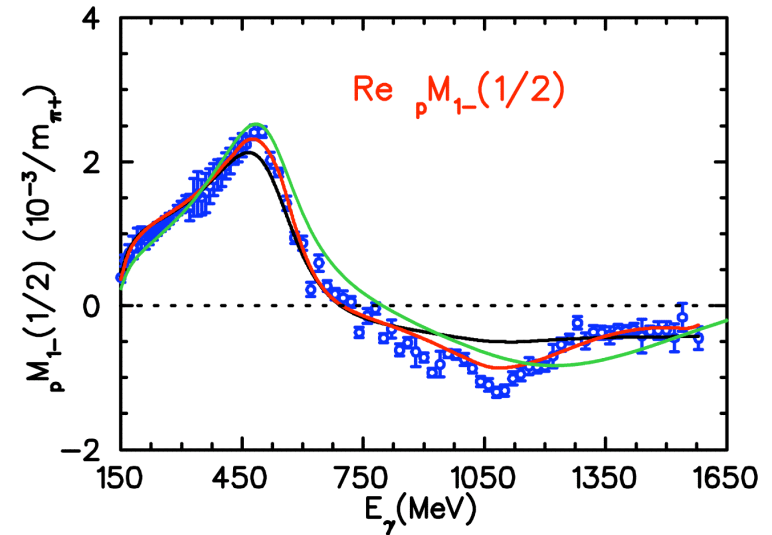
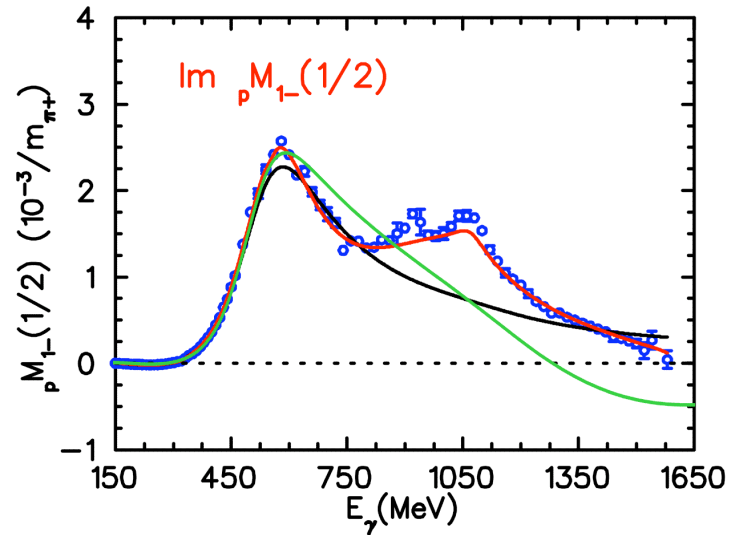
- new L+P fit
- MAID2007
- SAID-SN11



