

# Constraining Single-Energy PWAs Using Energy-Dependent PWAs

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- Analyses of  $\pi$ N Scattering
- Discussion of Fitting Errors
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7<sup>th</sup> International Wksp. On Pion-Nucleon  
Partial-Wave Analysis and the  
Interpretation of Baryon Resonances

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## Overview: Reactions Included in Recent PWAs

$$K^- p \rightarrow K^- p$$

$$K^- p \rightarrow \bar{K}^0 n$$

$$K^- p \rightarrow \pi^0 \Lambda$$

$$K^- p \rightarrow \pi^+ \Sigma^-$$

$$K^- p \rightarrow \pi^0 \Sigma^0$$

$$K^- p \rightarrow \pi^- \Sigma^+$$

$$\pi^- p \rightarrow \eta n$$

$$\pi^- p \rightarrow K^0 \Lambda$$

## Constraints on Single-Energy PWAs of KN Scattering

- Small amplitudes ( $|\mathbf{T}(E_0)| < 0.05$ ) held fixed using unitarized solution
- Selected data bins of 20 MeV width
- Introduce penalty term in  $\chi^2$  to constrain fitted amplitudes
- Parameterize each amplitude in bin by:

$$T(E) = T(E_0) + T'(E_0)(E - E_0)$$

where  $E$  is c. m. energy of data point in bin,

$E_0$  is central energy in bin,

$T(E_0)$  is the complex  $\mathbf{T}$ -matrix amplitude at c. m. energy  $E_0$ ,

$T'(E_0)$  is “slope parameter” which is fixed at value from unitarized solution.

## Multichannel Parametrization of Amplitudes

The “KSU Model” satisfies unitarity and time-reversal invariance. The total partial-wave S-matrix has the form

$$\mathbf{S} = \mathbf{B}^T \mathbf{R} \mathbf{B}$$

where the background matrix  $\mathbf{B}$  is unitary but not generally symmetric:

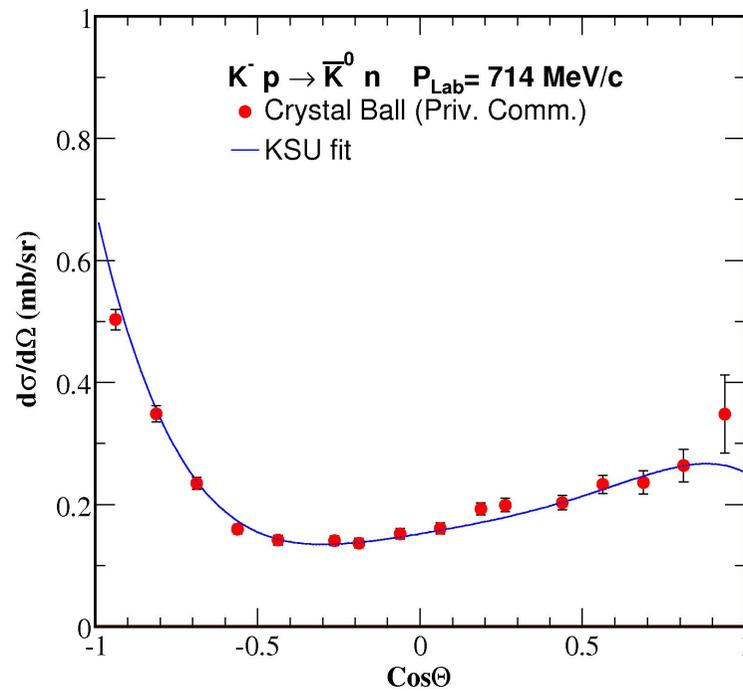
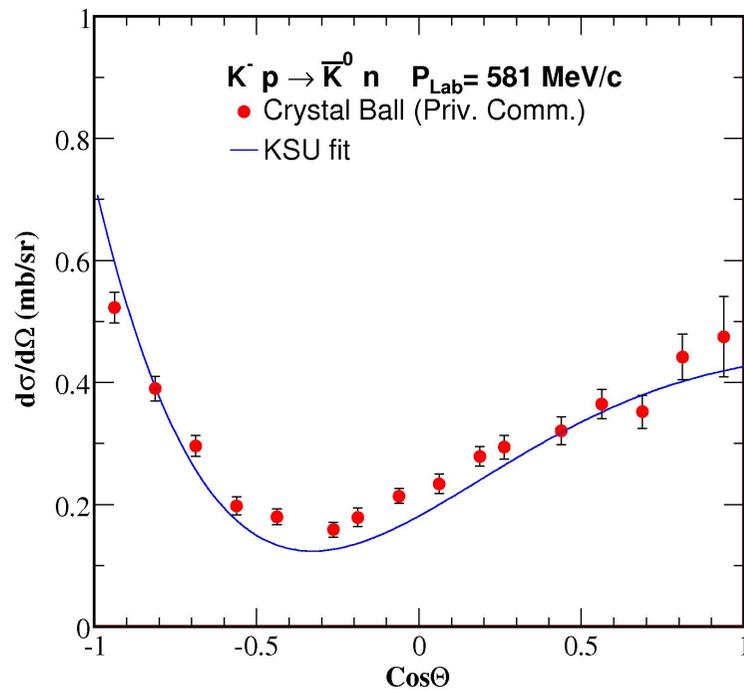
$$\mathbf{B} = \mathbf{B}_1 \mathbf{B}_2 \cdots \mathbf{B}_n$$

The matrix  $\mathbf{R}$  is both unitary and symmetric. It is a generalization of the multichannel BW form to include multiple resonances. It is constructed from a K-matrix:

$$\mathbf{R} = \mathbf{I} + 2i\mathbf{K}(1 - i\mathbf{K})^{-1}$$

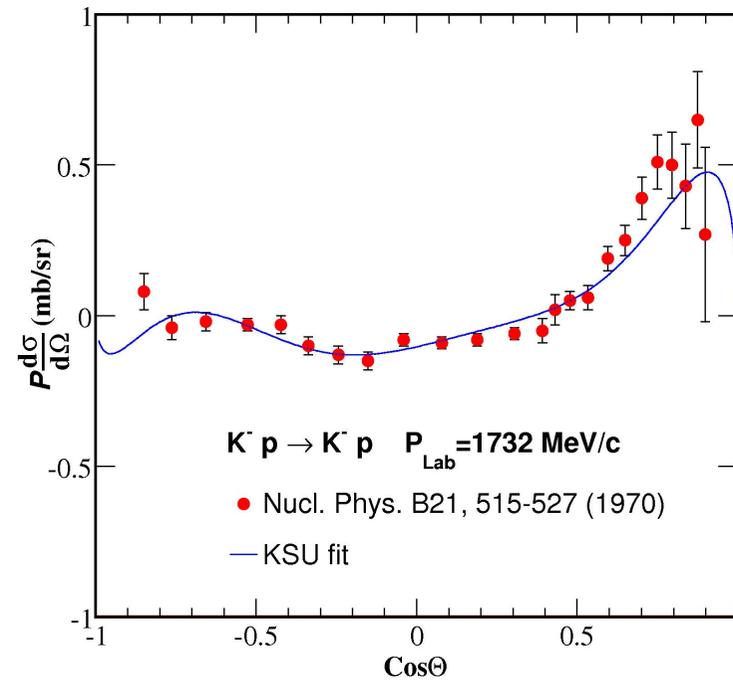
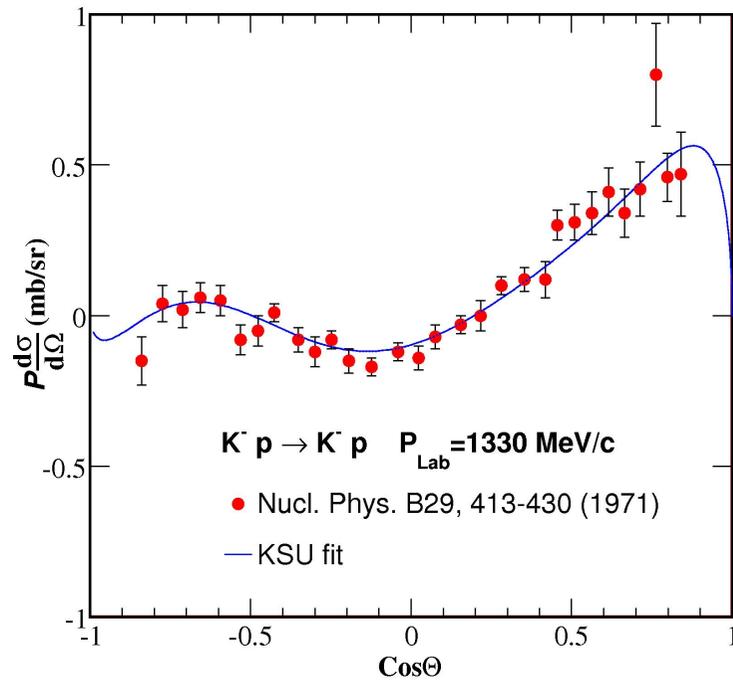
# Results of Energy-Dependent Fits

