

Extraction and Interpretation of the Resonance Parameters

Saša Ceci

Milorad Korolija

Branimir Zauner

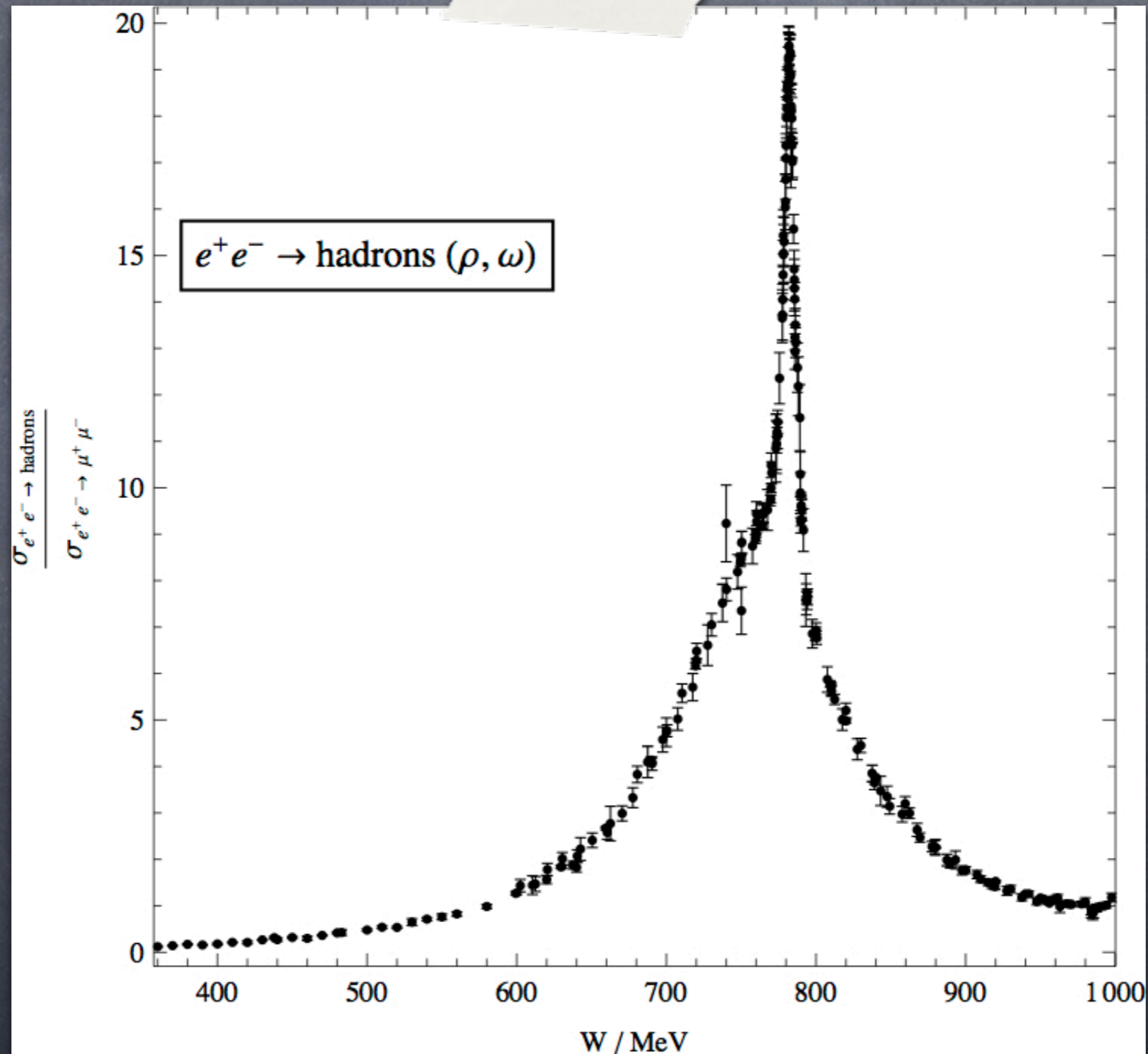


7TH INTERNATIONAL WORKSHOP ON PION-NUCLEON PARTIAL WAVE
ANALYSIS AND THE INTERPRETATION OF BARYON RESONANCES

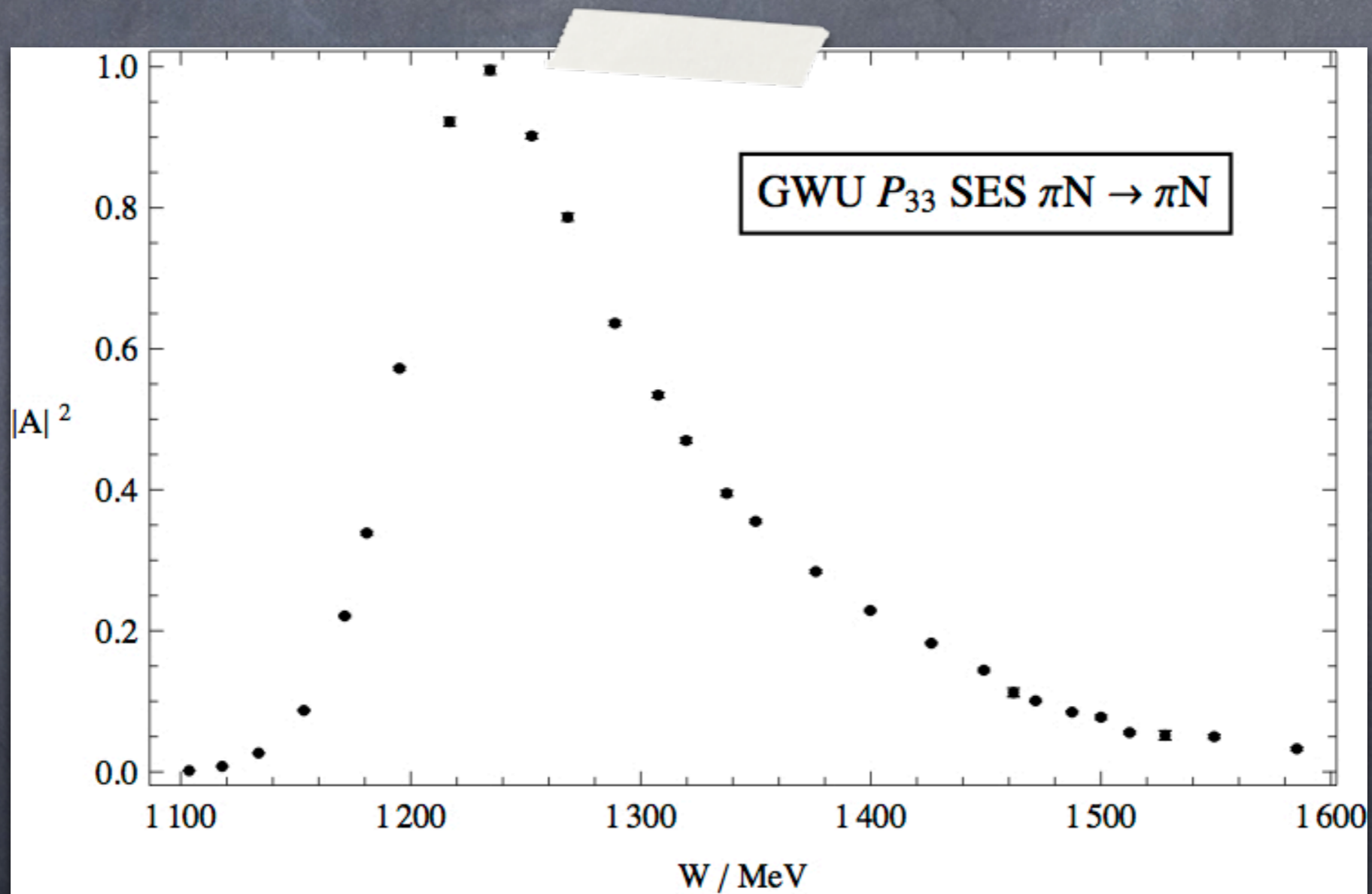
23 - 27 SEPTEMBER 2013, CAMOGLI (ITALY)



