

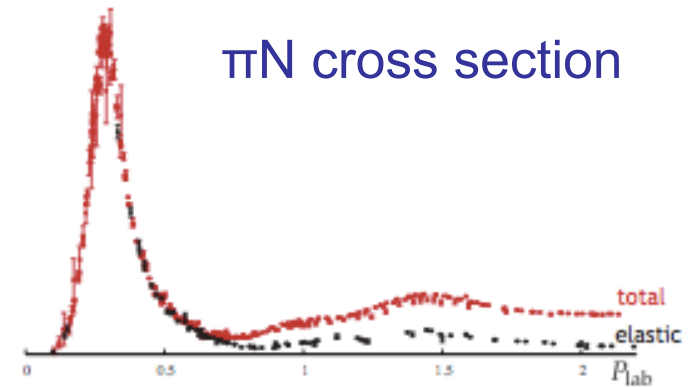
Resonance Determination from LQCD

Robert Edwards
Jefferson Lab

PWA 2013

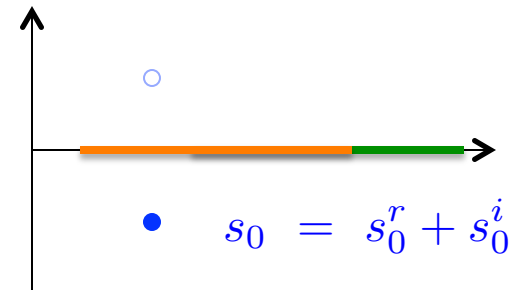
We want to determine resonances

- Most hadrons are resonances
 - E.g., a bump in elastic hadron-hadron scattering



- Formally defined as a pole in partial-wave projected scattering amplitude

$$t_l(s) \sim \frac{R}{s_0 - s} + \dots$$

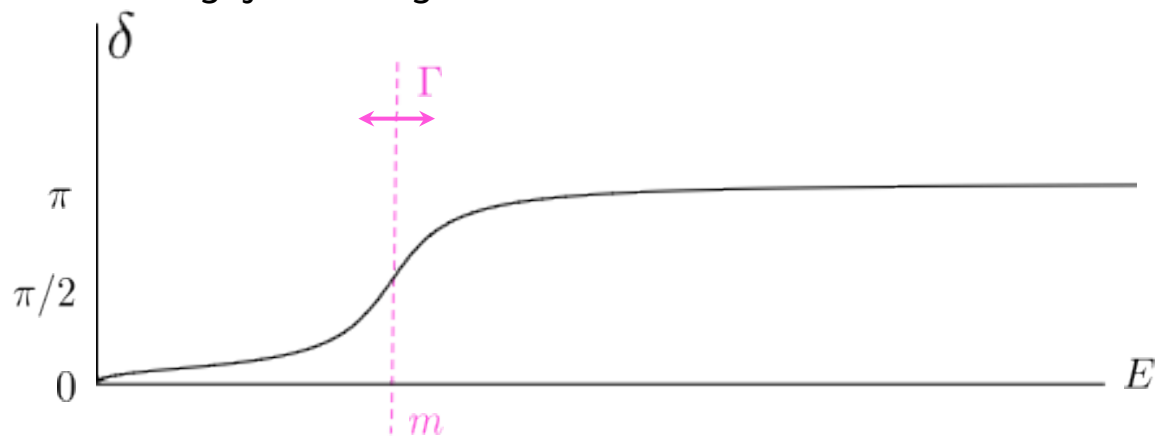


- Will appear as a pole in a production amplitude like $\gamma N \rightarrow \pi N$

Scattering

Experimentally - determine amplitudes as function of energy E

E.g. just a single elastic resonance



e.g.

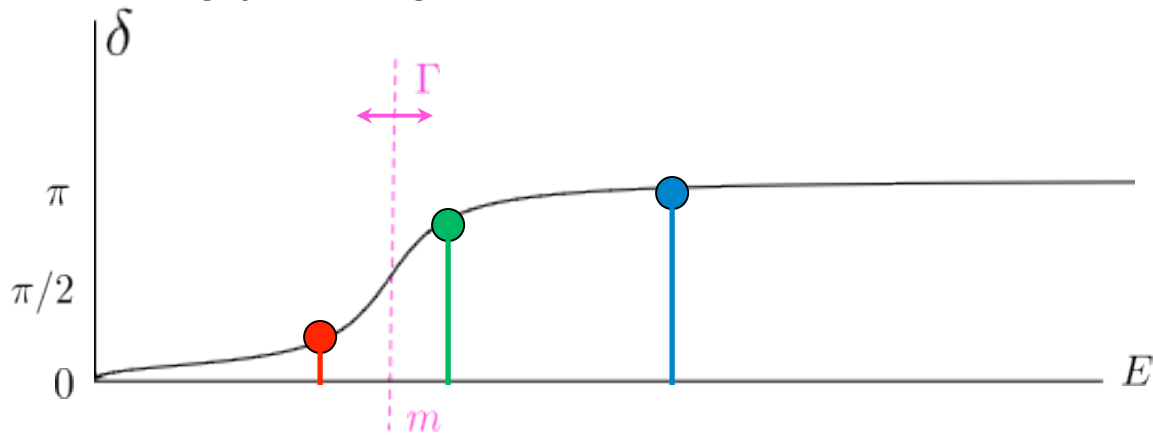
$$\pi\pi \rightarrow \rho \rightarrow \pi\pi$$

$$\pi N \rightarrow \Delta \rightarrow \pi N$$

Scattering - in finite volume!

Scattering in a periodic cubic box (length L)

E.g. just a single elastic resonance



e.g.

$$\pi\pi \rightarrow \rho \rightarrow \pi\pi$$

$$\pi N \rightarrow \Delta \rightarrow \pi N$$

At some L , have discrete excited energies

Scattering in finite volume field theory

The idea: 1 dim quantum mechanics

Two spin-less bosons: $\psi(x,y) = f(x-y) \rightarrow f(z)$ $\left[-\frac{1}{m} \frac{d^2}{dz^2} + V(z) \right] f(z) = E f(z)$

Solutions $f(p) \rightarrow \cos[p|z| + \delta(p)], \quad E = p^2/m$

Quantization condition when $-L/2 < z < L/2$ $pL + 2\delta(p) = n\pi \pmod{2\pi}$

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

non-int mom dynamical shift

Scattering amplitudes from finite volume

- In a 4-dimensional quantum field theory

$$\cot \delta_l(p) = \mathcal{M}_l \left(\frac{pL}{2\pi} \right)$$

Known functions of l
(actually, in cubic irreps)

4-momentum, e.g. from lattice

Scattering amplitudes from finite volume

- Method generalizes to higher partial waves (elastic case)

$$0 = \det \left[\begin{pmatrix} \cot \delta_1(p) & & & \\ & \cot \delta_3(p) & & \\ & & \dots & \\ & & & \dots \end{pmatrix} - \mathcal{M}_l \left(\frac{pL}{2\pi} \right) \right]$$

Matrix of known functions (actually, in cubic irreps Λ)

4-momentum from lattice

- Idea:
 - In whatever formalism, compute discrete energies (4-momentum)
 - Here, we will use a lattice formalism
 - From these energies can obtain scattering amplitudes

Spectrum from variational method

Two-point correlator

$$C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle$$

$$C_{ij}(t) = \sum_{\mathbf{n}} e^{-E_{\mathbf{n}} t} \langle 0 | \Phi_i(0) | \mathbf{n} \rangle \langle \mathbf{n} | \Phi_j^\dagger(0) | 0 \rangle$$

$$Z_i^{\mathbf{n}} \equiv \langle \mathbf{n} | \Phi_i^\dagger | 0 \rangle$$

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Matrix of correlators

$$C(t) = \begin{pmatrix} \langle 0 | \Phi_1(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_1(t) \Phi_2^\dagger(0) | 0 \rangle & \dots \\ \langle 0 | \Phi_2(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_2(t) \Phi_2^\dagger(0) | 0 \rangle & \dots \\ \vdots & & \ddots \end{pmatrix}$$

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“Rayleigh-Ritz method”

Diagonalize:

eigenvalues \rightarrow spectrum

eigenvectors \rightarrow spectral “overlaps” $Z_i^{\mathbf{n}}$

Spectrum from variational method

Two-point correlator

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“Rayleigh-Ritz method”

Diagonalize:

eigenvalues \rightarrow spectrum

eigenvectors \rightarrow spectral “overlaps” $Z_i^{\mathbf{n}}$

Each state optimal combination of Φ_i

$$\Omega^{(\mathbf{n})} = \sum_i v_i^{(\mathbf{n})} \Phi_i$$

Operators

- Quark fields act on vacuum to produce states with some quantum numbers

$$\Omega = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{q}(\vec{x}, t) \gamma_5 q(\vec{x}, t) \sim \hat{\pi}(\vec{p})$$

- Can have combinations of composite-operators

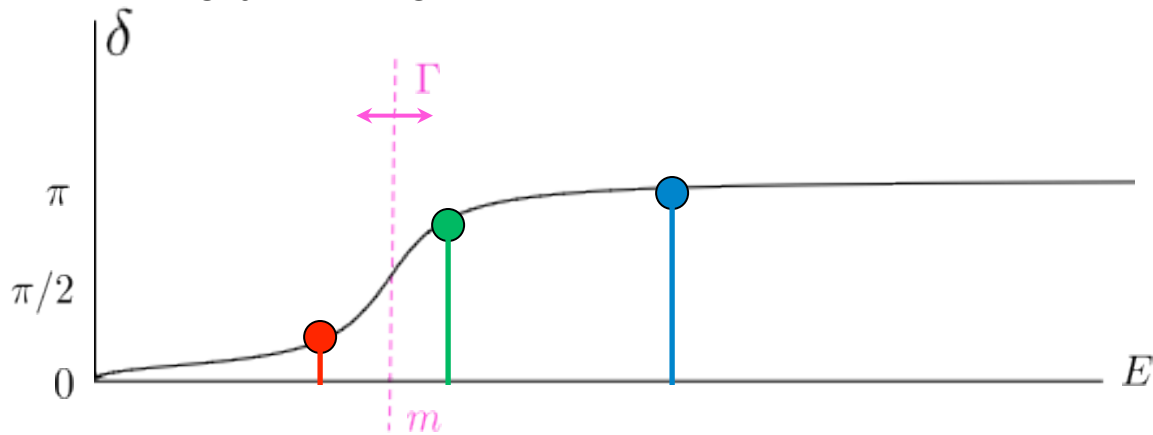
$$\Omega = \sum_{\vec{p}=\vec{p}_1+\vec{p}_2} w_{\vec{p}_1, \vec{p}_2} \hat{\pi}(\vec{p}_1) \hat{\pi}(\vec{p}_2)$$

- Can form different meson & baryon operator constructions to overlap with desired J^{PC} and J^P of interest

Reminder - scattering in a finite volume

Scattering in a periodic cubic box (length L)

E.g. just a single elastic resonance



e.g.

$$\pi\pi \rightarrow \rho \rightarrow \pi\pi$$

$$\pi N \rightarrow \Delta \rightarrow \pi N$$

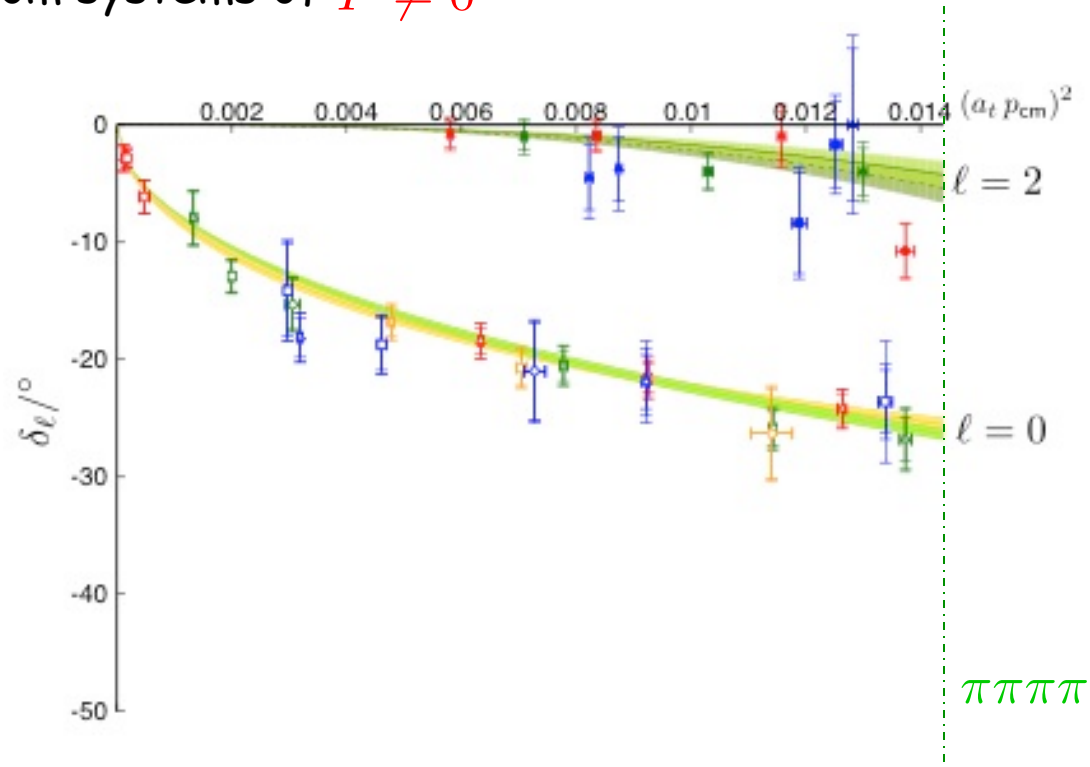
At some L , have discrete excited energies

