

Nucleon resonances in a covariant quark model

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and the Interpretation of Baryon Resonances**

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Covariant spectator quark model

- Wave functions
- Quark current
- Transition current \Rightarrow form factors/helicity amplitudes

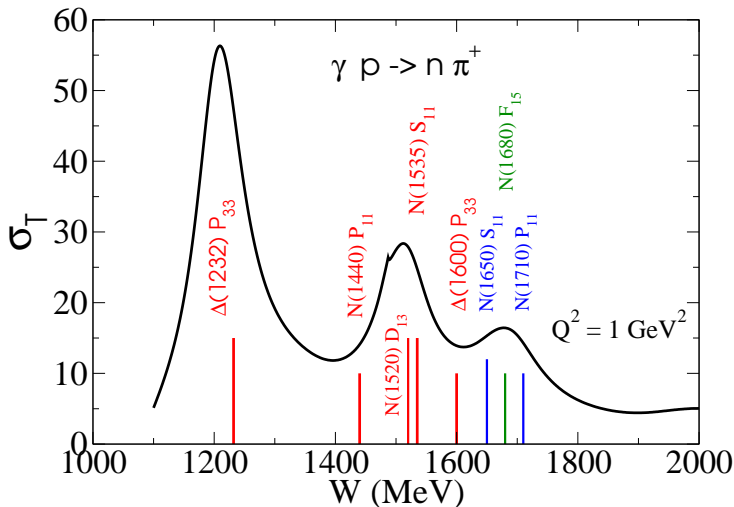
$$F = F_{qqq} + \underbrace{F_{qqq(q\bar{q})}}_{\approx \frac{1}{Q^4} F_{qqq}}$$

Form factors: **quark core** plus **meson cloud** (suppressed at high Q^2)

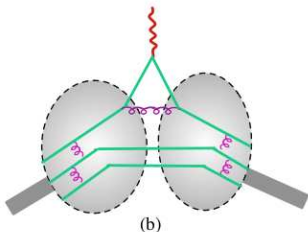
\Rightarrow Results for $\gamma^* N \rightarrow N^*$ form factors/helicity amplitudes

$N^* = N(939), N(1440), N(1520), N(1535), \Delta(1232), \Delta(1600)$

Nucleon Resonance Structure

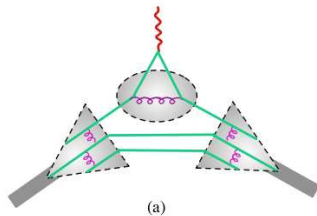


Formalism (Light Front vs CSQM)



Light Front formalism

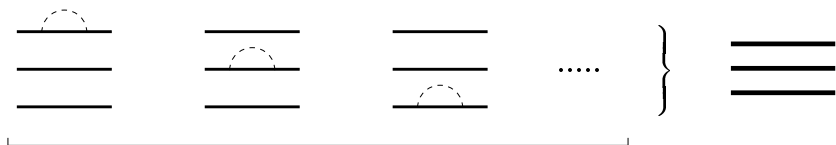
- Pointlike quarks
- Baryon states as a sum of Fock states:
 $qqq, qqqg, qqq(q\bar{q}), \dots$
- **Light quarks**
 $\kappa_u, \kappa_d = 0$



Covariant Spectator QM formal.

- **Gluon** interactions between $q\bar{q}$
 \Rightarrow **quark form factors**
- **Baryon**: system of **dressed quarks** (**gluons** and $q\bar{q}$)
- **Massive quarks** with anomalous magnetic moments κ_u, κ_d

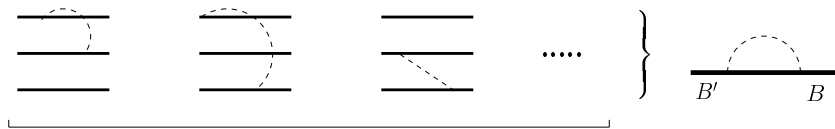
Quark structure and electromagnetic interaction (I) †



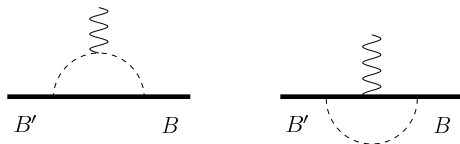
γ coupling:



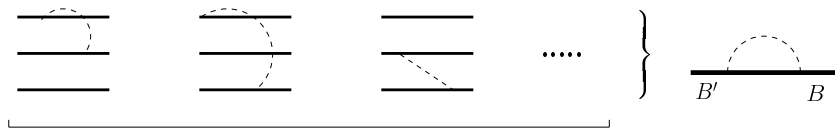
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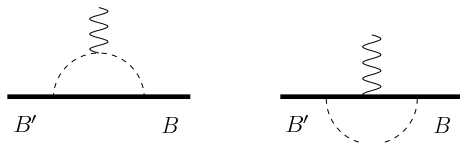
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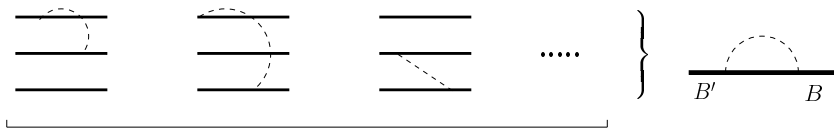


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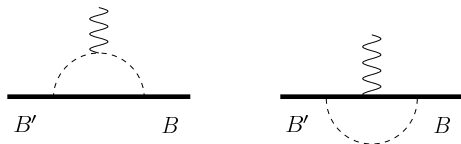


- **Not important** at high Q^2 : pQCD: supression $1/Q^4$
 CE Carlson FBS Sup 11, 10 (1999): $F \propto \frac{1}{(Q^2)^{(N-1)}$, $N = 3 + 2$
Very important at low Q^2

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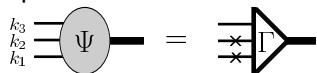
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 Very important at low Q^2
- Assume NO interference with quark dressing processes

$$F = F^B + F^{mc}$$

(bare \oplus meson cloud)

Spectator QM: Baryon wave functions (I)

- Baryon: 3 constituent quark system
- **Covariant Spectator Theory**: wave function Ψ defined in terms of a 3-quark vertex Γ with 2 on-mass-shell quarks



$$\Psi_\alpha(P, k_3) = \left(\frac{1}{m_q - \not{k}_3 - i\varepsilon} \right)_{\alpha\beta} \Gamma^\beta(P, k_1, k_2)$$

Gross and Agbakpe PRC 73, 015203 (2006); Gross, GR and Peña PRC 77, 015202 (2008)

- Ψ is **free** of singularities ($3q$ on-shell $\Gamma \equiv 0$) \Rightarrow parametrize Ψ
Stadler, Gross and Frank PRC 56, 2396 (1998); Savkli and Gross PRC 63, 035208 (2001)
- On-shell integration (k_1, k_2) \Rightarrow $k = k_1 + k_2$, $r = \frac{1}{2}(k_1 - k_2)$
 \Rightarrow integration in \mathbf{k} and $s = (k_1 + k_2)^2$
Gross, GR and Peña, PRC 77, 015202 (2008); PRD 85, 093005 (2012)

$$\int_{k_1} \int_{k_2} = \frac{\pi}{4} \int d\Omega_{\hat{\mathbf{r}}} \int_{4m_q^2}^{+\infty} ds \sqrt{\frac{s - 4m_q^2}{s}} \int \frac{d^3\mathbf{k}}{2\sqrt{s + \mathbf{k}^2}} \rightarrow \int \frac{d^3\mathbf{k}}{2\sqrt{m_D^2 + \mathbf{k}^2}}$$

Mean value theorem: $\sqrt{s} \rightarrow m_D$; cov. int. in diquark **on-shell** mom.

Spectator QM: Baryon wave functions (II)

Ψ_B not determined by a dynamical equation \Leftarrow phenomenology $M_B = M_B^{\text{exp}}$

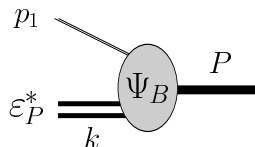
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- Baryon wave functions: $B = \text{diquark} \oplus \text{quark}$

Combination of **diquark** (12) and single **quark** (3) states, using $SU(6) \otimes O(3)$:

$$\Psi_B = \sum (\text{color}) \otimes (\text{flavor}) \otimes (\text{spin}) \otimes (\text{orbital}) \otimes \underbrace{\psi_B(P, k)}_{\text{radial}}$$

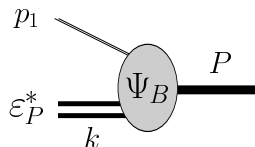


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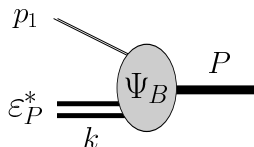
- Ψ_B in **rest frame** using **quark states**

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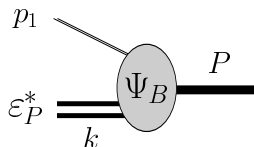
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- **Covariant** generalization of Ψ_B in terms **baryon** properties

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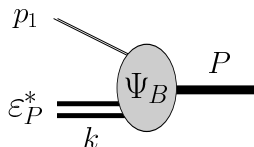
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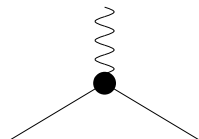


- Ψ_B in **rest frame** using **quark** states
- **Covariant** generalization of Ψ_B in terms **baryon** properties
- Ψ_B can be used on **any** frame and/or Q^2 regime
- Phenomenology in the **radial wf** (momentum scale parameters)

