

Quark core and pion cloud effects in resonant meson electro-production

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Outline

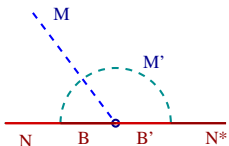
- ▶ Aims
- ▶ A short survey of the coupled channel approach incorporating quasi-bound quark-model states and meson clouds
- ▶ The underlying quark models
- ▶ Meson cloud effects in the region of the $\Delta(1232)$ and $N(1440)$ resonances
- ▶ Analysis of the S11 wave resonances
- ▶ D-wave resonances: D13, D33 and D15
- ▶ Summary

Aims

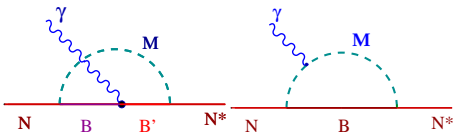
- ▶ An attempt to answer the long standing question: what is the principal mechanism that explains the structure of individual resonances: **excitation of the quark core** or **dynamical generation** through baryon-meson coupling?
- ▶ To use **explicit quark degrees of freedom** that provide "natural" constraints on the baryon meson **coupling constants** as well as the **range of the interaction** (cut-off); the resulting number of free parameters thus becomes considerably smaller.
- ▶ To assess the importance of the meson cloud in scattering and electroproduction.
- ▶ *Fil rouge* of the talk: to identify the processes where the meson cloud effects are (more or less) clearly manifested.

Meson-cloud effects, formally:

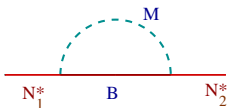
- ▶ MNN^* vertex renormalization



- ▶ γNN^* renormalization



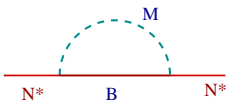
- ▶ resonance mixing



e.g. P11 $N^0(939) + N^0(1440)$,
 S11 $N^0(1535) + N^0(1650)$,
 D13 $N^0(1520) + N^0(1700)$

$$N^* = \cos \theta N_1^{(0)} + \sin \theta N_2^{(0)}, \quad \theta = \theta(W).$$

- ▶ self energy



Quark models with mesons

A natural environment: the model in which the mesons are linearly coupled to the quark core:

$$H = H_{\text{quark}} + \int dk \sum_{lmt} \left\{ \omega_k a_{lmt}^\dagger(k) a_{lmt}(k) + [V_{lmt}(k) a_{lmt}(k) + \text{h. c.}] \right.$$

$$T(k, k_0) = \pi \sqrt{\frac{\omega_k}{k}} \langle \Psi^-(k_0) || V(k) || \Phi_N \rangle$$

$$|\Psi(W)^\pm\rangle = a^\dagger(k_0) |\Phi_N\rangle + |\chi^\pm\rangle, \quad |\chi^-\rangle = -\frac{1}{H - W \mp i\epsilon} V(k) |\Phi_N\rangle$$

Generalized to

$$K^{JT}(k, k_0) = -\pi \sqrt{\frac{\omega_k E_N}{kW}} \langle \Psi_{JT}^P(W) || V(k) || \Phi_N \rangle$$

The principle value (PV) states

$$|\Psi_{JT}^P\rangle = \sqrt{\frac{\omega_0 E_N}{k_0 W}} \left\{ [a^\dagger(k_0) |\Phi_N\rangle]^{JT} - \frac{\mathcal{P}}{H - W} [V(k_0) |\Phi_N\rangle]^{JT} \right\}$$

Construction of the K-matrix

Aim: to include many-body states of quarks in the scattering formalism (Chew-Low type approach)

Construct K-matrix in the spin-isospin (JI) basis:

$$K_{M'B' MB}^{JI} = -\pi \sqrt{\frac{\omega_M E_B}{k_M W}} \langle \Psi_{JI}^{MB}(W) || V_{M'}(k) || \Psi_{B'} \rangle$$

dressed states

by using principal-value (PV) states

$$|\Psi_{JI}^{MB}(W)\rangle = \sqrt{\frac{\omega_M E_B}{k_M W}} \left\{ [a^+(k_M) |\Psi_B\rangle]^{JI} - \frac{\mathcal{P}}{H - W} [V(k_M) |\Psi_B\rangle]^{JI} \right\}$$

normalized as

$$\langle \Psi^{MB}(W) | \Psi^{M'B'}(W') \rangle = \delta(W - W') \delta_{MB, M'B'} (1 + \mathbf{K}^2)_{MB, MB}$$

Ansatz for the channel PV states

$$\begin{aligned}
 |\Psi_{II}^{MB}\rangle = & \sqrt{\frac{\omega_M E_B}{k_M W}} \left\{ [a^\dagger(k_M) |\Psi_B\rangle]^{II} \right. \\
 & + \sum_{\mathcal{R}} c_{\mathcal{R}}^{MB} |\Phi_{\mathcal{R}}\rangle \\
 & \left. + \sum_{M'B'} \int \frac{dk \chi^{M'B'MB}(k, k_M)}{\omega_k + E_{B'}(k) - W} [a^\dagger(k) |\Psi_{B'}\rangle]^{II} \right\}
 \end{aligned}$$

free meson
(defines the channel)

bare (genuine) baryons (3q)

meson "clouds"
with amplitudes χ

Above the meson-baryon (MB) threshold:

$$K_{M'B'MB}(k, k_M) = \pi \sqrt{\frac{\omega_M E_B}{k_M W}} \sqrt{\frac{\omega_{M'} E_{B'}}{k_{M'} W}} \chi^{M'B'MB}(k, k_M)$$