Isospin=2 elastic $\pi\pi$ -scattering

- Example, non-resonant I=2 $\pi\pi$ in S & D-wave

$$p_{cm} \cot \delta_0(p_{cm}) = \frac{1}{a_0} + \frac{1}{2}r_0 p_{cm}^2 + \dots$$

$$p_{cm} \cot \delta_2(p_{cm}) = \frac{1}{a_2}$$
- Large number of points come from systems of $\vec{P} \neq 0$

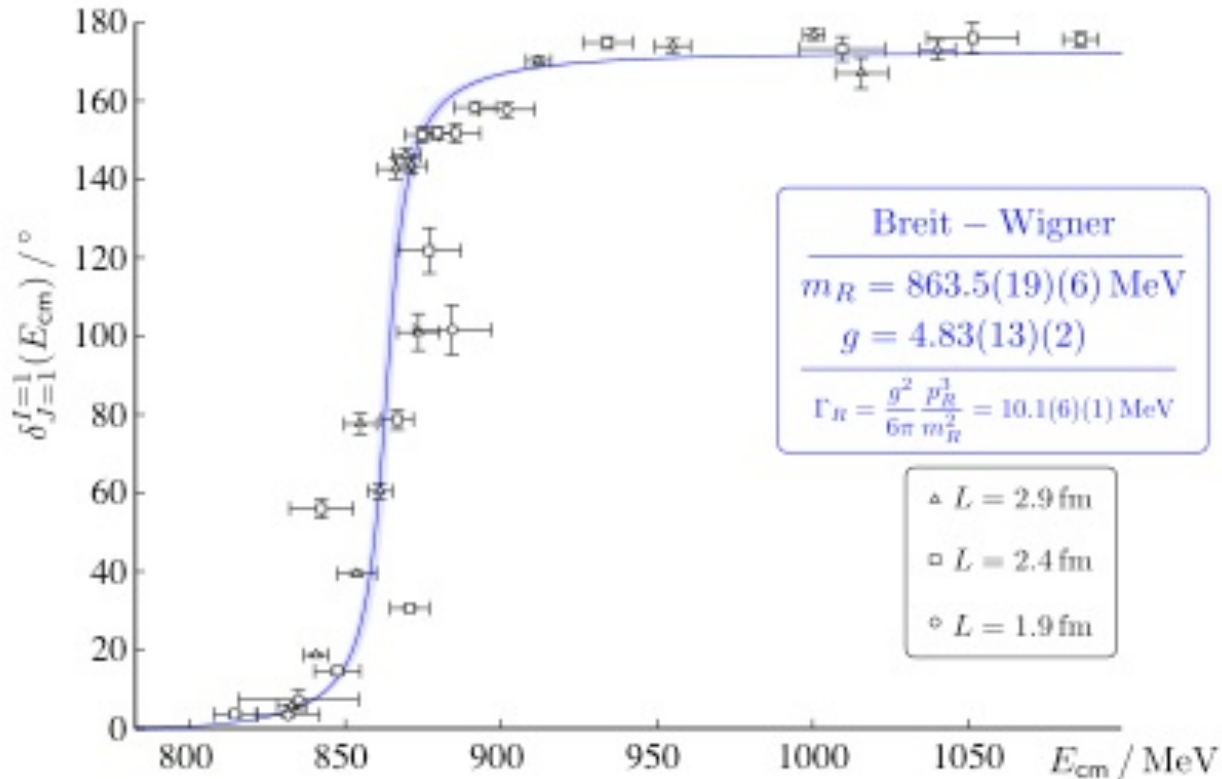


arXiv:1203.6041

Single channel elastic scattering

Isospin=1: $\pi\pi$

$\pi\pi \rightarrow \rho \rightarrow \pi\pi$

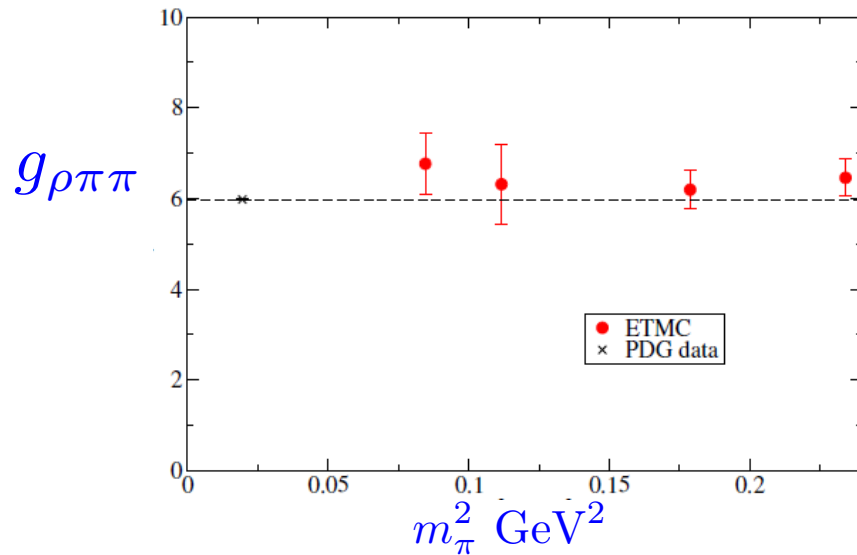


arXiv:1212.0830

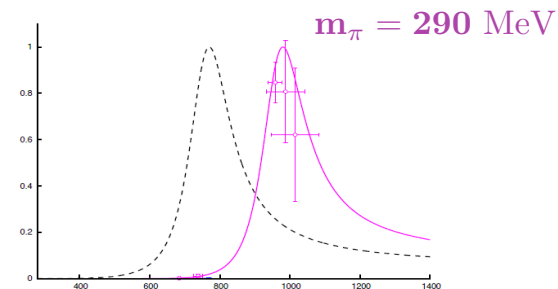
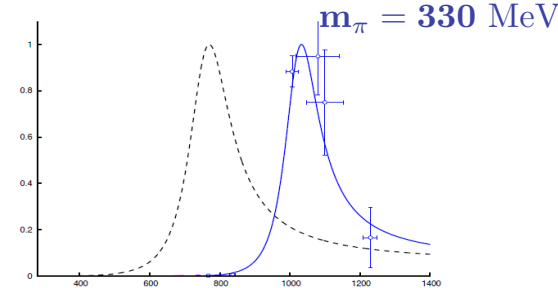
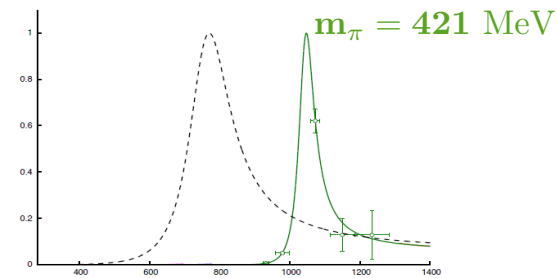
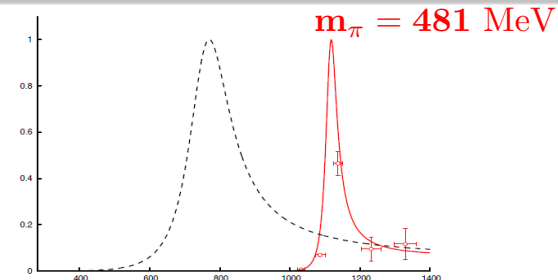
Coupling in Isospin = 1 $\pi\pi$

Comparison to other calculations: Feng, et.al, 1011.5288

Extracted coupling: stable in pion mass



Stability a generic feature of couplings??



Extension to inelastic scattering

Elastic case: method extends to higher partial waves

$$0 = \det \left[\begin{pmatrix} \cot \delta_1(p) & & & \\ & \cot \delta_3(p) & & \\ & & \ddots & \\ & & & \end{pmatrix} - \mathcal{M}_\Lambda \left(\frac{pL}{2\pi} \right) \right]$$

Diagram annotations:

- A blue box labeled "4-momentum from lattice" has arrows pointing to the p in $\cot \delta_1(p)$, $\cot \delta_3(p)$, and the p in $\frac{pL}{2\pi}$.
- An orange box labeled "Matrix of known functions (in cubic irreps Λ)" has an arrow pointing to the \mathcal{M}_Λ term.

Extension to inelastic scattering

- Can generalize to a scattering t-matrix

$$0 = \det \left[\left(\left[t^{(l)}(E_{cm}) \right]_{ij}^{-1} + i\rho_i(E_{cm})\delta_{ij} \right) - \delta_{ij} \mathcal{M}_l \left(\frac{p_i(E_{cm})L}{2\pi} \right) \right] \quad \text{Channels labelled by } i,j$$

where $t_{ij}^{(l)}(E_{cm})$ is the scattering **t**-matrix $\text{Im}(t_{ij}^{-1}) = -\delta_{ij}\rho_i \Theta(E_{cm} - E_{\text{threshold}}^{(i)})$

and $\rho_i(E_{cm}) = 2\frac{p_i(E_{cm})}{E_{cm}}$ is the phase-space for channel i

E.g.: isospin=0, $J^P=0^+$ channels $i = (\pi\pi, KK, \eta\eta, \dots)$

E.g.: baryon $\frac{1}{2}^-$ channels $i = (\pi N, \eta N, \dots)$

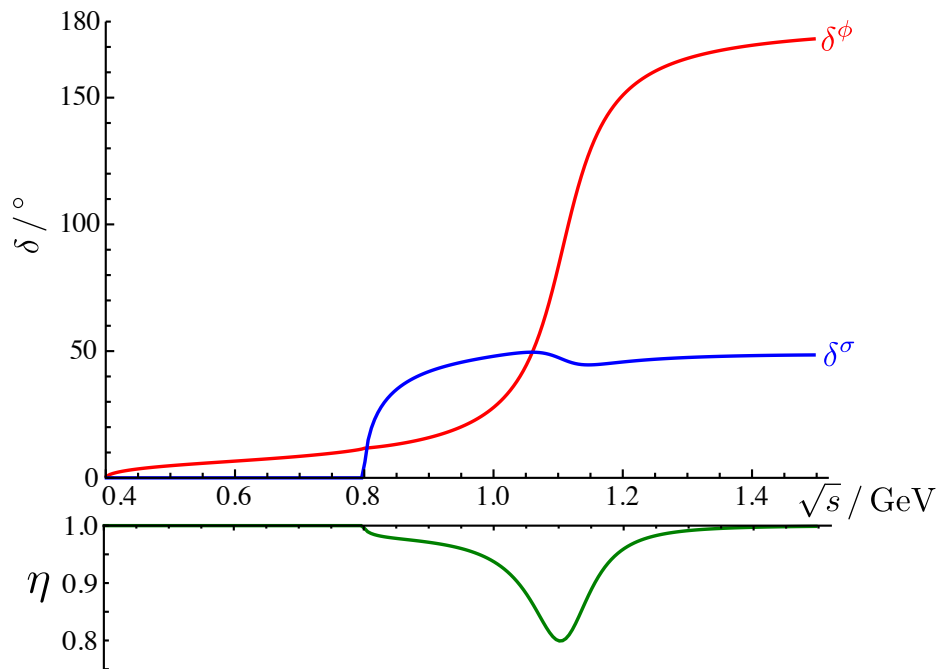
- **Underconstrained** problem: one energy level - many scatt. amps to determine
 - Already showed you an example approach
 - Parameterize t-matrix
 - » "Energy dependent" analysis