Spectator QM: Quark current (VMD at quark level) (I)

- **Quark current** [$f_{i\pm}$ quark form factors]

$$j_q^\mu = \left[\frac{1}{6} f_{1+} + \frac{1}{2} f_{1-} \tau_3 \right] \gamma^\mu + \left[\frac{1}{6} f_{2+} + \frac{1}{2} f_{2-} \tau_3 \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}$$

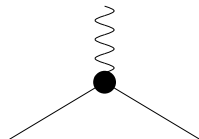


Quarks with **anomalous** magnetic moments κ_u, κ_d

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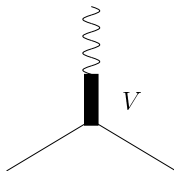
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Quarks with **anomalous** magnetic moments κ_u, κ_d

- **Vector meson dominance parameterization:**



Spectator QM: Quark current (VMD at quark level) (II)

- Vector meson dominance parameterization: PRC77 015202 (2008)



$$f_{1\pm} = \lambda_q + (1 - \lambda_q) \frac{m_v^2}{m_v^2 + Q^2} + c_{\pm} \frac{M_h^2 Q^2}{(M_h^2 + Q^2)^2}$$

$$f_{2\pm} = \kappa_{\pm} \left\{ d_{\pm} \frac{m_v^2}{m_v^2 + Q^2} + (1 - d_{\pm}) \frac{M_h^2}{M_h^2 + Q^2} \right\}$$

2 poles:

- Light vector meson: $m_v \simeq m_{\rho} (\approx m_{\omega})$
- Effective heavy meson: $M_h (= 2M_N)$ ← short range

Nucleon magnetic moments $\Rightarrow \kappa_{\pm}$

4 parameters: $\lambda_q, c_{\pm}, d_{\pm}$ (mixture coefficients) $\oplus d_+ = d_-$

↑ Fitted to nucleon form factors data

Spectator QM: Transition currents ($\gamma N \rightarrow N^*$)

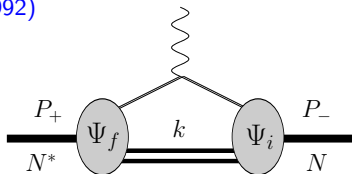
Quark current $j_q^\mu \oplus$ Baryon wave function $\Psi_B \Rightarrow J^\mu$

Transition current J^μ in **spectator formalism**

F Gross et al PR 186 (1969); PRC 45, 2094 (1992)

Relativistic impulse approximation:

$$J^\mu = 3 \sum_\lambda \int \bar{\Psi}_f(P_+, k) j_q^\mu \Psi_i(P_-, k)$$



diquark on-shell

$$q = P_+ - P_-, \quad P = \frac{1}{2}(P_+ + P_-), \quad Q^2 = -q^2$$

$q \cdot J \neq 0$: Landau prescription: $J^\mu \rightarrow J^\mu - \frac{q \cdot J}{q^2} q^\mu$

JJ Kelly, PRC 56, 2672 (1997); Z Batiz and F Gross, PRC 58, 2963 (1998)

Spin 1/2 resonances: transition currents †

Nucleon:

$$J^\mu = \bar{u}(P_+) \left[F_1 \gamma^\mu + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \right] u(P_-)$$

$$\gamma N \rightarrow N(1440) \text{ (R): } J^P = \frac{1}{2}^+$$

$$J^\mu = \bar{u}_R(P_+) \left[F_1^* \left(\gamma^\mu - \frac{\not{q}q^\mu}{q^2} \right) + F_2^* \frac{i\sigma^{\mu\nu} q_\nu}{M_R + M_N} \right] u(P_-)$$

$$\gamma N \rightarrow N(1535) \text{ (S): } J^P = \frac{1}{2}^-$$

$$J^\mu = \bar{u}_S(P_+) \left[F_1^* \left(\gamma^\mu - \frac{\not{q}q^\mu}{q^2} \right) + F_2^* \frac{i\sigma^{\mu\nu} q_\nu}{M_S + M_N} \right] \gamma_5 u(P_-)$$

Form factors $F_1^*, F_2^* \rightarrow A_{1/2}, S_{1/2}$ exclusive functions of Q^2

Spin 3/2 resonances: transition currents †

$\gamma N \rightarrow \Delta(1232), N^*(1520), \Delta(1600)$:

$$J^\mu = \bar{u}_\beta(P_+) \left[G_1 q^\beta \gamma^\mu + G_2 q^\beta P^\mu + G_3 q^\beta q^\mu - G_4 g^{\beta\mu} \right] \begin{pmatrix} \gamma_5 \\ 1 \end{pmatrix} u(P_-)$$

u_β Rarita-Schwinger spinor

Only 3 independent form factors:

$$q_\mu J^\mu = 0 \Rightarrow G_4 = (M_R \pm M_N) G_1 + \frac{1}{2} (M_R^2 - M_N^2) G_2 - Q^2 G_3$$

$$G_1, G_2, G_3 \Rightarrow G_M, G_E, G_C \text{ or } A_{1/2}, A_{3/2}, S_{1/2}$$

Definition of the helicity amplitudes $(\frac{1}{2} \rightarrow \frac{1}{2}, \frac{3}{2})$ ††

Resonance R rest frame

$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{K}} \langle R, S'_z = +\frac{3}{2} | \epsilon_+ \cdot J | N, S_z = +\frac{1}{2} \rangle$$

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \langle R, S'_z = +\frac{1}{2} | \epsilon_+ \cdot J | N, S_z = -\frac{1}{2} \rangle$$

$$S_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \langle R, S'_z = +\frac{1}{2} | \epsilon_0 \cdot J | N, S_z = +\frac{1}{2} \rangle \frac{|\mathbf{q}|}{Q},$$

$$\alpha = \frac{e^2}{4\pi}$$

$$K = \frac{M_R^2 - M^2}{2M_R}$$

Spectator QM: Nucleon wave function

Nucleon wave function: [PRC 77,015202 (2008); EPJA 36, 329 (2008)]

Simplest structure –**S-state** in quark-diquark system

$$\Psi_N(P, k) = \frac{1}{\sqrt{2}} [\Phi_I^0 \Phi_S^0 + \Phi_I^1 \Phi_S^1] \psi_N(P, k)$$

Isospin states: $\Phi_I^{0,1}$

Spin states:

$$\Phi_S^0(s) \equiv u(P, s) \quad \Phi_S^1(s) \equiv -(\varepsilon_\lambda^*)_\alpha U^\alpha(P, s)$$

$$U^\alpha(P, s) = \sum_{\lambda s'} \langle \frac{1}{2} s'; 1\lambda | \frac{1}{2} s \rangle \varepsilon_\lambda^\alpha u(P, s') \rightarrow \frac{1}{\sqrt{3}} \gamma_5 \left(\gamma^\alpha - \frac{P^\alpha}{M} \right) u(P, s)$$

$\varepsilon_\lambda = \varepsilon_{\lambda P}$ **function of nucleon momentum**

Fixed-Axis polarization states; PRC 77, 035203 (2008)

$\Rightarrow \Psi_N$ **pure S-state**

Radial (scalar) wave function: Nucleon

Scalar wave functions dependent of $(P - k)^2 = (\text{quark momentum})^2$

$$\chi_B = \frac{(M_B - m_D)^2 - (P - k)^2}{M_B m_D} \xrightarrow{NR} \frac{\mathbf{k}^2}{m_D^2}$$

$M_B = \text{baryon mass}; m_D = \text{diquark mass}$

Nucleon scalar wave function:

$$\begin{aligned} \psi_N(P, k) &= \frac{N_0}{m_D} \frac{1}{(\beta_1 + \chi_N)(\beta_2 + \chi_N)} = \frac{N_0}{m_D} \frac{1}{\beta_2 - \beta_1} \left[\frac{1}{\beta_1 + \chi_N} - \frac{1}{\beta_2 + \chi_N} \right] \\ &\xrightarrow{NR} \frac{N_0}{m_D} \frac{1}{\beta_2 - \beta_1} \left[\frac{1}{\beta_1 + \frac{\mathbf{k}^2}{m_D^2}} - \frac{1}{\beta_2 + \frac{\mathbf{k}^2}{m_D^2}} \right] \end{aligned}$$

Position space:

$$\psi_N(P, k) \xrightarrow{FT} \frac{e^{-m_D \sqrt{\beta_1} r}}{r} - \frac{e^{-m_D \sqrt{\beta_2} r}}{r}$$

β_1, β_2 momentum range parameters; $\beta_2 > \beta_1$:

β_1 long spatial range; β_2 short spatial range

Nucleon form factors (I) ††

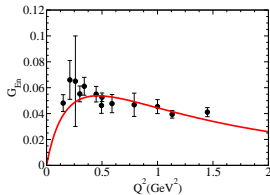
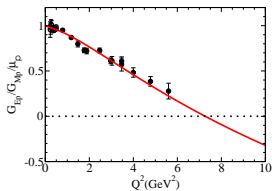
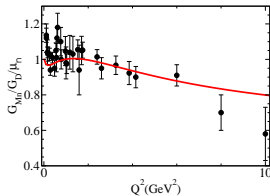
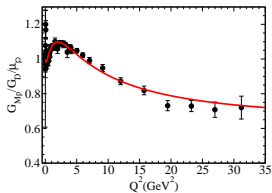
F Gross, GR and MT Peña, PRC 77, 015202 (2008)