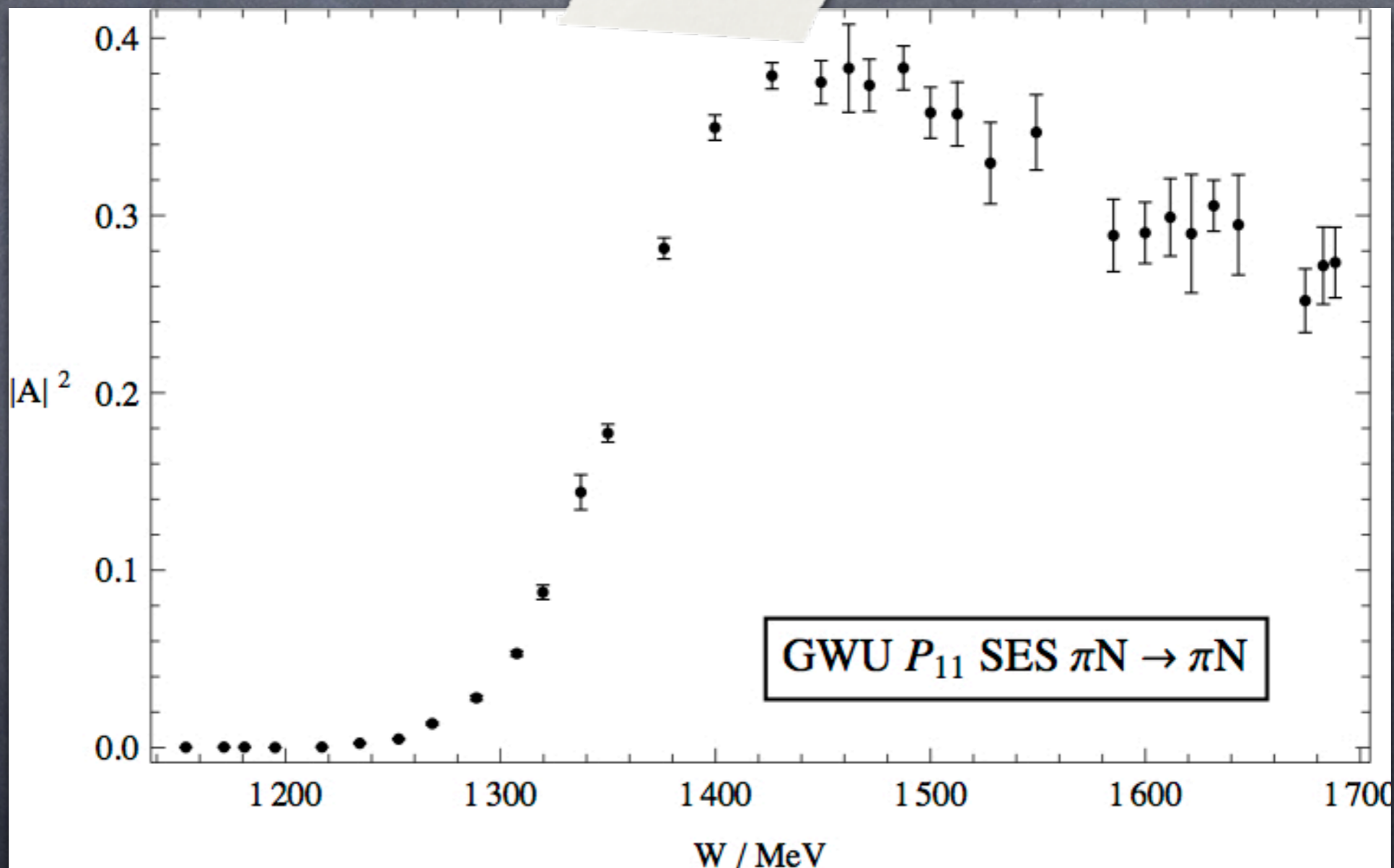
The Data: $e^+e^- \rightarrow \rho(770), \omega(782)$



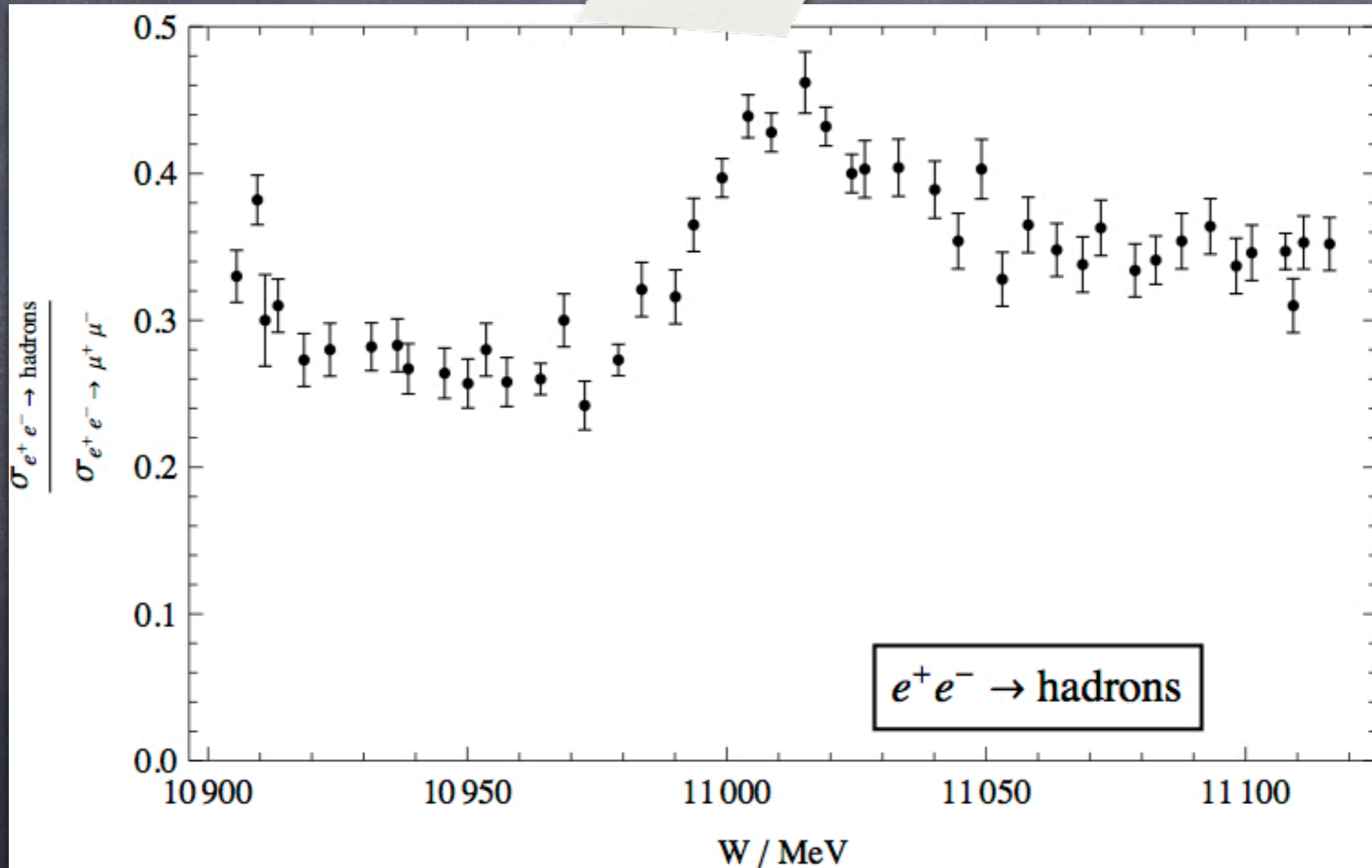
The Data: $\pi N \rightarrow \Delta(1232)$



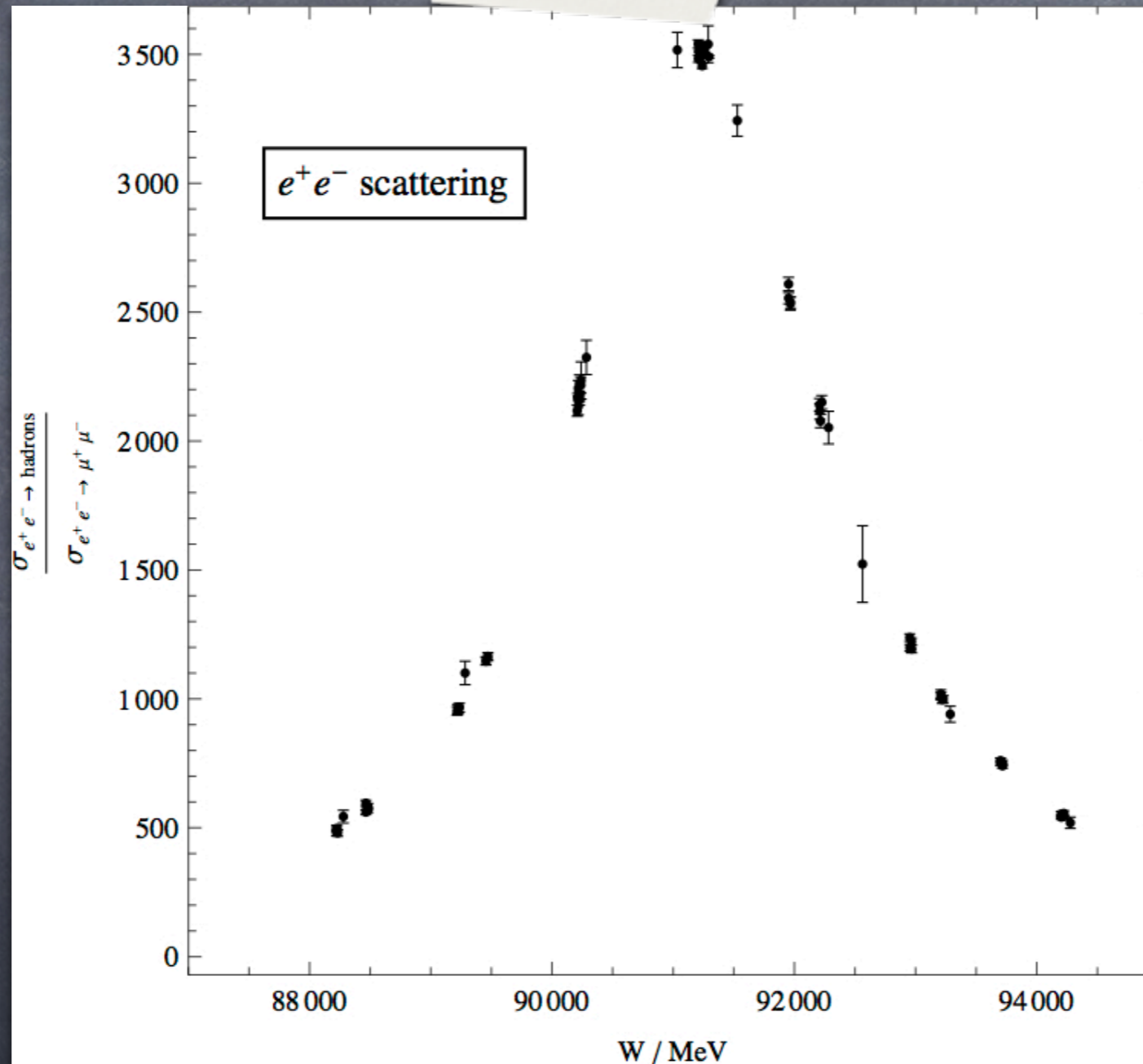
The Data: $\pi N \rightarrow N(1440)$



The Data: $e^+e^- \rightarrow Y(11020)$



The Data: $e^+e^- \rightarrow Z$ boson



The Parametrization (I)