Extension to inelastic scattering

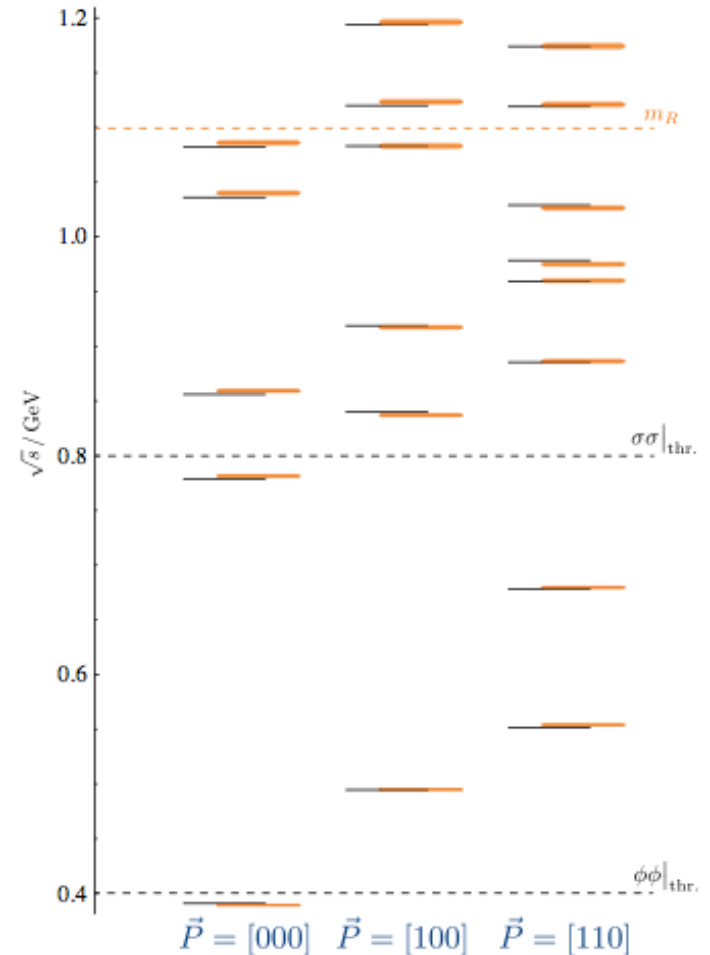
Toy model of two channel scattering:

$\phi\phi, \sigma\sigma$

K-matrix: single pole + polynomial in s



Spectrum in a 3.2fm box



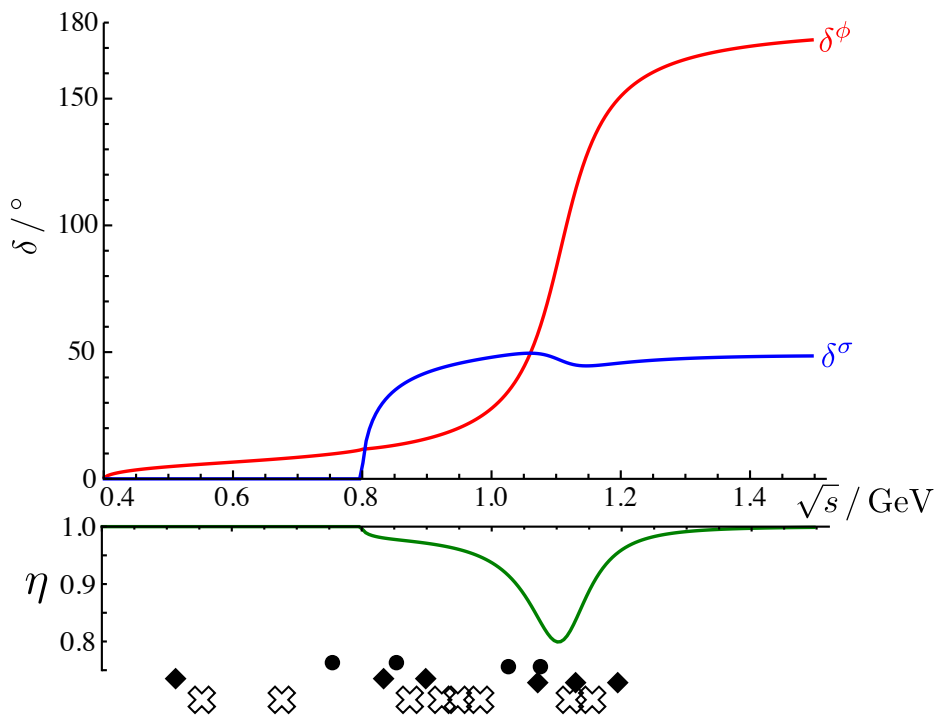
arXiv:1211.0929

Extension to inelastic scattering

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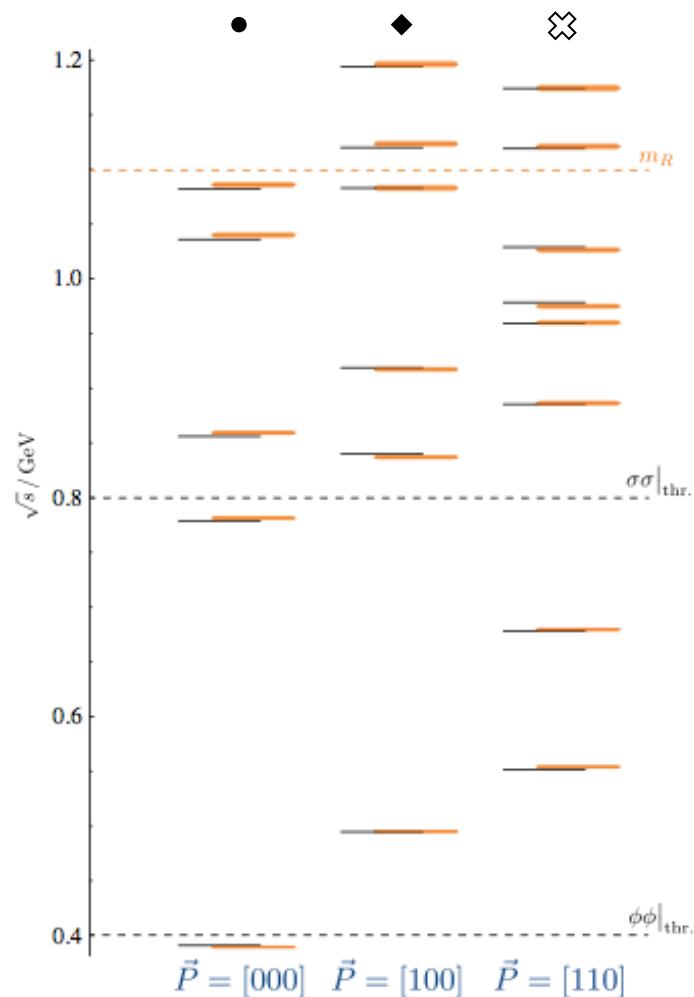
K-matrix: single pole + polynomial in s



Different in-flight momenta allow mapping of energy dependence

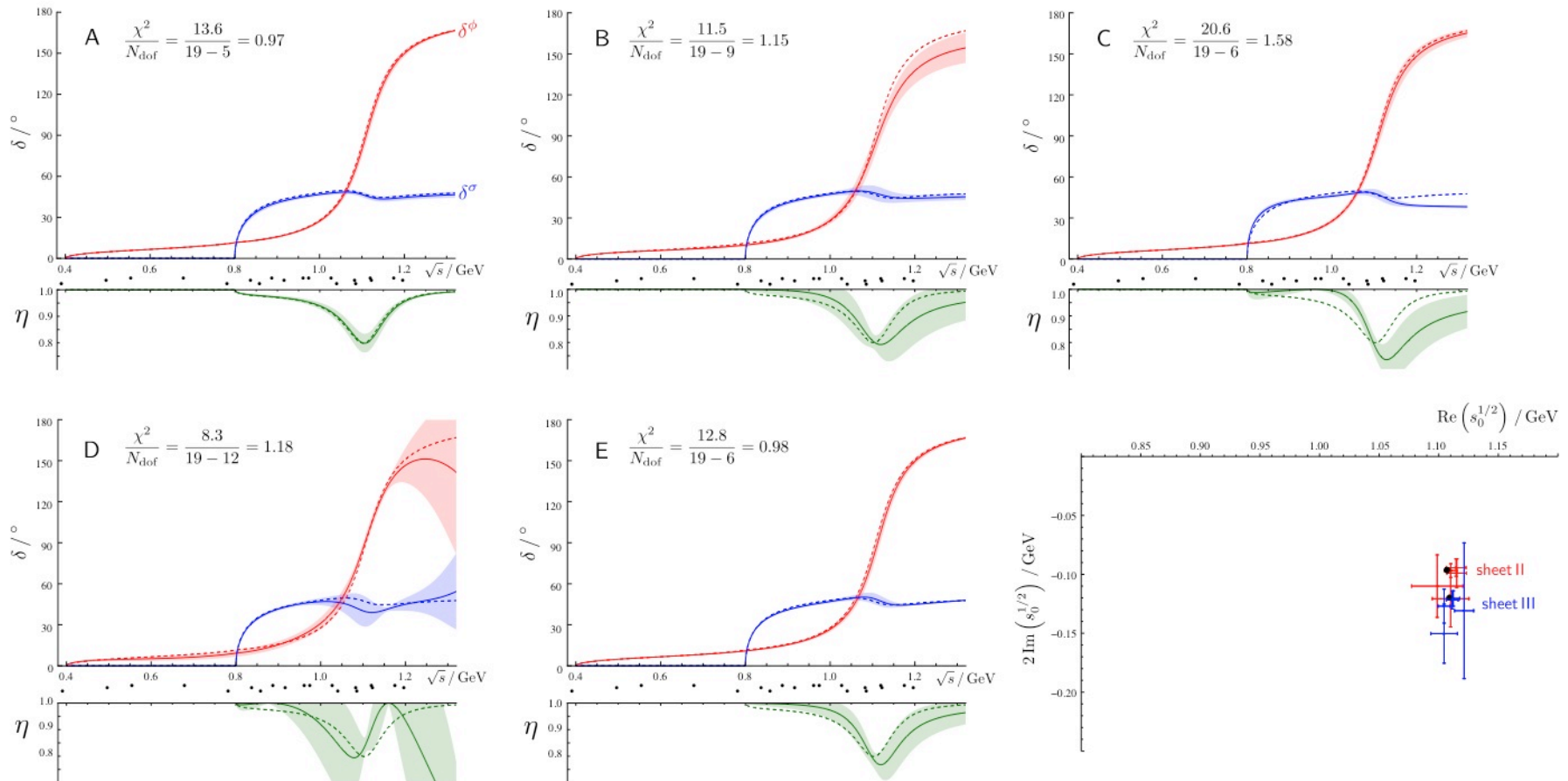
arXiv:1211.0929

Spectrum in a 3.2fm box



Coupled channel - toy model

Various parameterizations can describe the spectrum



→ agree on the t-matrix pole position`

This is a PWA workshop - where are the baryons?

