

Data Analysis from the Finite Volume

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Bethe Center for
Theoretical Physics

7TH INTERNATIONAL WORKSHOP ON PION-NUCLEON PARTIAL WAVE
ANALYSIS AND THE INTERPRETATION OF BARYON RESONANCES

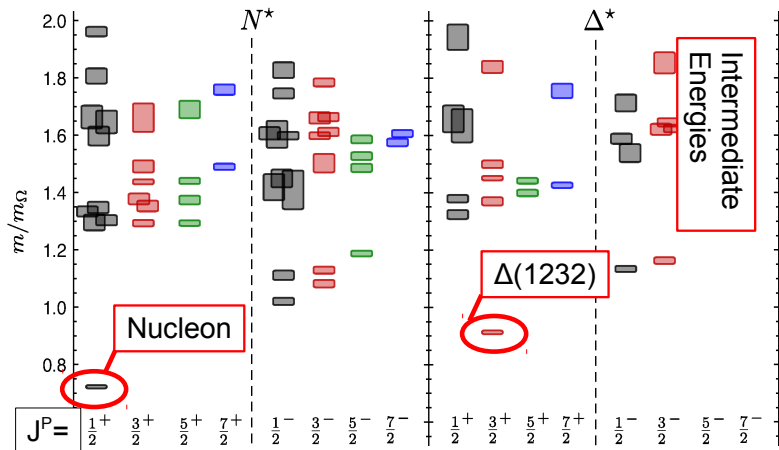
23 - 27 SEPTEMBER 2013, CAMOGLI (ITALY)



The baryon spectrum: N^* and Δ resonances

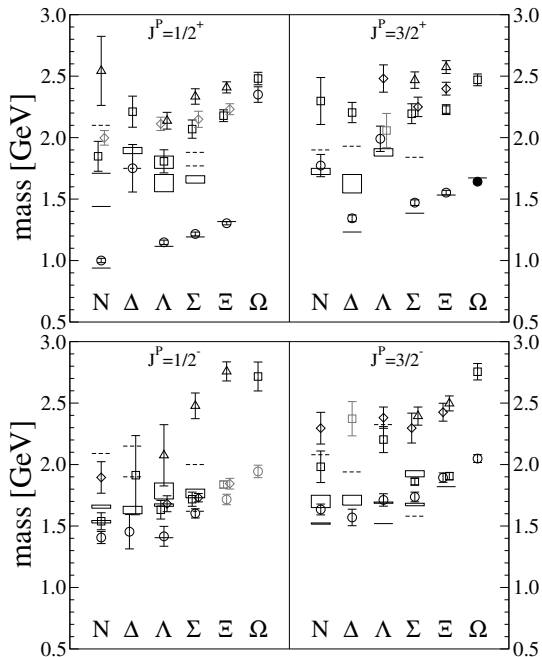
- Many resonances predicted in lattice calculations —
Missing resonance problem from quark model reappears

[Edwards *et al.*, Phys.Rev. D84 (2011)]:



$m_\pi = 396 \text{ MeV (!)}$

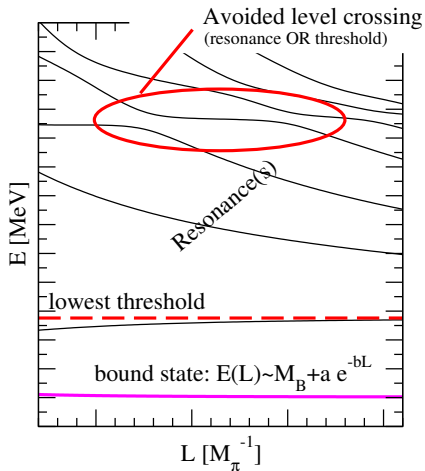
New results from baryon spectroscopy



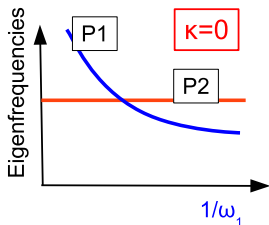
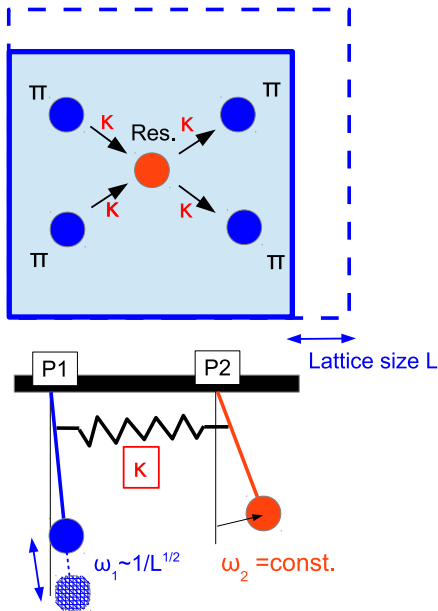
- Seven ensembles
- 200-300 gauge configurations each
- M_π from 255 to 596 MeV
- Lattice size
 $16^3 \times 32$,
 $12^3 \times 24$, $24^3 \times 48$
- Lattice spacing
between 0.1324
and 0.1398 fm
- Extrapolated to the physical pion mass; shown plot: not yet finite-volume corrected

[Engel, Lang, Mohler, Schäfer, PRD (2013)]

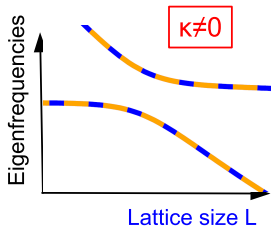
Eigenvalues in the finite volume



Understanding avoided level crossing



Resonance (system P2) decouples from pions in a box (system P1)



Resonance couples to boxed pions

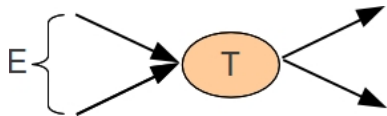
Understanding avoided level crossing

Two-body scattering

Scattering in the infinite volume limit

- Unitarity of the scattering matrix S : $SS^\dagger = \mathbb{1}$ $[S = \mathbb{1} - i \frac{p}{4\pi E} T]$.

$$\text{Im } T^{-1}(E) = \sigma \equiv \frac{p}{8\pi E}$$



- \rightarrow Generic (Lippman-Schwinger) equation for unitarizing the T -matrix:

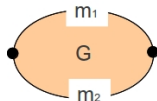
$$T = V + V G T \quad \text{Im } G = -\sigma$$

V : (Pseudo)potential, σ : phase space.