# new L+P fit to new polarization data from Mainz and Bonn

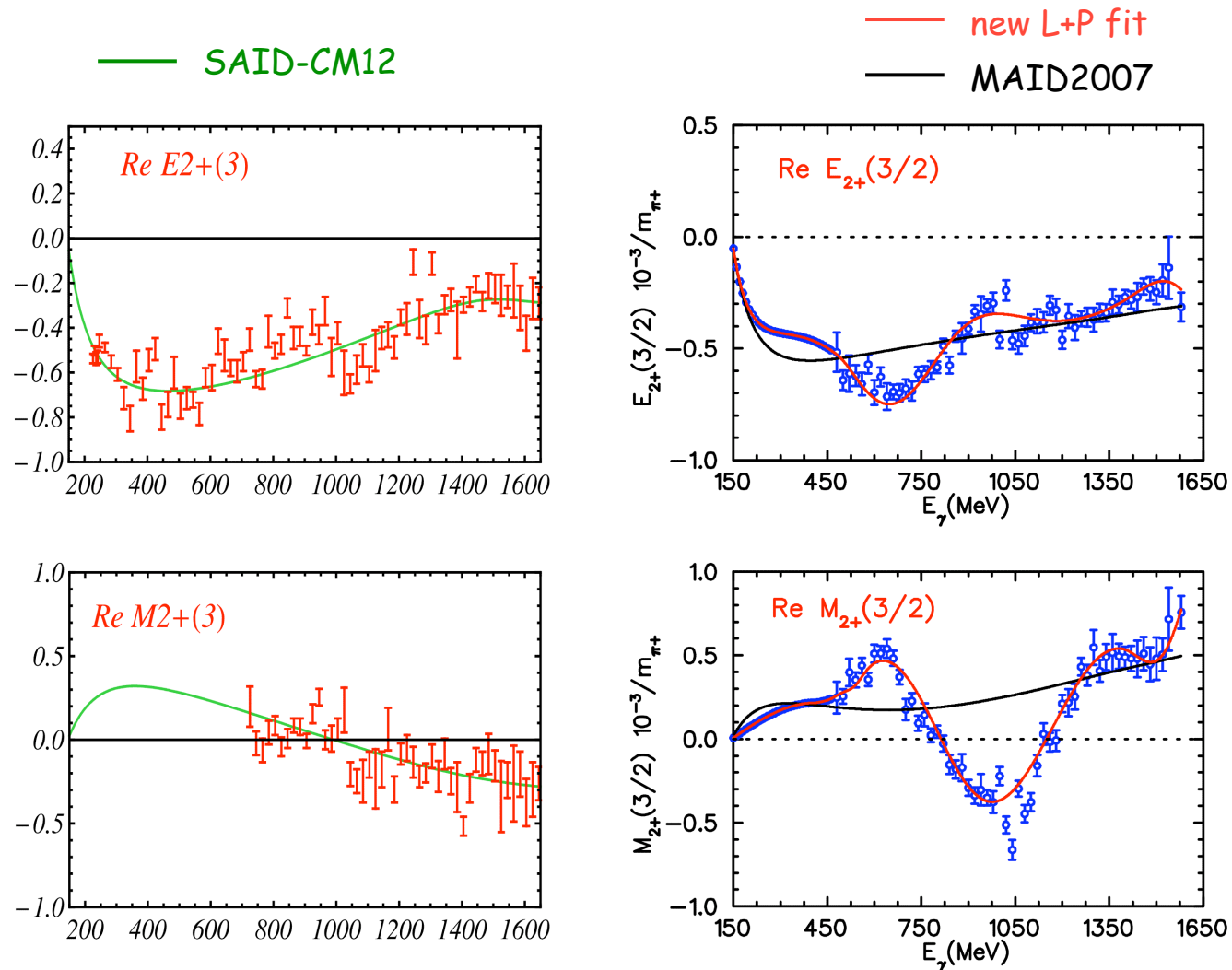


- new L+P fit
- MAID2007
- SAID-CM12





# Comparison between SAID-CM12 and MAID-LP13



some structures seen in the new fits could be physical

some could just be one out of many possible solutions, biased by the ED solution

# summary and conclusion



- with plenty of new polarization observables for  $\gamma, \pi^0$   $\gamma, \pi^+$   $\gamma, \eta$   
 $d\sigma, \Sigma, T, P, E, F, G, H, Cx', Ox'$   
the time is right to start a model-independent PWA
- for a PWA with S+P+D+F waves:  
24 complex multipoles have to be fixed at each energy (SE analysis)
- we have started with a set of 5 observables  $d\sigma, \Sigma, T, F, G$   
and have searched for 24 moduli, keeping all phases fixed
- this new SE analysis is parametrized in a Laurent+Pietarinen (L+P),  
that will hopefully lead us in a new improved MAID model

this work is still in progress  
it has just started

# $\chi^2$ compared for MAID2007 and new L+P expansion



## MAID2007

$E_\gamma$ [MeV]	proton			
	$N_{\text{mult}}$	$N_{\text{data}}$	$\chi_{\text{se}}^2/N$	$\chi_{\text{gl}}^2/N$
140—200	10	990	0.79	2.37
200—450	10	5622	0.97	2.53
450—850	24	6403	1.09	3.55
850—1210	24	2965	1.39	8.43
1210—1610	24	1454	3.26	8.67
total		17434	1.27	4.41