Current reality:

Most baryon results limited to single-particle operator constructions

No in principle limitation:

However, contraction cost for baryon+multi-meson systems is high

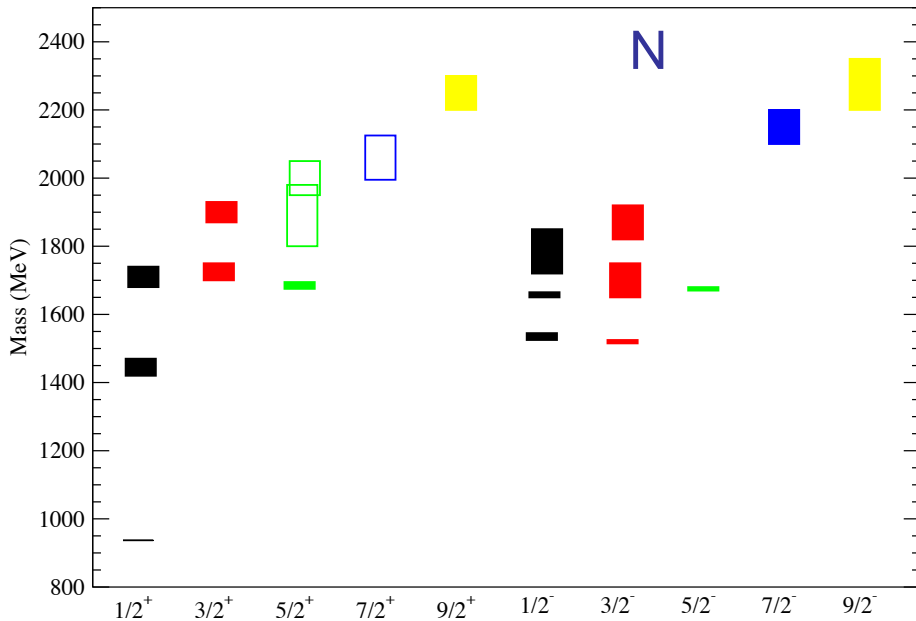
Do have issue how to systematically parameterize 3-particle scattering

With caveats, will show results restricted to single-particle operator constructions

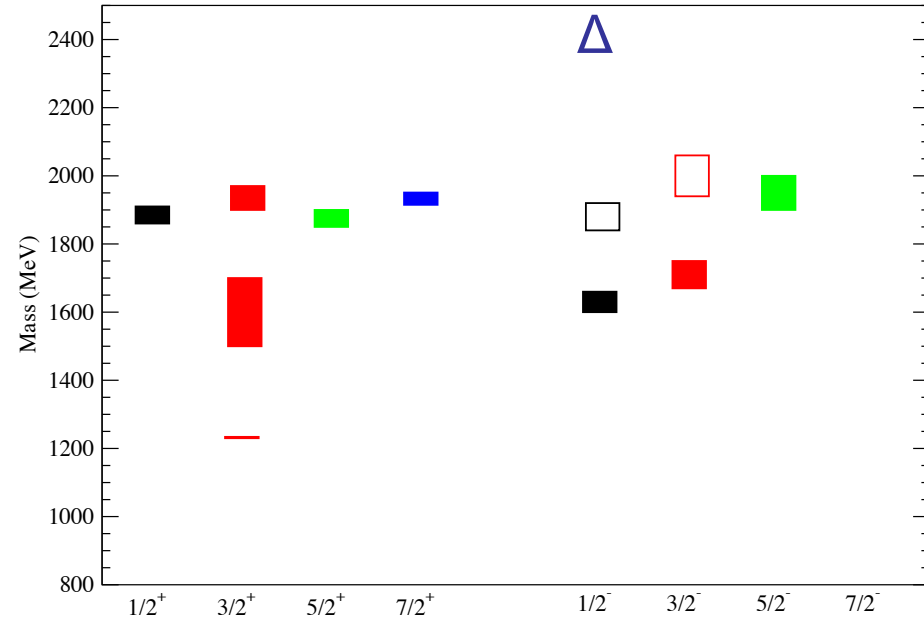
Where are the "Missing" Baryon Resonances?

Nucleon & Delta spectrum
PDG uncertainty on
B-W mass

Nucleon (Exp): 4*, 3*, some 2*



Delta (Exp): 4*, 3*, some 2*



Where are the "Missing" Baryon Resonances?

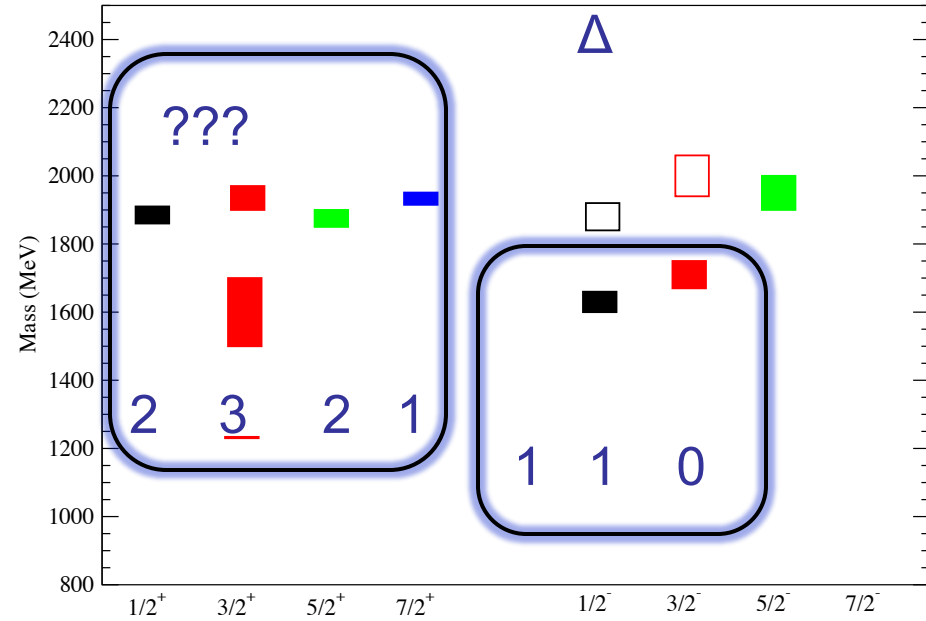
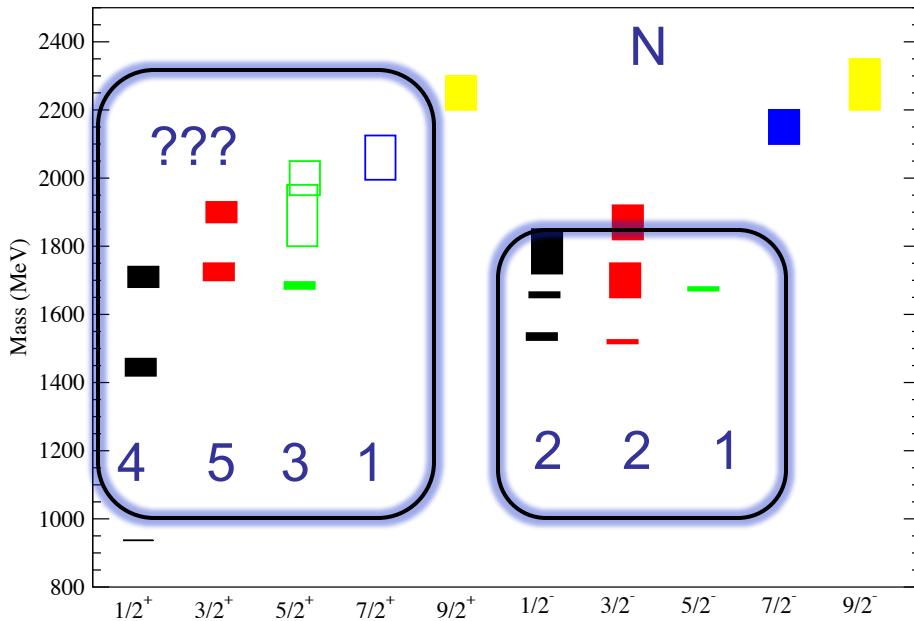
Do not see the expected QM counting

QM predictions

Nucleon & Delta spectrum
 PDG uncertainty on
 B-W mass

Nucleon (Exp): 4*, 3*, some 2*

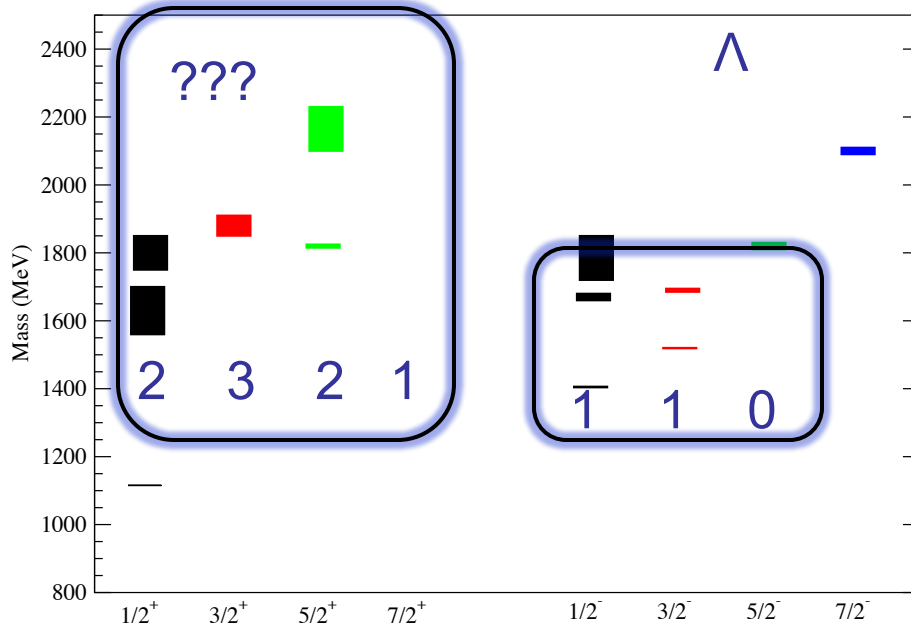
Delta (Exp): 4*, 3*, some 2*



Strange Quark Baryon Spectrum

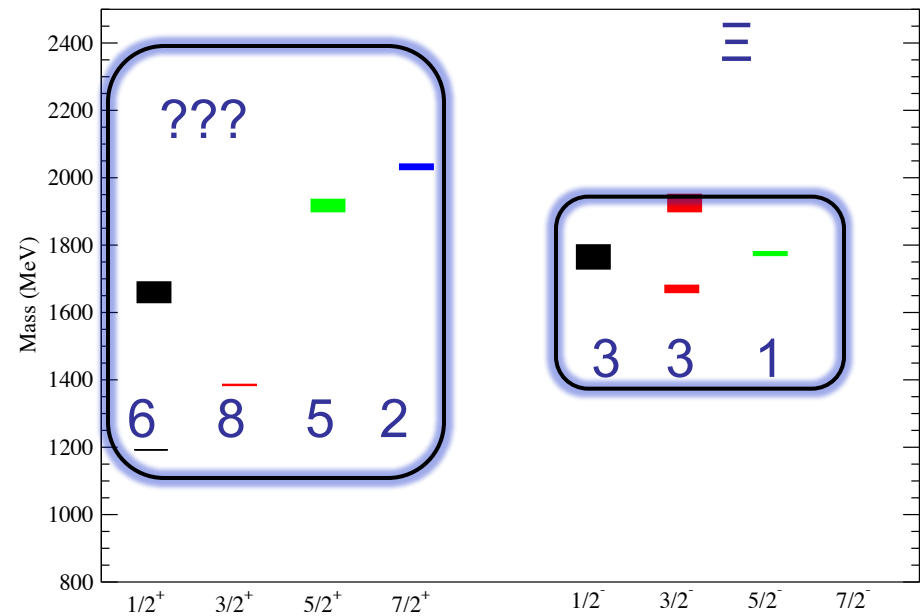
Strange quark baryon spectrum even sparser