Nucleon form factors: $G_E = F_1 - \tau F_2$, $G_M = F_1 + F_2$; $\tau = \frac{Q^2}{4M_N^2}$

$$G_E(Q^2) = \frac{1}{2} [(f_{1+} + f_{1-\tau_3}) - \tau(f_{1+} + f_{2-\tau_3})] \int_k \psi_N(P_+, k) \psi_N(P_-, k)$$

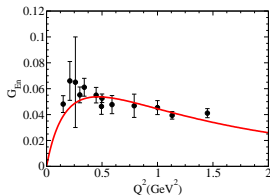
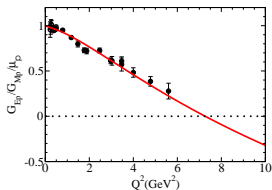
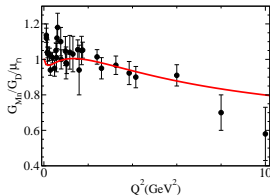
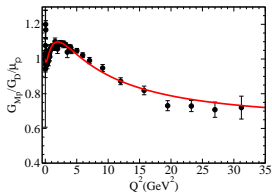
$$G_M(Q^2) = \frac{1}{6} [(f_{1+} + 5f_{1-\tau_3}) + (f_{1+} + 5f_{2-\tau_3})] \int_k \psi_N(P_+, k) \psi_N(P_-, k)$$

Nucleon form factors (II) [PRC 77, 015202 (2008)] - model II



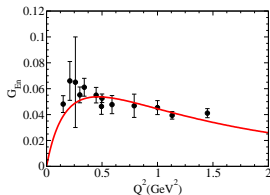
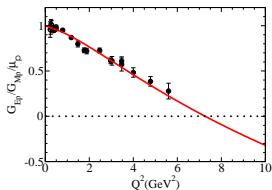
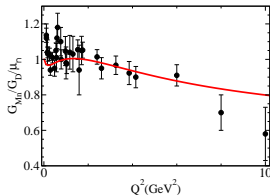
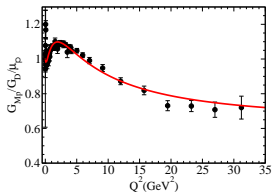
- Quark current fix 4 parameters; Scalar wave function [2]

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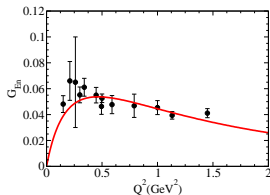
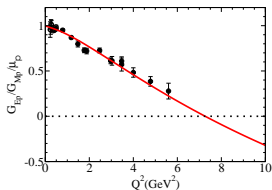
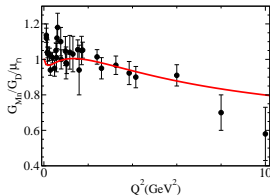
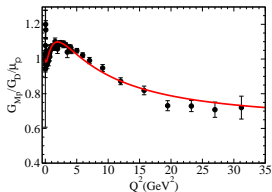
- Quark current fix 4 parameters; Scalar wave function [2]
- No pion cloud (explicit)

Nucleon form factors (II) [PRC 77, 015202 (2008)] - model II



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Nucleon form factors (II) [PRC 77, 015202 (2008)] - model II



- Quark current fix 4 parameters; Scalar wave function [2]
- No pion cloud (explicit)
- How can we test the valence quark parametrization? **Lattice**

GR and MT Peña JPG 36, 115011 (2009)

- Quark current (VMD): $j_q^\mu = j_1 \gamma^\mu + j_2 \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}$
Replace hadron masses by lattice masses ($M_N, m_\rho, M_h = 2M_N$)

$$j_q^\mu(M_N; m_\rho, M_h = 2M_N) \rightarrow j_q^\mu(M_N^{\text{latt}}; m_\rho^{\text{latt}}, 2M_N^{\text{latt}})$$

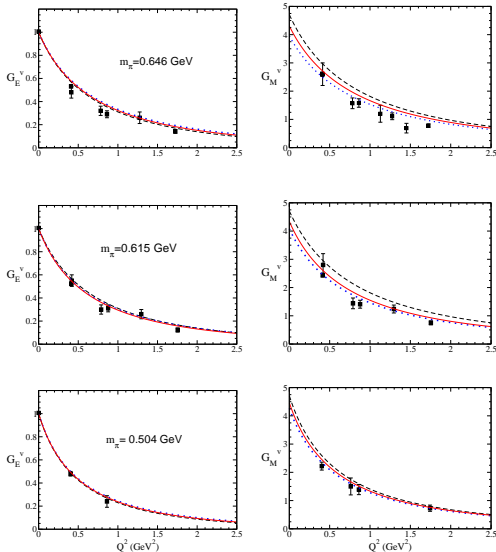
- Wave functions:

$$\Psi_B(\{M_B\}) \rightarrow \Psi_B(\{M_B^{\text{latt}}\})$$

$$\Rightarrow \text{Form factors } G_X(m_\pi^{\text{latt}}, Q^2)$$

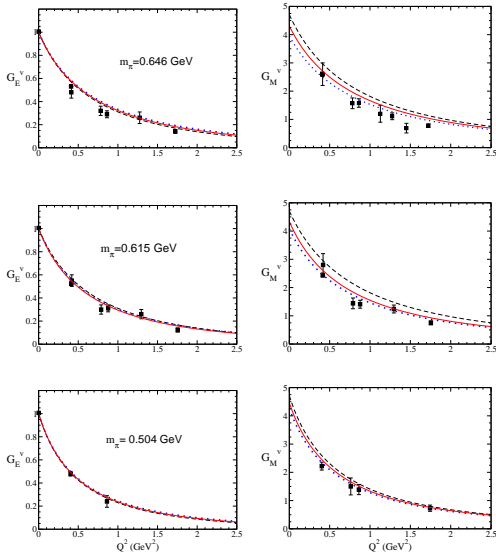
If meson cloud are suppressed: $G_X \equiv G_X^B$

Compare G_X^B -model with lattice data



Data from Gockeler et al, PRD 71, 034508 (2005) - - Model II

Nucleon form factors on lattice [JPG 36, 115011 (2009)] G_X^{p-n}



Describes physical and lattice QCD regimes - - - Model II

- $N(1440)$ is the **1st radial excitation** of the nucleon
Same spin and isospin structure as the nucleon; $\psi_R \neq \psi_N$
- Ψ_R orthogonal to Ψ_N
Orthogonality given by scalar wave functions

$$\int_k \psi_R(P_+, k) \psi_N(P_-, k) \Big|_{Q^2=0} = 0$$

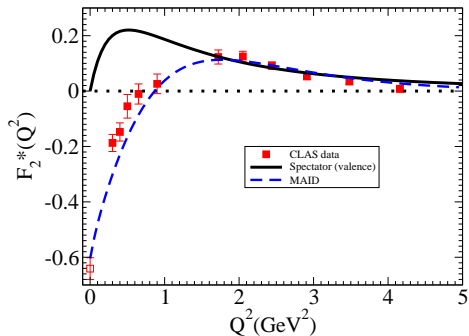
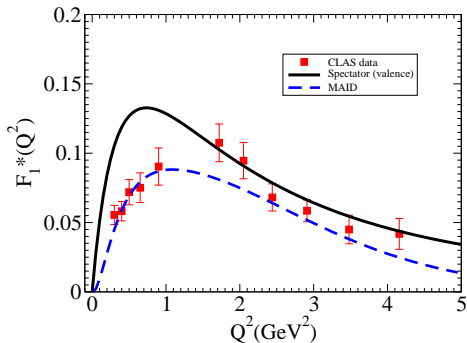
Wave function:

$$\psi_R(\chi_R) = N_1 \frac{\overbrace{\beta_3 - \chi_R}^{\text{radial excitation}}}{\beta_1 + \chi_R} \psi_N(\chi_R)$$

β_1 fixed by ψ_N ; β_3 determined by the **orthogonality condition**

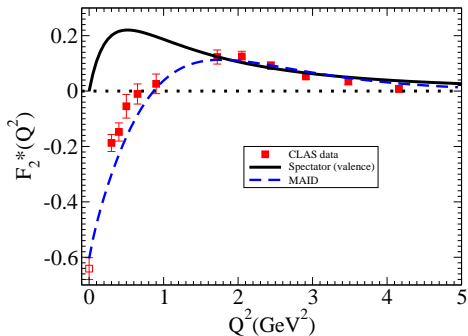
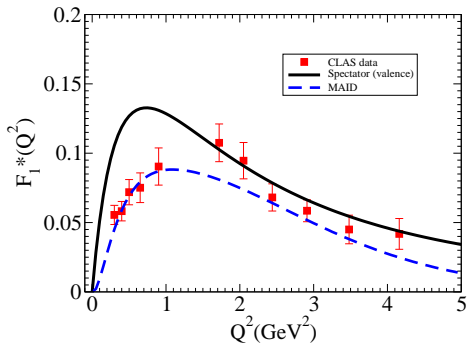
No adjustable parameters \rightarrow predictions

$\gamma N \rightarrow N(1440)$ form factors [PRD 81, 074020 (2010)]