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$$\sigma = \frac{4\pi}{q^2} \frac{2J + 1}{(2s_1 + 1)(2s_2 + 1)} |A|^2,$$

Cross section σ

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$$|A|^2 = x_p^2 \frac{[(M_p - W) \sin \delta_p + \Gamma_p/2 \cos \delta_p]^2}{(M_p - W)^2 + \Gamma_p^2/4}.$$

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$$|A|^2 \rightarrow |A|^2 + \sum_{k=0}^n B_k W^{2k}.$$

Background B

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Parameters

M_p Γ_p x_p δ_p

$B_0, B_1, B_2, B_3, B_4, B_5, B_6, \dots$

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$$|A(\delta_p = 0)|^2 = \frac{x_p^2 \Gamma_p^2/4}{(M_p - W)^2 + \Gamma_p^2/4},$$

$\delta_p = 0 \rightarrow$ simple BW formula

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The Fits

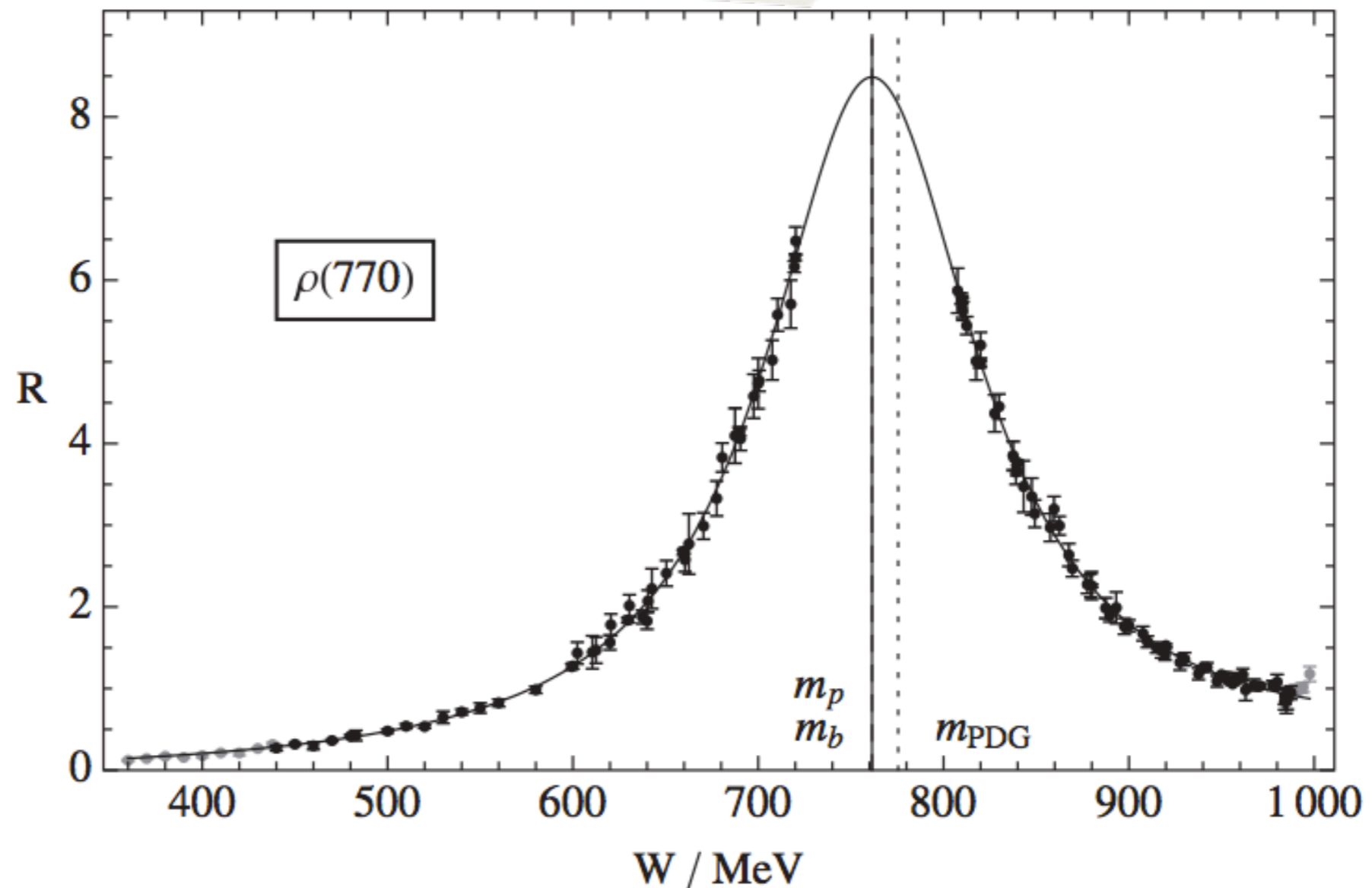


FIG. 1. Fitting pole parametrization, Eq. (11), to the data [5]. The pole and BW masses have almost the same value, which is quite different from PDG estimate (dotted line). [We removed the data from the peak to eliminate the influence of the $\omega(782)$ resonance.]

The Fits

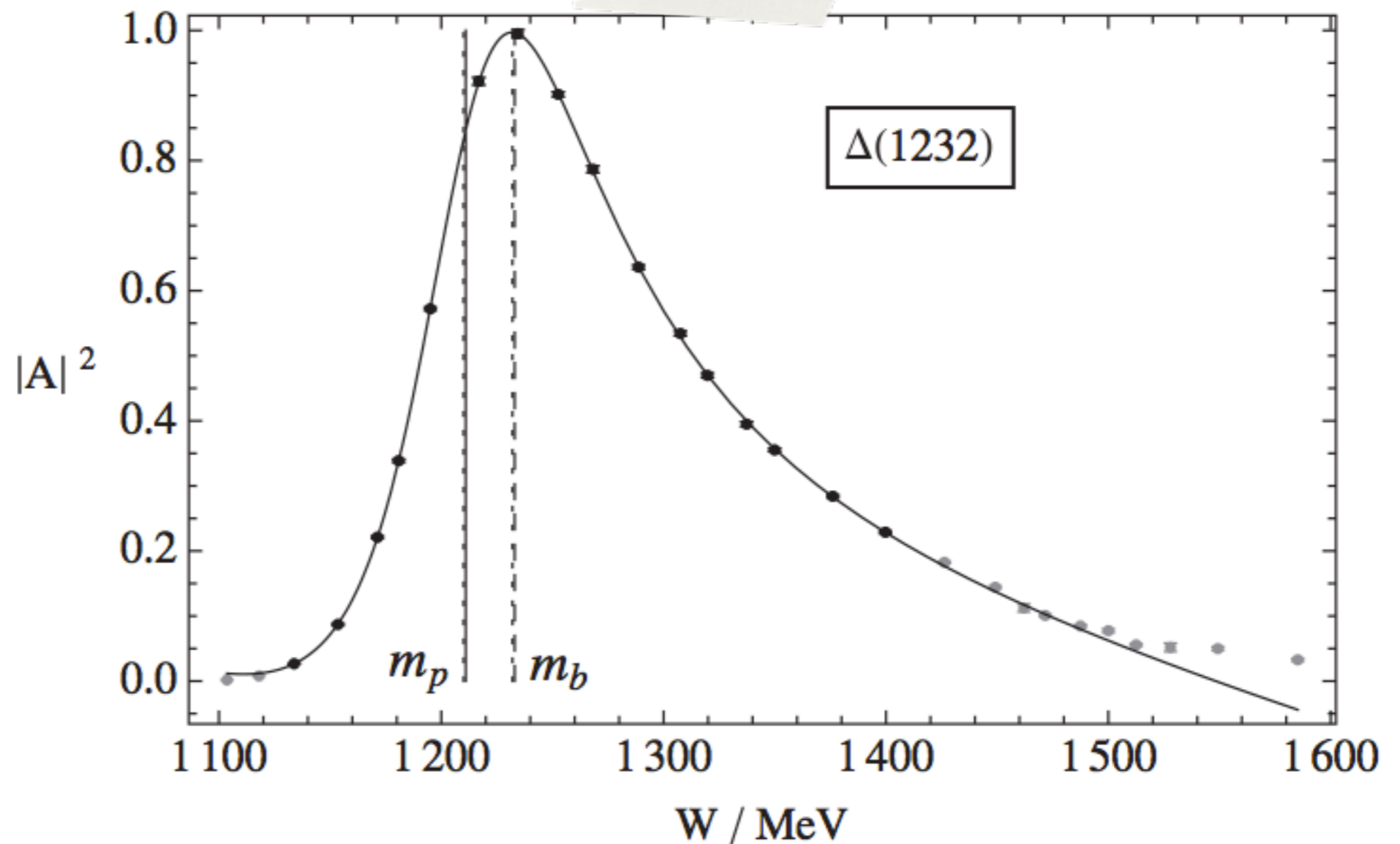


FIG. 2. Fitting pole parametrization, Eq. (11), to the SAID data [7]. Pole (solid line) and BW mass (dashed line) are clearly distinct, while the PDG estimates (dotted lines) are indistinguishable from them. All the data in this figure are analyzed, but only the black data points are used in the best fit.