Lambda Mass Spectrum (Exp): 4*, 3*



Since SU(3) flavor symmetry broken, expect mixing of 8_F & 10_F

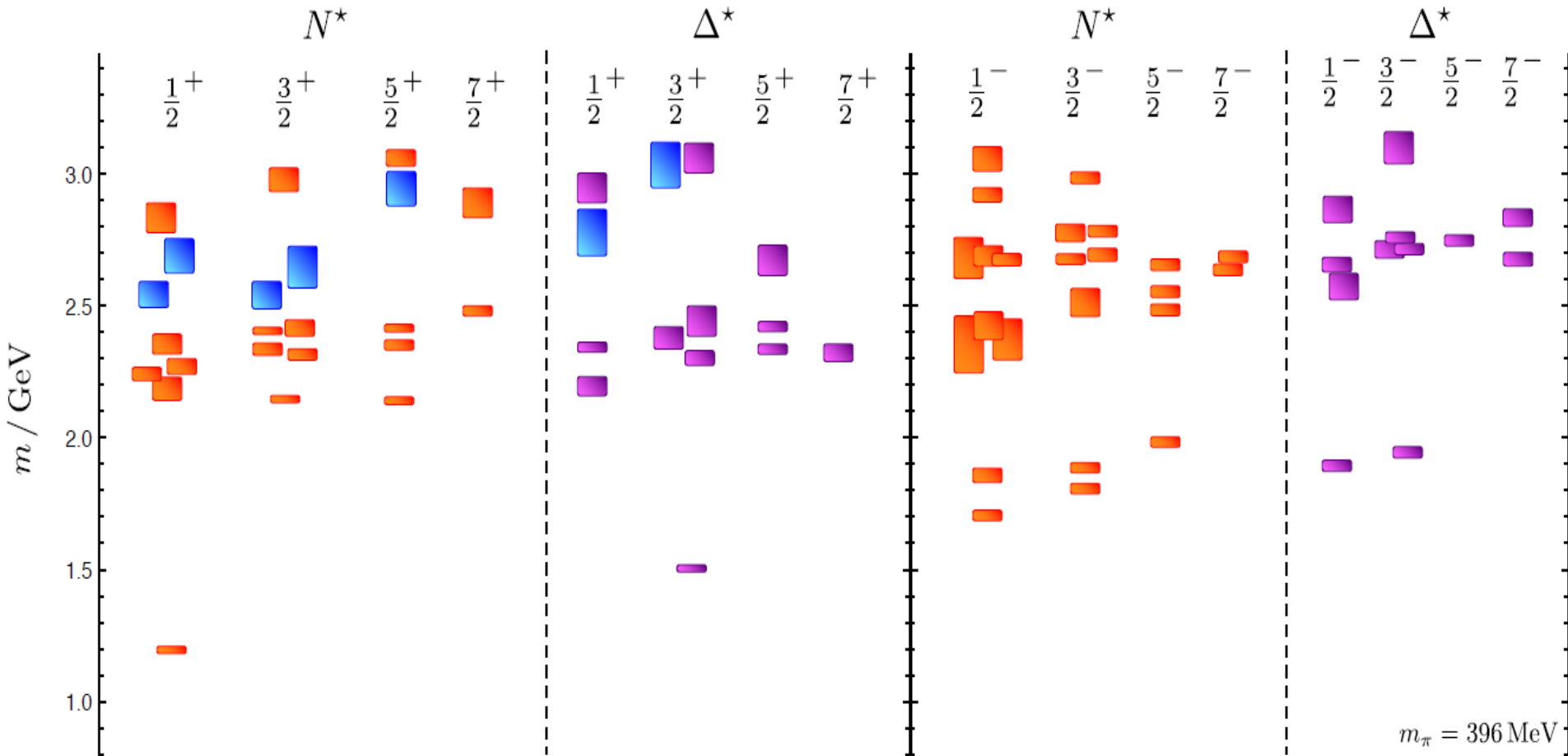
Sigma (Exp): 4*, 3*



Even less known states in Ξ & Ω

Spin identified Nucleon & Delta spectrum

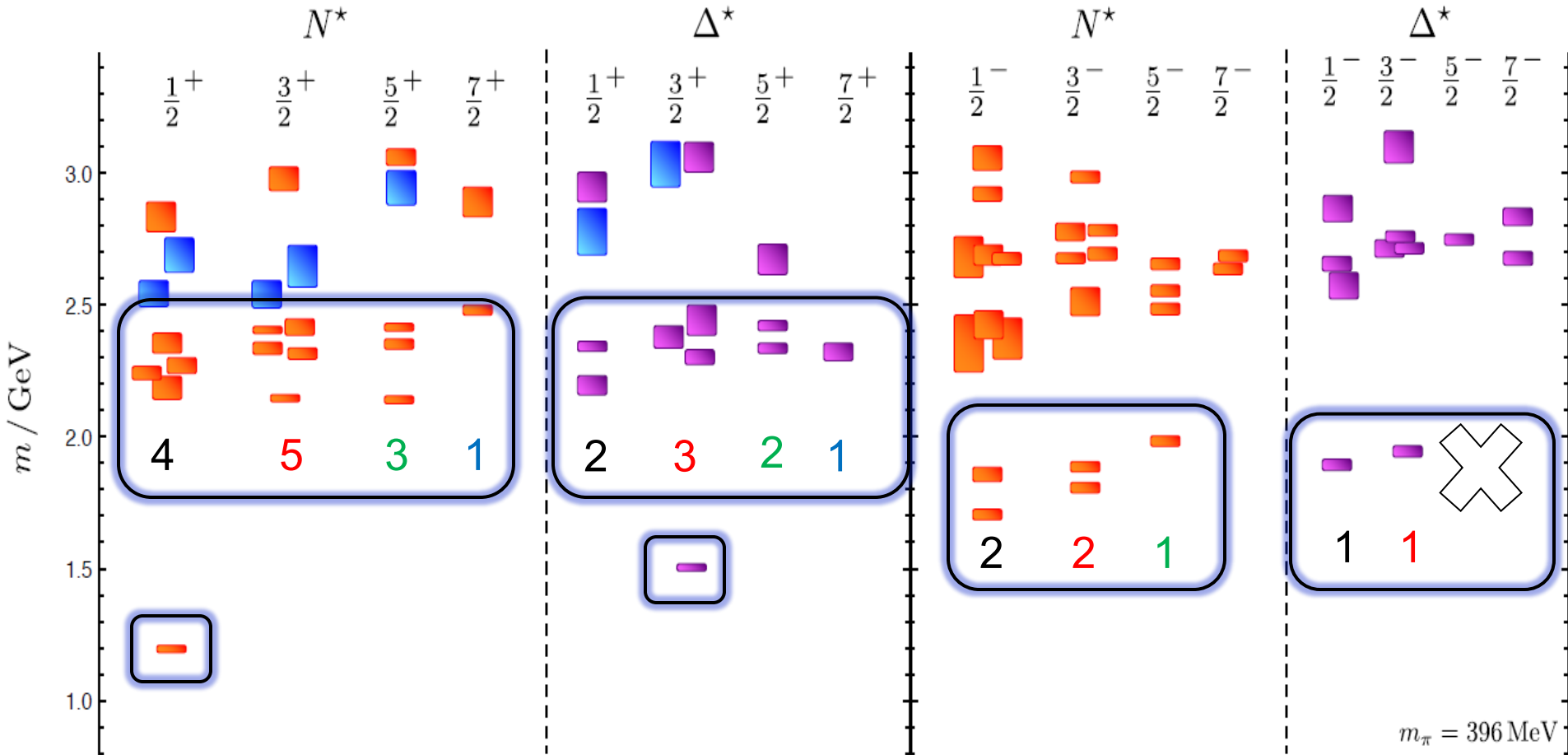
arXiv:1104.5152, 1201.2349



Spin identified Nucleon & Delta spectrum

Full non-relativistic quark model counting

arXiv:1104.5152, 1201.2349

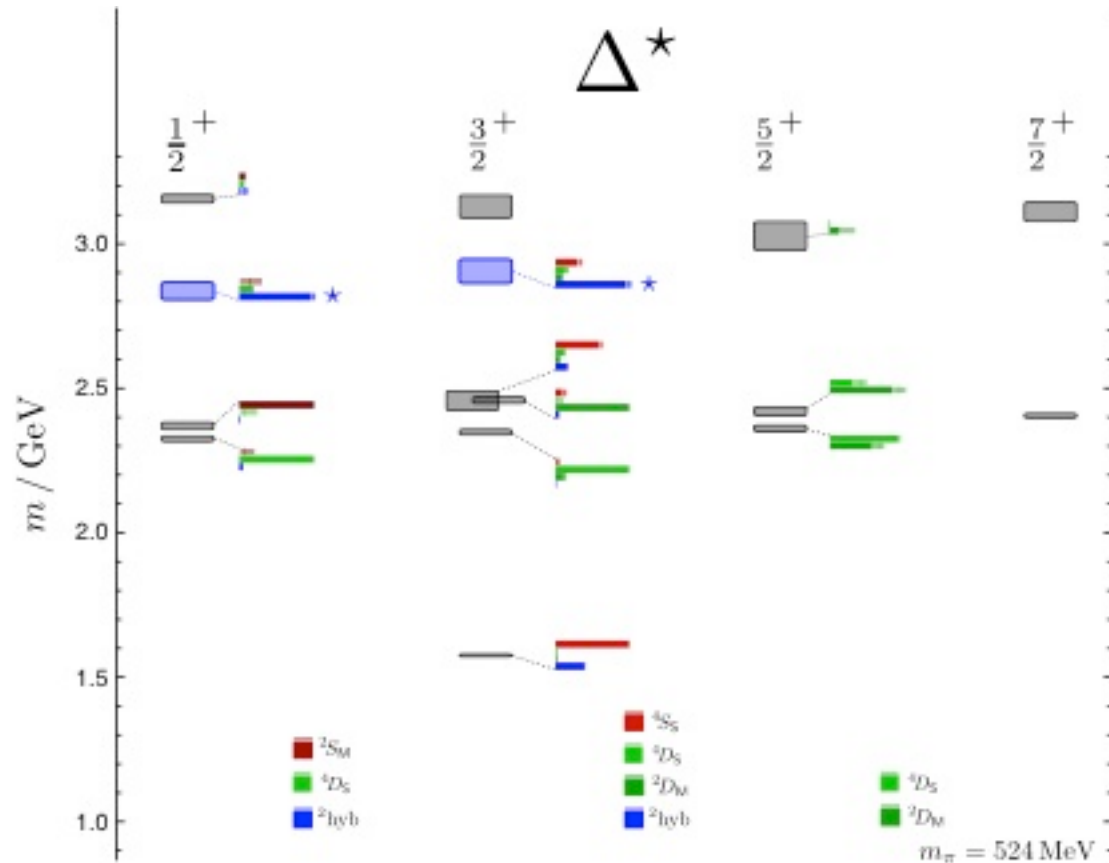


Interpreting content

“Spectral overlaps” give clue as to content of states

$$Z_i^n \equiv \langle \mathbf{n} | \Phi_i^\dagger | 0 \rangle$$

Large contribution from gluonic-based operators on states identified as having “hybrid” content

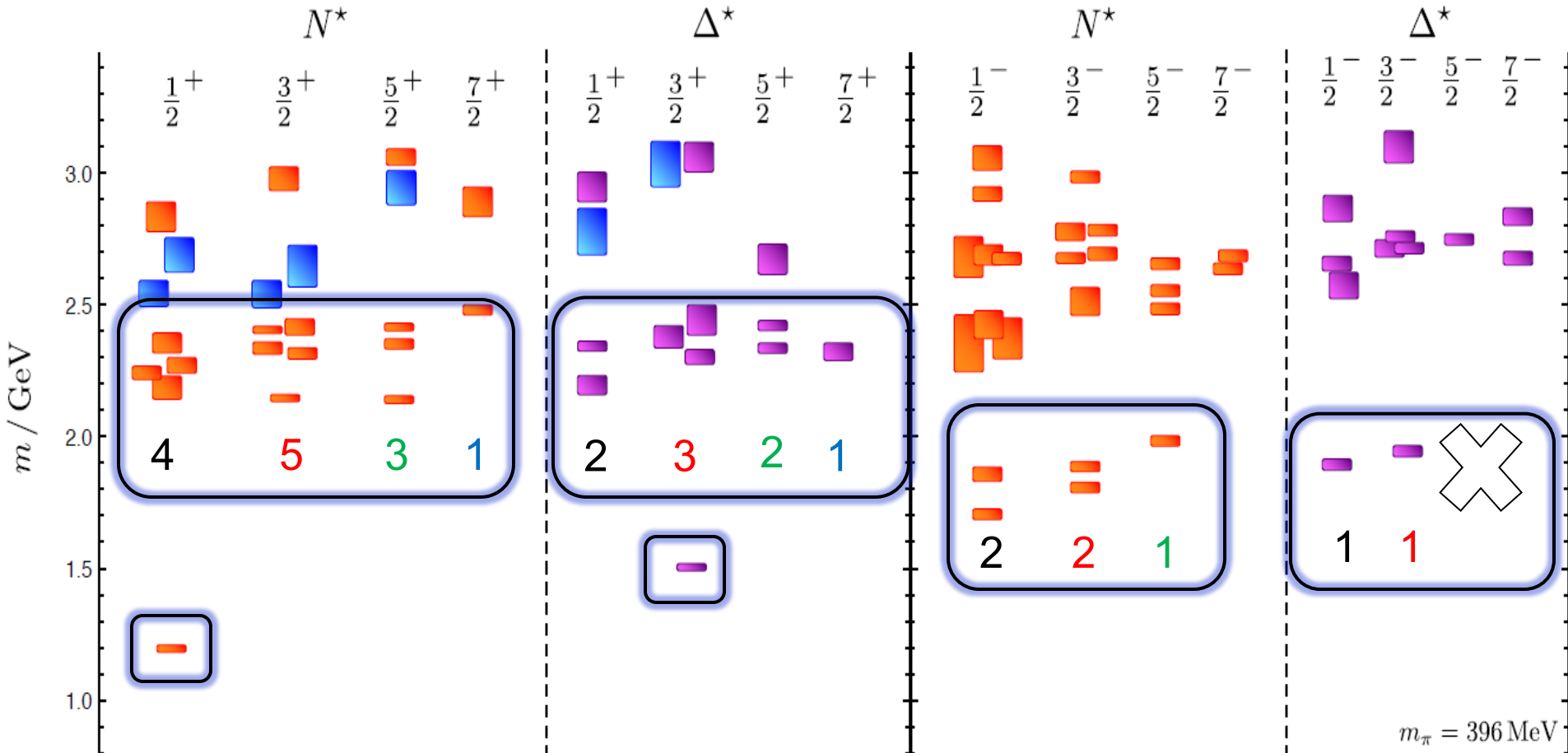


Spin identified Nucleon & Delta spectrum

Interpretation of level content from “spectral overlaps”