- G : Green's function:

$$G = \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon},$$

$$\omega_{1,2}^2 = m_{1,2}^2 + \vec{q}^2$$



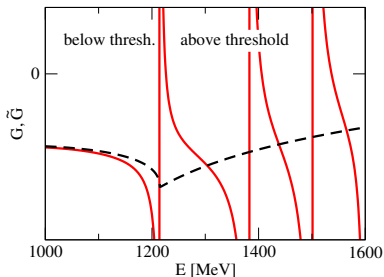
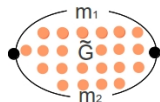
Discretization

Discretized momenta in the finite volume with periodic boundary conditions

$$\Psi(x) \stackrel{!}{=} \Psi(x + \hat{e}_i L) = \exp(i L q_i) \Psi(x) \implies q_i = \frac{2\pi}{L} n_i, \quad n_i \in \mathbb{Z}, \quad i = 1, 2, 3$$

$$\int \frac{d^3 \vec{q}}{(2\pi)^3} g(|\vec{q}|^2) \rightarrow \frac{1}{L^3} \sum_{\vec{n}} g(|\vec{q}|^2), \quad \vec{q} = \frac{2\pi}{L} \vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

$$G \rightarrow \tilde{G} = \frac{1}{L^3} \sum_{\vec{q}} \frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2}$$



- $E > m_1 + m_2$: \tilde{G} has poles at *free energies in the box*, $E = \omega_1 + \omega_2$
- $E < m_1 + m_2$: $\tilde{G} \rightarrow G$ exponentially with L (regular summation theorem).
- Formalism can be mapped to Lüscher's $\mathcal{Z}_{\ell m}$.

Finite \rightarrow infinite volume: the Lüscher equation

Warning: Very crude re-derivation.

- Measured eigenvalues of the Hamiltonian (tower of *lattice levels* $E(L)$)
 \rightarrow Poles of scattering equation \tilde{T} in the finite volume \rightarrow determines V :

$$\tilde{T} = (1 - V\tilde{G})^{-1}V \rightarrow V^{-1} - \tilde{G} \stackrel{!}{=} 0 \rightarrow V^{-1} = \tilde{G}$$

- The interaction V determines the T -matrix in the infinite volume limit:

$$T = (V^{-1} - G)^{-1} = (\tilde{G} - G)^{-1}$$

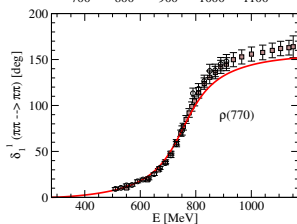
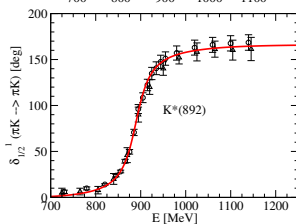
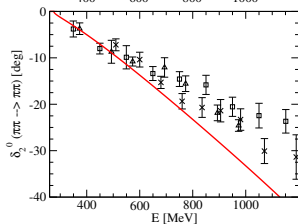
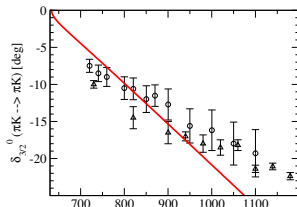
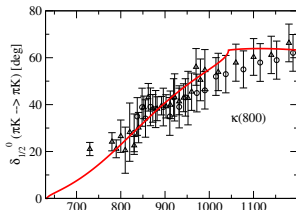
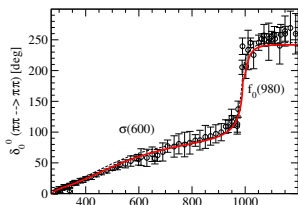
- Re-derivation of Lüscher's equation (T determines the phase shift δ):

$$p \cot \delta(p) = -8\pi\sqrt{s} (\tilde{G}(E) - \text{Re } G(E))$$

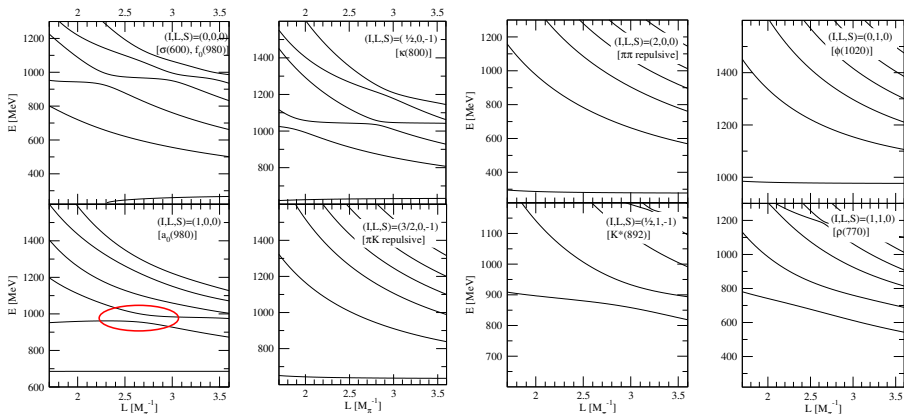
- V and dependence on renormalization have disappeared (!)