## Differential Cross Section



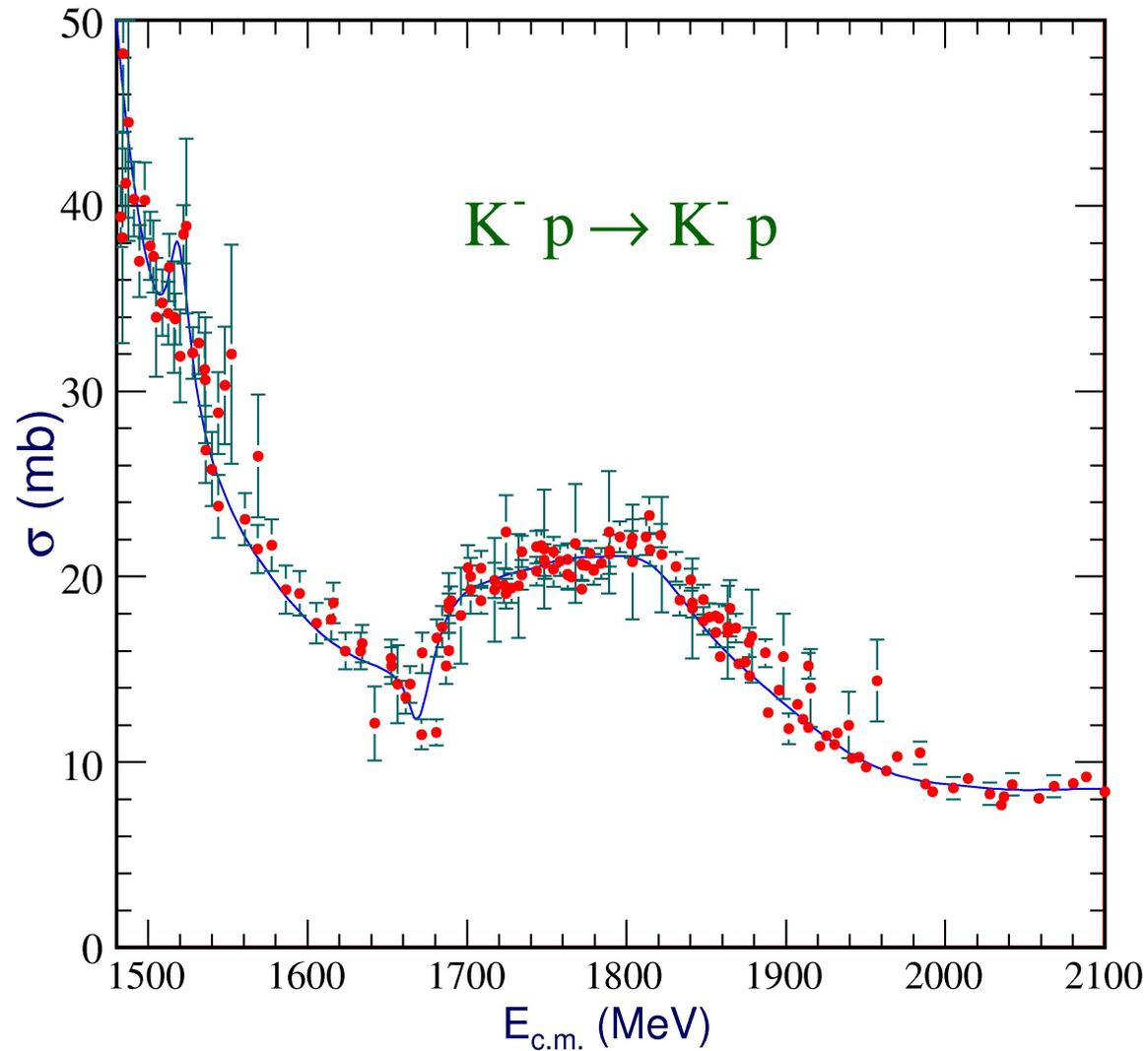
# Results of Energy-Dependent Fits

## Polarization



# Results of Energy-Dependent Fits

## Cross Section



## Analyses of $\pi N$ Scattering

- Results will now be presented for multichannel PWA of  $\pi N$  scattering in c.m. energy range 1.08-2.10 GeV.
- Includes new solutions for  $\pi N \rightarrow \eta N$  and  $\pi N \rightarrow K\Lambda$  amplitudes determined from single-energy analyses.
- Global energy-dependent fits resulted in smooth amplitudes consistent with S-matrix unitarity.
- Resonance parameters were obtained as results of global energy-dependent fits.

## Single-Energy Fits for $\pi N$ Reactions

- We began by assembling database of all available data for  $\pi^-p \rightarrow \eta n$  and  $\pi^-p \rightarrow K^0 \Lambda$ .
- Data were analyzed in bins of width 30 MeV.
- Data for  $\pi^-p \rightarrow \eta n$  spanned c.m. energies  $1530 \pm 15$  MeV to  $2070 \pm 15$  MeV.
- Data for  $\pi^-p \rightarrow K^0 \Lambda$  spanned c.m. energies  $1648 \pm 15$  MeV to  $2068 \pm 15$  MeV.
- In each energy bin, PW amplitudes were fitted as complex constants.

## Single-Energy Fits (Cont' d)

- We began by fitting the lowest energy bins, which were dominated by the  $S_{11}$  wave. As we moved to higher energy bins, additional partial waves were added as needed until we reached the highest energy bin.
- The initial single-energy amplitudes provided excellent fits to the data but were noisy. These were then included in global energy-dependent fits with amplitudes for  $\pi N \rightarrow \pi N$ ,  $\pi N \rightarrow \pi\pi N$ , and  $\gamma N \rightarrow \pi N$ .
- The global fits resulted in smooth amplitudes consistent with S-matrix unitarity, but the smoothed amplitudes no longer provided excellent agreement with  $d\sigma/d\Omega$  and polarization data.

## Single-Energy Fits (Cont' d)

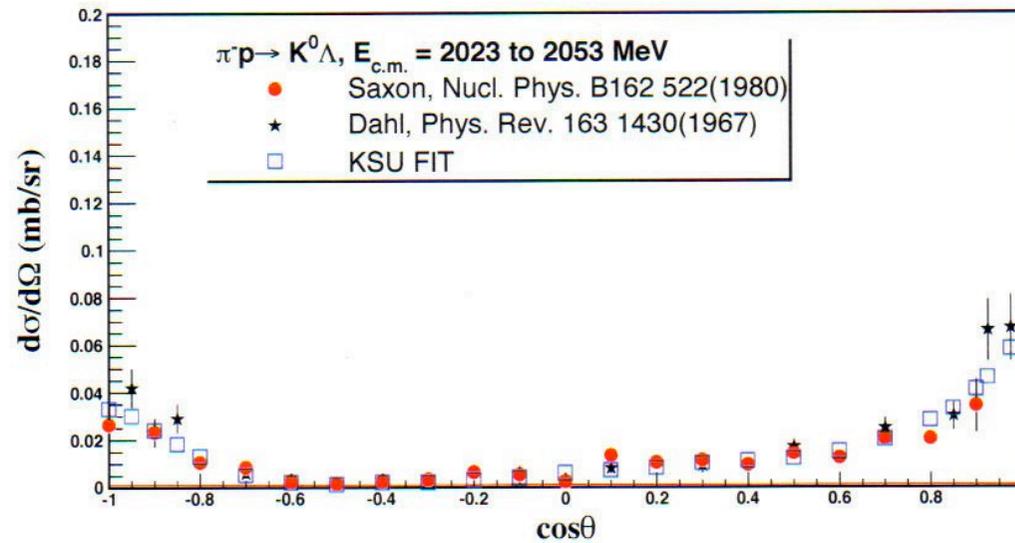
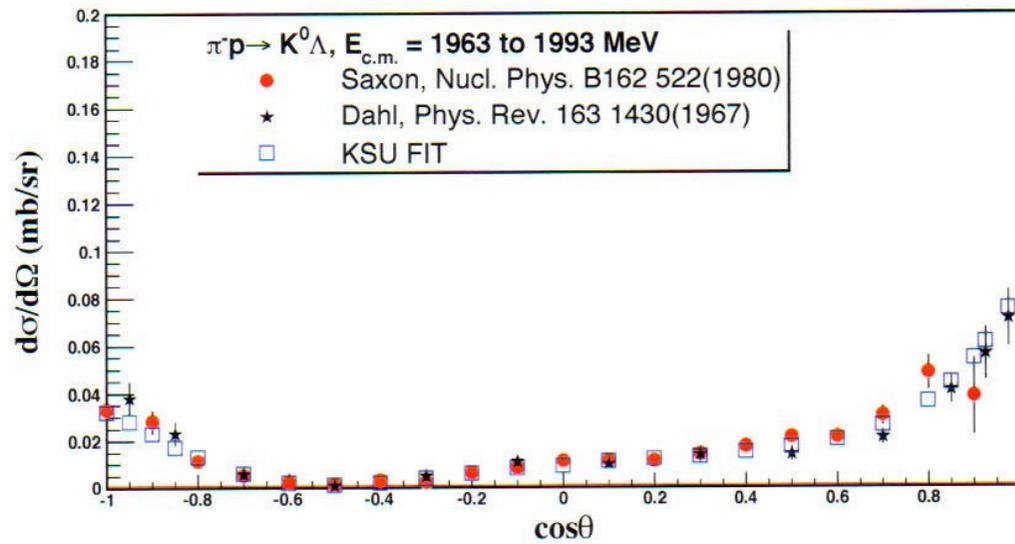
- We next iterated the single-energy fits. For the  $\eta N$  fits, small amplitudes (with  $|T| < 0.02$ ) were held fixed at values from the global fits. This constraint resulted in somewhat smoother energy-dependent amplitudes.
- The  $D_{13}$  amplitudes were found to be small over the entire energy range, as expected since  $\pi\pi N$  channels are known to saturate the unitary bound for  $D_{13}$ . The  $D_{13}$  amplitudes were omitted in subsequent fits.
- For  $\pi N \rightarrow \eta N$ , the important partial waves were found to be  $S_{11}$ ,  $P_{11}$ ,  $P_{13}$ ,  $D_{15}$ ,  $F_{15}$ , and  $G_{17}$ .
- For  $\pi N \rightarrow K\Lambda$ , the important partial waves were found to be  $S_{11}$ ,  $P_{11}$ ,  $P_{13}$ ,  $D_{15}$ , and  $F_{15}$ . Some small higher waves were also included.