The Fits

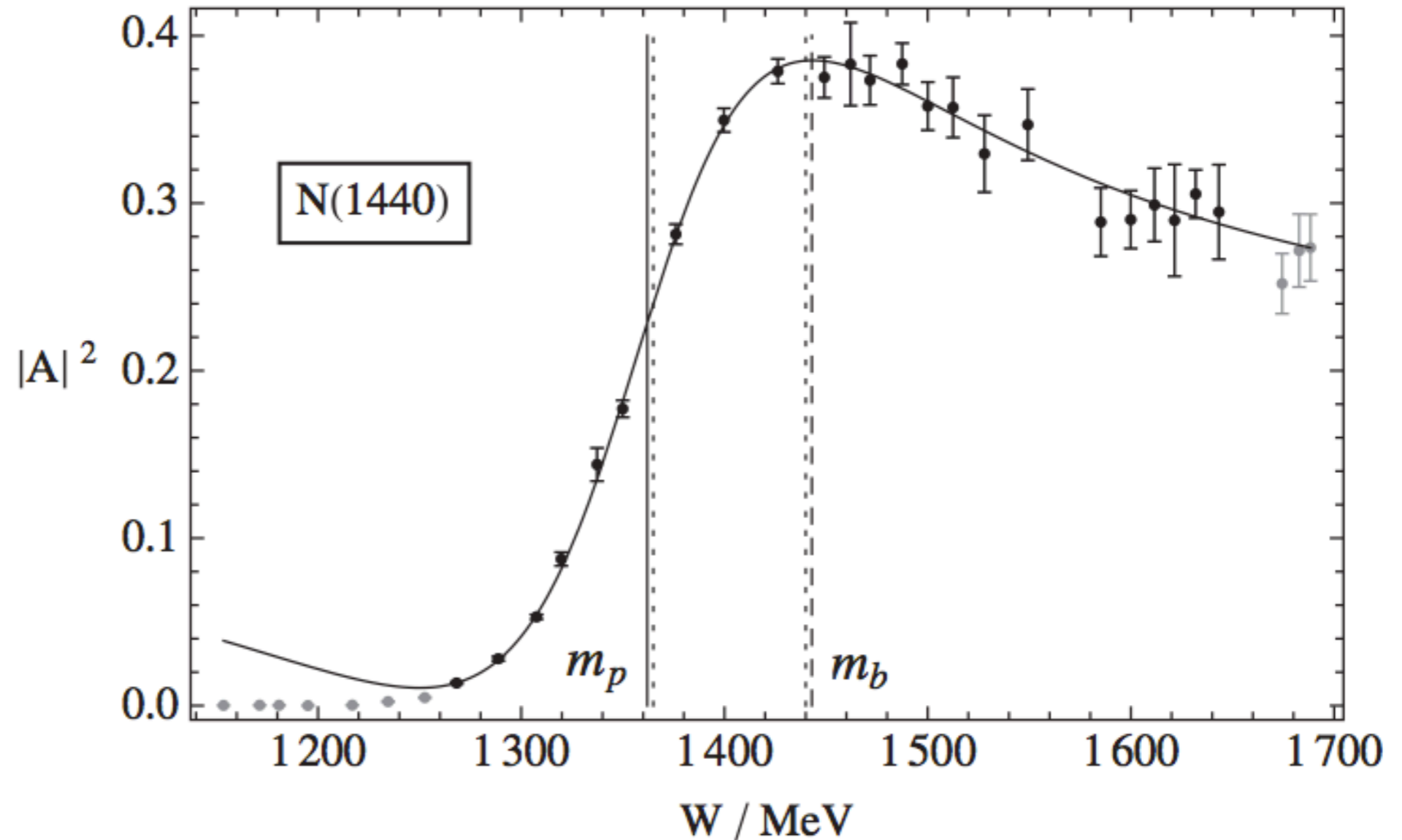


FIG. 3. Fitting pole parametrization, Eq. (11), to the SAID data [7]. This resonance has the largest difference between pole and BW mass. PDG estimates (dotted lines) are consistent with pole (solid line) and BW (dashed line) parameters.

The Fits

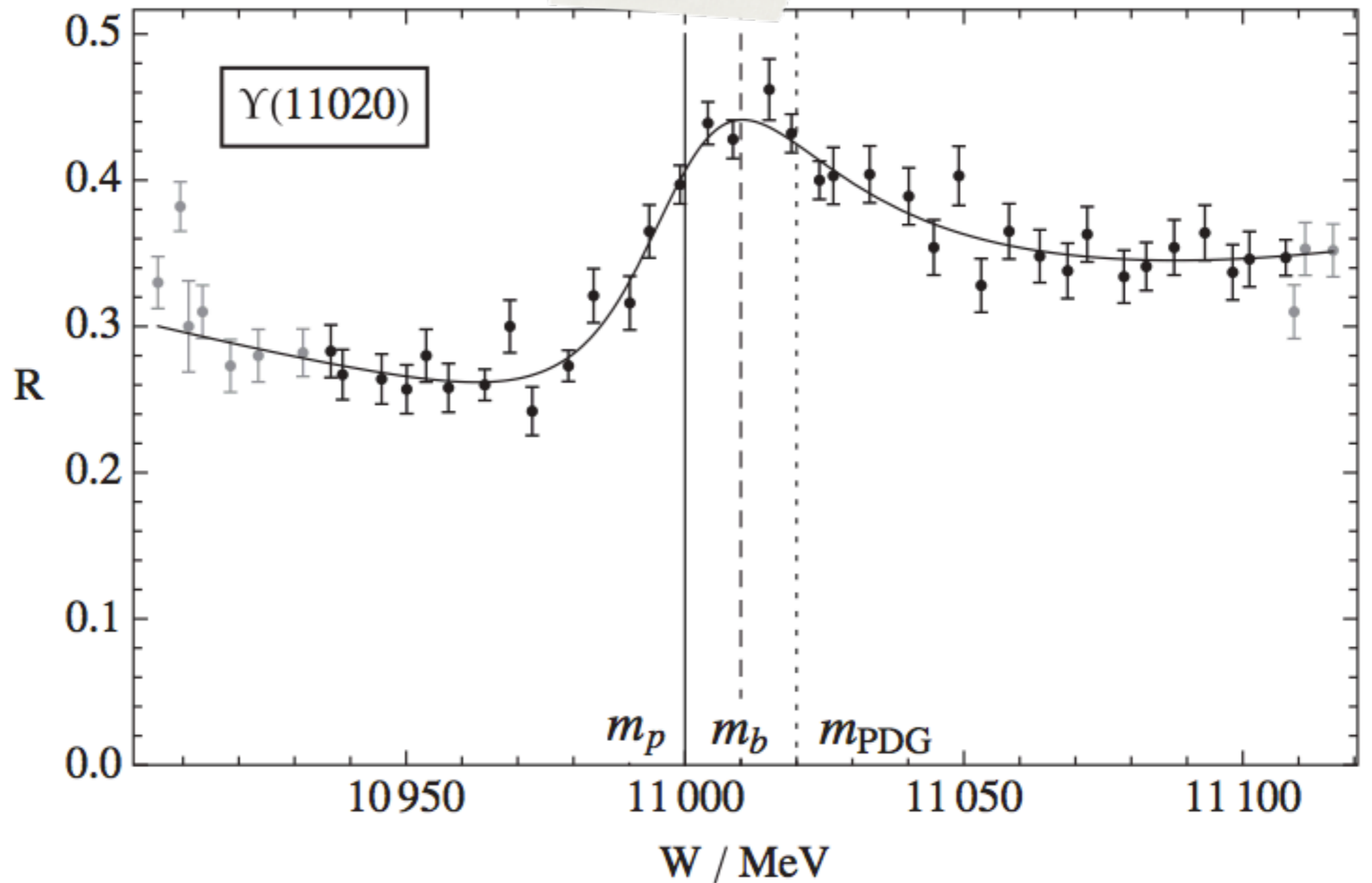


FIG. 4. Fitting pole parametrization, Eq. (11), to the *BABAR* data [8]. Pole and BW masses are clearly distinct, but PDG estimate coincides with neither of them.