Fit to meson-meson PW data using unitary ChPT with NLO terms

[M.D., Meißner, JHEP (2012)] using IAM [Oller, Oset, Peláez, PRC (1999)]



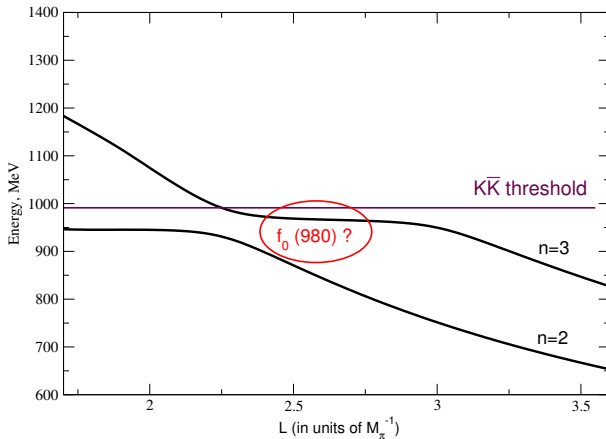
Prediction of levels (also for $M_\pi \neq M_\pi^{\text{phys.}}$)



[M.D., Meißner, JHEP (2012)]

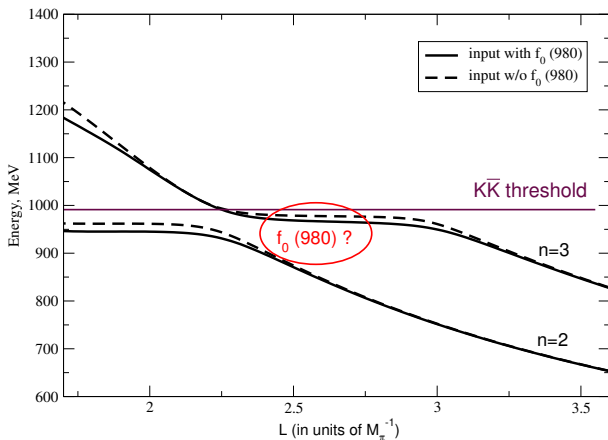
Loops in t - and u -channel (1-loop calculation): [Albaladejo, Rios, Oller, Roca, arXiv: 1307.5169; Albaladejo, Oller, Oset, Rios, Roca, JHEP (2013)]

Avoided level crossing in the energy levels



Using Unitarized Chiral Perturbation Theory to produce energy levels

Changing the input ...



- Weaker coupling to the $K\bar{K}$ channel, $f_0(980)$ disappears
- Avoided level crossing still occurs at the same place: **threshold!**
- The threshold can be moved by using **twisted b.c.!**

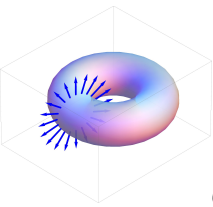
Twisting the boundary conditions (B.C.)

[Bernard, Lage, Meißner, Rusetsky, JHEP (2011), M.D., Meißner, Oset, Rusetsky, EPJA (2011)]

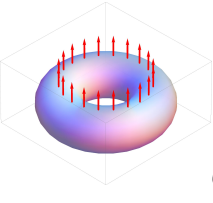
- Periodic B.C.:

$$\Psi(\vec{x} + \hat{e}_i L) = \Psi(\vec{x})$$

- Periodic in 2 dim.:



$$\theta_1 = 0$$

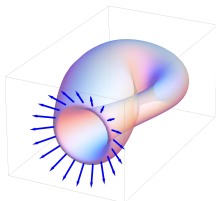


$$\theta_2 = 0$$

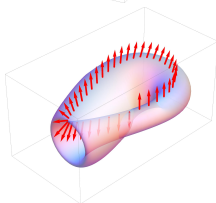
- Twisted B.C.:

$$\Psi(\vec{x} + \hat{e}_i L) = e^{i\theta_i} \Psi(\vec{x})$$

- Periodic/antiperiodic:



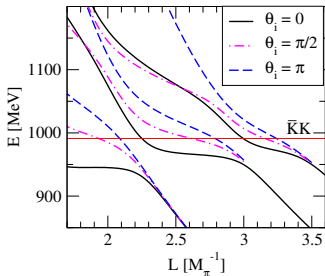
$$\theta_1 = 0$$



$$\theta_2 = \pi$$

Example: the $f_0(980)$

- S -wave, coupled-channels $\pi\pi$, $\bar{K}K$.
- Twisted B.C. for the s -quark:
 $u(\vec{x} + \hat{e}_i L) = u(\vec{x})$
 $d(\vec{x} + \hat{e}_i L) = d(\vec{x})$
 $s(\vec{x} + \hat{e}_i L) = e^{i\theta_i} s(\vec{x})$
- Three unknown transitions
 - $V(\pi\pi \rightarrow \pi\pi)$
 - $V(\pi\pi \rightarrow \bar{K}K)$
 - $V(\bar{K}K \rightarrow \bar{K}K)$



Extracting resonances from lattice data

[M.D., Meißner, Oset/Rusetsky, EPJA 47 (2011)]

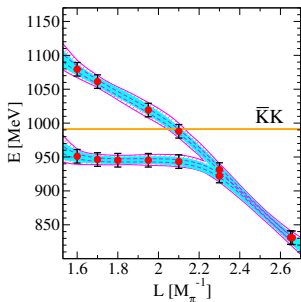
- Need for an interpolation in energy (\rightarrow Unitarized ChPT, ...)
- Expand a **two-channel** transition V in energy ($i, j: \pi\pi, \bar{K}K$):

$$V_{ij}(E) = a_{ij} + b_{ij}(E^2 - 4M_K^2)$$

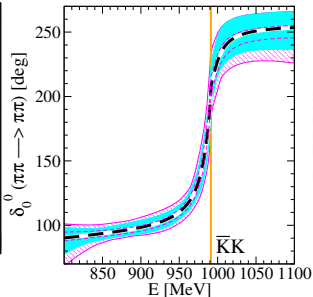
- Include model-independently known LO contribution in a, b .
- Or even NLO contributions (more fit parameters)

[M.D., Meißner, JHEP (2012)].