## Single-Energy Fits (Cont' d)

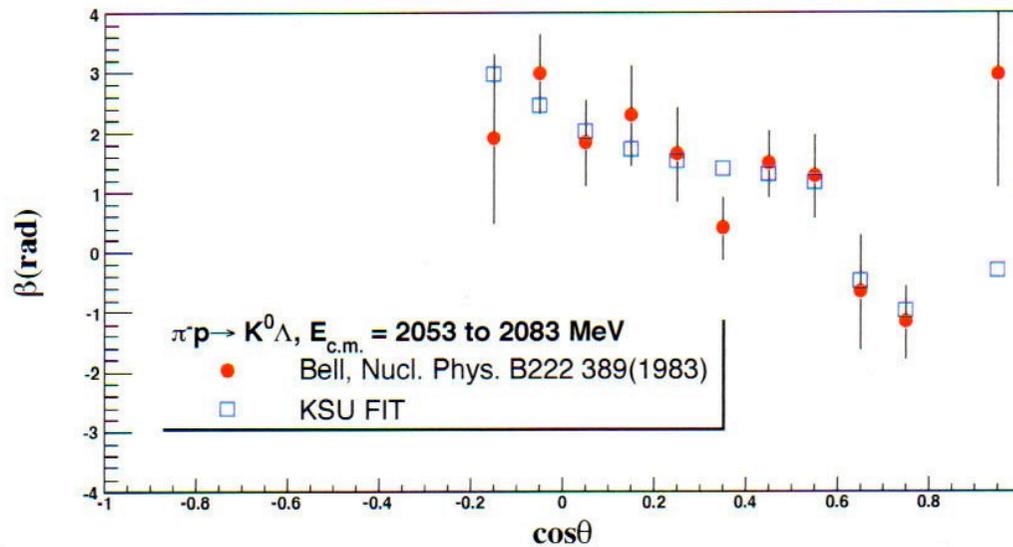
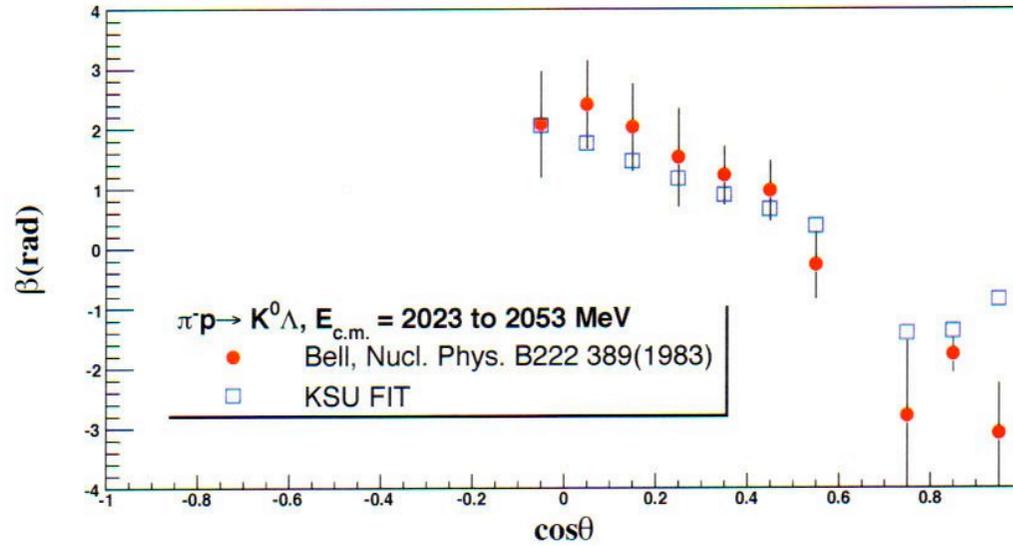
- We iterated between the single-energy fits and the global energy-dependent fits. In subsequent iterations, we added additional constraints to the single-energy fits by holding some of the larger amplitudes fixed at the values from the global energy-dependent fits.
- Initially only  $S_{11}$  amplitudes were held fixed, so the single-energy fits became more constrained and resulted in smoother values for the smaller amplitudes.
- In the final round,  $S_{11}$  and  $P_{11}$  amplitudes were held fixed while the other amplitudes were varied.

## Single-Energy Fits:

$d\sigma/d\Omega$  for  
 $\pi^-p \rightarrow K^0\Lambda$

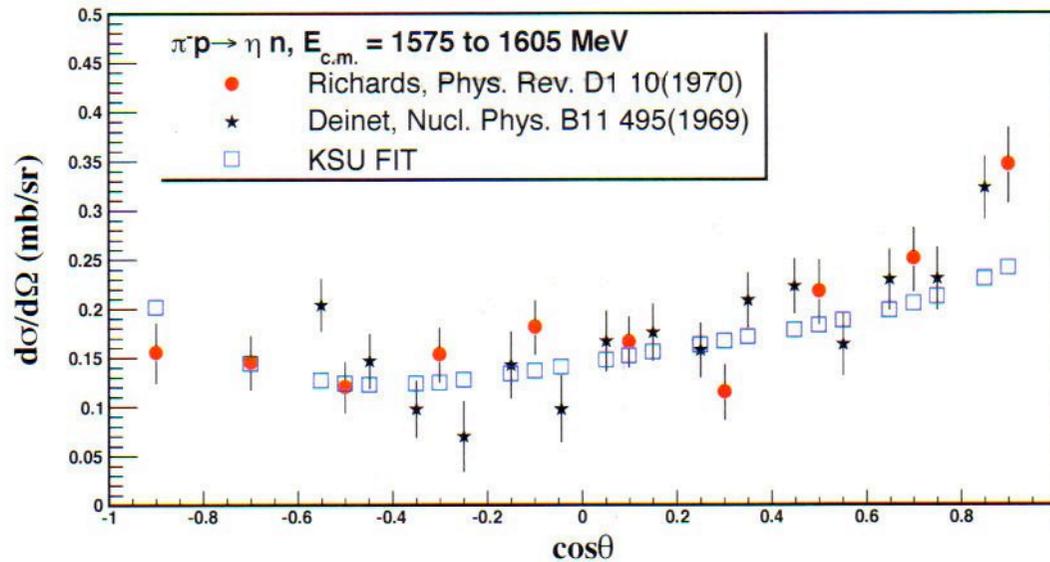
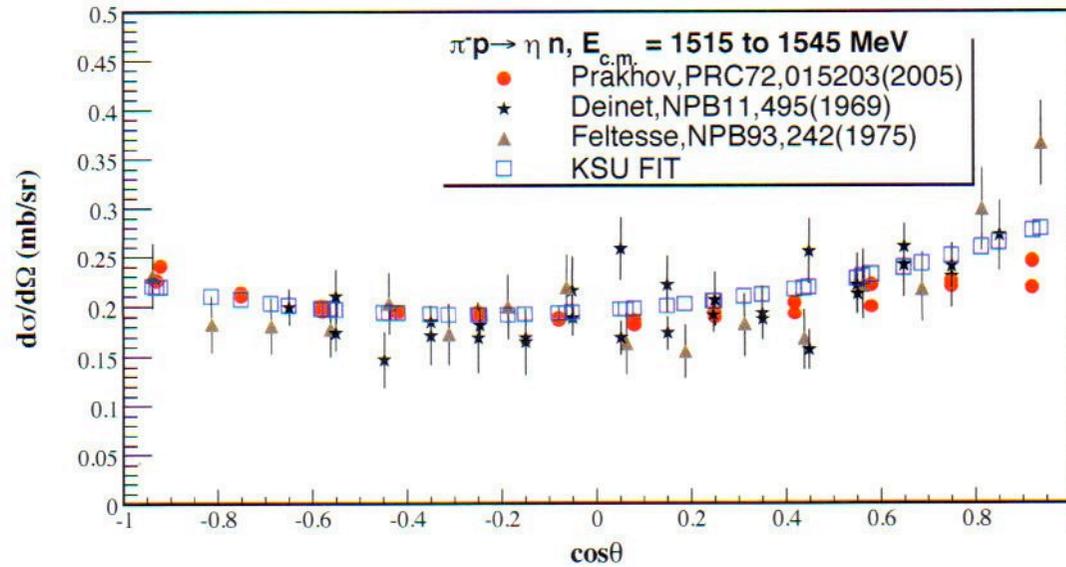