T matrix for scattering and electroproduction

Solution for the K matrix

$$K_{MB,M'B'} = \pi \sqrt{\frac{\omega_M E_B}{k_M W}} \sqrt{\frac{\omega_{M'} E_{B'}}{k_{M'} W}} \left[\sum_{\mathcal{R}} \frac{\tilde{V}_{BR}^M \tilde{V}_{B'R}^{M'}}{(M_{\mathcal{R}} - W)} + \mathcal{D}_{MB,M'B'} \right]$$

$$|\tilde{\Phi}_{\mathcal{R}}\rangle = \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'} |\Phi_{\mathcal{R}}\rangle \quad \tilde{V}_{BR} = \frac{1}{\sqrt{Z_{\mathcal{R}}(W)}} \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'} V_{BR'}$$

and for the T matrix

$$T_{MB,M'B'} = K_{MB,M'B'} + i \sum_{M''K''} T_{MB,M''B''} K_{M''B'',M'B'}$$

Including the γN channel

$$T_{MB,\gamma N} = K_{MB,\gamma N} + i \sum_{M'K'} T_{MB,M'B'} K_{M'B',\gamma N}$$

$$K_{M'B',\gamma N} = -\pi \sqrt{\frac{\omega_{\gamma} E_N}{k_{\gamma} W}} \langle \Psi_{JI}^{M'B'} || V_{\gamma} || \Psi_N \rangle$$

Separation of the electroproduction amplitudes

$$\mathcal{M}_{MB \gamma N} = \mathcal{M}_{MB \gamma N}^{(\text{res})} + \mathcal{M}_{MB \gamma N}^{(\text{bkg})}$$

The resonant part of the amplitude can then be written as:

$$\mathcal{M}_{MB \gamma N}^{(\text{res})} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{\omega_\pi E_N}} \frac{1}{\pi \mathcal{V}_{NN^*}} \underbrace{\langle \Psi_{N^*}^{(\text{res})}(W) | \tilde{V}_\gamma | \Psi_N \rangle}_{A_{N^*}} T_{MB \pi N}$$

The helicity amplitude A_{N^*} for electro-excitation:

$$A_{N^*} \equiv \langle \Psi_{N^*}^{(\text{res})}(W) | \tilde{V}_\gamma | \tilde{\Psi}_N \rangle$$

The resonant state takes the form:

$$|\Psi_{N^*}^{(\text{res})}(W)\rangle = \frac{1}{\sqrt{Z_{N^*}}} \left\{ |\tilde{\Phi}_{N^*}\rangle - \sum_{MB} \int \frac{dk}{\omega_k + E_B - W} \frac{\tilde{\mathcal{V}}_{BN^*}^M(k)}{[a^\dagger(k) |\Psi_B\rangle]^{II}} \right\}$$

The underlying quark models

- ▶ σ -model with quarks
(bosonized version of the NJL model) (early work, only for $\Delta(1232)$)
- ▶ Chromodielectric model
dynamical confinement (early work, only for $\Delta(1232)$)
- ▶ Cloudy Bag Model (current work)
extended to pseudo-scalar SU(3) octet and the ρ meson:

$$\mathcal{L}_{\text{CBM}}^{(\text{quark-meson})} = -\frac{i}{2f} \bar{q} \lambda_a (\gamma_5 \phi_a + \boldsymbol{\gamma} \cdot \mathbf{A}_a) q \delta \left(r - R_{\text{bag}} \right), \quad a = 1, 2, \dots, 8$$

parameterizes the baryon-meson and baryon-photon coupling constants and form factors in terms of " f_π " and the bag radius R_{bag} .

Parameters:

$R_{\text{bag}} = 0.83$ fm (from the ground state calculations)

$f_\pi = 76$ MeV (reproducing the experimental value of $g_{\pi NN}$)

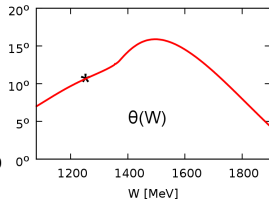
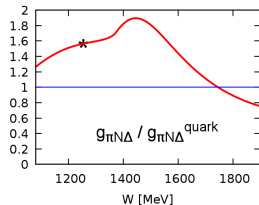
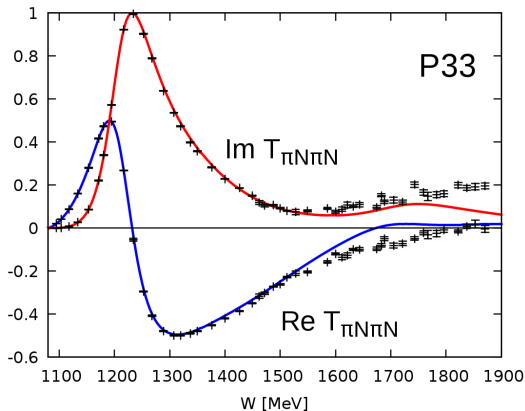
$f_K = 1.2 f_\pi$, $f_\eta = f_\pi$ or $1.2 f_\pi$, $f_\pi < f_\rho < 2 f_\pi$

similar results for 0.75 fm $< R_{\text{bag}} < 1.0$ fm

Free parameters: bare masses of the resonances

P33 scattering

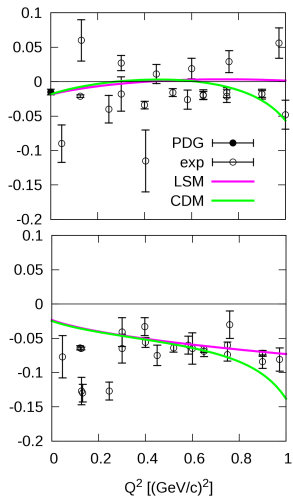
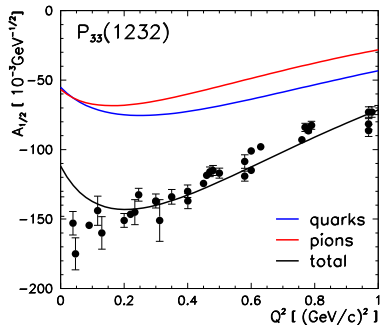
Cloudy Bag Model, $R_{\text{bag}} = 0.83 \text{ fm}$, $f_{\pi} = 76 \text{ MeV}$



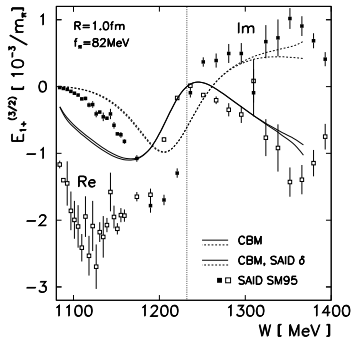
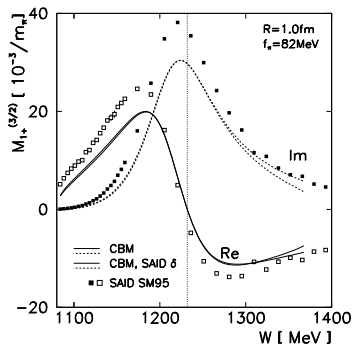
$$\Delta(1232) = \cos \theta (1s)^3 + \sin \theta (1s)^2 (2s)^1 \equiv \cos \theta \Delta^0(1232) + \sin \theta \Delta^0(1600)$$

$\Delta(1232)$ helicity amplitude

Linear σ -model with quarks (LSM)
 Chromodielectric model (CDM)
 P.Alberto, M.Fiolhais, S.Širca, B.G.
 PLB 373 (1996) 229.