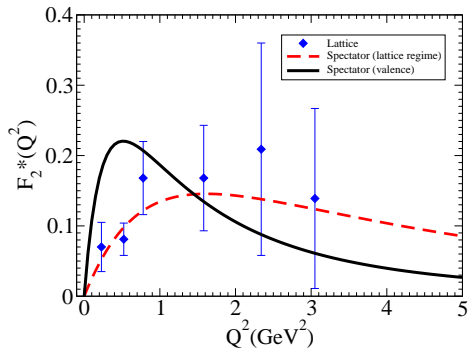
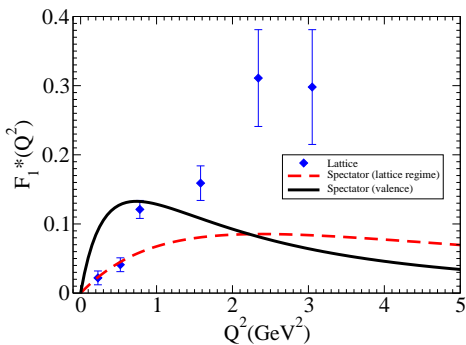
- **CLAS data** - Aznauryan et al PRC 80, 055203 (2009), **MAID fit**
- **Good agreement for $Q^2 > 1.5$ GeV²**
- **Difference for $Q^2 < 1.5$ GeV²** –manifestation of meson cloud
- **Good description also of lattice data** (H.W. Lin et al)

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$\gamma N \rightarrow N(1440)$ on lattice [PRD 81, 074020 (2010)] ††



Data: H.W. Lin et al PRD 78, 114508 (2008)

Good agreement with Lattice data

GR and K Tsushima, PRD 81, 074020 (2010)

- Pointlike diquark $k_1 - k_2 = 0$ [no diquark w/ P-states] $J^P = \frac{1}{2}^-$
- Pure spin 1/2 core: [Karl-Isgur model: $\cos \theta_S \approx 0.85$]

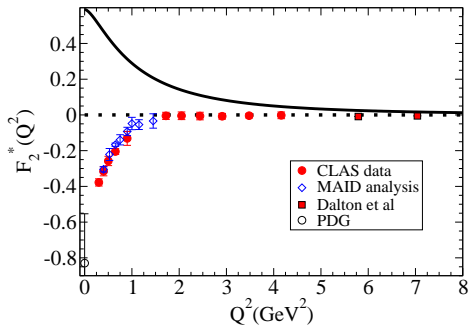
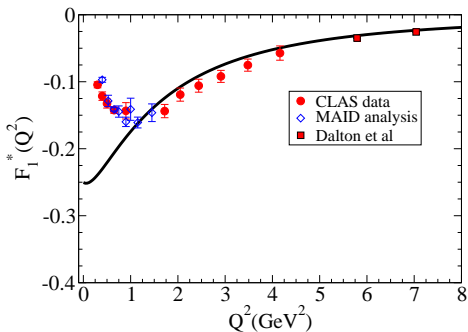
$$\begin{aligned} |N(1535)\rangle &= \cos \theta_S |S = 1/2\rangle - \sin \theta_S |S = 3/2\rangle \\ &\rightarrow |S = 1/2\rangle \end{aligned}$$

- Radial wave function: $\psi_{S11}(\chi_{S11}) \equiv \psi_N(\chi_{S11})$
- Form factors: (\mathcal{I} = overlap integral - S11 rest frame)

$$F_i^* \propto \mathcal{I}(Q^2) = \int_k \frac{k_z}{|\mathbf{k}|} \psi_{S11}(P_{S11}, k) \psi_N(P_N, k)$$

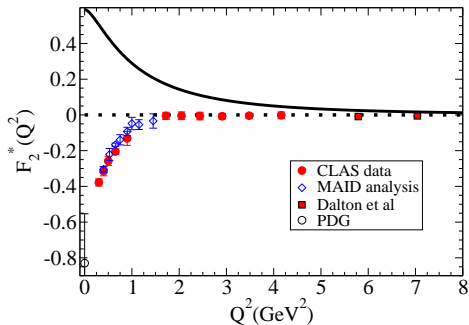
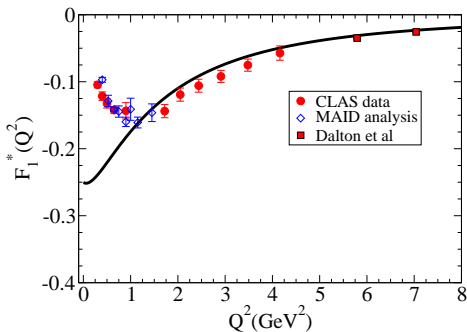
- At $Q^2 = 0$: $\mathcal{I}(0) \propto |\mathbf{q}|_0 = \frac{M_R^2 - M^2}{2M_R} \neq 0$
- No exact orthogonality ($\mathcal{I}(0) \neq 0$)
Approximated orthogonality $Q^2 \gg |\mathbf{q}|_0^2 \approx 0.23 \text{ GeV}^2$
Model valid for $Q^2 > 2.3 \text{ GeV}^2$

$\gamma^* N \rightarrow N^*(1535)$ form factors [PRD 84, 051301 (2011)]



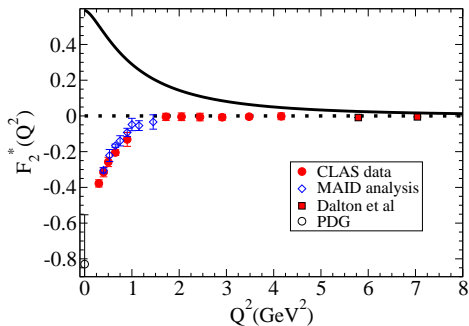
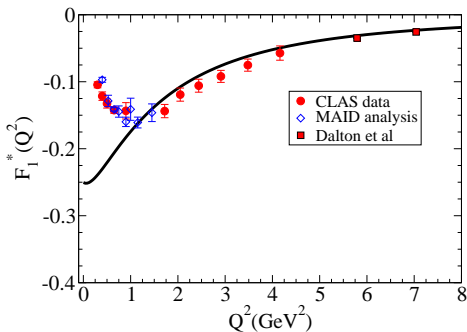
- Model compared with CLAS and MAID data

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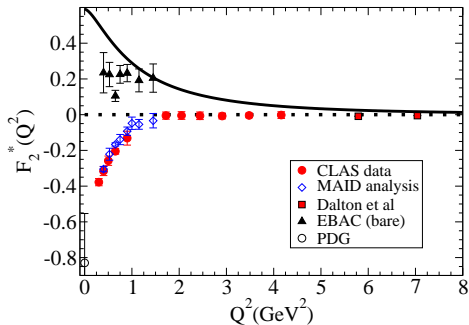
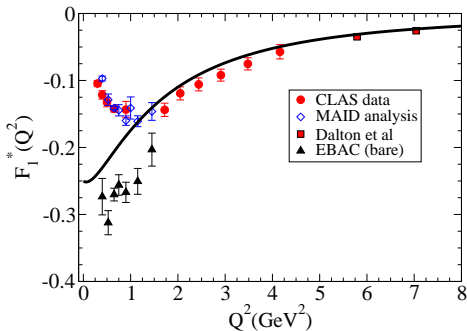
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- F_1^* OK; F_2^* wrong sign

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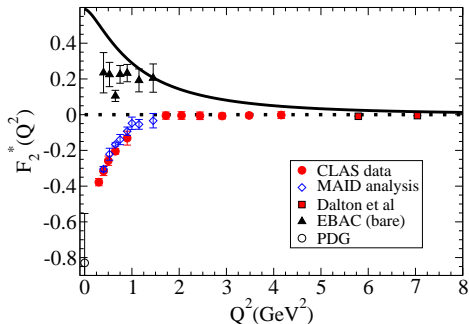
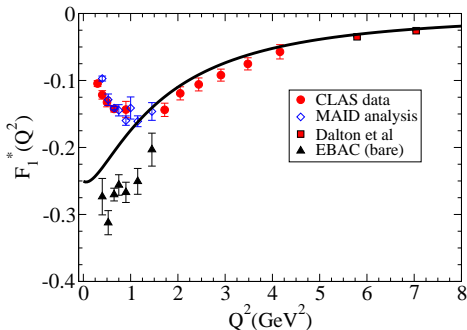
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- ...

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- **Valence quark effects under control**

Implications of $F_2^* = 0$?

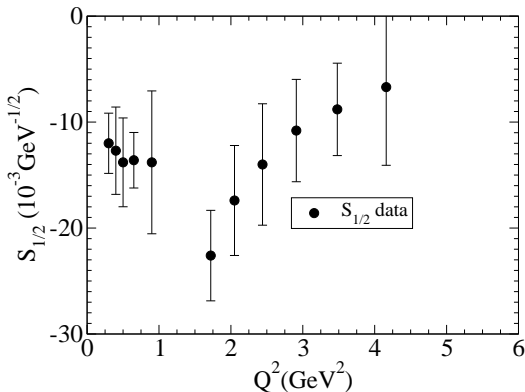
Cancellation between
valence and meson cloud

GR, K Tsushima
PRD 84, 051301 (2011)

GR, D Jido, K Tsushima
PRD 85, 093014 (2012)

$$\tau = \frac{Q^2}{(M_R + M)^2} \quad Q^2 > 1.5 \text{ GeV}^2$$

$$S_{1/2} \simeq -\frac{\sqrt{1+\tau}}{\sqrt{2}} \frac{M_S^2 - M^2}{2M_S Q} A_{1/2}$$