# Single-Energy Fits: Spin Rotation Parameter for $\pi^-p \rightarrow K^0\Lambda$



## Single-Energy Fits:

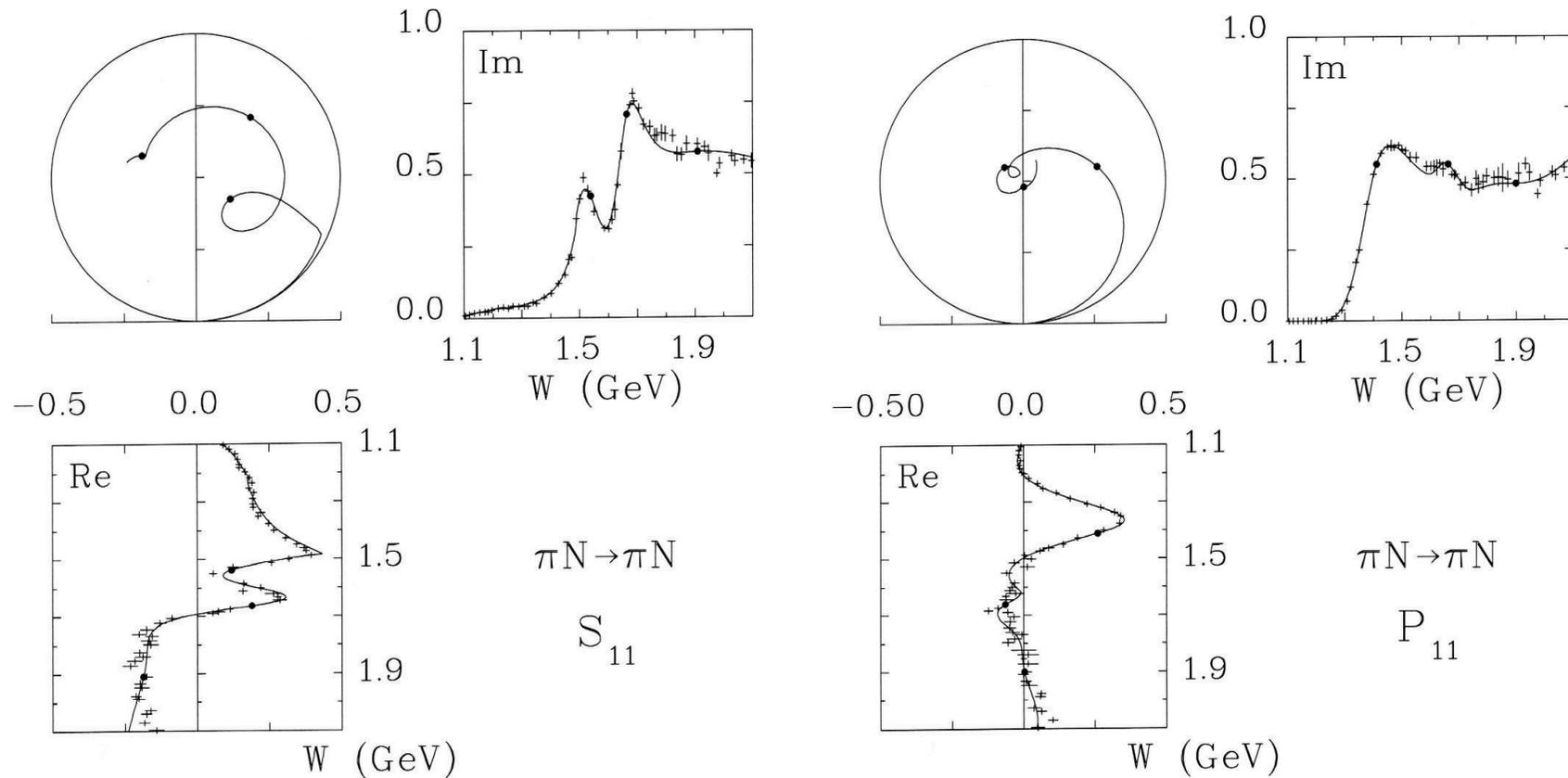
$d\sigma/d\Omega$  for  
 $\pi^-p \rightarrow \eta n$



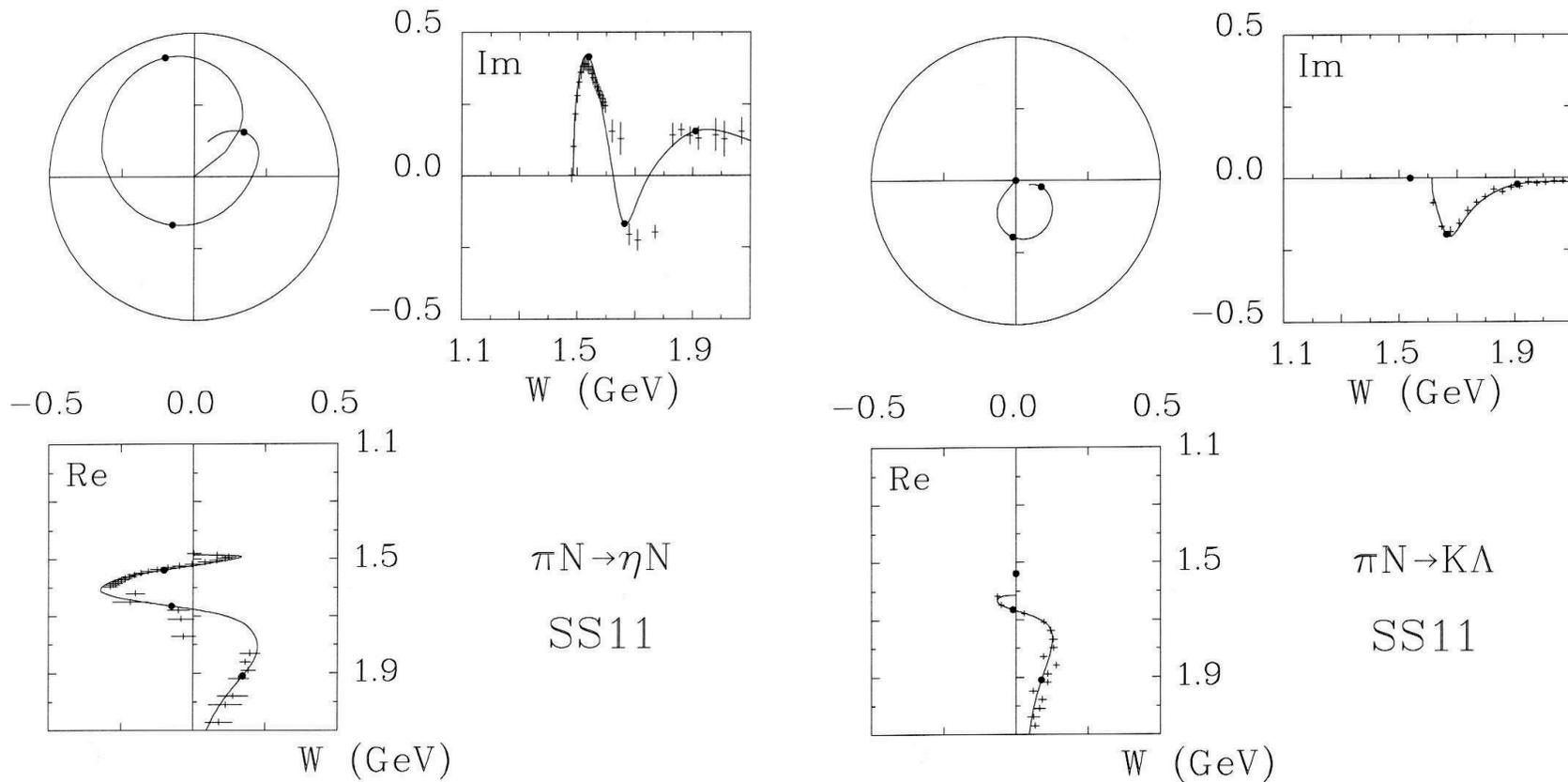
## Multichannel Energy-Dependent Fits

- $\pi N \rightarrow \pi N$  amplitudes (SAID SP06 sol' n)
- $\gamma N \rightarrow \pi N$  amplitudes (SAID FA07 sol' n)
- $\pi N \rightarrow \pi \pi N$  amplitudes (DMM *et al.*, PRD **30**, 904 (1984))
- $\pi N \rightarrow \eta N$  (this work)
- $\pi N \rightarrow K \Lambda$  (this work)
- “KSU Model” was used to provide a unitary, multichannel parametrization of partial-wave amplitudes