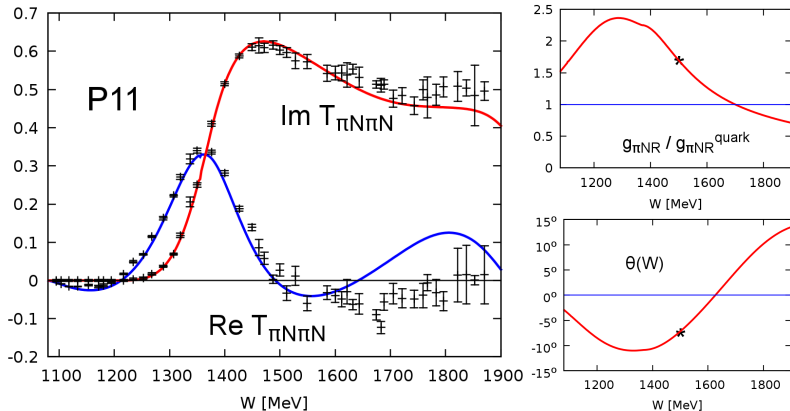
$\Delta(1232)$ photoproduction



P. Alberto, L. Amoreira, M. Fiolhais, B. G., S. Širca, Eur. Phys. J. A **26** (2005) 99.

P11 scattering

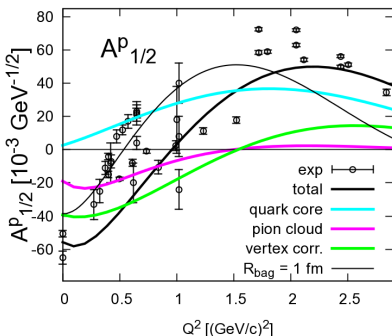
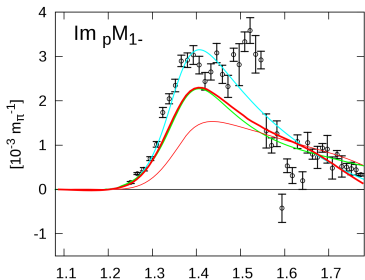
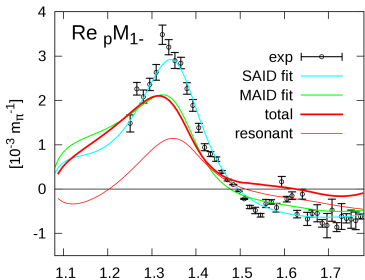
Cloudy Bag Model, $R_{\text{bag}} = 0.83 \text{ fm}$, $f_{\pi} = 76 \text{ MeV}$



$$N(1440) = \cos \theta (1s)^3 + \sin \theta (1s)^2(2s)^1 = \cos \theta N^0(939) + \sin \theta N^0(1440)$$

Included $N(1710)$, $g_{\pi BN(1710)} \approx \frac{1}{5} g_{\pi BN^*}$

P11 photoproduction and helicity amplitudes on proton



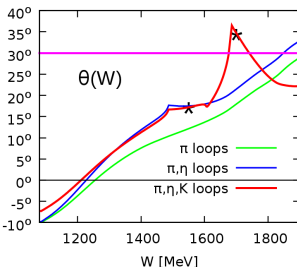
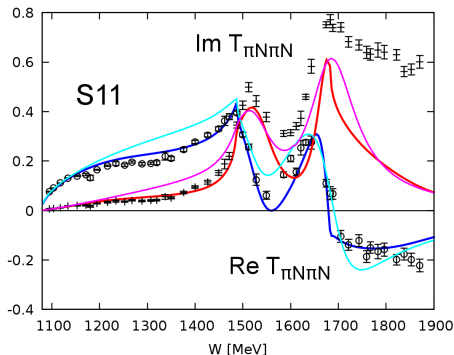
P. Alberto, M. Fiolhais, B. G.,
J. Marques, PLB **523**, 273 (2001),
B. G. and S. Širca, Eur. Phys. J. A
38, 271 (2008),
B. G., S. Širca, and M. Fiolhais,
Eur. Phys. J. A **42**, 185 (2009)

S11 scattering

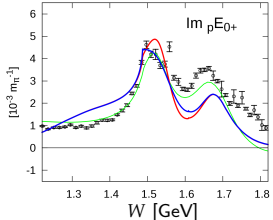
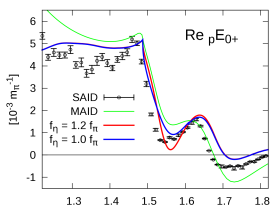
Single-quark excitations $1s \rightarrow 1p_{1/2}$ and $1s \rightarrow 1p_{1/3}$

$$N(1535) = -\sin\theta |^4 \mathbf{8}_{1/2} \rangle + \cos\theta |^2 \mathbf{8}_{1/2} \rangle$$

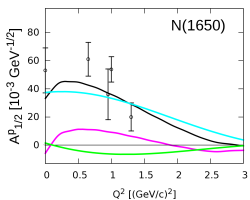
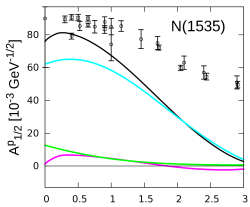
$$N(1650) = \cos\theta |^4 \mathbf{8}_{1/2} \rangle + \sin\theta |^2 \mathbf{8}_{1/2} \rangle$$