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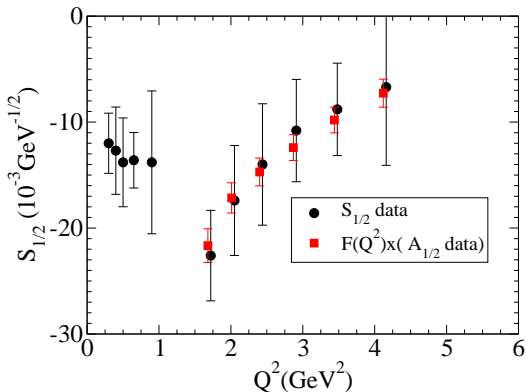
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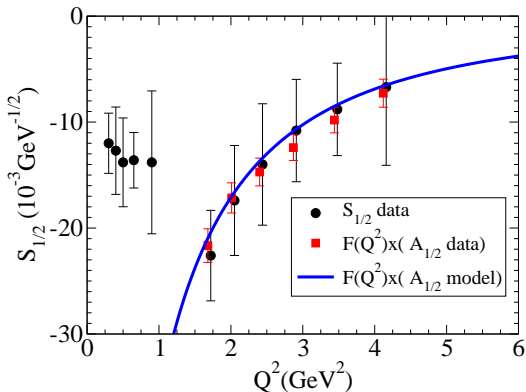
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Spin 3/2 resonances: $\Delta(1232)$ wave functions

- Δ wave function: [EPJA 36, 329 \(2008\)](#); [PRD 78, 114017 \(2008\)](#); [PRD 80, 013008 \(2009\)](#)

$$\Psi_{\Delta} = N [\Psi_S + a\Psi_{D3} + b\Psi_{D1}]$$

$$S = 0 \oplus \frac{3}{2}, \quad D3 = 2 \oplus \frac{3}{2}, \quad D1 = 2 \oplus \frac{1}{2}$$

- Wave functions fitted to form factor data [G_M^* , G_E^* , G_C^*]
(Radial wave functions: $\alpha_S, \alpha_{D3}, \alpha_{D1}, \alpha'_{D1}$)

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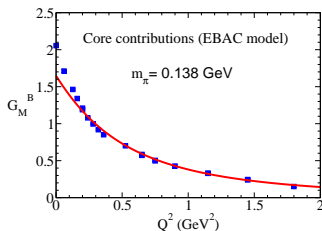
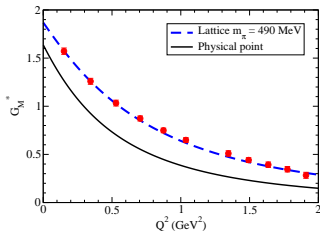
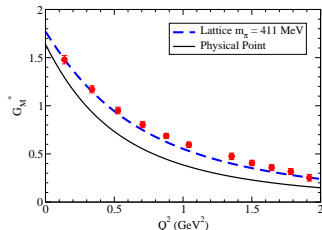
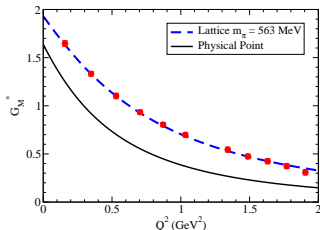
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- With D-states: **small effect**
- S-state model: $\Rightarrow G_M^*$:

$$G_M^B = \frac{8}{3\sqrt{3}} \frac{M}{M + M_{\Delta}} f_v \int_k \psi_{\Delta} \psi_N, \quad f_v = f_{1-} + \frac{M + M_{\Delta}}{2M} f_{2-}$$

Fit EBAC (core): J Diaz et al, PRC 75, 015205 (2007)

$\gamma N \rightarrow \Delta$: $G_M^*(Q^2)$ on lattice [PRD 80, 013008 (2009)]

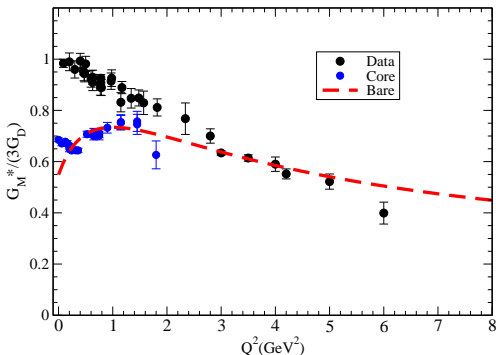


EBAC: J. Diaz et al, PRC 75, 015205 (2007) \oplus

Lattice: Alexandrou et al, PRD 77, 085012 (2008)

$\gamma N \rightarrow \Delta: G_M^*(Q^2)$ (valence)

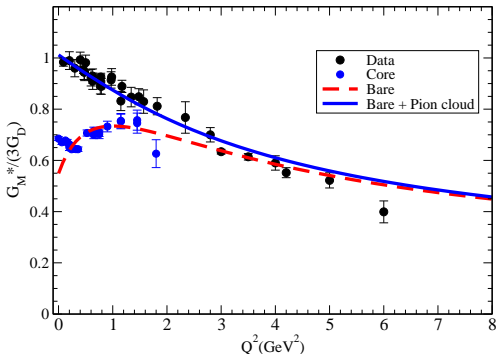
GR and MT Peña PRD 80, 013008 (2009) $G_D = 1/(1 + Q^2/0.71)^2$



• Bare \approx EBAC model

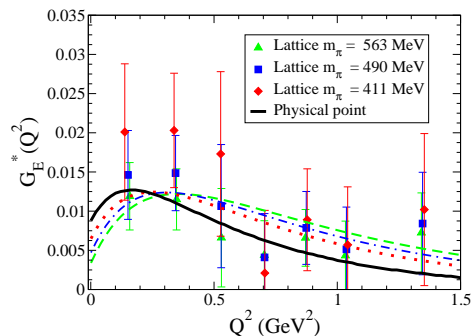
$\gamma N \rightarrow \Delta: G_M^*(Q^2)$ (valence + pion cloud [phenomenological])

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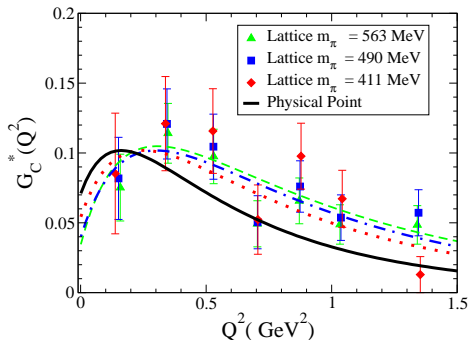


- Bare \approx EBAC model $\oplus G_M^\pi = \lambda_\pi \left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2} \right)^2 (3G_D)$ $\frac{G_M^B(0)}{3G_D} \leq 0.7$

Fit to lattice QCD data (bare contribution)
 Alexandrou et al, PRD, 77, 085012 (2008)

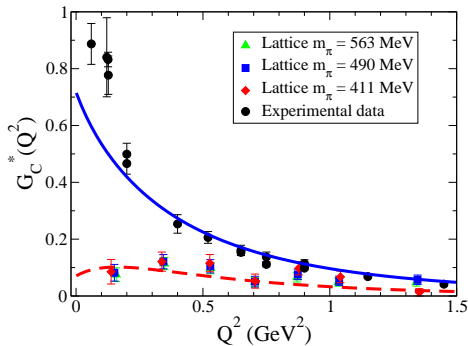
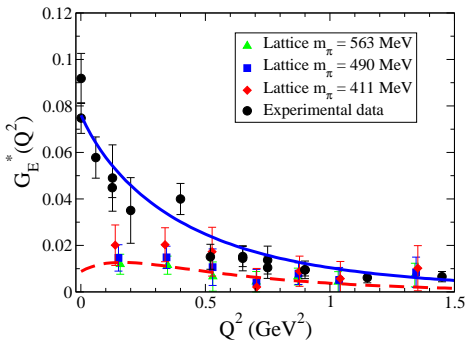


D3 state: 0.72%



D1 state: 0.72%

$\gamma N \rightarrow \Delta: G_E^*(Q^2), G_C^*(Q^2)$ (bare + pion cloud) ††



--- Lattice extrapolation \rightarrow Physical regime

\oplus Pion cloud [Large N_c ; no additional parameters]

AJ Buchmann et al, PRD 66, 056002 (2002); V Pascalutsa and M Vanderhaeghen, PRD 76, 111501 (2007)

Pion cloud dominant; Good global description ($Q^2 < 1.5 \text{ GeV}^2$)

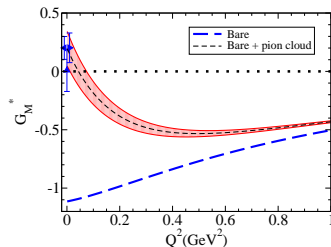
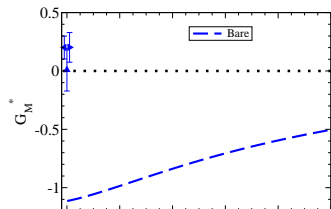
$\Delta(1600)$ as the 1st radial excitation
of $\Delta(1232)$ EPJA, 36, 329 (2008) [S-state]

$$G_E^* \equiv 0, G_C^* \equiv 0$$

$$\text{Bare : } G_M^B(0) = -1.113$$

$SU(3)$ symmetry \Rightarrow π cloud effects

Decay	BR
$\Delta(1600) \rightarrow \pi N$	0.153 ± 0.019
$\Delta(1600) \rightarrow \pi \Delta$	0.590 ± 0.100
$\Delta(1600) \rightarrow \pi N(1440)$	0.130 ± 0.040



- More general case (no pointlike diquark limit) $J^P = \frac{3}{2}^-$
- Orthogonality assured by radial wave function ψ_R

$$\int_k \frac{k_z}{|\mathbf{k}|} \psi_R(P_R, k) \psi_N(P_N, k) \Big|_{Q^2=0} = 0 \quad (R \text{ rest frame})$$