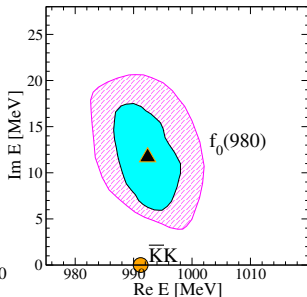
lattice data & fit



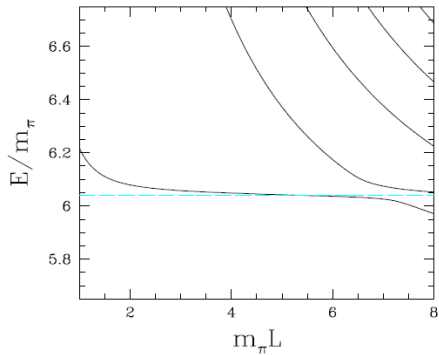
extracted phase shift



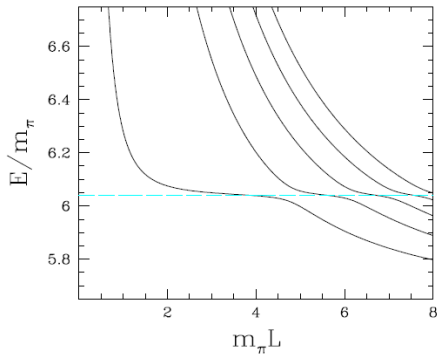
$f_0(980)$ pole position



Moving frames to scan narrow resonances ($\Sigma^* \rightarrow \pi\Lambda$)



$$P = 0$$



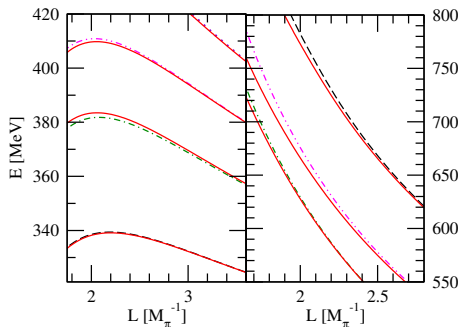
$$P = \frac{2\pi}{L}(0, 0, 1)$$

Breaking of cubic symmetry through boost

Example: Lattice points \vec{q}^* boosted with $P = (0, 0, 0) \rightarrow \frac{2\pi}{L} (0, 0, 2)$:

Mixing of partial waves in boosted multiple channels: $\sigma(600)$

[M.D., E. Oset, A. Rusetsky, EPJA (2012)]



Solid: Levels from A_1^+ .

Non-solid: Neglecting the D -wave.

- $\pi\pi$ & $\bar{K}K$ in S -wave, $\pi\pi$ in D -wave.
- Organization in Matrices (A_1^+), e.g. $\vec{P} = (2\pi/L)(0, 0, 1)$, $(2\pi/L)(1, 1, 1)$, and $(2\pi/L)(0, 0, 2)$:

$$V = \begin{pmatrix} V_S^{(11)} & V_S^{(12)} & 0 \\ V_S^{(21)} & V_S^{(22)} & 0 \\ 0 & 0 & V_D^{(22)} \end{pmatrix}$$

$$\tilde{G} = \begin{pmatrix} \tilde{G}_{00,00}^{R(1)} & 0 & 0 \\ 0 & \tilde{G}_{00,00}^{R(2)} & \tilde{G}_{00,20}^{R(2)} \\ 0 & \tilde{G}_{20,00}^{R(2)} & \tilde{G}_{20,20}^{R(2)} \end{pmatrix}$$

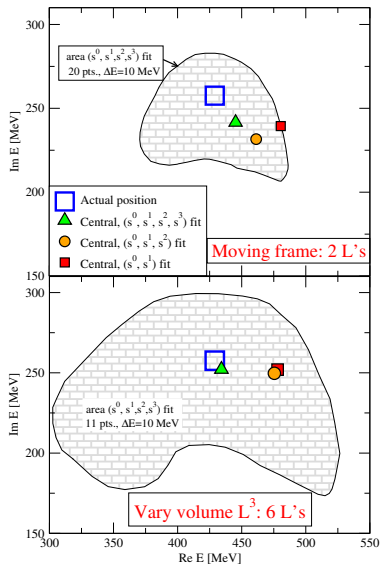
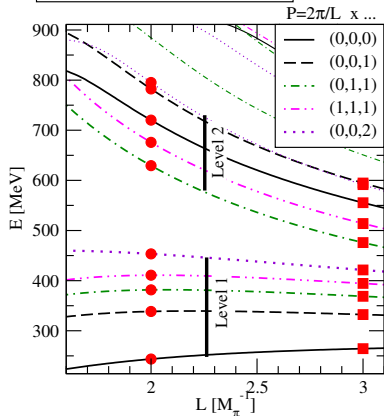
- Phase extraction (κ): Expand and fit V_S , V_P *simultaneously* to different representations instead of
 1. P -wave from B_1 , B_2 , E
 2. S -wave from P and A_1 (reduction of error).