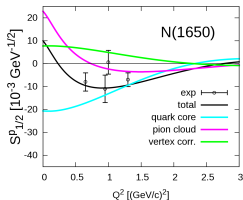
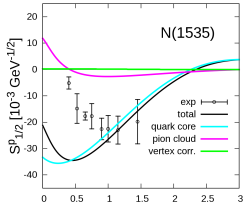
S11 photoproduction and helicity amplitudes on proton



photoproduction



helicity amplitudes



B. G., S. Širca, Eur. Phys. J. A **47**, 61 (2011)

D-wave resonances in quark models

Single-quark excitations $1s \rightarrow 1p_{1/2}$ and $1s \rightarrow 1p_{3/2}$

D13 resonances: $N(1520) = -\sin\theta |^4\mathbf{8}_{3/2}\rangle + \cos\theta |^2\mathbf{8}_{3/2}\rangle$

$N(1700) = \cos\theta |^4\mathbf{8}_{3/2}\rangle + \sin\theta |^2\mathbf{8}_{3/2}\rangle$

$${}^{2S+1}\mathbf{8}_{3/2} = c_{MS}^S |(1s)^2 1p_{3/2}\rangle_{MS} + c_{MA}^S |(1s)^2 1p_{3/2}\rangle_{MA} + c_M^S |(1s)^2 1p_{1/2}\rangle$$

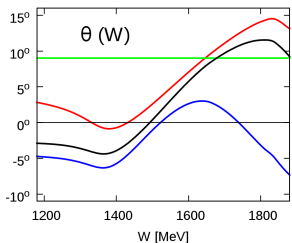
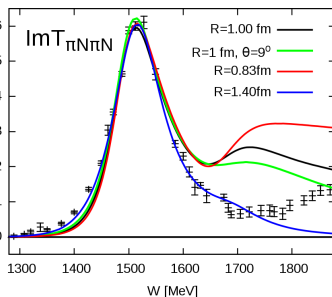
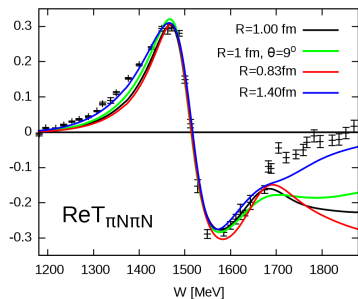
D33 resonance: $\Delta(1700) = |^2\mathbf{10}_{3/2}\rangle = \frac{\sqrt{5}}{3} |(1s)^2 1p_{3/2}\rangle - \frac{2}{3} |(1s)^2 1p_{1/2}\rangle,$

D15 resonance: $N(1675) = |^4\mathbf{8}_{5/2}\rangle = |(1s)^2 1p_{3/2}\rangle.$

Considerable modification of quark-meson coupling constants with respect to the corresponding quark-model values:

B. G., S. Širca, Eur. Phys. J. A **49**, 111 (2013)

D13: elastic scattering amplitudes

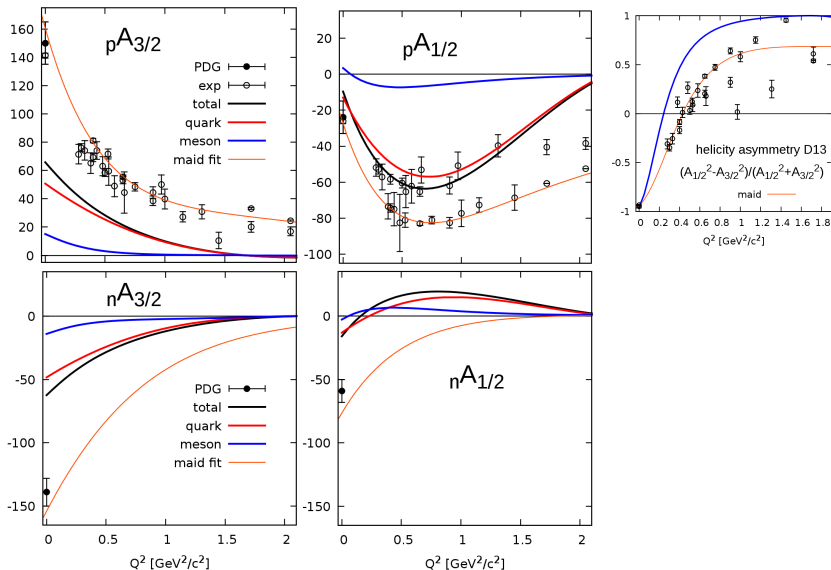


$R = 0.83$ fm: $g_{\pi NN^*} = 1.7 g_{\pi NN^*}$ (QM)
 $g_{\pi \Delta N^*}^{s\text{-wave}} = 0.50 g_{\pi \Delta N^*}^{s\text{-wave}}$ (QM).

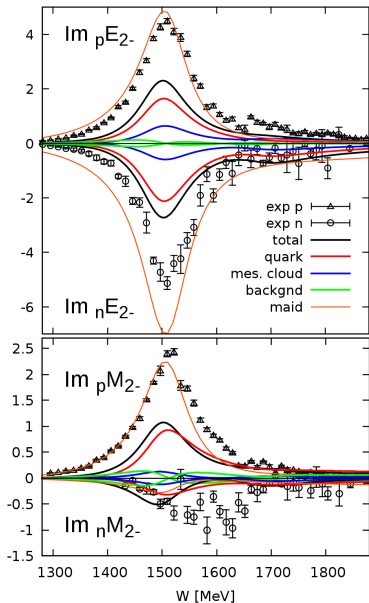
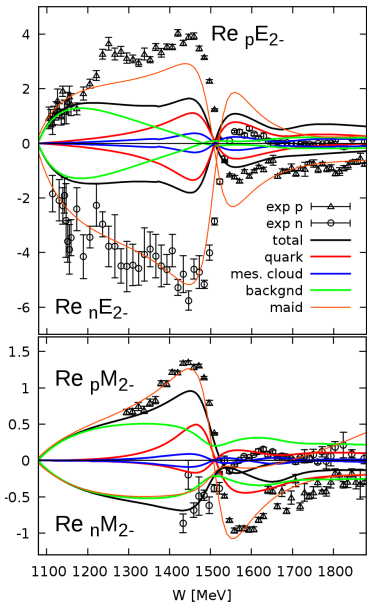
$R = 1.0$ fm: $g_{\pi NN^*} = 1.43 g_{\pi NN^*}$ (QM),
 $g_{\pi \Delta N^*}^{s\text{-wave}} = 0.58 g_{\pi \Delta N^*}^{s\text{-wave}}$ (QM).