arXiv:1104.5152, 1201.2349

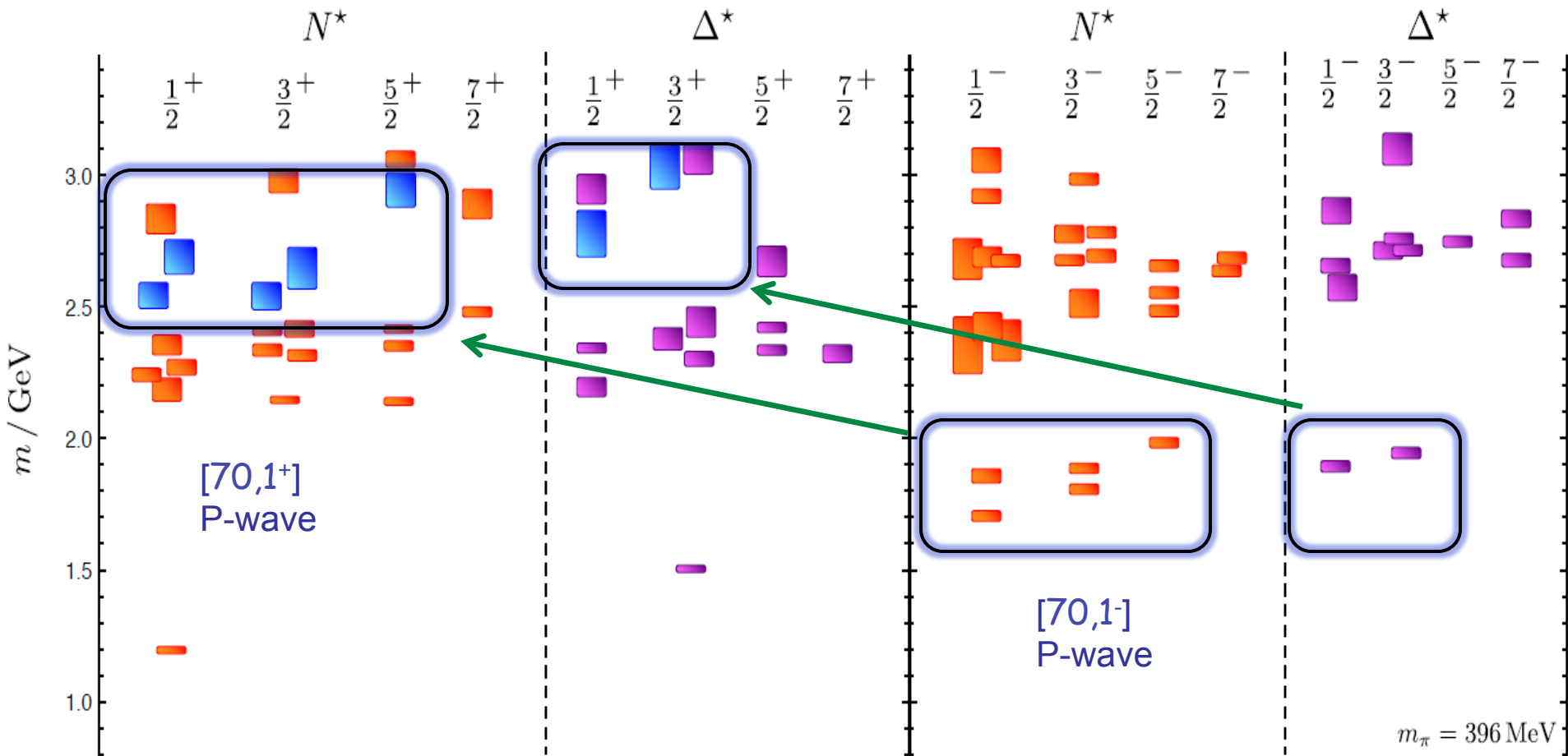
$$Z_i^n \equiv \langle \mathbf{n} | \Phi_i^\dagger | 0 \rangle$$



Hybrid baryons

Negative parity structure replicated: gluonic components (*hybrid* baryons)

$$Z_i^n \equiv \langle n | \Phi_i^\dagger | 0 \rangle$$



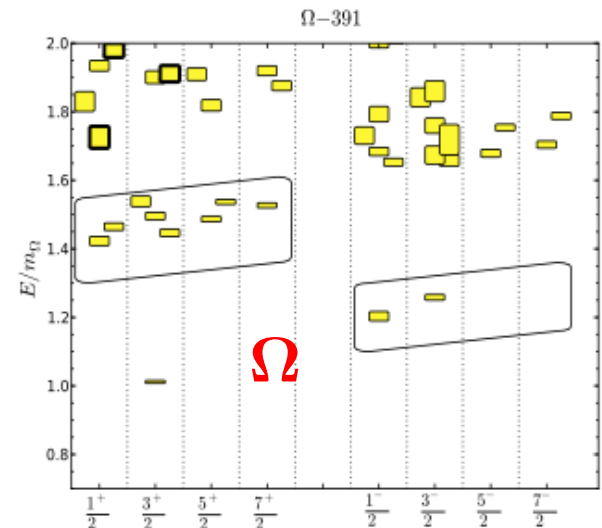
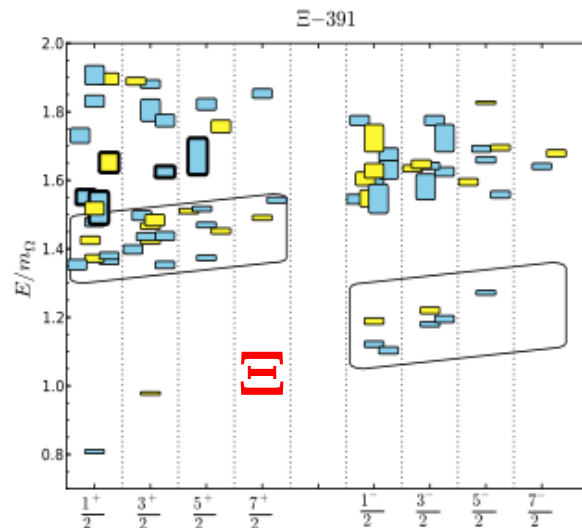
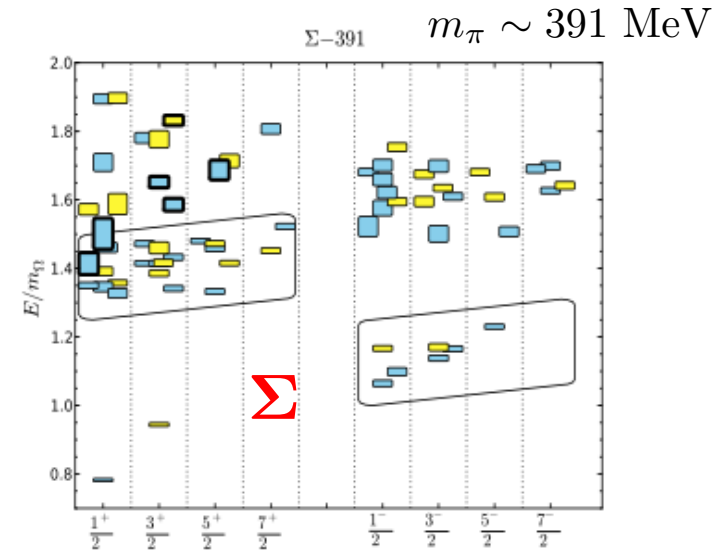
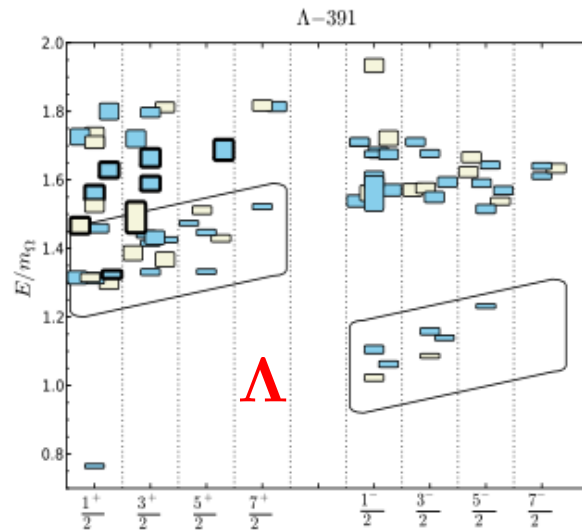
Light quarks - other isospins

Light quarks – SU(3)
flavor broken

Full non-relativistic
quark model counting

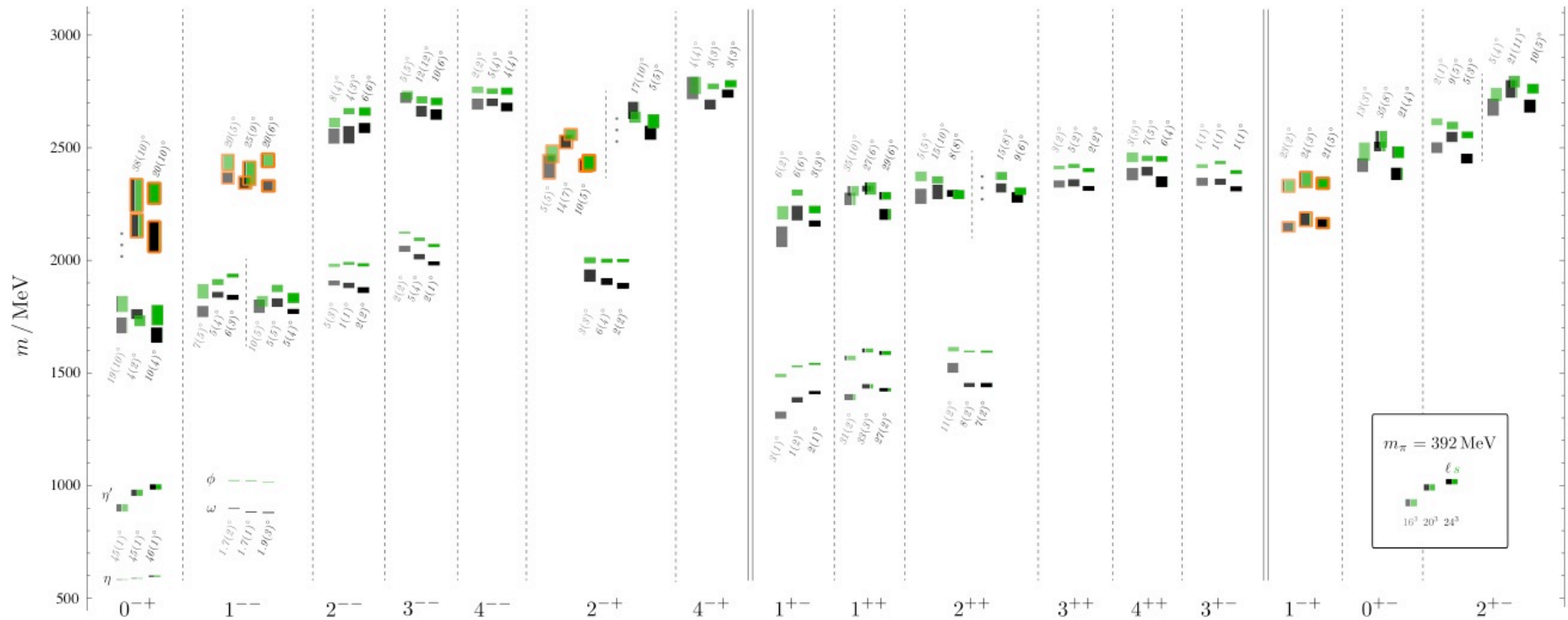
Some mixing of SU(3)
flavor irreps

arXiv:1212.5236



Volume dependence: isoscalar mesons

Energies determined from single-particle operators:
 Range of J^{PC} - color indicates light-strange flavor mixing



Some volume dependence:

Interpretation: energies determined up to a hadronic width

arXiv:1309.2608

Summary & prospects

Spectrum of eigenstates of QCD in a finite-box can be related to scattering amplitudes

Using lattice QCD - first steps in this direction:

- Showed you “simple” (elastic) cases of scattering
- First glimpses at full excited spectrum, but without scattering studies

Path forward: resonance determination!