Phase shifts from a moving frame: the $\sigma(600)$

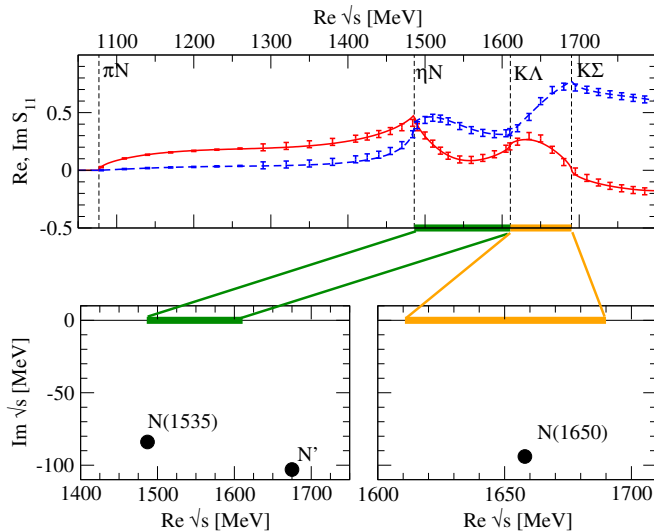
Comparison: *Variation of L vs moving frames*

- The first two levels for the first five boosts:

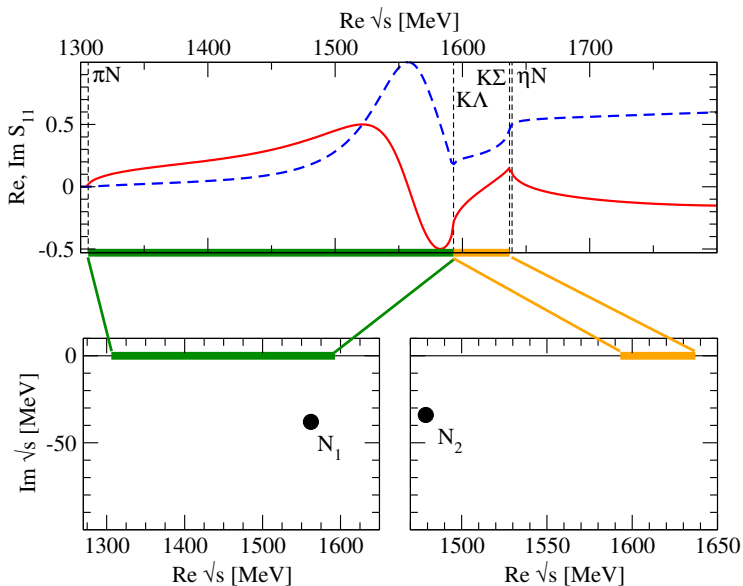
● ■ : Pseudo-data [10 MeV error]



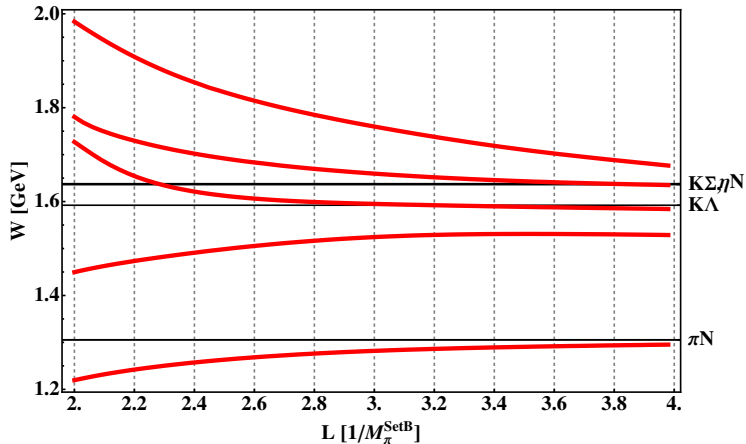
- Unitarized chiral interaction with NLO contact terms (full BS)



Chiral extrapolation to a QCDSF lattice setup



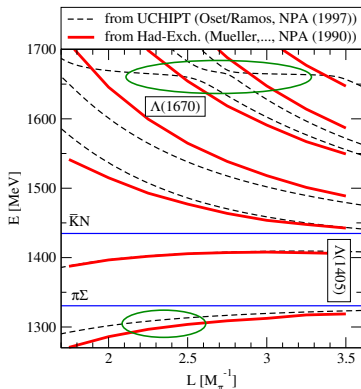
Prediction of the lattice spectrum



- No one-to-one mapping of levels to resonances;
- Need coupled channel analysis;
- Lattice might see hidden poles.

- (Non-factorizing/off-shell) Lippman-Schwinger equation in the finite volume,

$$T^{(P)}(q'', q') = V(q'', q') + \frac{2\pi^2}{L^3} \sum_{i=0}^{\infty} \vartheta^{(P)}(i) \frac{V(q'', q_i) T^{(P)}(q_i, q')}{\sqrt{s} - E_a(q_i) - E_b(q_i)}, \quad q_i = \frac{2\pi}{L} \sqrt{i}.$$



- Access to sub- $\bar{K}N$ -threshold dynamics:
- Discrepancies of lowest levels: levels sensitive to different $\Lambda(1405)$ dynamics.
- One- or two-pole structure:
 - Will NOT lead to additional level.
 - but shifted threshold levels.

$N\pi$ ($1/2^-$) channel

$m_\pi=266$ MeV; distillation method; variational analysis using a basis of N (3 quarks) and $N\pi$ (5 quarks) interpolators;

$$(N_{\pm}^{(i)})_{\mu}(\vec{p}=0) = \sum_{\vec{x}} \epsilon_{abc} \left(P_{\pm} \Gamma_1^{(i)} u_a(\vec{x}) \right)_{\mu} \left(u_b^T(\vec{x}) \Gamma_2^{(i)} d_c(\vec{x}) \right)$$



$$\pi^+(\vec{p}=0) = \sum_{\vec{x}} \bar{d}_a(\vec{x}) \gamma_5 u_a(\vec{x}),$$

$$\pi^0(\vec{p}=0) = \sum_{\vec{x}} \frac{1}{\sqrt{2}} (\bar{u}_a(\vec{x}) \gamma_5 u_a(\vec{x}) - \bar{d}_a(\vec{x}) \gamma_5 d_a(\vec{x}))$$