One parameter to fit to high Q^2 data: β_4 (small mc effects)

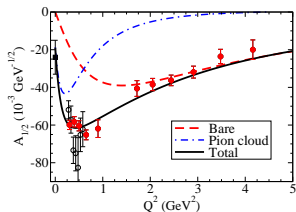
$$\psi_R \approx \frac{1}{m_D(\beta_2 + \chi)} \left\{ \frac{1}{\beta_1 + \chi} - \frac{\lambda_R}{\beta_4 + \chi} \right\}$$

- Valence quarks **are not** sufficient to explain the data ($A_{3/2} \neq 0$): exclusive of CSQM
- \Rightarrow Include phenomenological parametrizations of the meson cloud at low Q^2

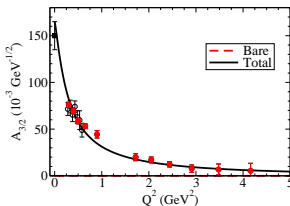


$\gamma^* N \rightarrow N(1520)$ form factors [arXiv:1309.0730]

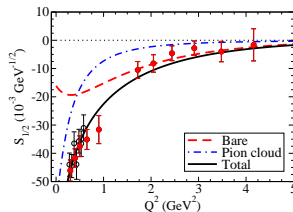
$A_{1/2}$



$A_{3/2}$



$S_{1/2}$



----- Bare; ——— Bare plus meson cloud

$A_{1/2}, S_{1/2}$: Valence quarks \Rightarrow good description for $Q^2 > 1.5 \text{ GeV}^2$

$A_{3/2} \Leftarrow$ Meson cloud

Conclusions

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Valence quark degrees of freedom under control

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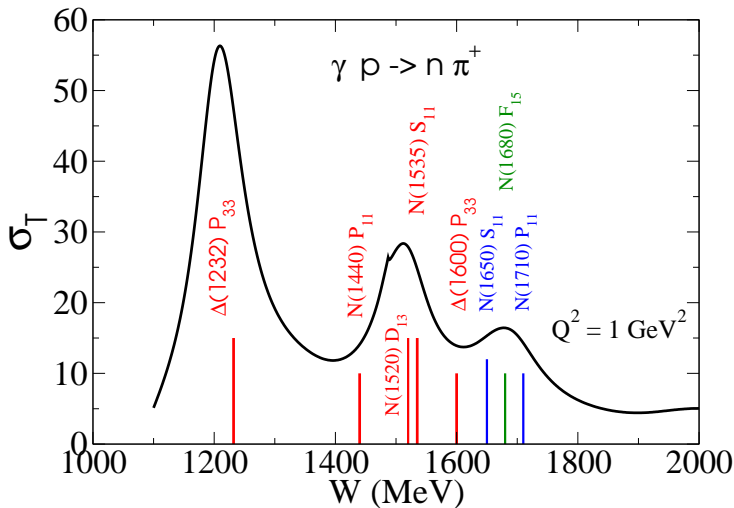
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- **Model applied** to several electromagnetic reactions
(including baryons with **strange quarks**): octet, decuplet (Δ , Ω^-),
octet-decuplet, $\gamma^* \Lambda \rightarrow \Lambda(1670)$, Δ Dalitz decay ($\Delta \rightarrow e^+e^- N$), DIS

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Pion cloud **very** important
Lattice data [$\gamma^* N \rightarrow \Delta(1232)$ and also $\Delta\Delta$]
Dominance of valence quark for $Q^2 > 2 \text{ GeV}^2$
- Perspective of extension to other resonances
- **Model applied** to several electromagnetic reactions (including baryons with **strange quarks**): octet, decuplet (Δ , Ω^-), octet-decuplet, $\gamma^* \Lambda \rightarrow \Lambda(1670)$, Δ Dalitz decay ($\Delta \rightarrow e^+ e^- N$), DIS

Thank you 

Nucleon Resonance Structure



Selected bibliography (part 1)

- **A pure S-wave covariant model for the nucleon**,
F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C **77**, 015202 (2008)
[arXiv:nucl-th/0606029].
- **Fixed-axis polarization states: covariance and comparisons**,
F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C **77**, 035203 (2008).
- **Covariant nucleon wave function with S, D, and P-state components**,
F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. D **85**, 093005 (2012)
[arXiv:1201.6336 [hep-ph]].
- **A covariant formalism for the N^* electroproduction at high momentum transfer**, **Review**
G. Ramalho, F. Gross, M. T. Peña and K. Tsushima,
Exclusive Reactions and High Momentum Transfer IV, 287 (2011)
[arXiv:1008.0371 [hep-ph]].
- **Studies of Nucleon Resonance Structure in Exclusive Meson Electroproduction**, **Review (pages 87-92)**
I. G. Aznauryan et al, Int. J. Mod. Phys. E **22**, 1330015 (2013)
[arXiv:1212.4891 [nucl-th]].

Selected bibliography (part 2)

- **A Covariant model for the nucleon and the Δ ,**
G. Ramalho, M. T. Peña and F. Gross, Eur. Phys. J. A **36**, 329 (2008)
[arXiv:0803.3034 [hep-ph]].
- **D-state effects in the electromagnetic $N\Delta$ transition,**
G. Ramalho, M. T. Peña and F. Gross, Phys. Rev. D **78**, 114017 (2008)
[arXiv:0810.4126 [hep-ph]].
- **Valence quark contribution for the $\gamma N \rightarrow \Delta$ quadrupole transition extracted from lattice QCD,**
G. Ramalho and M. T. Peña, Phys. Rev. D **80**, 013008 (2009)
[arXiv:0901.4310 [hep-ph]].
- **Nucleon and $\gamma N \rightarrow \Delta$ lattice form factors in a constituent quark model,**
G. Ramalho and M. T. Peña, J. Phys. G **36**, 115011 (2009)
[arXiv:0812.0187 [hep-ph]].

Selected bibliography (part 3)

- **Valence quark contributions for the $\gamma N \rightarrow P_{11}(1440)$ form factors**,
G. Ramalho and K. Tsushima, Phys. Rev. D **81**, 074020 (2010)
[arXiv:1002.3386 [hep-ph]].
- **A model for the $\Delta(1600)$ resonance and $\gamma N \rightarrow \Delta(1600)$ transition**,
G. Ramalho and K. Tsushima, Phys. Rev. D **82**, 073007 (2010)
[arXiv:1008.3822 [hep-ph]].
- **A covariant model for the $\gamma N \rightarrow N(1535)$ transition at high momentum transfer**,
G. Ramalho and M. T. Peña, Phys. Rev. D **84**, 033007 (2011)
[arXiv:1105.2223 [hep-ph]].
- **A simple relation between the $\gamma N \rightarrow N(1535)$ helicity amplitudes**,
G. Ramalho and K. Tsushima, Phys. Rev. D **84**, 051301 (2011)
[arXiv:1105.2484 [hep-ph]].
- **Valence quark and meson cloud contributions to the $\gamma^* N \rightarrow N^*(1520)$ form factors**,
G. Ramalho and M. T. Pea, arXiv:1309.0730 [hep-ph].

- **Valence quark and meson cloud contributions for the $\gamma^*\Lambda \rightarrow \Lambda^*$ and $\gamma^*\Sigma^0 \rightarrow \Lambda^*$ reactions,**
G. Ramalho, D. Jido and K. Tsushima, Phys. Rev. D **85**, 093014 (2012) [arXiv:1202.2299 [hep-ph]].
- **Electromagnetic form factors of the Δ with D-waves,**
G. Ramalho, M. T. Peña and F. Gross, Phys. Rev. D **81**, 113011 (2010) [arXiv:1002.4170 [hep-ph]].
- **The shape of the Δ baryon in a covariant spectator quark model,**
G. Ramalho, M. T. Peña and A. Stadler, Phys. Rev. D **86**, 093022 (2012) [arXiv:1207.4392 [nucl-th]].
- **A Relativistic quark model for the Ω^- electromagnetic form factors,**
G. Ramalho, K. Tsushima and F. Gross, Phys. Rev. D **80**, 033004 (2009) [arXiv:0907.1060 [hep-ph]].

More information

Strange baryons: octet, decuplet (Δ, Ω) Δ Dalitz decay;
Nucleon DIS, ...