## new L+P expansion method

$E_\gamma$ [MeV]	proton			
	$N_{\text{mult}}$	$N_{\text{data}}$	$\chi_{\text{se}}^2/N$	$\chi_{\text{gl}}^2/N$
145—465	10	6821	1.81	2.96
450—850	24	7887	1.47	2.74
850—1210	24	4221	2.64	5.12
1210—1590	24	1958	3.00	4.14
total		20887	1.96	3.42

some improvement is visible,  
but the new solution fails for some observables, which are not fitted  
this method has **less predictive power than the original unitary isobar model**,  
however, it is perhaps a good method to solve the **Complete Experiment**

**the work is in progress**

# complete and overcomplete analysis

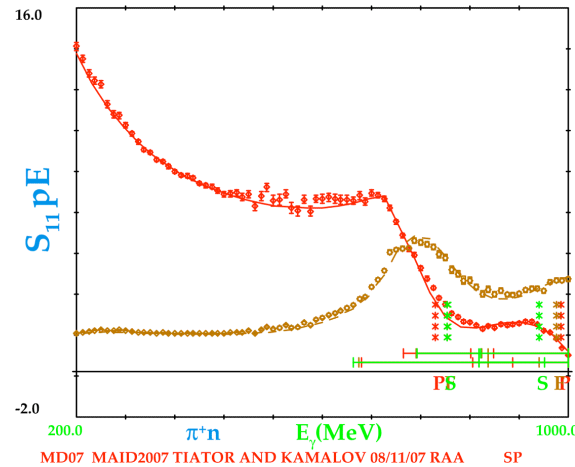


results of a  
truncated partial  
wave analysis with  
 $L_{\max}=3$   
of the MAID pseudo  
data

performed in  
collaboration with  
SAID group

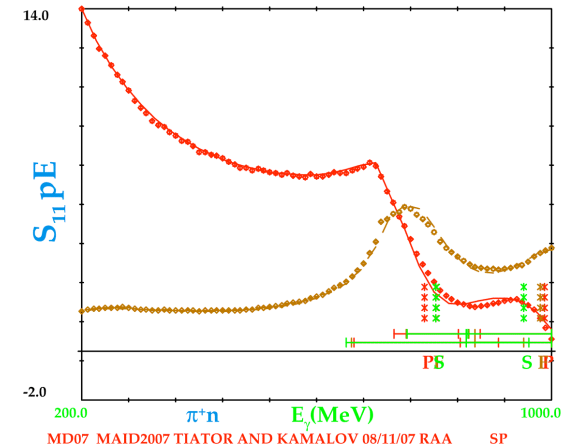
Workman, Paris, Briscoe,  
Schumann, Ostrick,  
Tiator, Kamalov,  
Eur. Phys. J. A47, 143 (2011)

4 type S ( $d\sigma, \Sigma, T, P$ )

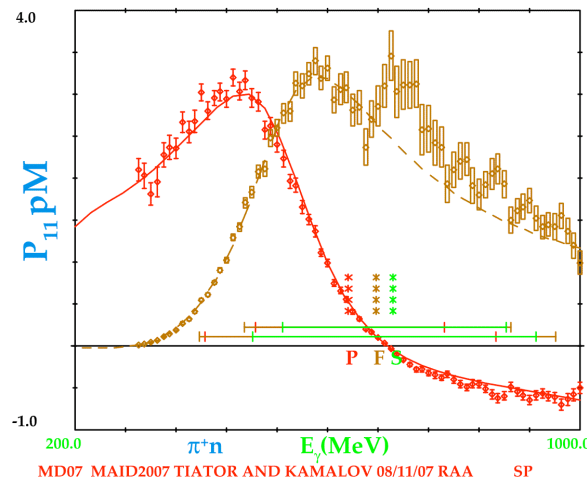


12 type S,BT,BR

( $d\sigma, \Sigma, T, P, E, F, G, H, Cx', Cz', Ox', Oz'$ )



4 type S ( $d\sigma, \Sigma, T, P$ )



12 type S,BT,BR

( $d\sigma, \Sigma, T, P, E, F, G, H, Cx', Cz', Ox', Oz'$ )