The Fits

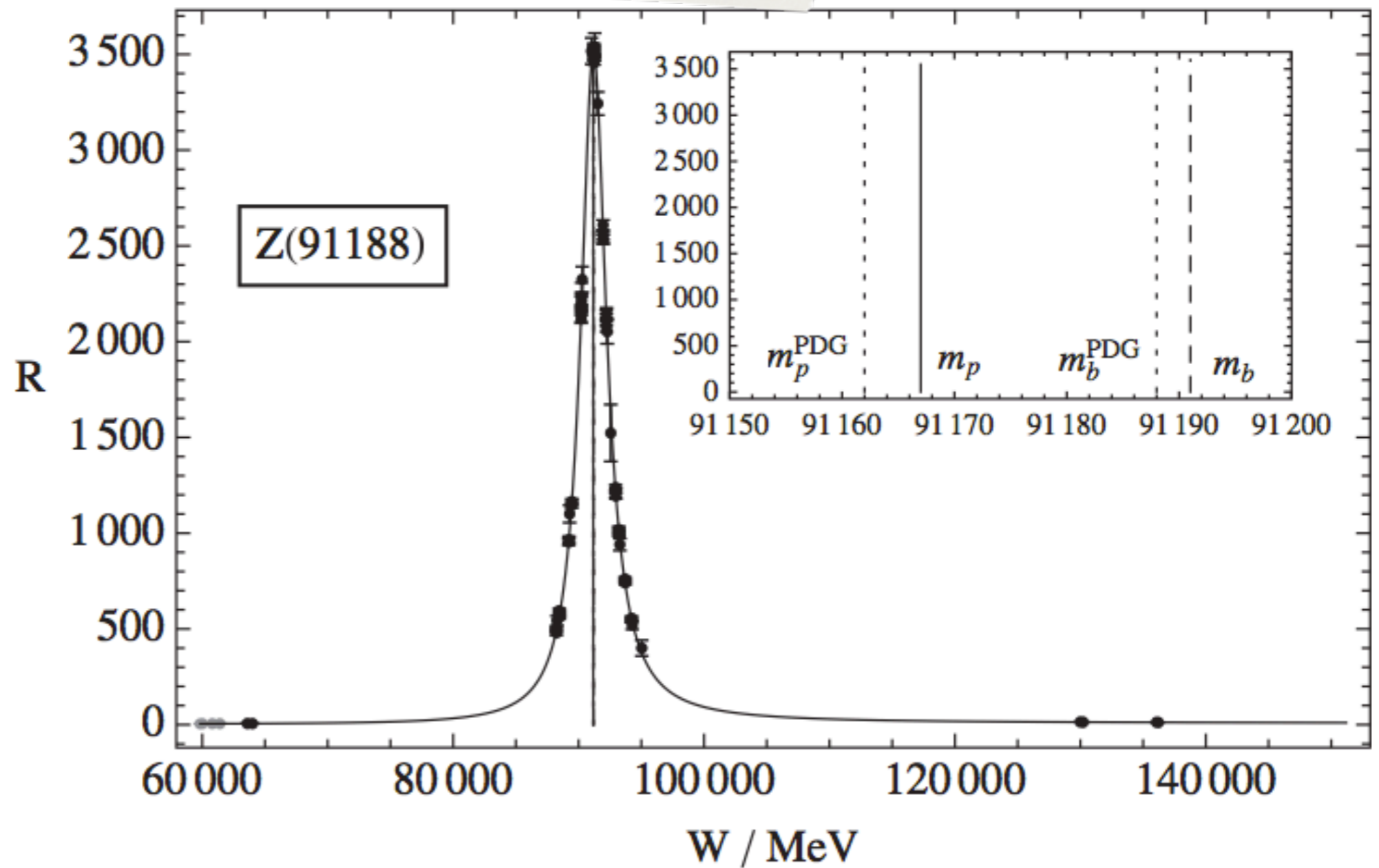


FIG. 5. Fitting pole parametrization, Eq. (11), to the data [5]. The pole and BW parameters are consistent with their PDG estimates (dotted lines).

Resonance Parameters

TABLE I. Resonance pole parameters extracted by using the pole formula, Eq. (11). Our $2\delta_p$ is compared to the residue phase θ_p from the literature. The PDG pole estimates are from Ref. [5]. The ρ meson pole and the Z boson residue phase are estimated by an analytic continuation of the Gounaris-Sakurai [2] and Breit-Wigner [5] parametrizations, respectively.

Resonance	M_p/MeV	Γ_p/MeV	$x_p/\%$	$2\delta_p/^\circ$
$\rho(770)$	762 ± 1	138 ± 2	0.71	1 ± 1
Pole	763	144
$\Delta(1232)$	1211 ± 1	102 ± 1	103 ± 1	-47 ± 1
PDG pole	1210 ± 1	100 ± 2	104 ± 2	-47 ± 1
$N(1440)$	1362 ± 5	191 ± 10	61 ± 4	-81 ± 10
PDG pole	1365 ± 15	190 ± 30	65 ± 10	$-85 \pm_{10}^{15}$
$Y(11020)$	$11\,000 \pm 2$	43 ± 6	0.10	-52 ± 8
<i>BABAR</i> [8]	$10\,996 \pm 2$	37 ± 3
$Z(91188)$	$91\,167 \pm 6$	2493 ± 5	15.4	-2.2 ± 0.2
PDG pole	$91\,162 \pm 2$	2494 ± 2	...	-2.35

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consistent
with
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TEST

Poles

$$M_p = 1211 \text{ MeV}$$

$$\Gamma_p = 102 \text{ MeV}$$

$$x_p = 103\%$$

$$2\delta_p = -47^\circ$$

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Breit-Wigners?

$$M_b = 1233 \text{ MeV}$$

$$\Gamma_b = 121 \text{ MeV}$$

$$x_b = 103\%$$

$$2\delta_b = -47^\circ$$

Resonance Parameters II

TABLE II. Resonance parameters extracted by using new Breit-Wigner formula, Eq. (14). PDG estimates are from Ref. [5].

Resonance	M_b/MeV	Γ_b/MeV	$x_b/\%$	$2\delta_b/^\circ$
$\rho(770)$	761 ± 1	139 ± 2	0.71	0 ± 1
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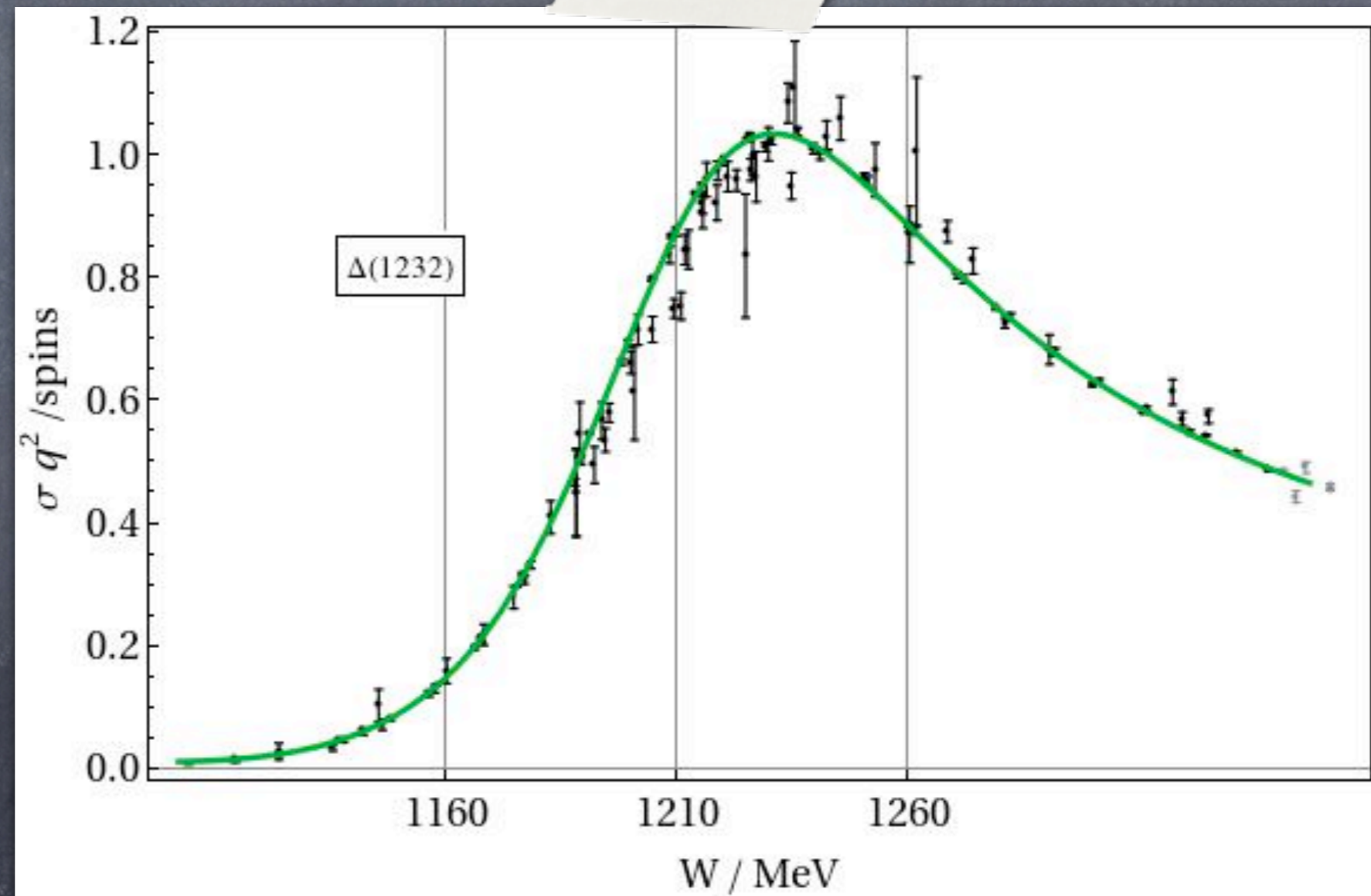
consistent
with the
Breit-
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$\pi N \rightarrow \pi N$ Scattering Data (PDG)

Preliminary!