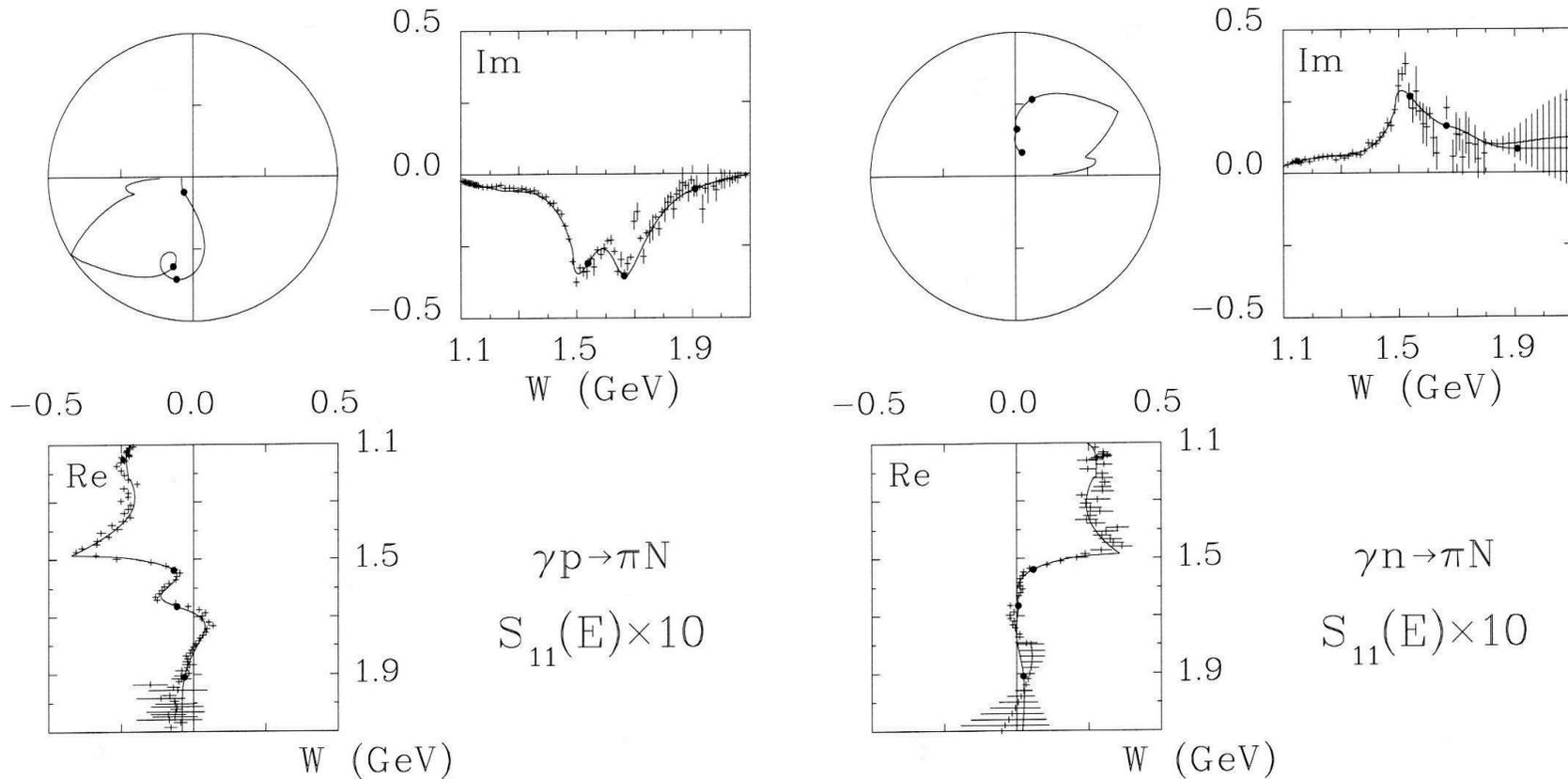
# Argand Diagrams: $S_{11}$ and $P_{11}$ amplitudes



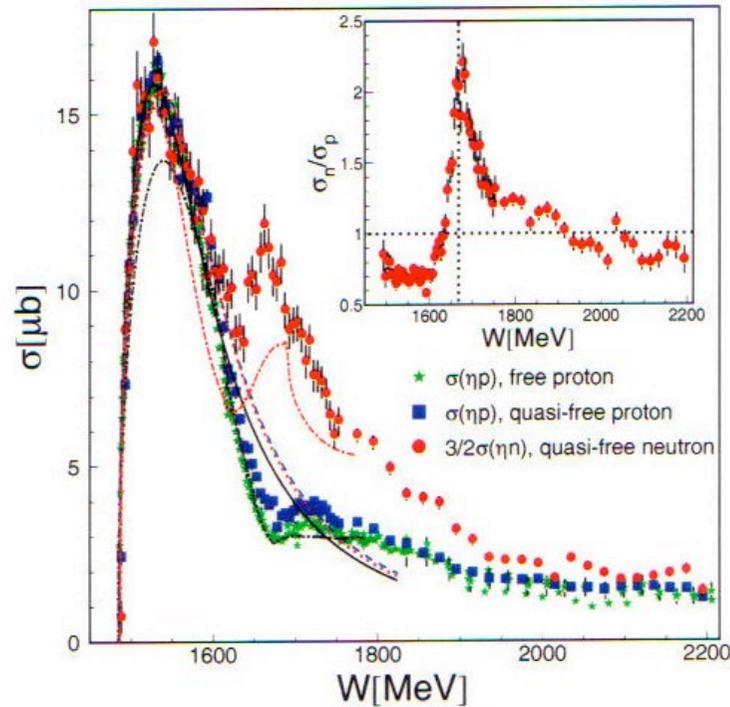
# Argand Diagrams: $S_{11}$ amplitudes



# Argand Diagrams: $S_{11}$ amplitudes



# Narrow peak in $\eta$ photoproduction



**Fig. 23.** Total cross sections as function of final state invariant mass  $W$  without cut on spectator momentum. (Red) dots: quasi-free neutron, (blue) squares: quasi-free proton, (green) stars: free proton data. Curves: fitted (up to  $W = 1600$  MeV)  $S_{11}(1535)$  line shapes. (Black) solid: free proton, (blue) dashed: quasi-free proton, (red) dotted: quasi-free neutron. Dash-dotted curves: model results from [51]. Insert: ratio of quasi-free neutron - proton data.

$D_{13}(1700)$

Mass = 1665(3) MeV

Width = 56(8) MeV

Pole Mass = 1662 MeV

Pole Width = 55 MeV

$$A_{1/2}(p\gamma) = +0.021(5) \text{ GeV}^{-1/2}$$

$$A_{1/2}(n\gamma) = -0.049(8) \text{ GeV}^{-1/2}$$

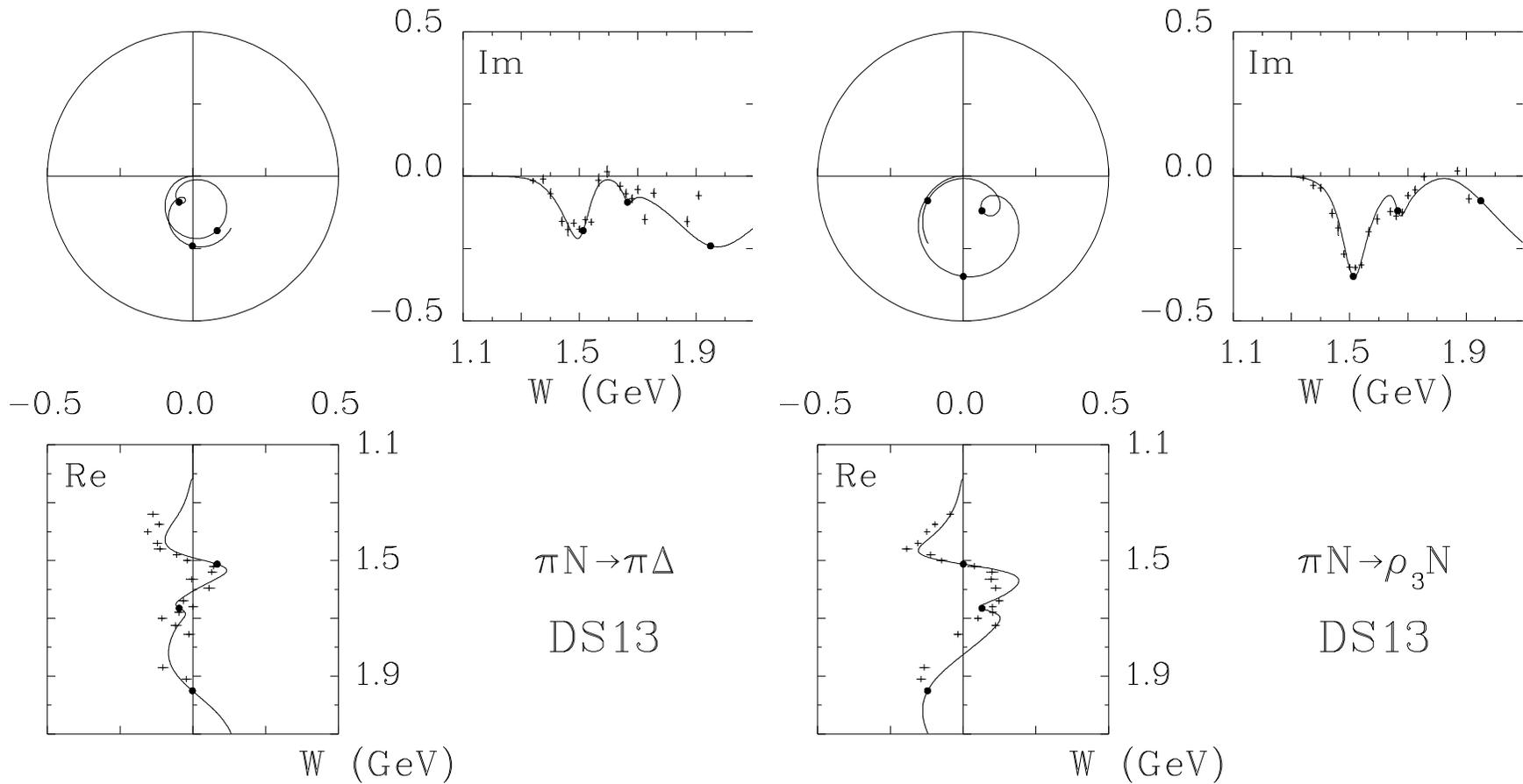
$$A_{3/2}(p\gamma) = +0.050(9) \text{ GeV}^{-1/2}$$