$R = 1.4$ fm: $g_{\pi NN^*} = 1.3 g_{\pi NN^*}$ (QM),
 $g_{\pi \Delta N^*}^{s\text{-wave}} = 0.8 g_{\pi \Delta N^*}^{s\text{-wave}}$ (QM).

D13: helicity amplitudes ($R_{\text{bag}} = 1.0 \text{ fm}$)

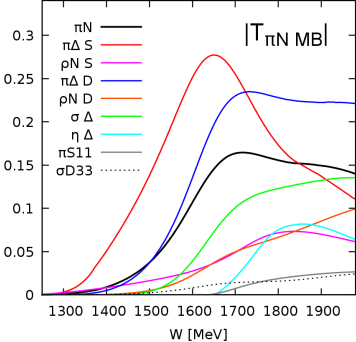
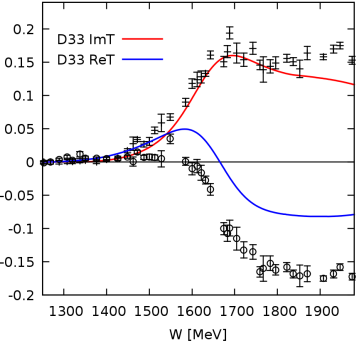


D13: photoproduction amplitudes



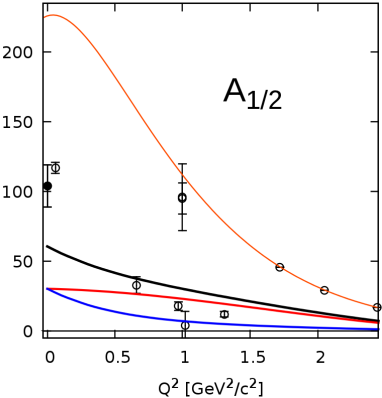
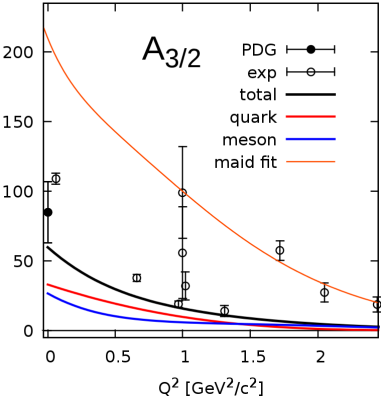
D33: scattering amplitudes

$R_{\text{bag}} = 0.83 \text{ fm}, \quad g_{\pi NN^*} = 2.4 g_{\pi NN^*} \text{ (QM)}$

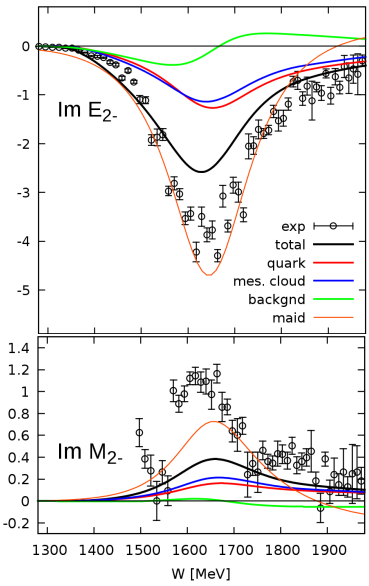
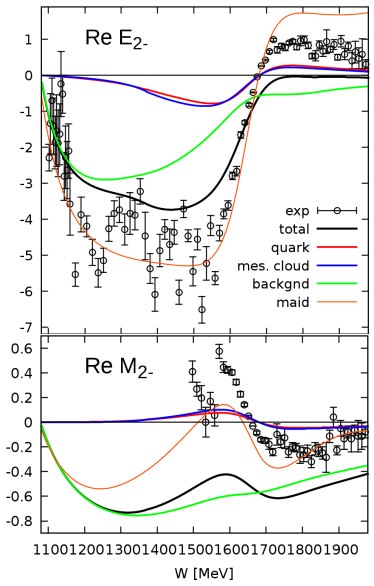


Res.	πN	$\pi \Delta (S)$	$\pi \Delta (D)$	ρN	σ
$\Delta(1700)$	15 %	50 %	29 %	4 % (S)	4 %
PDG	10–20 %	25–50 %	5–15 %	5–20 %	

D33: helicity amplitudes



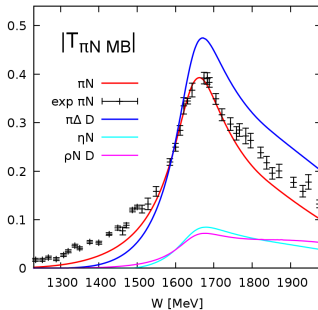
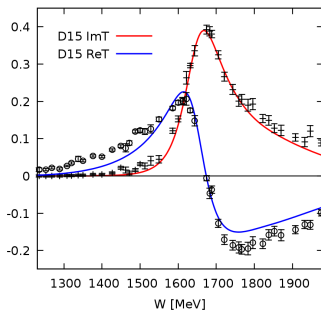
D33: photo-production amplitudes