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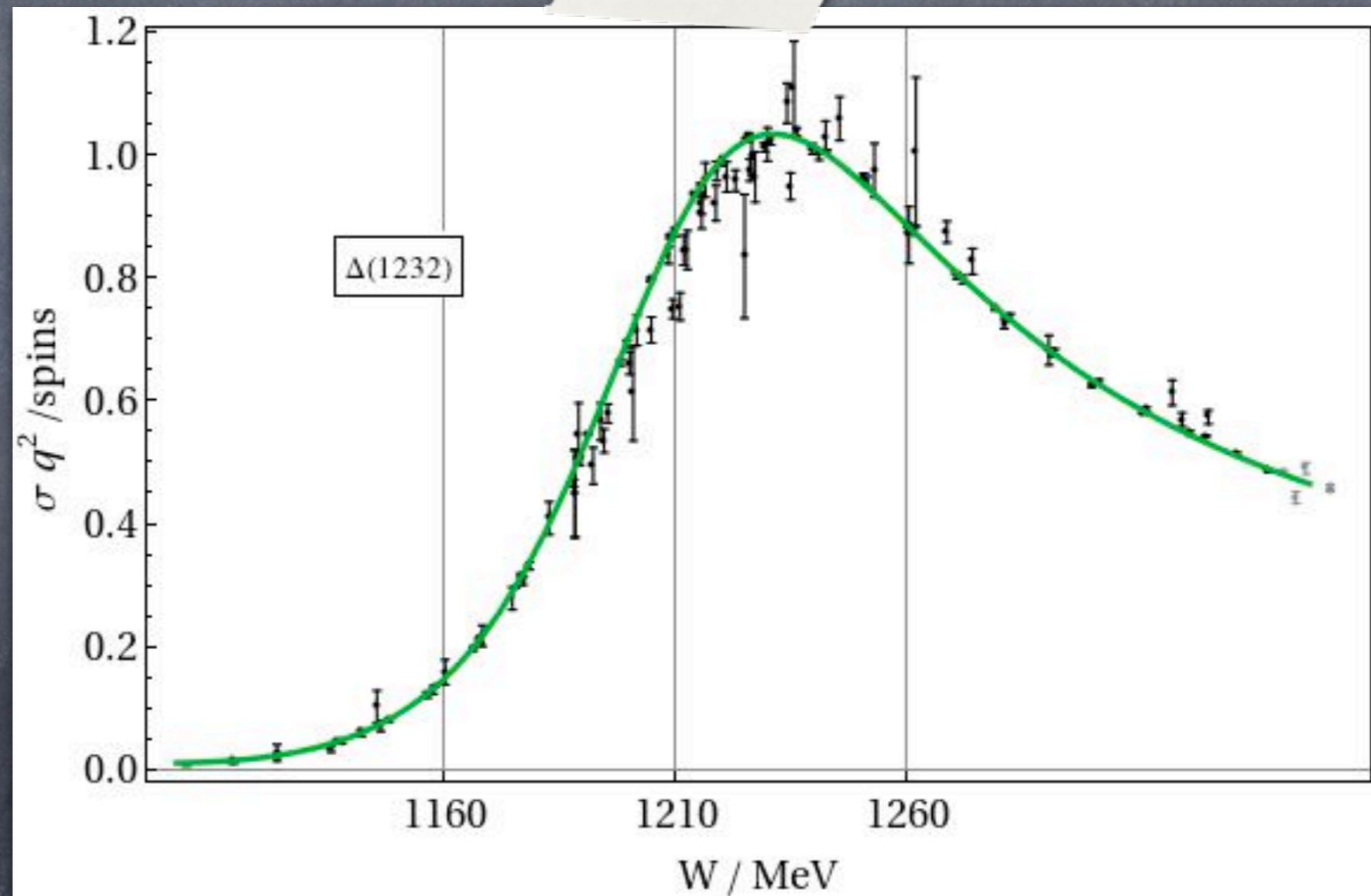
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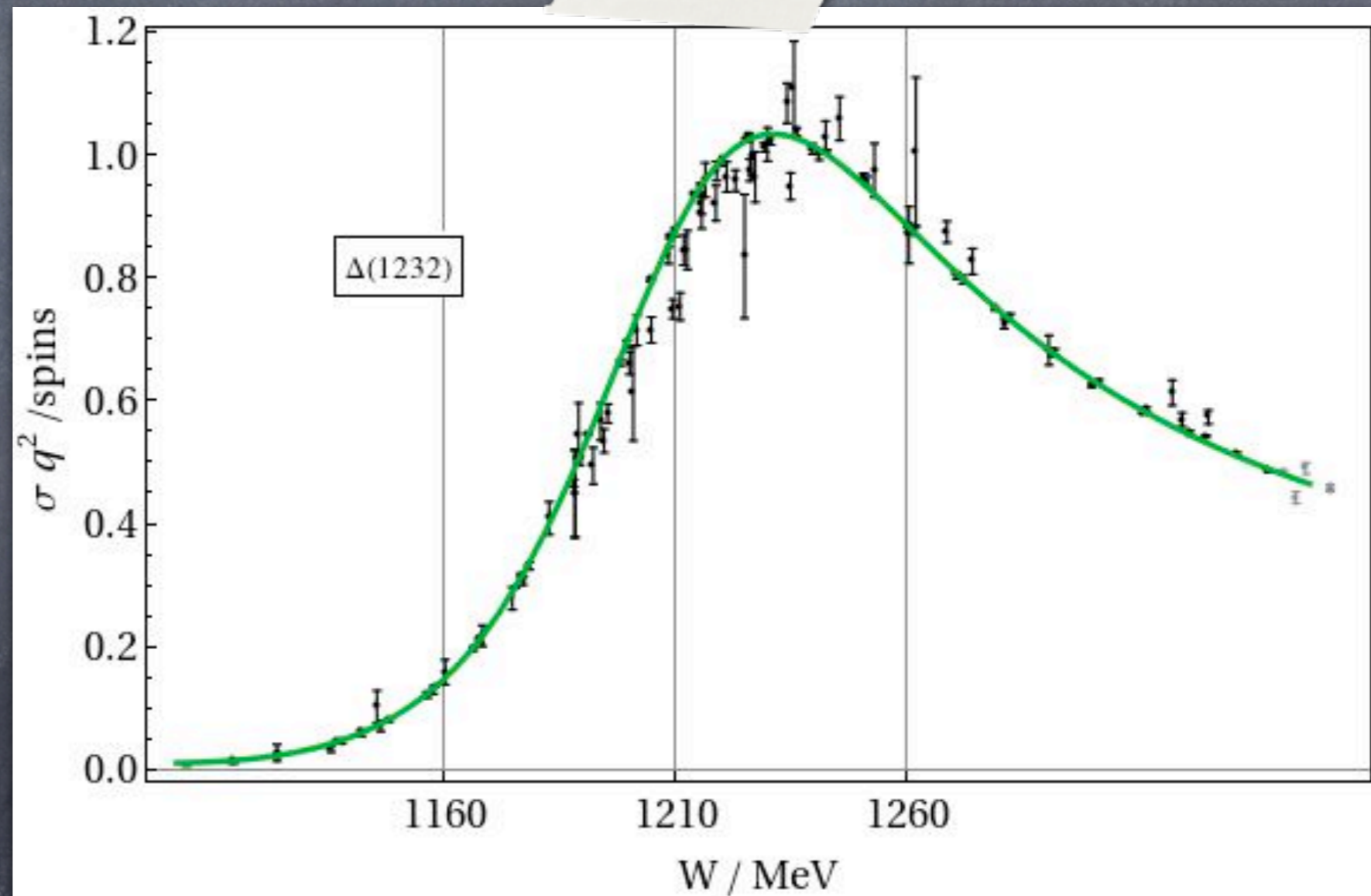
Breit-Wigners

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Summary

- There are **two parametrizations** that fit the cross section data well
 - Parametrization I that produces **the pole parameters**
 - Parametrization II that produces **the Breit-Wigner parameters**
 - They are mathematically **equivalent** (hence longstanding pole-BW duality, aka **pole-mics**)
- There is curious connection between parameter δ and the complex residue phase θ ($\theta=2\delta$)
- There are resonance parameters in PDG book that are **neither pole, nor Breit-Wigner** parameters (potential problem for LQCD)