$$A_{3/2}(n\gamma) = -0.092(14) \text{ GeV}^{-1/2}$$

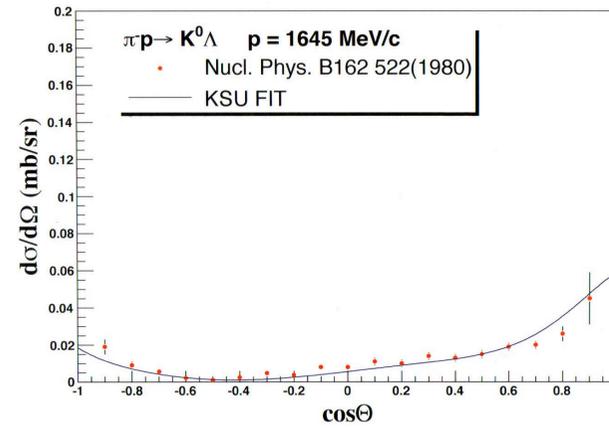
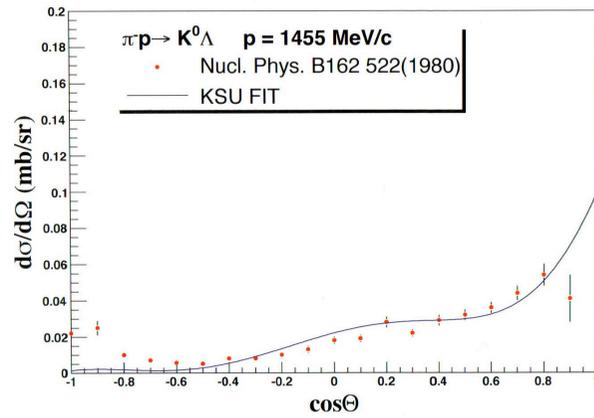
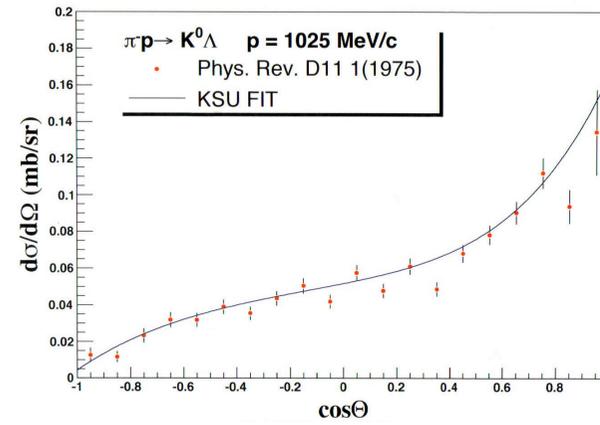
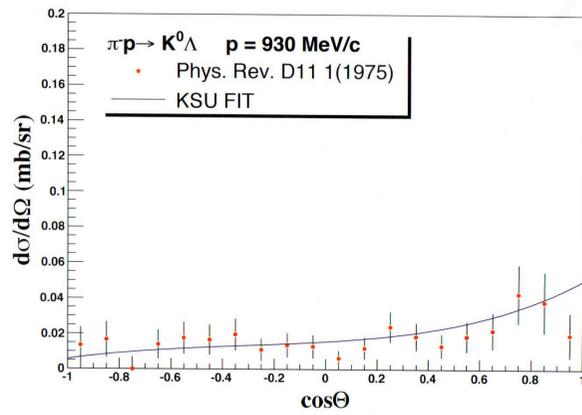
I. Jaegle *et al.*, *Eur. Phys. J. A* **47**, 89 (2011).

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*Phys. Rev. C* **86**, 055203 (2012).

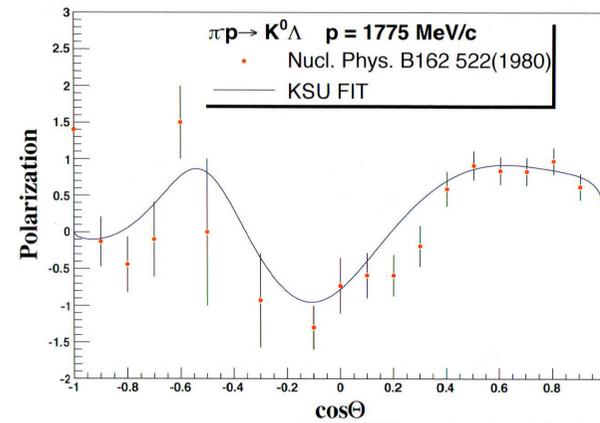
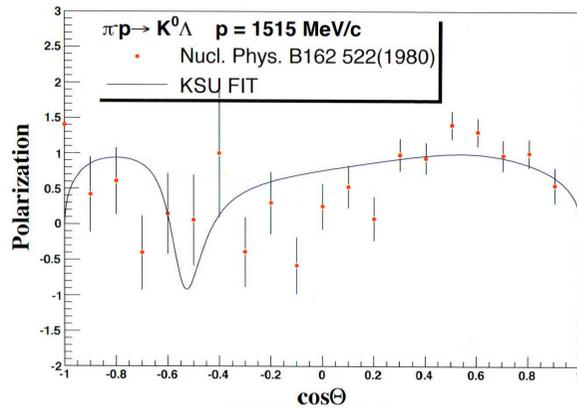
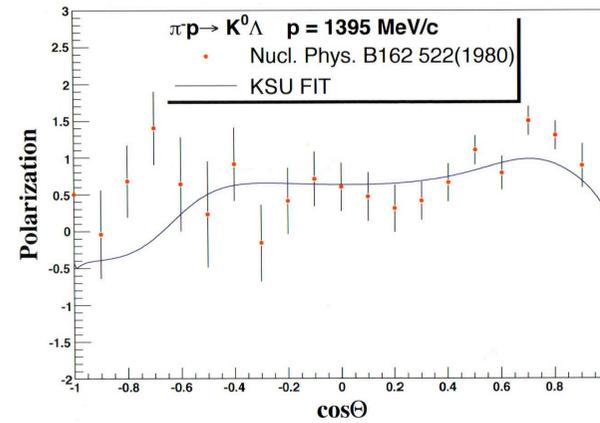
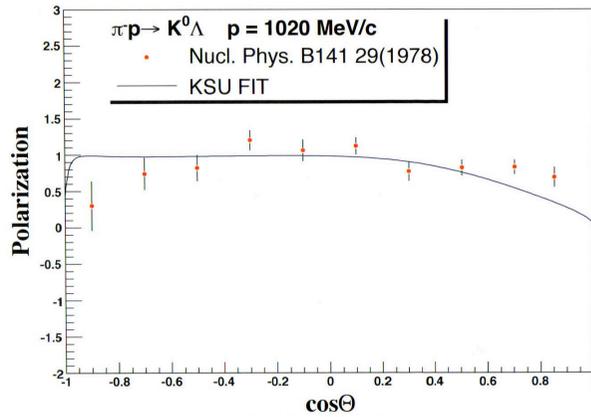
# Argand Diagrams: $D_{13}$ amplitudes