- Calculations underway at 230 MeV pion masses
- Currently investigating multi-channel scattering in different systems

Challenges:

- Must develop reliable 3-body formalism (hard enough in infinite volume)
- Large number of open channels in physical pion mass limit – it’s the real world!
- Can QCD allow simplifications (e.g., isobars?)

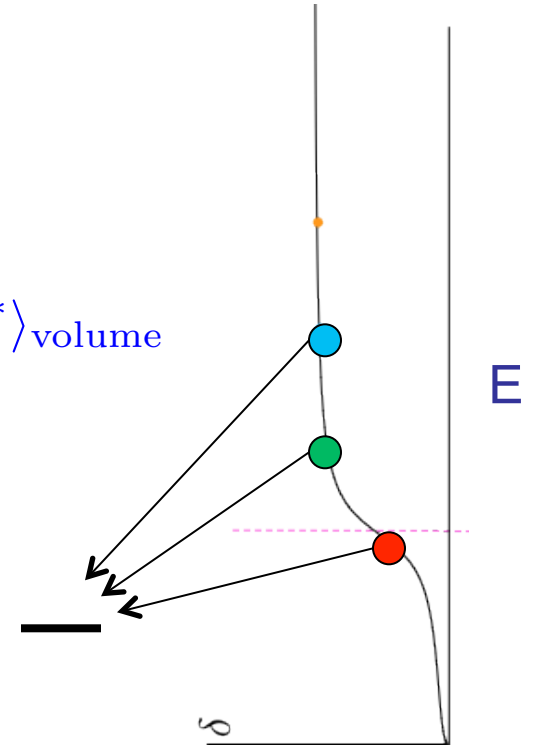
The details...

- The end

Form Factors

- What is a form-factor off of a resonance?
- What is a resonance? Spectrum first!
- Extension of scattering techniques:
 - Finite volume matrix element modified

$$\langle N | J_\mu | N^* \rangle_\infty(Q^2, E) \leftarrow [\underbrace{\delta'(E)}_{\text{Phase shift}} + \underbrace{\Phi'(E)}_{\text{Kinematic factor}}] \langle N | J_\mu | N^* \rangle_{\text{volume}}$$

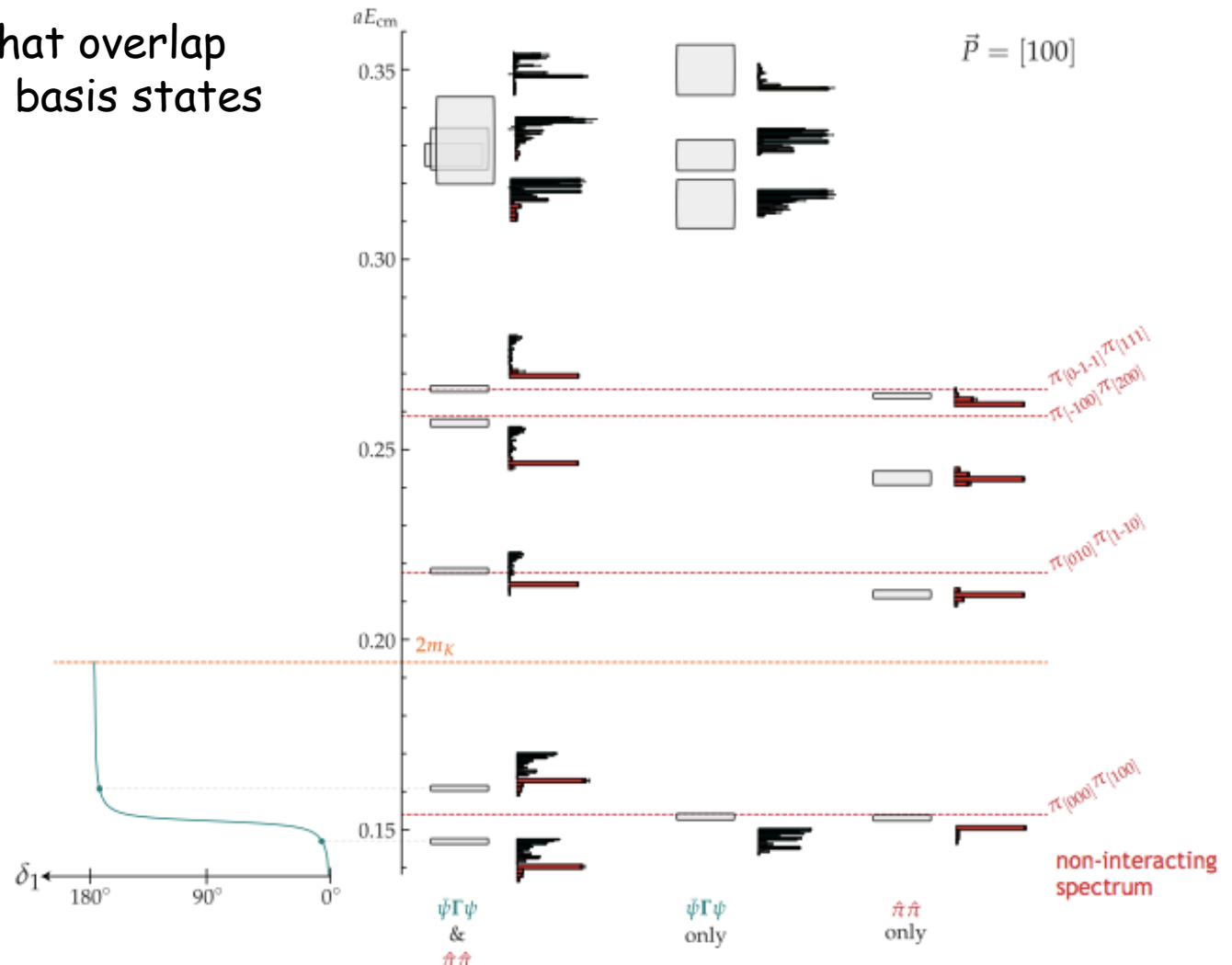


- Requires excited level transition FF's: some experience
 - Charmonium E&M transition FF's (1004.4930)
 - Nucleon 1st attempt: "Roper" -> N (0803.3020)

Need "broad" operator basis

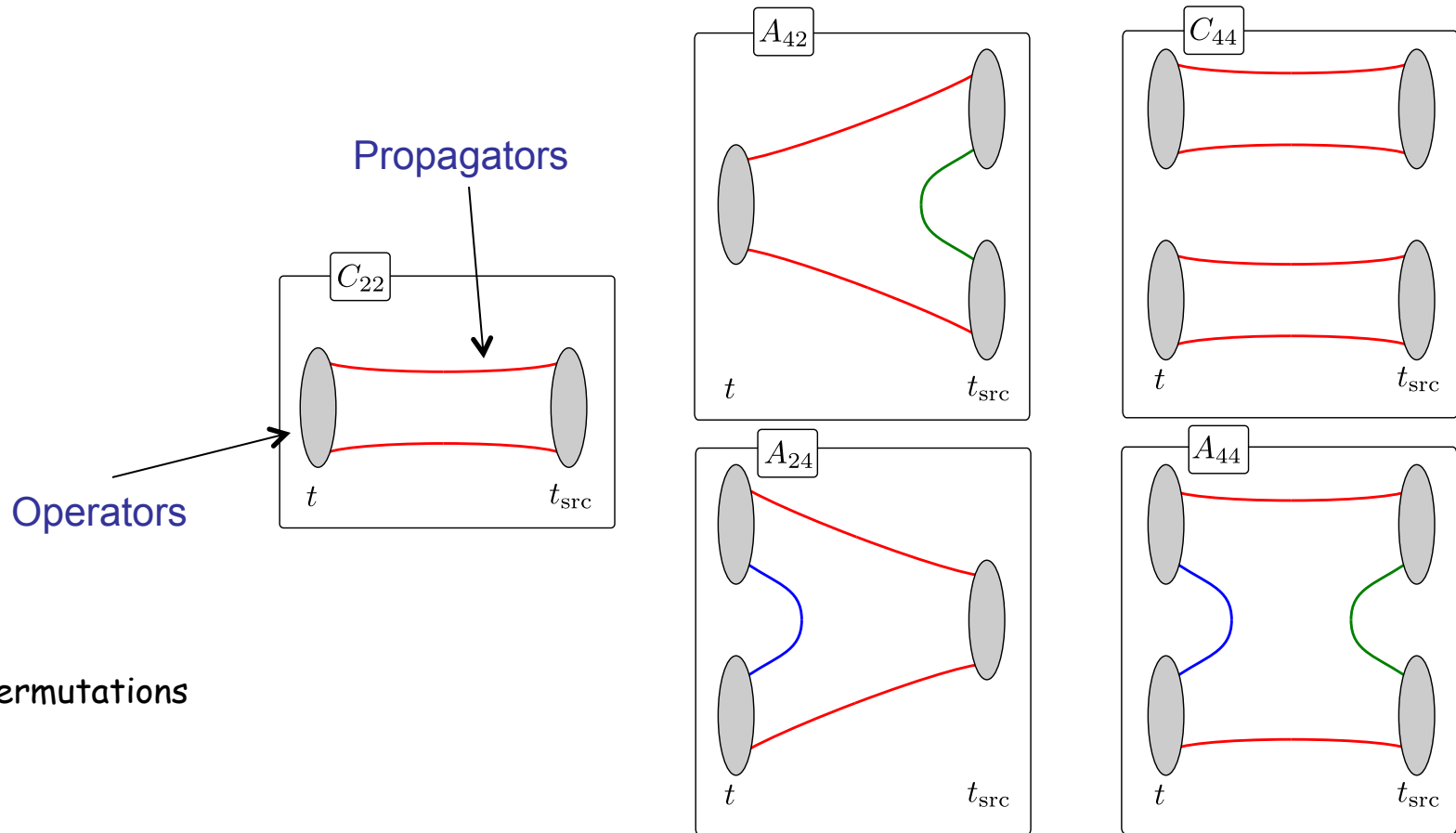
For variational method

- Need operators that overlap well with relevant basis states



Contractions

Cost to produce correlators driven by contractions

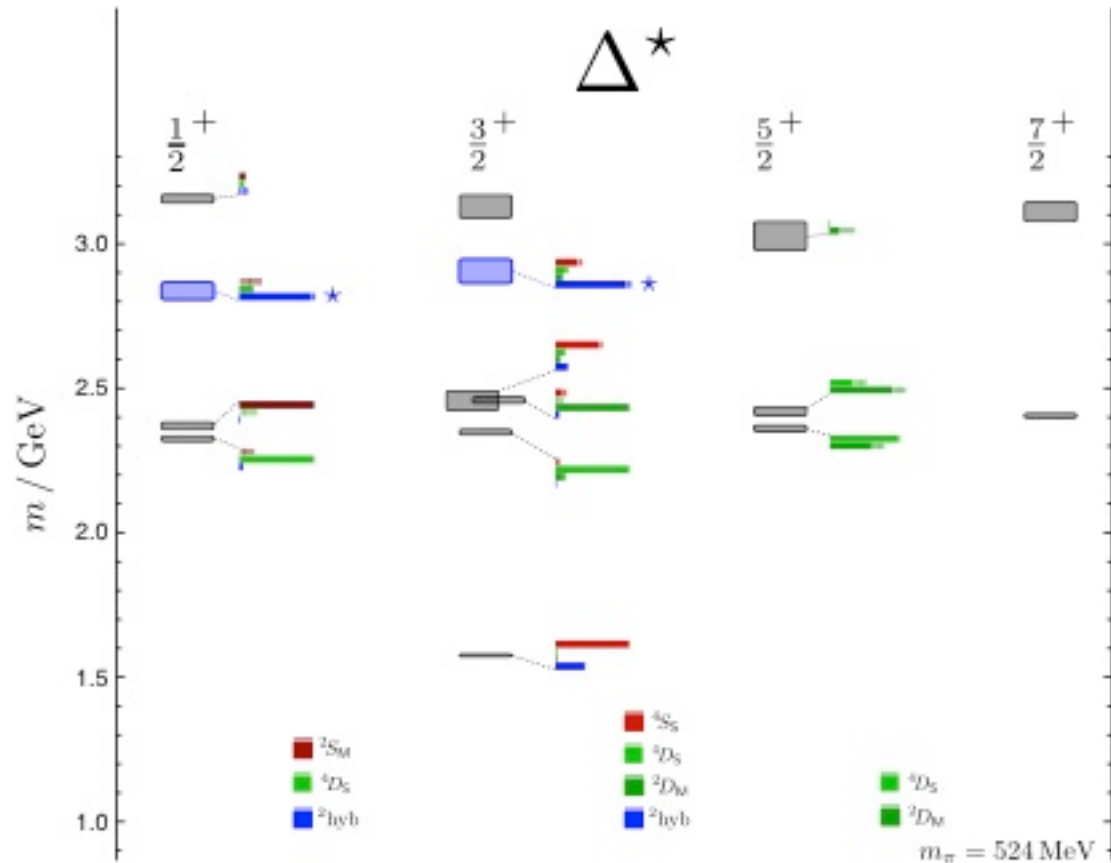


Interpreting content

“Spectral overlaps” give clue as to content of states

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Large contribution from gluonic-based operators on states identified as having “hybrid” content