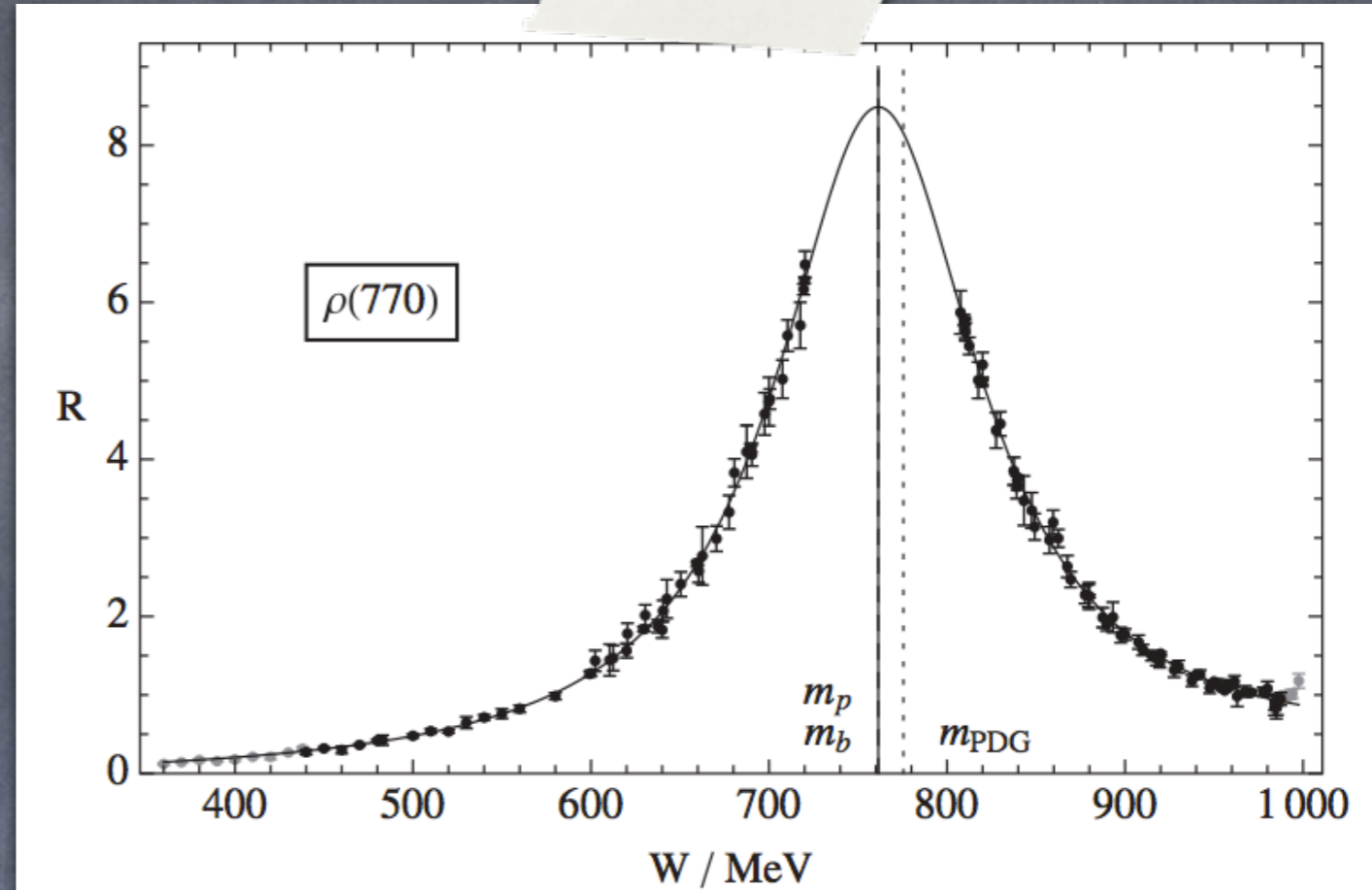
Thank you!