# $d\sigma/d\Omega$ for $\pi^- p \rightarrow K^0 \Lambda$

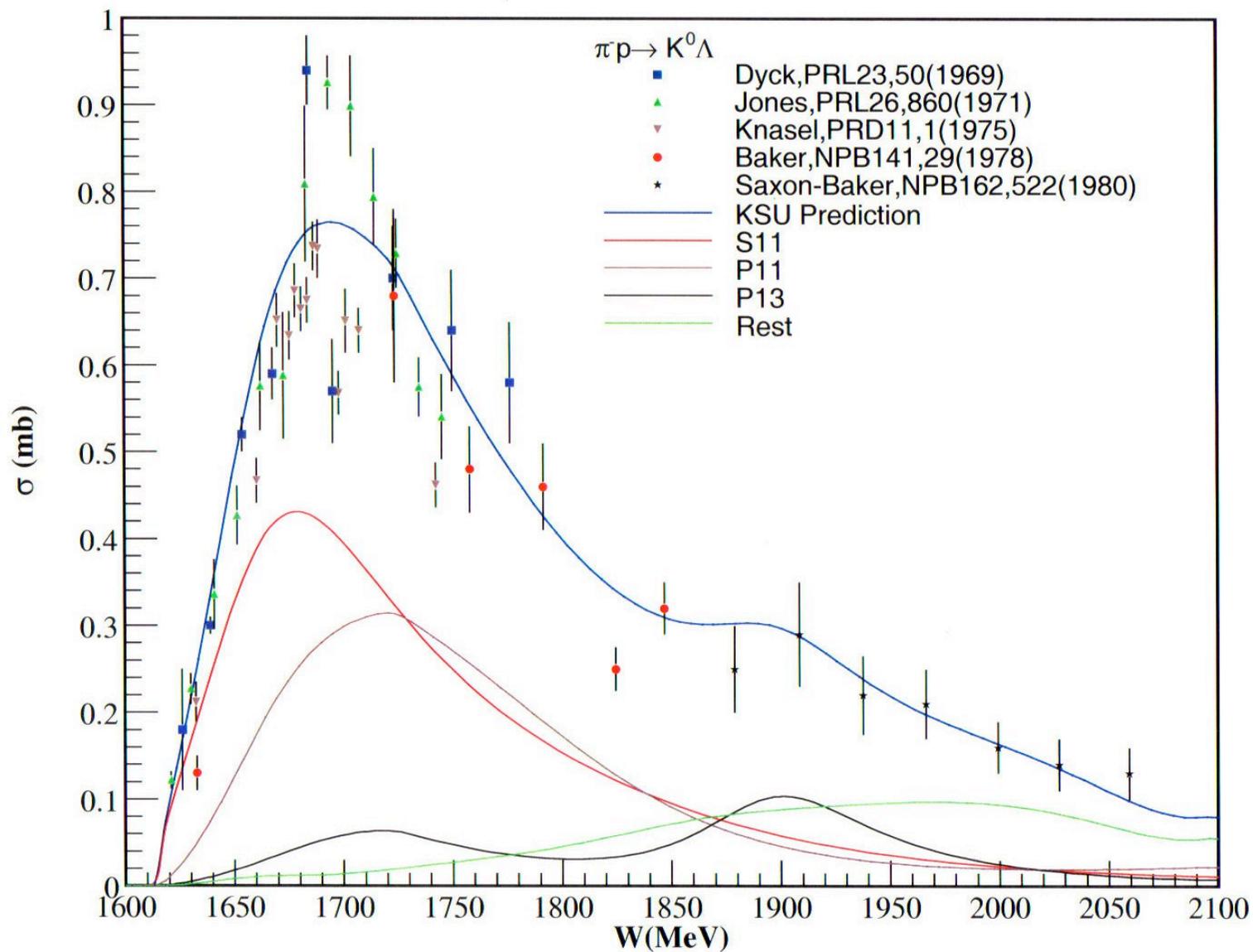


# Polarization for $\pi^- p \rightarrow K^0 \Lambda$

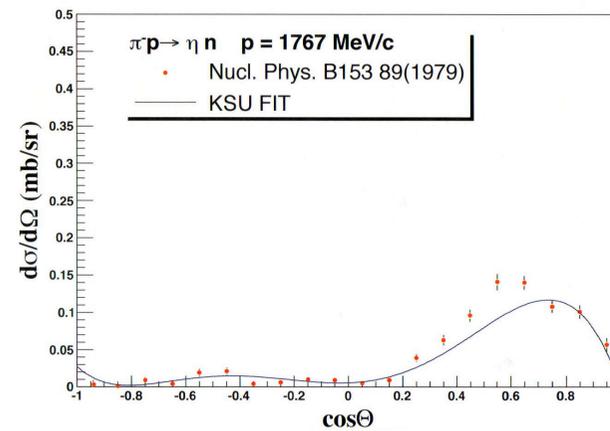
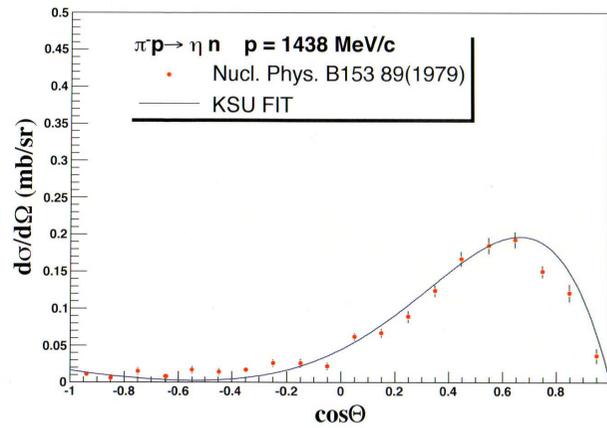
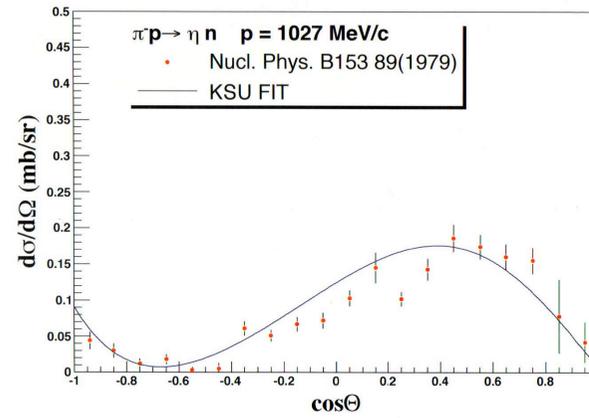
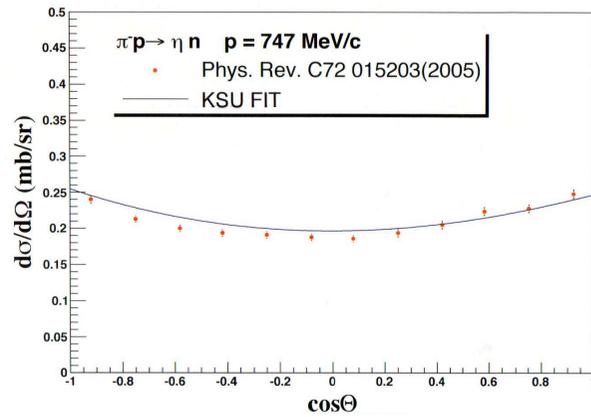


# $\sigma(\pi^-p \rightarrow K^0\Lambda)$

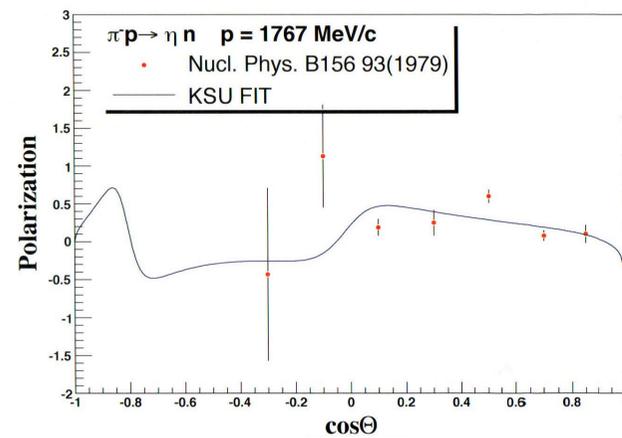
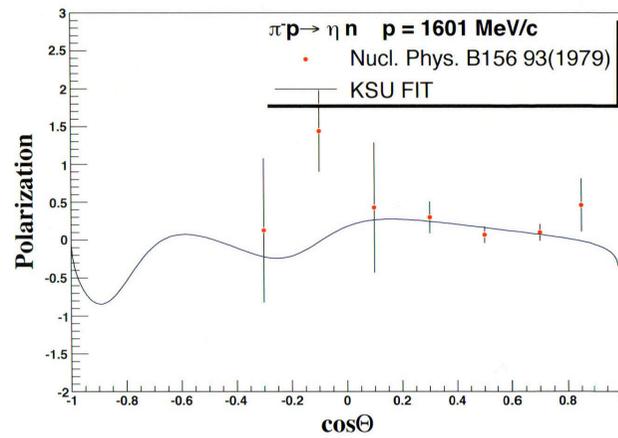
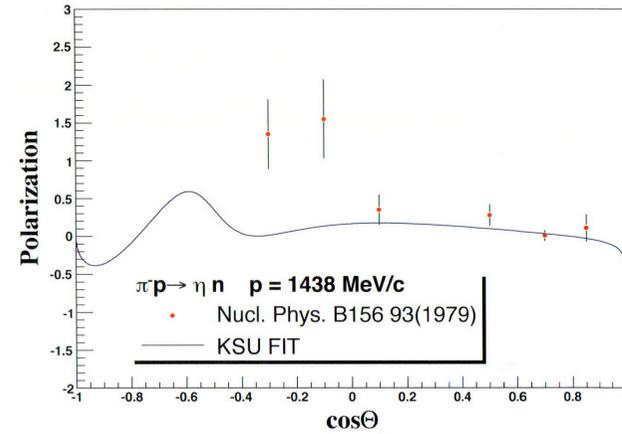
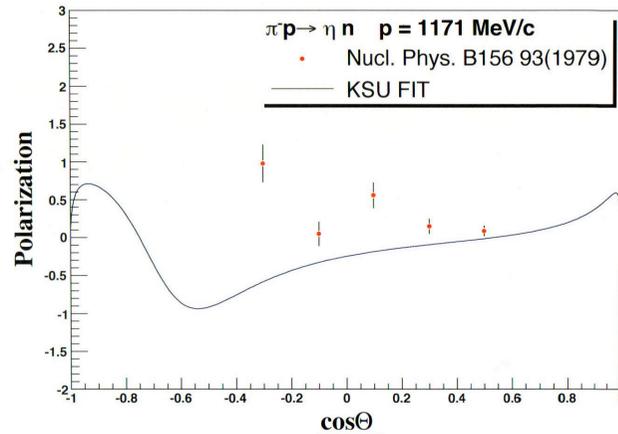
Prediction