D15: scattering amplitudes

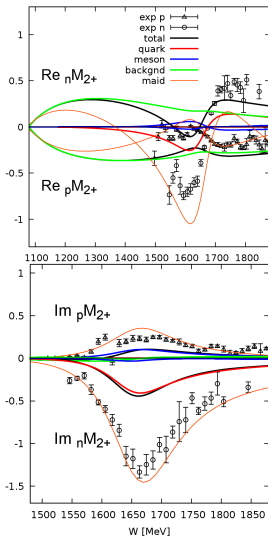
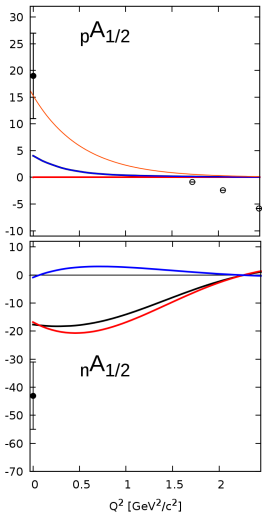
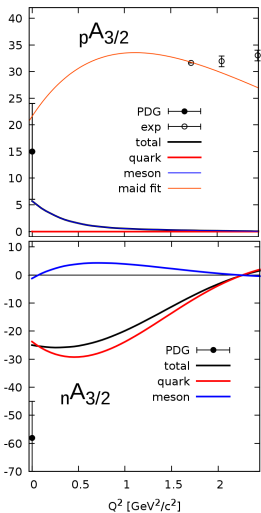
$$R_{\text{bag}} = 1.0 \text{ fm}$$

$$g_{\pi NN^*} = 2.25 g_{\pi NN^*}(\text{QM}), \quad g_{MBN^*}^{d\text{-wave}} = 1.45 g_{MBN^*}^{d\text{-wave}}(\text{QM}) \quad MB \neq \pi N$$



Res.	πN	$\pi \Delta (D)$	$\rho N (D)$	$\eta N(D)$	$\sigma N (F)$
$N(1675)$	39 %	58 %	2 %	2 %	-
PDG	35–45 %	50 ± 15 %	1 ± 1 %	0 ± 1 %	7 ± 3 %

D15: helicity and photo-production amplitudes



Concluding remarks

- ▶ Pion cloud dominates (over other mesons)
 - ▶ *p-wave* pions: strong contribution of for P11 and P33 resonances; similar results in different chiral quark models
 - ▶ *s-wave* and *d-wave* pions: comparable contribution for the S and D resonances; because of possible cancellations, the results may be model dependent
- ▶ η -meson and kaon clouds less important: sizable effects in the region of the upper S11 resonance.
- ▶ Bare masses: typically 200 MeV to 500 MeV higher than the "physical" masses; strange-meson loops contribute $\sim 10\%$

Where to look for the manifestation of the meson cloud?

Manifestation of the meson cloud:

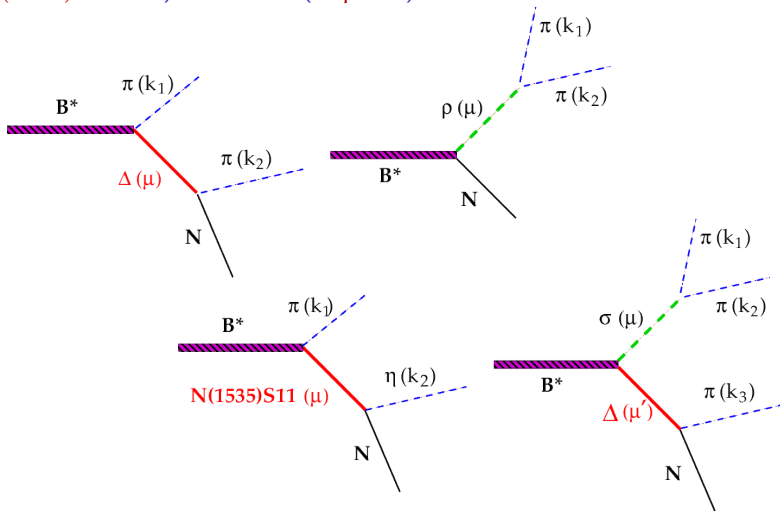
- ▶ Clear and "unmistakable" signal
 - ▶ E2/M1 and S2/M1 ratio of helicity amplitudes for $\Delta(1232)$
- ▶ Strong signal, commonly agreed upon
 - ▶ substantial enhancement of the $\pi N \Delta(1232)$ vertex
 - ▶ large enhancement of the M1 electro-production amplitude on the Δ
- ▶ Strong signal but still disputable (model dependent)
 - ▶ zero-crossing of the helicity amplitude for $N^*(1440)$
 - ▶ substantial enhancement of the the $\pi N N^*(1440)$ vertex
- ▶ Moderate evidence for a sizable contribution
 - ▶ helicity amplitudes for D33
- ▶ Possible explanation – more elaborate analysis needed
 - ▶ W-dependence of resonance mixing in P33, P11, S11, D13 partial waves
 - ▶ missing strength of the d-wave vertices and helicity amplitudes, in particular for the D13 and D15 partial waves

Self-energy correction (in MeV)

Resonance	pion loops	$\eta\Delta$ loops	$K\Lambda/K\Sigma$ loops	total
$\Delta(1232)P33$	201.5			201.5
$N(1440)P11$	66.8			66.8
$N(1535)S11$	60.9	24.5	25.2	110.6
$N(1650)S11$	133.0	-3.5	82.7	212.2
$N(1520)D13$	176.1	-	1.5	177.6
$N(1700)D13$	218.0	-	7.7	225.7
$\Delta(1700)D33$	484.3	-	9.3	493.6

Assumption about the two- and three-meson channels

2π , $\pi\eta$ and 3π decays through intermediate baryons ($\Delta(1232)$, $N(1535)S11$, ...) or mesons (σ , ρ , ...)



Intermediate baryon state

For $\mu \sim M_{\mathcal{R}}$, the intermediate baryon state can be written as

$$\begin{aligned}
 |\tilde{\Psi}^{\alpha}(\mu)\rangle &= \sum_{\beta} \left(\mathbf{1} + \mathbf{K}^2 \right)_{\beta\alpha}^{-\frac{1}{2}} |\Psi^{\beta}(\mu)\rangle \\
 &\approx \frac{1}{\sqrt{2\pi}} \frac{\sqrt{\Gamma_{MB}(\mu)}}{\sqrt{(M_{\mathcal{R}} - \mu)^2 + \frac{1}{4}\Gamma^2(\mu)}} |\hat{\Psi}_{\mathcal{R}}\rangle + \dots,
 \end{aligned}$$

μ is the invariant mass of the baryon-meson system into which the intermediate baryon decays.