Octet to decuplet electromagnetic transition $\gamma^* B \rightarrow B'$

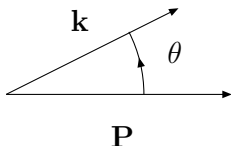
	$G_M^B(0)$	$G_M^{MC}(0)$	$G_M(0)$	$ G_M(0) _{exp}$
$\gamma^* N \rightarrow \Delta$	1.63	1.38	3.02	3.04 ± 0.11
$\gamma^* \Lambda \rightarrow \Sigma^{*0}$	1.68	1.40	3.08	3.26 ± 0.37
				3.35 ± 0.57
$\gamma^* \Sigma^+ \rightarrow \Sigma^{*+}$	2.09	1.13	3.22	4.10 ± 0.57
$\gamma^* \Sigma^0 \rightarrow \Sigma^{*0}$	0.97	0.49	1.46	
$\gamma^* \Sigma^- \rightarrow \Sigma^{*-}$	-0.16	-0.15	-0.31	< 0.8
$\gamma^* \Xi^0 \rightarrow \Xi^{*0}$	2.19	1.10	3.29	
$\gamma^* \Xi^- \rightarrow \Xi^{*-}$	-0.19	-0.21	-0.38	

GR and K Tsushima PRD 87, 093011 (2013) [$G_M^B(0)$];

GR and K Tsushima PRD 88, 053002 (2013) [$G_M^{MC}(0)$]

Diquark polarization states

- **Helicity states** defined in terms of $k = (E_k, k \sin \theta, 0, k \cos \theta)$



$\varepsilon_k(\lambda)$ dependent of θ

- **Fixed-axis:** vector particle is bound to a system with $P = (P, 0, 0, P)$

$$\varepsilon_P(0) = \begin{bmatrix} P \\ 0 \\ 0 \\ P_0 \end{bmatrix}, \quad \varepsilon_P(\pm) = \mp \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ \pm 1 \\ 0 \end{bmatrix}$$

\Rightarrow wave functions with **No** angular dependence;
F. Gross, GR and M. T. Peña, PRC 77, 035203 (2008)

Electromagnetic current: an example

Nucleon wave function:

$$\Psi_N(P_-, k) = \frac{1}{\sqrt{2}} \left[\Phi_I^0 u(P_-) - (\varepsilon_{P_-}^*)_\beta U^\beta(P_-) \Phi_I^1 \right] \psi_N(P_-, k)$$

Δ wave function:

$$\Psi_\Delta(P_+, k) = -(\varepsilon_{P_+}^*)_\alpha u^\alpha(P_+) \tilde{\Phi}_I^1 \psi_\Delta(P_+, k)$$

Transition current:

$$J^\mu \propto [\bar{u}_\alpha(P_+) j_q^\mu U_\beta(P_-)] \Delta^{\alpha\beta} \int_k \psi_\Delta(P_+, k) \psi_N(P_-, k)$$

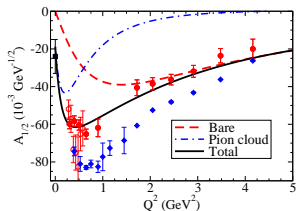
$$\Delta^{\alpha\beta} = \sum_{\lambda_D} \varepsilon_{P_+}^\alpha(\lambda_D) \varepsilon_{P_-}^{\beta*}(\lambda_D)$$

$$= - \left(g^{\alpha\beta} - \frac{P_-^\alpha P_+^\beta}{P_+ \cdot P_-} \right) - a \left(P_- - \frac{P_+ \cdot P_-}{M_\Delta^2} P_+ \right)^\alpha \left(P_+ - \frac{P_+ \cdot P_-}{M^2} P_- \right)^\beta$$

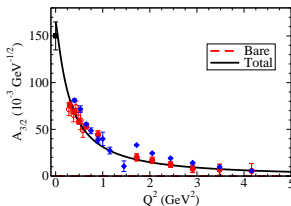
$$a = \frac{M_\Delta M}{P_+ \cdot P_- (M_\Delta M + P_+ \cdot P_-)}$$

$\gamma^* N \rightarrow N(1520)$ amplitudes – Results (II) MAID

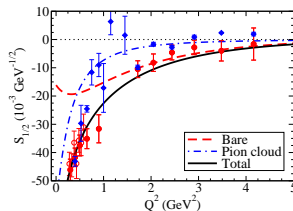
$A_{1/2}$



$A_{3/2}$



$S_{1/2}$



--- Bare; — Bare plus meson cloud

$A_{1/2}, S_{1/2}, A_{3/2}$: good description

Discrepance between CLAS and MAID analysis

Baryon wave function -example: Nucleon isospin

S-state in quark-diquark [PRC 77,015202 (2008)]:

$$\Psi_N(P, k) = \frac{1}{\sqrt{2}} [\Phi_I^0 \Phi_S^0 + \Phi_I^1 \Phi_S^1] \psi_N(P, k)$$

Example $|p \uparrow\rangle$:

$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \chi_I = p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Phi_I^0 = \frac{1}{\sqrt{2}} [ud - du] u$$

$$\Phi_I^1 = \frac{1}{\sqrt{6}} [2uud - (ud + du)u]$$

$$\xi^s = \frac{1}{\sqrt{2}} [ud - du]$$

$$\xi^m = \begin{cases} uu & m = +1 \\ \frac{1}{\sqrt{2}} [ud + du] & m = 0 \\ dd & m = -1 \end{cases}$$

$$\Phi_I^0 = \xi^s u \equiv \xi^s \chi_I$$

$$\Phi_I^1 = -\frac{1}{\sqrt{3}} (\tau \cdot \xi) u \equiv -\frac{1}{\sqrt{3}} (\tau \cdot \xi) \chi_I$$

Baryon wave function -example: Nucleon spin (I)

Example $|p \uparrow\rangle$: $\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\chi_S = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Spin-0: $\Phi_S^0 = \overbrace{\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)}^{\varepsilon^s} \uparrow = \varepsilon^s \chi_S$
Relativistic generalization $\rightarrow \varepsilon^s u(P, \uparrow)$

Spin-1: $\Phi_S^1 = \frac{1}{\sqrt{6}} [2 \uparrow\uparrow\downarrow - (\downarrow\uparrow + \uparrow\downarrow) \uparrow] = -\frac{1}{\sqrt{3}} (\sigma \cdot \varepsilon_P^*) \chi_S$
Relativistic generalization $\rightarrow -(\varepsilon_P^*)_\alpha U^\alpha(P, \uparrow)$

ε_P^* in rest frame: $\varepsilon_P^\alpha(0) = (0, 0, 0, 1)$ $\varepsilon_P^\alpha(\pm) = \mp \frac{1}{\sqrt{2}} (0, 1, \pm i, 0)$

\Rightarrow Fixed-axis polarization base

$\Phi_S^{0,1}$ in terms of baryon properties

Baryon wave function -example: Nucleon spin (II)

Covariant form to $U^\alpha(P, s)$

$$U^\alpha(P, s) = \frac{1}{\sqrt{3}} \gamma_5 \left(\gamma^\alpha - \frac{P^\alpha}{M_N} \right) u(P, s) \xrightarrow{NR} -\frac{1}{\sqrt{3}} (\sigma \cdot \varepsilon_P^*) \chi_s$$

Fixed-axis polarization base [F Gross, GR and MT Peña, PRC 77, 015202 (2008)]

$$\varepsilon_P^\alpha(0) = \frac{1}{M_N} \left(P, 0, 0, \sqrt{M_N^2 + P^2} \right) \quad \varepsilon_P^\alpha(\pm) = \mp \frac{1}{\sqrt{2}} (0, 1, \pm i, 0)$$

ε_P^α = function of baryon momentum

Pure angular momentum states; Covariant representation

S-state:

$$\Psi_S(P, k; s) = -\tilde{\Phi}_I^1 \psi_S(P, k) [(\varepsilon_P^*)^\alpha u_\alpha(P, s)]$$

D-state: $S = 1/2, 3/2$

$$\Psi_D(P, k; s) = -\tilde{\Phi}_I^1 \psi_D(P, k) \left[3(\varepsilon_P^*)_\alpha (\mathcal{P}_S)^{\alpha\beta} W_\beta(P, k, s) \right]$$

$$W^\beta(P, k, s) = \mathcal{D}^{\beta\sigma}(P, k) u_\sigma(P, s), \quad \mathcal{D} = \text{D-state operator}$$

$N^*(1520)$ wave function

$$\Psi_R = \cos \theta_D \Psi_{P1} - \sin \theta_D \Psi_{P3}, \quad \Psi_P = N_P [\phi_I^0 X_\rho + \phi_I^1 X_\lambda] \tilde{\psi}_P(r, k)$$

- Nonrelativistic form (CM): $k_\rho \rightarrow r = \frac{1}{2}(k_1 - k_2)$, $k_\lambda \rightarrow k = k_1 + k_2$
S Capstick and W Roberts, Prog. Part. Nucl. Phys. **45** S241 (2000)

Case $S = 1/2$

$$\begin{aligned} X_\rho(s) &= \sum_{m s'} \left\langle 1\frac{1}{2}; m s' \middle| \frac{3}{2} s \right\rangle \left[Y_{1m}(r) |s'\rangle_\lambda + Y_{1m}(k) |s'\rangle_\rho \right] \\ X_\lambda(s) &= \sum_{m s'} \left\langle 1\frac{1}{2}; m s' \middle| \frac{3}{2} s \right\rangle \left[Y_{1m}(r) |s'\rangle_\rho + Y_{1m}(k) |s'\rangle_\lambda \right] \end{aligned}$$

Case $S = 3/2$

$$\begin{aligned} X_\rho(s) &= \sum_{m s'} \left\langle 1\frac{1}{2}; m s' \middle| \frac{3}{2} s \right\rangle Y_{1m}(r) \chi_{s'}^S \\ X_\lambda(s) &= \sum_{m s'} \left\langle 1\frac{1}{2}; m s' \middle| \frac{3}{2} s \right\rangle Y_{1m}(k) \chi_{s'}^S \end{aligned}$$

- Relativistic generalization: $U_R^\alpha(P, s) = \frac{1}{\sqrt{3}} \gamma_5 \left(\gamma^\alpha - \frac{P^\alpha}{M_R} \right) u_R(P, s)$

$$k \rightarrow \tilde{k} = k - \frac{P \cdot k}{M_R^2} P \quad |s\rangle_\rho \rightarrow u_R(P, s)$$

$$Y_{1m}(k) \rightarrow -\frac{1}{\sqrt{-\tilde{k}^2}} (\varepsilon_{mP} \cdot \tilde{k}) \quad |s\rangle_\lambda \rightarrow -(\varepsilon_{\Lambda P}^*)_\alpha U_R^\alpha(P, s)$$

$$Y_{1m}(r) \rightarrow \zeta_m^\nu \quad \chi_s^S \rightarrow -(\varepsilon_{\Lambda P}^*)^\beta u_\beta(P, s)$$