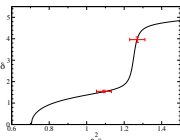
$$O_{N\pi}(I = \frac{1}{2}, I_3 = \frac{1}{2}) = p\pi^0 + \sqrt{2}n\pi^+$$

$$N\pi(\vec{p}=0) = \gamma_5 N_+(\vec{p}=0)\pi(\vec{p}=0)$$



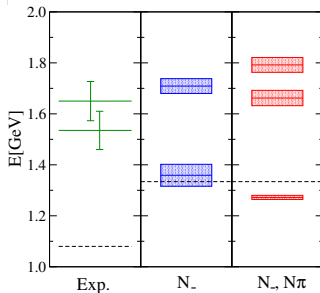
Lüscher relation

→ phase shift:

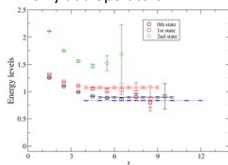


Assuming 2 elastic resonances with identical coupling we get $m_1=1.678$ GeV
 $m_2=1.873$ GeV

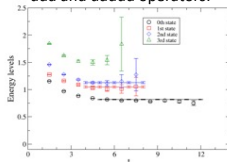
C.B. Lang and V. Verduci, *this conf. and Phys.Rev. D 87, 054502 (2013)*

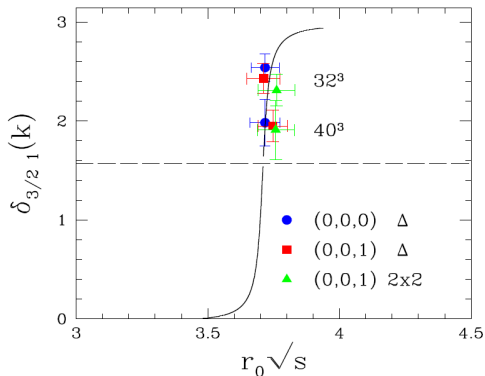


only odd operators:



odd and uddud operators:





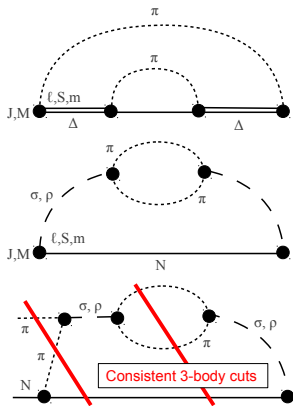
Effective range formula

$$\frac{k^3}{E} \cot \delta_{3/2,1}(k) = \frac{24\pi}{g_{\Delta N\pi}^2} (m_{\Delta}^2 - E^2)$$

$$\Gamma_{\Delta} = \frac{g_{\Delta N\pi}^2}{24\pi} \frac{k_{\Delta}^3}{m_{\Delta}^2} \quad \frac{g_{\Delta N\pi}^2}{4\pi} = 14.6$$

- $\Lambda(1405)$: Eigenlevels found, one of them below $\bar{K}N$ threshold.
- Suppression of the strange quark electromagnetic form factor
 \rightarrow large $\bar{K}N$ component.

Three-particle intermediate states



- πN scattering: Known large inelasticities
 $\pi\pi N [\pi\Delta, \sigma N, \rho N, \dots]$

- $\pi\pi/\pi N$ boosted subsystems.

- Is it enough to include (boosted) 2-particle subsystems in the propagator?

No.

- Three-body s -channel dynamics requires particle exchange transitions. \Rightarrow Three-body unitarity

[Aaron, Almodo, Young, PR 174 (1968) 2022,
 Aitchison, Brehm, PLB 84 (1979) 349, PRD 25 (1982) 3069;
 Hansen, Sharpe, Davoudi, Briceño...]

- Rapid progress in the actual ab-initio calculations of resonances/phase shifts: $\rho(770)$, $a_0(980)$, $K^*(s, p, d)$, $N(1535)$, $N(1650)$, $\Delta(1232)$,
- Close to the physical point, finite volume effects dominate the spectrum.
- Use finite volume effects in your favor: Lüscher & extensions (coupled channels, moving frames, twisted boundary conditions,)
- Energy interpolation needed in many aspects —Unitarized ChPT, coupled-channel, K-matrix approaches can provide a framework.

- Thank you to the Organizers!
- Thank you for slides: R. Briceño, G. Engel, C. Lang, B. Menadue, M. Petschlies, A. Rusetsky, G. Schierholz, M. Wagner, D. Wilson.

Fit to meson-meson data using unitary ChPT with NLO terms

[M.D., Meißner, JHEP (2012)] using IAM [Oller, Oset, Peláez, PRC (1999)]

Unitary extension of ChPT, can be matched to ChPT order-by-order.

Table: Fitted values for the L_i [$\times 10^{-3}$] and q_{\max} [MeV].

L_1	L_2	L_3	L_4
$0.873^{+0.017}_{-0.028}$	$0.627^{+0.028}_{-0.014}$	-3.5 [fixed]	$-0.710^{+0.022}_{-0.026}$
L_5	$L_6 + L_8$	L_7	q_{\max} [MeV]
$2.937^{+0.048}_{-0.094}$	$1.386^{+0.026}_{-0.050}$	$0.749^{+0.106}_{-0.074}$	981 [fixed]

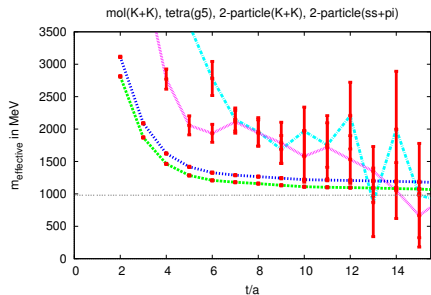
- A resonance is characterized by its (complex) pole position and residues, corresponding to resonance mass, width, and branching ratio.

Table: Pole positions z_0 [MeV] and residues $a_{-1}[M_\pi]$ in different channels.