$\hat{\Psi}_{\mathcal{R}}$ is the three-quark state surrounded by the meson cloud:

$$|\hat{\Psi}_{\mathcal{R}}\rangle = Z_{\mathcal{R}}^{-\frac{1}{2}} \left[|\Phi_{\mathcal{R}}\rangle - \sum_{MB} \int \frac{dk}{\omega_k + E_B - W} \mathcal{V}_{BR}^M(k) [a^{\dagger}(k) |\tilde{\Psi}_B\rangle]^I \right]$$

Equations for meson amplitudes (Lippmann-Schwinger)

$$\chi^{M'B'MB}(k, k_M) = - \sum_{\mathcal{R}} c_{\mathcal{R}}^{MB} V_{B'\mathcal{R}}^{M'}(k) + \mathcal{K}^{M'B'MB}(k, k_M) + \sum_{M''B''} \int dk' \frac{\mathcal{K}^{M'B'M''B''}(k, k') \chi^{M''B''MB}(k', k_M)}{\omega'_k + E_{B''}(k') - W}$$

with kernels

$$\mathcal{K}^{M'B'MB}(k, k') = \sum_{B''} f_{BB''}^{B''} \frac{\mathcal{V}_{B''B'}^{M'}(k') \mathcal{V}_{B''B}^M(k)}{\omega_k + \omega'_k + E_{B''}(\bar{k}) - W}$$

($f_{BB''}^{B''}$ are spin-isospin coefficients)

The solution assumes the form

$$\chi^{M'B'MB}(k, k_M) = - \sum_{\mathcal{R}} c_{\mathcal{R}}^{MB} \mathcal{V}_{B'\mathcal{R}}^{M'}(k) + \mathcal{D}^{M'B'MB}(k, k_M)$$

Solving the coupled equations

Dressed vertices then satisfy:

$$\mathcal{V}_{BR}^M(k) = V_{BR}^M(k) + \sum_{M'B'} \int dk' \frac{\mathcal{K}^{MBM'B'}(k, k') \mathcal{V}_{B'R}^{M'}(k')}{\omega'_k + E_{B'}(k') - W}$$

and similarly the background part of the amplitude:

$$\begin{aligned} \mathcal{D}^{M'B'MB}(k, k_M) &= \mathcal{K}^{M'B'MB}(k, k_M) \\ &+ \sum_{M''B''} \int dk' \frac{\mathcal{K}^{M'B'M''B''}(k, k') \mathcal{D}^{M''B''MB}(k', k_M)}{\omega'_k + E_{B''}(k') - W} \end{aligned}$$

The coefficients $c_{\mathcal{R}'}^{MB}$ in front of the quasi-bound states satisfy a set of equations:

$$\sum_{\mathcal{R}'} A_{\mathcal{R}\mathcal{R}'}(W) c_{\mathcal{R}'}^{MB}(W) = \mathcal{V}_{BR}^M(k_M)$$

$$A_{\mathcal{R}\mathcal{R}'} = (W - M_{\mathcal{R}}^0) \delta_{\mathcal{R}\mathcal{R}'} + \sum_{B'} \int dk \frac{\mathcal{V}_{B'R}^{M'}(k) V_{B'R'}^{M'}(k)}{\omega_k + E_{B'}(k) - W}$$

Approximation: separable kernels

$$\frac{1}{\omega_k + \omega'_k + E_{B''} - W} \approx \frac{(\omega_M + \omega_{M'} + E_{B''} - W)}{(\omega_k + E_{B''} - E_{B'}) (\omega'_k + E_{B''} - E_B)}$$

$$W = E_B + \omega_M = E_{B'} + \omega_{M'}$$

The approximation has the property:

$$\mathcal{K}^{M'B'MB}(k, k_{M'})^{\text{approx}} = \mathcal{K}^{M'B'MB}(k, k_{M'})^{\text{exact}}$$

and **preserves** the symmetry of the K-matrix: $K_{MB, M'B'} = K_{M'B', MB}$ and, as a consequence, the **unitarity** of the S-matrix.

Further simplification; for $\mathcal{V}_{BB'}$ entering the kernel $\mathcal{K}^{M'B'MB}(k, k_M)$:

$$\mathcal{V}_{BB'}^M(\text{dressed}) = r_{BB'}^M \mathcal{V}_{BB'}^M(\text{bare}) \quad r_{BB'}^M = \text{const}$$

e.g. $r_{N\Delta}^{l=1 \text{ pions}} = 1.3$, as determined by solving the coupled equations in the **P33** partial wave. Most $r_{BB'}^M = 1$.

Mixing of bare resonances

To solve the set of equations, diagonalize A to obtain U , along with the poles of the K matrix, and wave-function normalization Z :

$$UAU^T = \begin{bmatrix} Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}}) & 0 & 0 \\ 0 & Z_{\mathcal{R}'}(W)(W - M_{\mathcal{R}'}) & 0 \\ 0 & 0 & Z_{\mathcal{R}''}(W)(W - M_{\mathcal{R}''}) \end{bmatrix}$$

As a consequence, $\Phi_{\mathcal{R}}$ mix:

$$|\tilde{\Phi}_{\mathcal{R}}\rangle = \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'} |\Phi_{\mathcal{R}}\rangle \quad \tilde{\mathcal{V}}_{BR} = \frac{1}{\sqrt{Z_{\mathcal{R}}(W)}} \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'} \mathcal{V}_{BR'}$$