# $d\sigma/d\Omega$ for $\pi^- p \rightarrow \eta n$

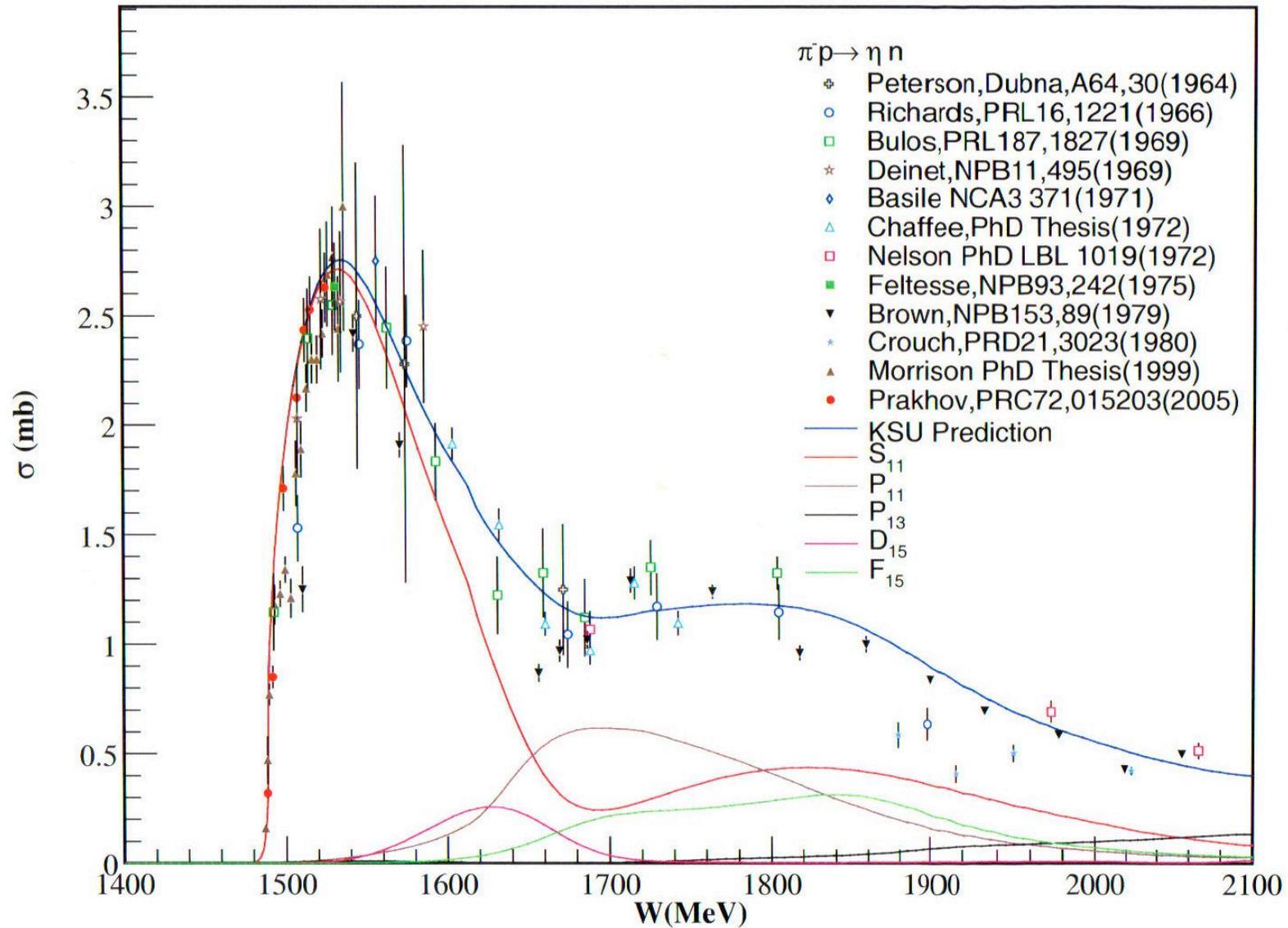


# Polarization for $\pi^- p \rightarrow \eta n$



# $\sigma(\pi^-p \rightarrow \eta n)$

Prediction



# Determination of Fitting Errors

- The fitting parameters for resonances are signed couplings ( $\pm\sqrt{\Gamma_i}$ ) and Breit-Wigner masses ( $M$ ), where  $\Gamma_i$  is the partial width for the  $i^{\text{th}}$  channel evaluated at c.m. energy  $M$
- Calculated resonance parameters are the partial widths, total width  $\Gamma = \sum \Gamma_i$ , branching ratios  $x_i = \Gamma_i/\Gamma$ , and “resonant amplitudes,  $\pm\sqrt{x_i}$ , where  $x$  is the elasticity or branching ratio for the elastic channel
- The full error matrix was used to propagate fitting errors into the calculated resonance parameters (errors were not determined for pole positions)
- During intermediate fits, some parameters were held fixed. For example, the mass of a high-mass resonance might be held fixed.
- After fit parameters had been determined, a final “zeroth-iteration” fit was done to determine fitting errors. This did not alter values of fit parameters but in this zeroth-iteration fit, all fitting parameters were treated as free parameters in order to take correlations into effect.

## Determination of Fitting Errors (continued)

- In addition, for the final fits, we increased error bars for some of the single-energy amplitudes included in the fits. If a given single-energy amplitude contributed  $\chi^2 > 10$ , we increased its error bars so that it contributed  $\chi^2 = 4$  instead. This was done because we felt some single-energy amplitudes had under-estimated error bars and we wanted to weaken their effect on the fits but did not want to remove the points outright.
- Finally, for our published resonance parameters (partial widths, branching ratios, BW masses and widths) we increased the error bars by scaling the fit errors by  $\sqrt{\chi^2/v}$ , where  $v$  is the number of degrees of freedom.

# Comments on Fitting Errors

- Some colleagues have remarked that our new error bars are “too small”. This comment may be based on a comparison with parameters from D.M. Manley and E.M. Saleski, PRD **45**, 4002 (1992).
- We note that in that publication (20 years ago), the  $\pi N$  amplitudes included in the fits were from the old 1979-1980 Carnegie Mellon-Berkeley and Karlsruhe-Helsinki PWAs. Resonance parameters from that paper were also scaled up by the square root of the reduced  $\chi^2$ . Differences and inconsistencies between these PWAs led to the larger uncertainties in our 1992 published resonance parameters.

## Comments on Fitting Errors (continued)

- Resonance parameters from our current work are more tightly constrained because we include in the fits pion photoproduction amplitudes and amplitudes for  $\eta N$  and  $K\Lambda$ , in addition to  $\pi N$  elastic and  $\pi\pi N$  amplitudes.
- Furthermore, our pion photoproduction amplitudes (SAID FA07) and  $\pi N$  elastic amplitudes (SAID SP06) are much more self consistent. When they can be compared, our new error bars are typically smaller than those in R.A. Arndt *et al.*, PRC **74**, 045205 (2006).

# Summary and Acknowledgments

- Single-energy amplitudes for several hadronic reactions have been determined by constrained analyses up to a c.m. energy of 2.1 GeV.
- Energy-dependent multichannel solutions consistent with S-matrix unitarity were obtained, allowing resonance parameters to be determined in a consistent way for many channels.
- These results provide tests of quark-model predictions and lattice gauge theory calculations.
- Work has begun on a PWA of  $\gamma p \rightarrow \eta p$ .
- Thanks to Mauro Giannini and the other organizers for inviting me to give this talk.
- This work was done in collaboration with my former Ph.D. students, John Tulpan, Hongyu Zhang, and Manoj Shrestha.
- Funding for this work was provided in part by U.S. DOE Grant DE-FG02-01ER41194.
- Thank you for your attention!