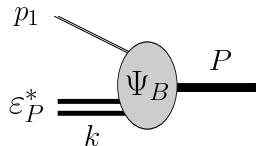
GR and MT Peña PRD **84**, 033007 (2011); arXiv:1309.0730

- \Rightarrow Reduction to quark-diquark system $\tilde{\psi}_P(r, k) \rightarrow \psi_P(P, k)$

Spectator quark model - Wave functions

- Wave functions: $B = \text{quark} \oplus \text{diquark}$

$$\Psi_B = \sum (\text{flavor}) \otimes (\text{spin}) \otimes (\text{orbital}) \otimes \overbrace{\psi_B(P, k)}^{\text{radial}}$$



Nucleon wave function: [PRC 77,015202 (2008)]

Simplest structure – **S-state** in quark-diquark system

$$\Psi_N(P, k) = \frac{1}{\sqrt{2}} [\Phi_I^0 \Phi_S^0 + \Phi_I^1 \Phi_S^1] \psi_N(P, k)$$

S11 wave function:

$$\Psi_{S11}(P, k) = \frac{1}{\sqrt{2}} \gamma_5 [\Phi_I^0 X_\rho - \Phi_I^1 X_\lambda] \psi_{S11}(P, k)$$

Ψ_N, Ψ_{S11} **covariant**; ψ_N, ψ_{S11} scalar wave function

$\Phi_I^{0,1}$ isospin; $\Phi_S^{0,1}, X_{\rho,\lambda}$ spin – combination of **quark states**
 $\Rightarrow \Psi_B$ **written** in terms of **baryon properties**

Spectator quark model – S11 wave function

$SU(6) \otimes O(3)$ QM:

$$|N^*(1535)\rangle = \cos\theta \underbrace{|N^2 P_{1/2}\rangle}_{S=1/2} - \sin\theta \underbrace{|N^4 P_{1/2}\rangle}_{S=3/2}$$

Approximations:

- No mixture between states (pure $S = 1/2$ state)
- Pointlike diquark

$$k_\rho = \frac{1}{\sqrt{2}}(k_1 - k_2) \rightarrow 0$$

No diquark internal P-states

Spectator quark model – S11 wave function

Symmetry in the exchange of quarks 1 and 2 $\left\{ \begin{array}{l} \rho = \text{anti-symmetric} \\ \lambda = \text{symmetric} \end{array} \right.$

Momentum: $k_\rho = \frac{1}{\sqrt{2}}(k_1 - k_2)$ $k_\lambda = \frac{1}{\sqrt{6}}(k_1 + k_2 - 2k_3)$

Spin states:

$$|\frac{1}{2}, +\rangle_\rho = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow \quad |\frac{1}{2}, +\rangle_\lambda = \frac{1}{\sqrt{6}}[2\uparrow\uparrow\downarrow - (\uparrow\downarrow + \downarrow\uparrow)\uparrow]$$

Using $SU(6)\otimes O(3)$ quark model $[1 \oplus \frac{1}{2}]$:

$$X_\rho(+)=\sum_m\langle 1 m; \frac{1}{2}, +\frac{1}{2} | \frac{1}{2}, +\frac{1}{2} \rangle Y_{1m}(\hat{k}_\lambda) | \frac{1}{2}, \frac{1}{2} - m \rangle_\rho + \sum_m (\dots) Y_{1m}(\hat{k}_\rho)$$

$$X_\lambda(+)=\sum_m\langle 1 m; \frac{1}{2}, +\frac{1}{2} | \frac{1}{2}, +\frac{1}{2} \rangle Y_{1m}(\hat{k}_\lambda) | \frac{1}{2}, \frac{1}{2} - m \rangle_\lambda + \sum_m (\dots) Y_{1m}(\hat{k}_\rho)$$

Pointlike diquark approximation $k_\rho \rightarrow 0 \Rightarrow Y_{1m}(\hat{k}_\rho) \equiv 0$

Spectator quark model – S11 wave function

Symmetry in the exchange of quarks 1 and 2 $\left\{ \begin{array}{l} \rho = \text{anti-symmetric} \\ \lambda = \text{symmetric} \end{array} \right.$

Momentum: $k_\rho = \frac{1}{\sqrt{2}}(k_1 - k_2)$ $k_\lambda = \frac{1}{\sqrt{6}}(k_1 + k_2 - 2k_3)$

Spin states:

$$|\frac{1}{2}, +\rangle_\rho = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow \quad |\frac{1}{2}, +\rangle_\lambda = \frac{1}{\sqrt{6}}[2\uparrow\uparrow\downarrow - (\uparrow\downarrow + \downarrow\uparrow)\uparrow]$$

Using $SU(6) \otimes O(3)$ quark model $[1 \oplus \frac{1}{2}]$:

$$X_\rho(+)=\sum_m\langle 1 m; \frac{1}{2}, +\frac{1}{2} | \frac{1}{2}, +\frac{1}{2} \rangle Y_{1m}(\hat{k}_\lambda) | \frac{1}{2}, \frac{1}{2} - m \rangle_\rho + \sum_m (\dots) Y_{1m}(\hat{k}_\rho)$$

$$X_\lambda(+)=\sum_m\langle 1 m; \frac{1}{2}, +\frac{1}{2} | \frac{1}{2}, +\frac{1}{2} \rangle Y_{1m}(\hat{k}_\lambda) | \frac{1}{2}, \frac{1}{2} - m \rangle_\lambda + \sum_m (\dots) Y_{1m}(\hat{k}_\rho)$$

Pointlike diquark approximation $k_\rho \rightarrow 0 \Rightarrow Y_{1m}(\hat{k}_\rho) \equiv 0$

Spectator quark model – S11 wave function (II)

$k_{\lambda 0, \pm}$ spherical components of k_λ , $N = 1/\sqrt{\mathbf{k}^2}$

$$X_\rho(+)=+N\left\{k_{\lambda 0}|+\rangle_\rho-\sqrt{2}k_{\lambda+}|-\rangle_\rho\right\}$$

$$X_\lambda(+)=+N\left\{k_{\lambda 0}|+\rangle_\lambda-\sqrt{2}k_{\lambda+}|-\rangle_\lambda\right\}$$

Relativistic generalization: $\epsilon_\lambda^\alpha, \tilde{k}$

Diquark polarization vector: ϵ_λ^α ($\lambda = 0, \pm$) [Fixed-Axis base]

4-momentum $\tilde{k} = k - \frac{P \cdot k}{M_S^2} P$ [diquark 3-momentum in rest frame]

$$X_\rho(+)= -N\left[(\tilde{k} \cdot \epsilon_0)u_S(+)-\sqrt{2}(\tilde{k} \cdot \epsilon_+)u_S(-)\right]$$

$$X_\lambda(+)= +N\left[(\tilde{k} \cdot \epsilon_0)\epsilon_\alpha U_S^\alpha(+)-\sqrt{2}(\tilde{k} \cdot \epsilon_+)\epsilon_\alpha U_S^\alpha(-)\right]$$

$$N \rightarrow \frac{1}{\sqrt{-\tilde{k}^2}} \quad U_S^\alpha(P, \pm) = \frac{1}{\sqrt{3}} \gamma_5 \left(\gamma^\alpha - \frac{P^\alpha}{M_S} \right) u(P, \pm) \quad [1 \oplus \frac{1}{2} \rightarrow \frac{1}{2}]$$

Current conservation (1)

- Quark current in a Feynman diagram (one-body current)

$$j^\mu(q) = j_1 \gamma^\mu + j_2 \frac{i\sigma^{\mu\nu} q_\nu}{2M}$$

on-shell: current conserved $q_\mu [\bar{u}(p') j^\mu u(p)] = 0$.

When the Dirac particle is off-shell, it is necessary to modify this current in order to maintain current conservation.

- If a dynamics exists, we can do this in the manner of Gross and Riska, [PRC 36, 1928 \(1987\)](#); [Adam *et. al*, PRC66, 044003 \(2002\)](#)

$$\tilde{S}(p) = \frac{h(p)}{m - \not{p}}, \quad j_R^\mu(p', p) = j_R^\mu(j^\mu, h(p'), h(p); p', p)$$

j_R^μ off-shell current, satisfies Ward-Takahashi identity

$$q_\mu j_R^\mu = \tilde{S}^{-1}(p') - \tilde{S}^{-1}(p)$$

Current conservation (2)

- When no dynamics exists a purely phenomenological treatment is needed

$$\begin{aligned}j^\mu &\rightarrow j^\mu - (q_\nu j^\nu) \frac{q^\mu}{q^2} \\ &= j_1 \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + j_2 \frac{i\sigma^{\mu\nu} q_\nu}{2M}\end{aligned}$$

On-shell: \not{q} term vanishes; current reduced to the 1st case

- Calculations of $\gamma^* N \rightarrow N^*$: equivalent to **Landau prescription**
Kelly, PRC 56, 2672 (1997); Batiz and Gross, PRC 58, 2963 (1998)

$$J^\mu \rightarrow J^\mu - (q \cdot J) \frac{q^\mu}{q^2}$$

Restores current conservation but does not affect the observables

- **DIS calculations**: subtraction term $-\frac{\not{q} q^\mu}{q^2}$ arises naturally from interaction currents neglected in impulse approximation
Batiz and Gross, PRC 58, 2963 (1998)