Solution for the K matrix

$$K_{MB,M'B'} = \pi \sqrt{\frac{\omega_M E_B}{k_M W}} \sqrt{\frac{\omega_{M'} E_{B'}}{k_{M'} W}} \left[\sum_{\mathcal{R}} \frac{\tilde{\mathcal{V}}_{BR}^M \tilde{\mathcal{V}}_{B'R}^{M'}}{(M_{\mathcal{R}} - W)} + \mathcal{D}_{MB,M'B'} \right]$$

and for the T matrix

$$T_{MB,M'B'} = K_{MB,M'B'} + i \sum_{M''K''} T_{MB,M''B''} K_{M''B'',M'B'}$$

Meson-quark vertexes

quark pion

$$V_{l=0,t}^{\pi}(k) = \frac{1}{2f_{\pi}} \sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}} \frac{1}{2\pi} \frac{k^2}{\sqrt{\omega_k}} \frac{j_0(kR)}{kR} \sum_{i=1}^3 \tau_t(i) \mathcal{P}_{sp}(i),$$

$$V_{1mt}^{\pi}(k) = \frac{1}{2f_{\pi}} \frac{\omega_s}{(\omega_s-1)} \frac{1}{2\pi} \frac{1}{\sqrt{3}} \frac{k^2}{\sqrt{\omega_k}} \frac{j_1(kR)}{kR} \sum_{i=1}^3 \tau_t(i) \left(\sigma_m(i) + r_{p_{1/2}} S_{1m}^{[\frac{1}{2}]}(i) + r_{p_{3/2}} S_{1m}^{[\frac{3}{2}]}(i) \right)$$

$$V_{2mt}^{\pi}(k) = \frac{1}{2f_{\pi}} \sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}} \frac{\sqrt{2}}{2\pi} \frac{k^2}{\sqrt{\omega_k}} \frac{j_2(kR)}{kR} \sum_{i=1}^3 \tau_t(i) \Sigma_{2m}^{[\frac{1}{2}\frac{3}{2}]}(i).$$

$$\mathcal{P}_{sp} = \sum_{m_j} |sm_j\rangle \langle p_{1/2}m_j|,$$

$$S_{1m}^{[\frac{1}{2}]} = \sqrt{3} \sum_{m_j m'_j} C_{\frac{1}{2}m'_j 1m}^{\frac{1}{2}m_j} |p_{1/2}m_j\rangle \langle p_{1/2}m'_j|,$$

$$\Sigma_{2m}^{[\frac{1}{2}\frac{3}{2}]} = \sum_{m_s m_j} C_{\frac{3}{2}m_j 2m}^{\frac{1}{2}m_s} |sm_s\rangle \langle p_{3/2}m_j|,$$

$$S_{1m}^{[\frac{3}{2}]} = \frac{\sqrt{15}}{2} \sum_{m_j m'_j} C_{\frac{3}{2}m'_j 1m}^{\frac{3}{2}m_j} |p_{3/2}m_j\rangle \langle p_{3/2}m'_j|,$$

$$r_{p_{1/2}} = \frac{\omega_{p_{1/2}}(\omega_s - 1)}{\omega_s(\omega_{p_{1/2}} + 1)}, \quad r_{p_{3/2}} = \frac{2\omega_{p_{3/2}}(\omega_s - 1)}{5\omega_s(\omega_{p_{3/2}} - 2)}.$$

s-wave η and *K* mesons

$$V^\eta(k) = \frac{1}{2f_\pi} \sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}} + 1)(\omega_s - 1)}} \frac{1}{2\pi} \frac{k^2}{\sqrt{\omega_k}} \frac{j_0(kR)}{kR} \sum_{i=1}^3 \lambda_8(i) \mathcal{P}_{sp}(i),$$

$$V_t^K(k) = \frac{1}{2f_K} \sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}} + 1)(\omega_s - 1)}} \frac{1}{2\pi} \frac{k^2}{\sqrt{\omega_k}} \frac{j_0(kR)}{kR} \sum_{i=1}^3 (V_t(i) + U_t(i)) \mathcal{P}_{sp}(i),$$

$\rho q q$ interaction involving the *s*-state and the $p_{j=1/2}$ -state quarks

$$H_{\rho J l}^{SP} = \int dk V_{J l}^{\rho SP}(k) \sum_{i=1}^3 \sum_{M l} \sigma_M(i) \tau_t(i) a_{1 l M l}(k) + \text{h.c.},$$

with possible values $J = 1$ and $l = 0, 2$:

$$V_{10}^{\rho SP}(k) = \frac{1}{4\pi\tilde{f}_\rho} \sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}} + 1)(\omega_s - 1)}} \frac{2}{3} \frac{k^2}{\sqrt{\omega_k}} \frac{j_0(kR)}{kR},$$

$$V_{12}^{\rho SP}(k) = \frac{1}{4\pi\tilde{f}_\rho} \sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}} + 1)(\omega_s - 1)}} \frac{\sqrt{2}}{3} \frac{k^2}{\sqrt{\omega_k}} \frac{j_2(kR)}{kR}.$$

$\rho q q$ interaction between the s -state and the $p_{j=3/2}$ -state quarks (SA):

$$H_{\rho J l}^{SA} = V_{J l}^{\rho SA}(k) \sum_{i=1}^3 \sum_{M t} \Sigma_{JM}^{[\frac{3}{2} \frac{1}{2}]}(i) \tau_t(i) a_{J M t}(k) + \text{h.c.}$$

with $J = 1, l = 0, 2$ and $J = 2, l = 2$:

$$V_{10}^{\rho SA}(k) = \frac{1}{4\pi \tilde{f}_{\rho}} \sqrt{\frac{\omega_{p_{3/2}} \omega_s}{(\omega_{p_{3/2}} - 2)(\omega_s - 1)}} \frac{1}{\sqrt{3}} \frac{k^2}{\sqrt{\omega_k}} \frac{j_0(kR)}{kR},$$

$$V_{12}^{\rho SA}(k) = -\frac{1}{4\pi \tilde{f}_{\rho}} \sqrt{\frac{\omega_{p_{3/2}} \omega_s}{(\omega_{p_{3/2}} - 2)(\omega_s - 1)}} \frac{1}{\sqrt{6}} \frac{k^2}{\sqrt{\omega_k}} \frac{j_2(kR)}{kR},$$

$$V_{22}^{\rho SA}(k) = -\frac{1}{4\pi \tilde{f}_{\rho}} \sqrt{\frac{\omega_{p_{3/2}} \omega_s}{(\omega_{p_{3/2}} - 2)(\omega_s - 1)}} \sqrt{\frac{3}{2}} \frac{k^2}{\sqrt{\omega_k}} \frac{j_2(kR)}{kR},$$

$$\langle \frac{3}{2} m_j | \Sigma_{JM}^{[\frac{3}{2} \frac{1}{2}]} | \frac{1}{2} m_s \rangle = C_{\frac{1}{2} m_s J M}^{\frac{3}{2} m_j}$$