Hybrid meson models

With minimal quark content, $q\bar{q}G$, gluonic field can be in a color singlet or octet

- ‘constituent’ gluon in S-wave

$G \sim 1_8^{--}$	$q\bar{q}_{L=0}$	$(0, 1, 2)^{++}, 1^{+-}$
	$q\bar{q}_{L=1}$	$0^{--}, (1^{-+})^3, 3^{-+} \dots$
- bag model

‘constituent’ gluon in P-wave

$G \sim 1_8^{+-}$	$q\bar{q}_{L=0}$	$(0, 1, 2)^{-+}, 1^{--}$
	$q\bar{q}_{L=1}$	$0^{+-}, (2^{+-})^2 \dots$
- flux-tube model

$(0, 1, 2)^{-+}, 1^{--}, (0, 1, 2)^{+-}, 1^{++}$

Hybrid meson models

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$(0, 1, 2)^{++}, 1^{+-}$
 $0^{--}, (1^{-+})^3, 3^{-+} \dots$
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$q\bar{q}_{L=0}$ $(0, 1, 2)^{-+}, 1^{--}$
 $q\bar{q}_{L=1}$ $0^{+-}, (2^{+-})^2 \dots$
- flux-tube model

$(0, 1, 2)^{+-}, (0, 1, 2)^{+-}, 1^{++}$

Hybrid baryon models

Minimal quark content, $qqqG$, gluonic field can be in color singlet, octet or decuplet

Now must take into account *permutation* symmetry of quarks and gluonic field

- bag model

$$G \sim 1_{8_c}^+$$

$$(N_{\frac{1}{2}^+})^2, (N_{\frac{3}{2}^+})^2, (\Delta_{\frac{1}{2}^+}), (\Delta_{\frac{3}{2}^+}), (N_{\frac{5}{2}^+})$$

- flux-tube model

$$(N_{\frac{1}{2}^+})^2, (N_{\frac{3}{2}^+})^2, (\Delta_{\frac{1}{2}^+}), (\Delta_{\frac{3}{2}^+}), (\Delta_{\frac{5}{2}^+})$$

Hybrid baryon models

Minimal quark content, $qqqG$, gluonic field can be in color singlet, octet or decuplet

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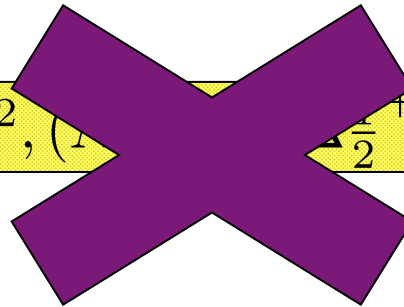
- bag model

$$G \sim 1_{8_c}^+$$

$$(N_{\frac{1}{2}}^+)^2, (N_{\frac{3}{2}}^+)^2, (\Delta_{\frac{1}{2}}^+), (\Delta_{\frac{3}{2}}^+), (N_{\frac{5}{2}}^+)$$

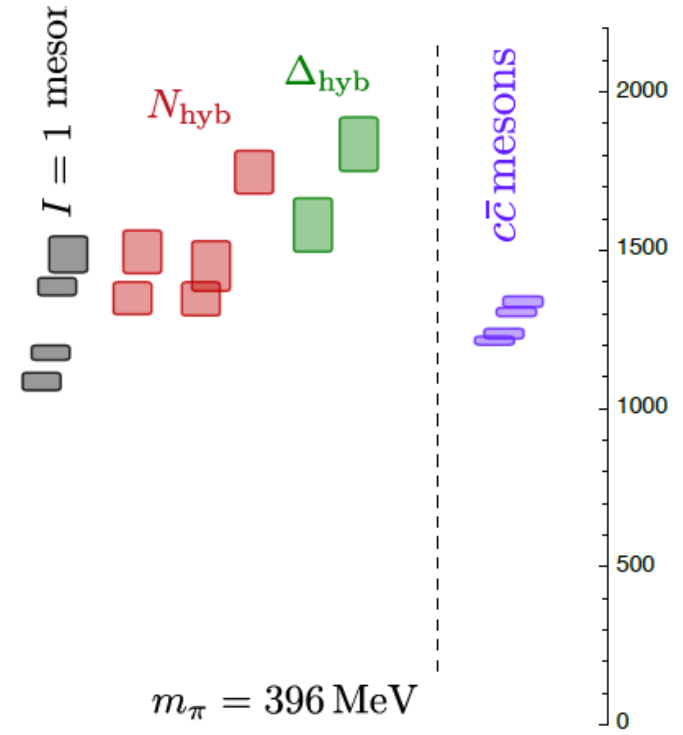
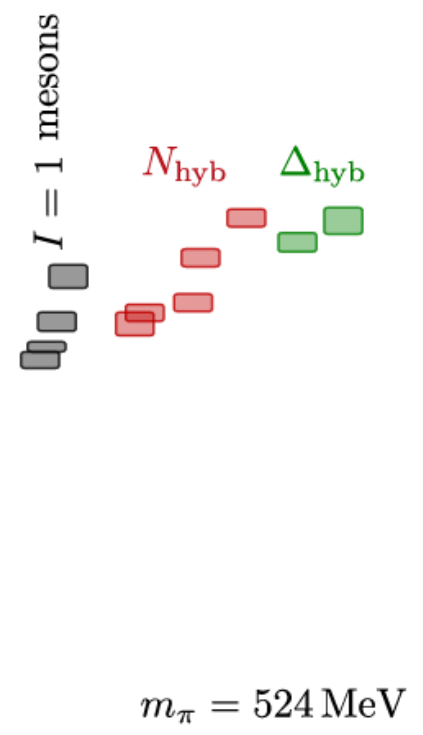
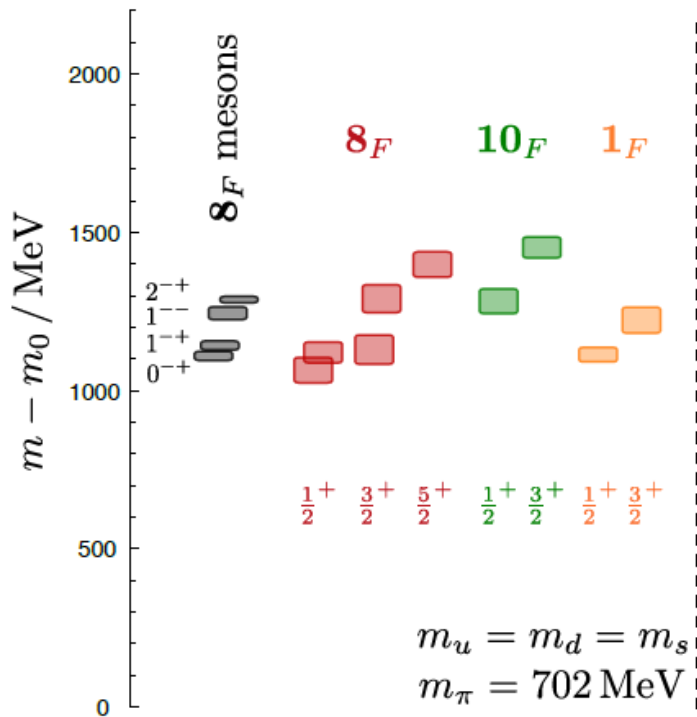
- flux-tube model

$$(N_{\frac{1}{2}}^+)^2, (\Delta_{\frac{1}{2}}^+), (\Delta_{\frac{3}{2}}^+), (\Delta_{\frac{5}{2}}^+)$$



Hybrid hadrons

“subtract off” the quark mass



$m_0 = \begin{cases} m_\rho & \text{light mesons} \\ m_N & \text{baryons} \\ m_{\eta_c} & \text{charmonium} \end{cases}$

Appears to be a single scale for gluonic excitations **~ 1.3 GeV**

Gluonic excitation transforming like a color **octet** with **$J^{PC} = 1^{+-}$**

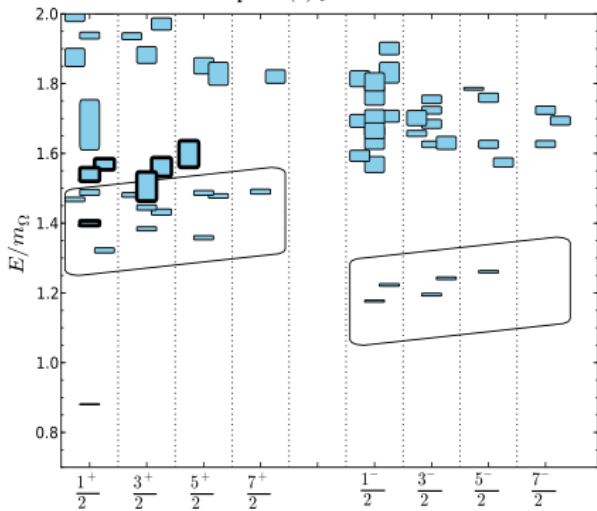
arXiv:1201.2349

SU(3) flavor limit

In SU(3) flavor limit – have exact flavor Octet, Decuplet and Singlet representations

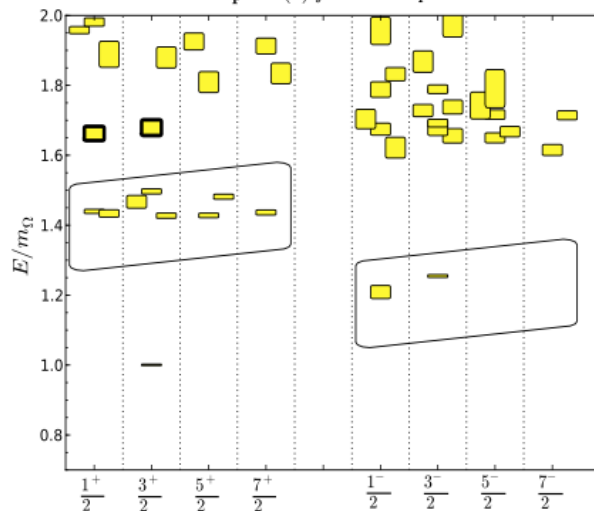
8_F

8_F SU(3) flavor octet



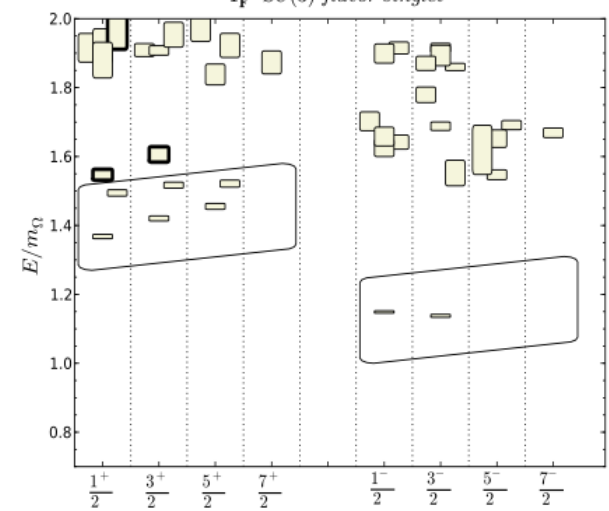
10_F

10_F SU(3) flavor decuplet



1_F

1_F SU(3) flavor singlet



$m_\pi \sim 700$ MeV

Full non-relativistic quark model counting

Additional levels with significant gluonic components

arXiv:1212.5236