The Fits

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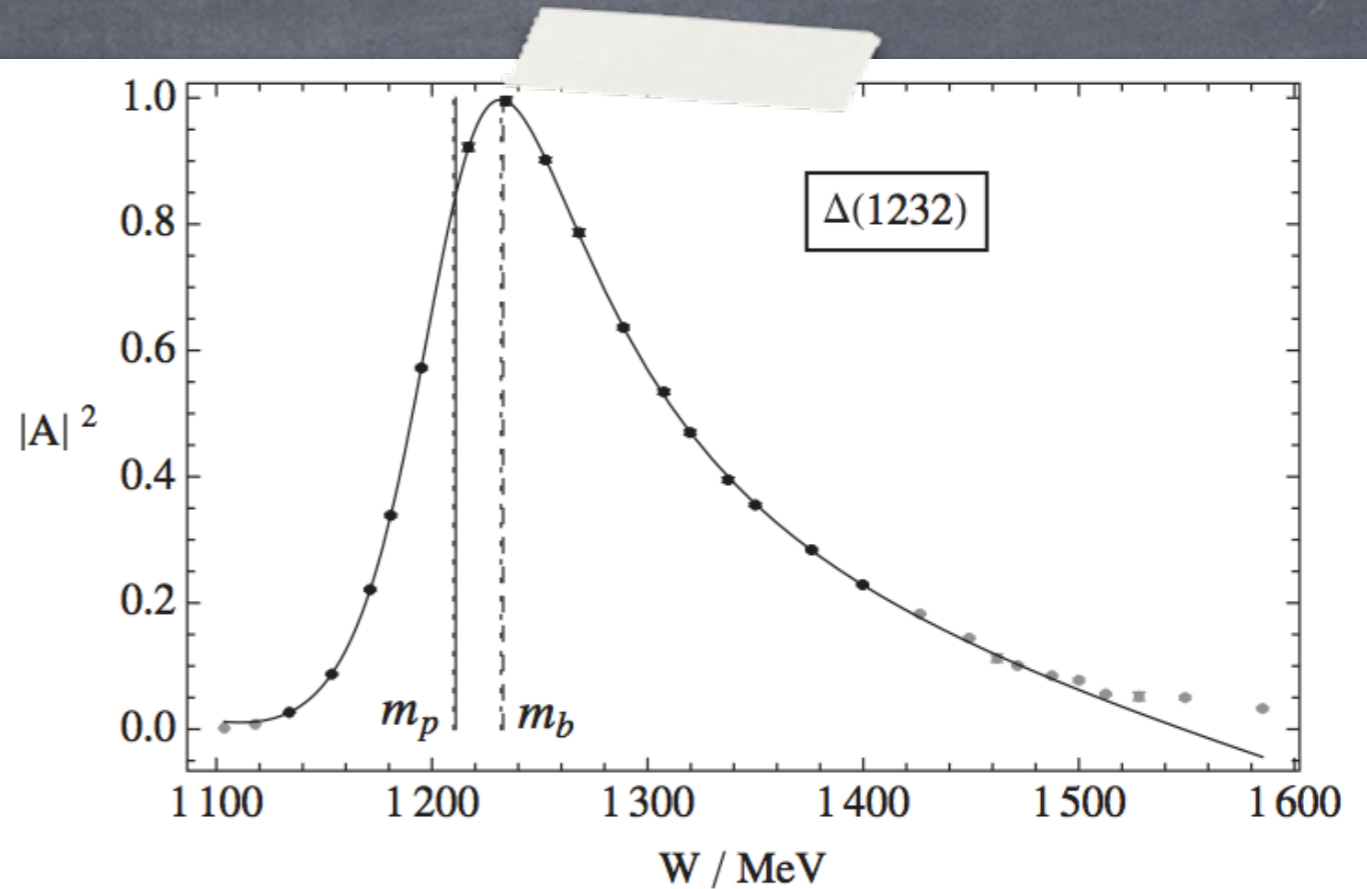
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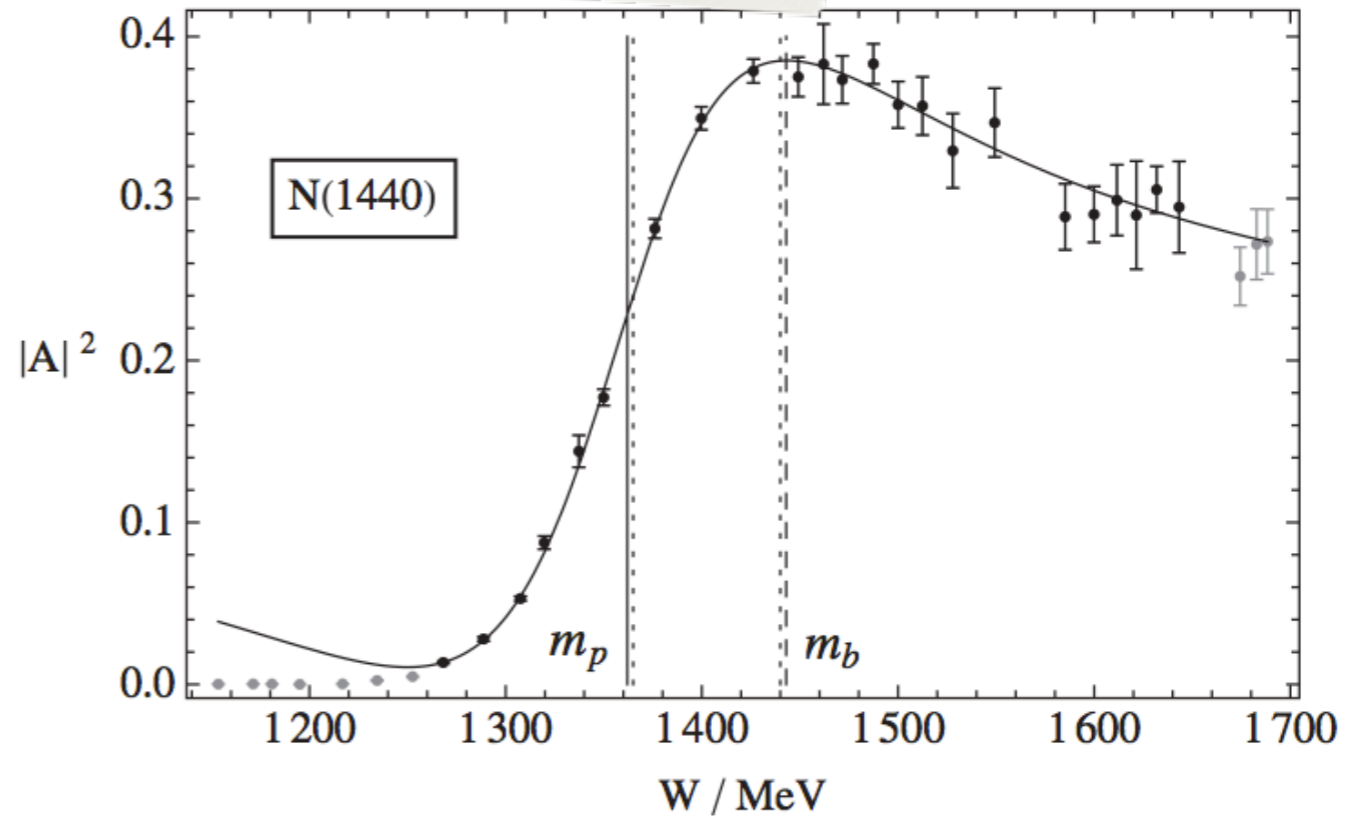
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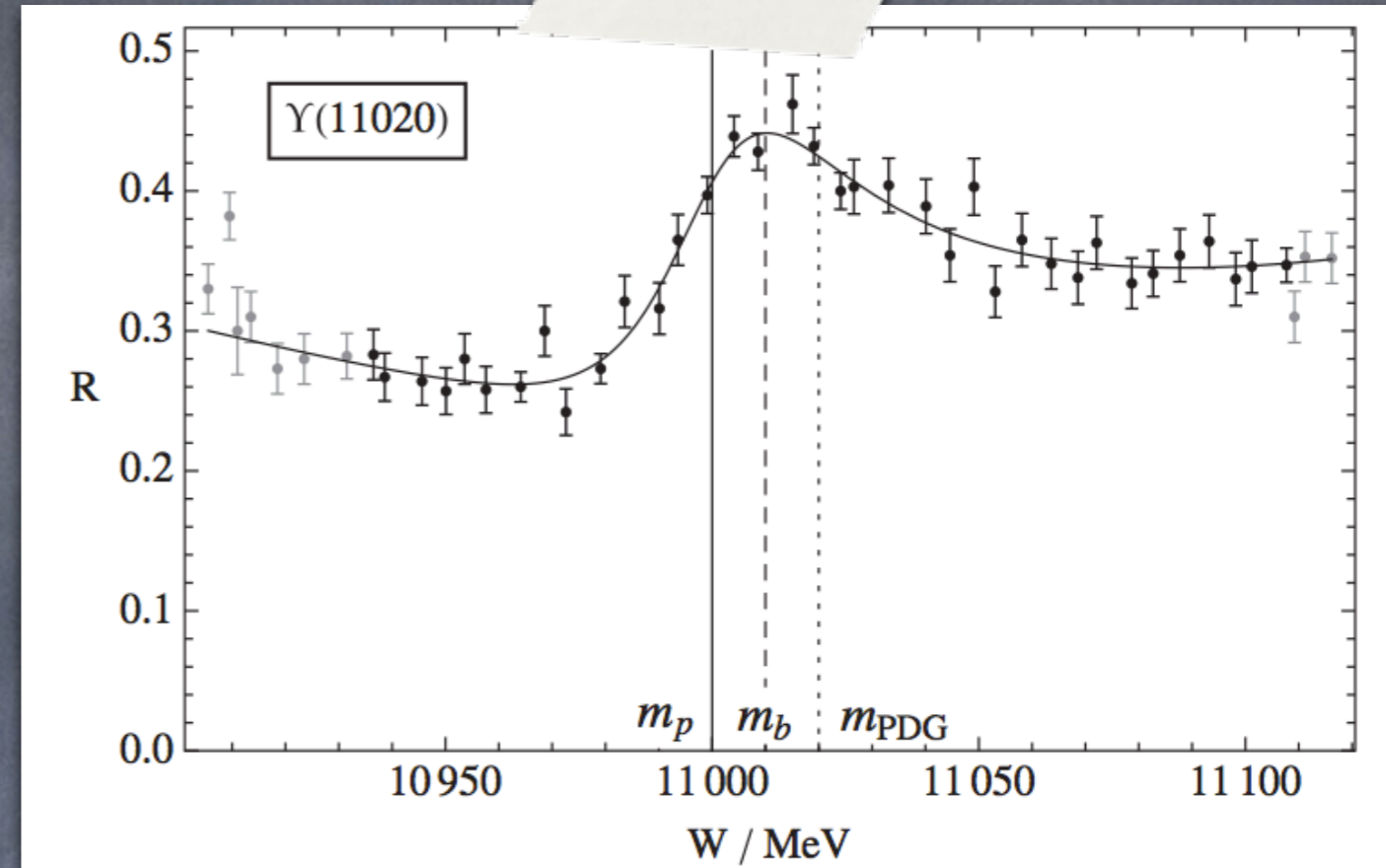
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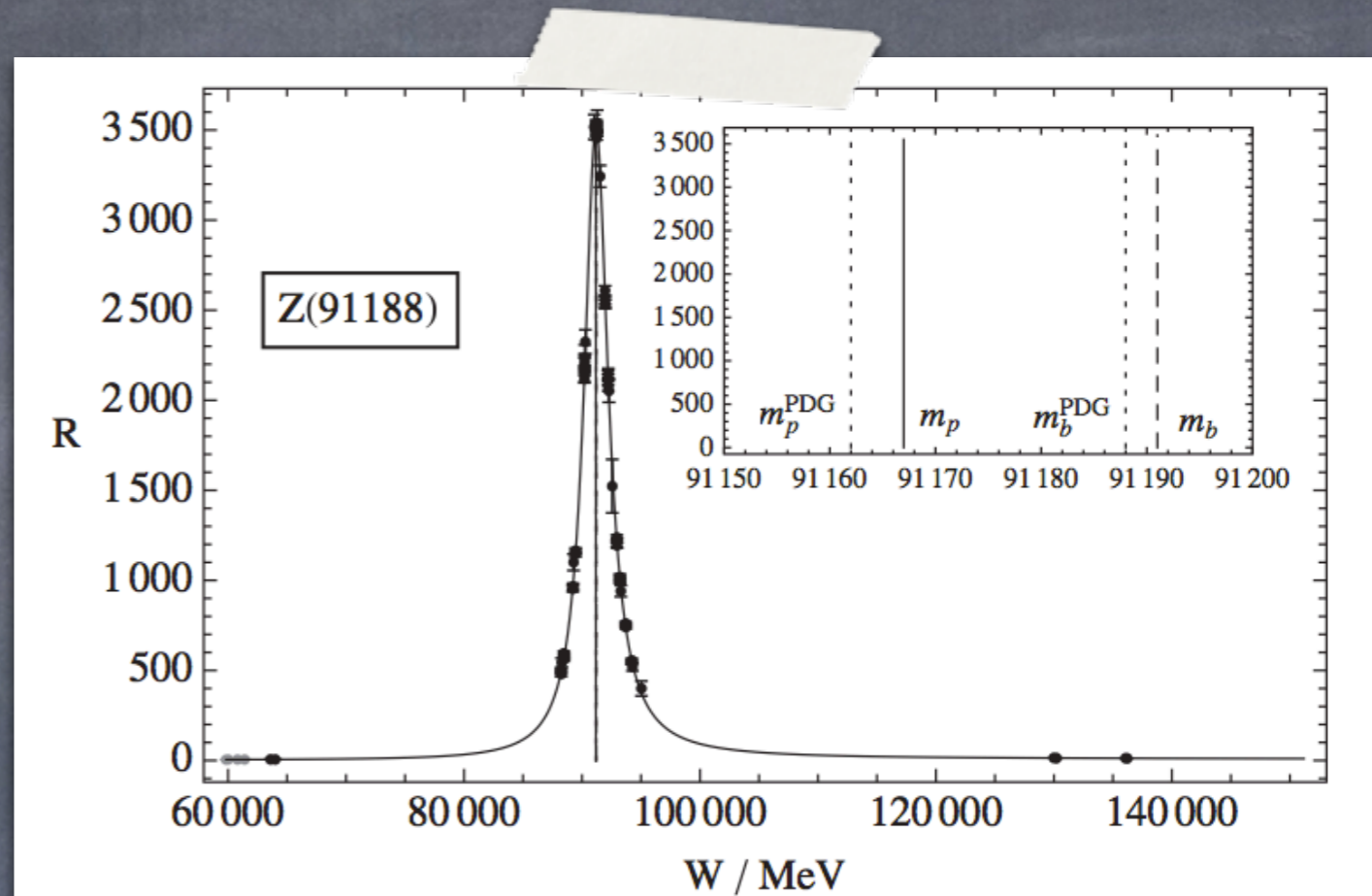
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Error estimates

We check two errors: fit error, and discrepancy between the 10 best results (with lowest $C_n = C_{n,n+1} + C_{n,n+2}$); we use the larger one

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