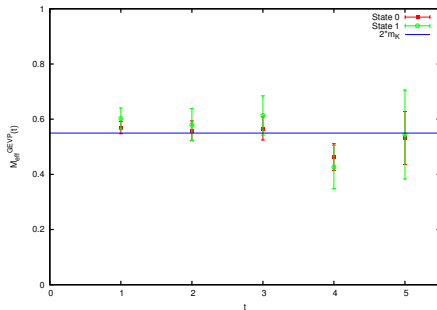
I, L, S : isospin, angular momentum, strangeness.

I	L	S	Resonance	sheet	z_0 [MeV]	a_{-1} [M_π]	a_{-1} [M_π]
0	0	0	$\sigma(600)$	pu	$434+i261$	$-31-i19$ ($\bar{K}K$)	$-30+i86$ ($\pi\pi$)
0	0	0	$f_0(980)$	pu	$1003+i15$	$16-i79$ ($\bar{K}K$)	$-12+i4$ ($\pi\pi$)
1/2	0	-1	$\kappa(800)$	pu	$815+i226$	$-36+i39$ (ηK)	$-30+i57$ (πK)
1	0	0	$a_0(980)$	pu	$1019-i4$	$-10-i107$ ($\bar{K}K$)	$21-i31$ ($\pi\eta$)
0	1	0	$\phi(1020)$	p	$976+i0$	$-2+i0$ ($\bar{K}K$)	—
1/2	1	-1	$K^*(892)$	pu	$889+i25$	$-10+i0.1$ (ηK)	$14+i4$ (πK)
1	1	0	$\rho(770)$	pu	$755+i95$	$-11+i2$ ($\bar{K}K$)	$33+i17$ ($\pi\pi$)

[Wagner, Daldrop, Abdel-Rehim, Urbach *et. al.* [ETMC], JHEP (2013) & new results]



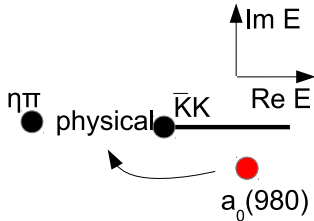
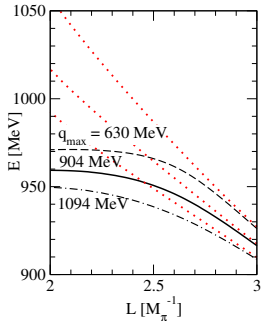
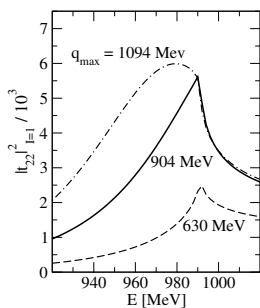
- $M_\pi \sim 300$ MeV, no singly disconnected diagrams.
- Operators: $\bar{K}K$ molecular, diquark-antidiquark, meson-meson.
- Two low-lying states, large overlap with meson-meson.



- $M_\pi \sim 300$ MeV, singly disconnected diagrams included.
- Operators: $q\bar{q}$, $\bar{K}K$ molecular.
- Again, two low lying states, no information on additional state.

The $a_0(980)$ in a multi-channel environment

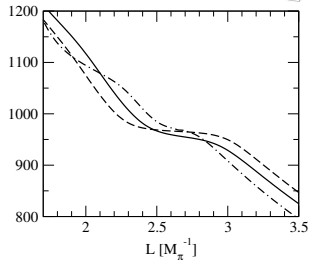
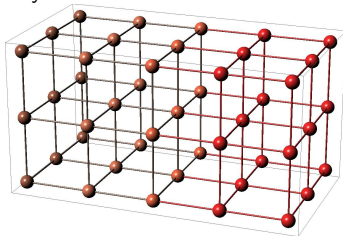
M.D., Meißner, Oset, Rusetsky, EPJA (2011); see also Lage, Meißner, Rusetsky, PLB (2009)



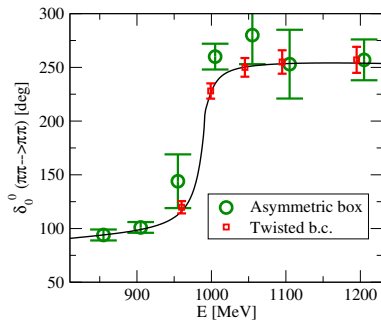
Disentangling coupled channels (the rigorous but impossible scenario)

Three measurements at the same E requires tuning of L

Asymmetric boxes



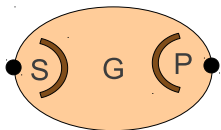
- $L_x = L, L_y = L, L_z = x L$,
where $x = 0.6$ (solid lines),
 $x = 1.0$ (dashed lines), $x = 1.4$
(dash-dotted lines)



Mixing of partial waves

Example: *S*- and *P*-waves

- Infinite volume limit: Rotational symmetry



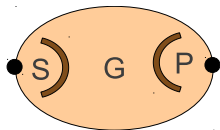
$$\int \frac{d^3\vec{q}}{(2\pi)^3} g(|\vec{q}|) Y_{\ell m}(\theta, \phi) Y_{\ell' m'}^*(\theta, \phi) \sim \delta_{\ell\ell'} \delta_{mm'}.$$

S → S	0	0	0
0	P ₋₁ → P ₋₁	0	0
0	0	P ₀ → P ₀	0
0	0	0	P ₁ → P ₁

Mixing of partial waves

Example: *S*- and *P*-waves

- Infinite volume limit: **Rotational symmetry**



$$\int \frac{d^3\vec{q}}{(2\pi)^3} g(|\vec{q}|) Y_{\ell m}(\theta, \phi) Y_{\ell' m'}^*(\theta, \phi) \sim \delta_{\ell\ell'} \delta_{mm'}.$$

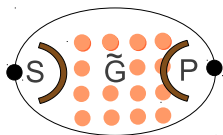
- Wigner-Eckart theorem:

$S \rightarrow S$	0	0	0
0	$P_{-1} \rightarrow P_{-1}$	0	0
0	0	Equal	0
0	0	0	$P_1 \rightarrow P_1$

Mixing of partial waves

Example: *S*- and *P*-waves

- Finite volume: Rotational symmetry \rightarrow Cubic symmetry



$$\frac{1}{L^3} \sum_{\vec{n}} g(|\vec{q}|) Y_{\ell m}(\theta, \phi) Y_{\ell' m'}^*(\theta, \phi) \sim A_{\ell \ell' m m'}$$

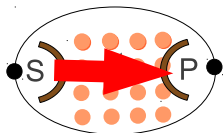
- *S* – *G*-wave mixing, but *S* – *P* waves still orthogonal:

S \rightarrow S	0	0	0
0	P ₋₁ \rightarrow P ₋₁	0	0
0	0	P ₀ \rightarrow P ₀	0
0	0	0	P ₁ \rightarrow P ₁

Mixing of partial waves

Example: S - and P -waves

- Finite volume & boost: Cubic symmetry \rightarrow subgroups of cubic symmetry



$$\frac{1}{L^3} \sum_{\vec{n}} g(|\vec{q}|) Y_{\ell m}(\theta, \phi) Y_{\ell' m'}^*(\theta, \phi) \sim A_{\ell \ell' m m'}.$$

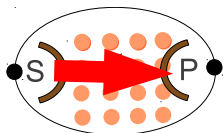
- For boost $P = \frac{2\pi}{L} (0,1,1)$:

S \rightarrow S	0	S \rightarrow P ₀	0
0	P ₋₁ \rightarrow P ₋₁	0	P ₋₁ \rightarrow P ₁
P ₀ \rightarrow S	0	P ₀ \rightarrow P ₀	0
0	P ₁ \rightarrow P ₋₁	0	P ₁ \rightarrow P ₁

Mixing of partial waves

Example: S - and P -waves

- Finite volume & boost: Cubic symmetry \rightarrow subgroups of cubic symmetry



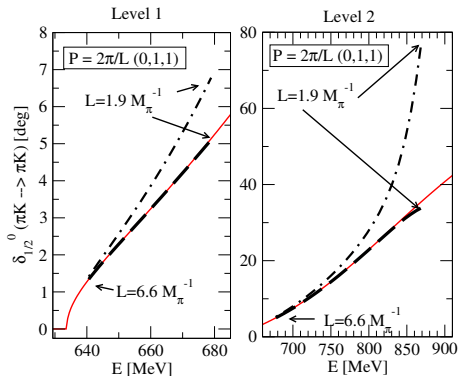
$$\frac{1}{L^3} \sum_{\vec{n}} g(|\vec{q}|) Y_{\ell m}(\theta, \phi) Y_{\ell' m'}^*(\theta, \phi) \sim A_{\ell \ell' m m'}.$$

- More complicated boosts:

$S \rightarrow S$	$S \rightarrow P_{-1}$	$S \rightarrow P_0$	$S \rightarrow P_1$
$P_{-1} \rightarrow S$	$P_{-1} \rightarrow P_{-1}$	$P_{-1} \rightarrow P_0$	$P_{-1} \rightarrow P_1$
$P_0 \rightarrow S$	$P_0 \rightarrow P_{-1}$	$P_0 \rightarrow P_0$	$P_0 \rightarrow P_1$
$P_1 \rightarrow S$	$P_1 \rightarrow P_{-1}$	$P_1 \rightarrow P_0$	$P_1 \rightarrow P_1$

Knowledge of P -wave (from separate analysis of lattice data) allows to disentangle the S -wave:

$$p \cot \delta_S = -8\pi E \frac{p \cot \delta_P \hat{G}_{SS} - 8\pi E (\hat{G}_{SP}^2 - \hat{G}_{SS} \hat{G}_{PP})}{p \cot \delta_P + 8\pi E \hat{G}_{PP}}$$

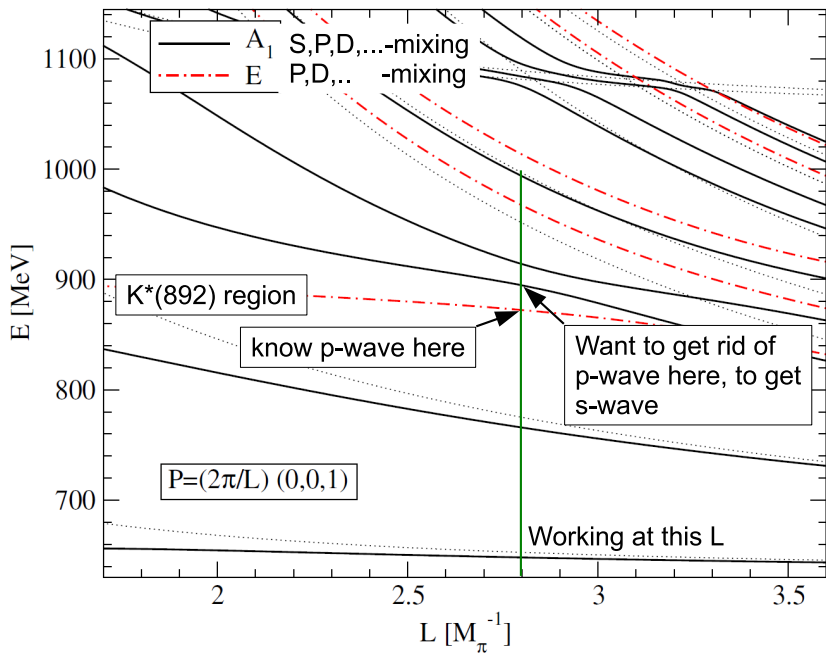


- $\delta_S \equiv \delta_{1/2}^0(\pi K \rightarrow \pi K)$
- Red solid: Actual S -wave phase shift.
- Dash-dotted: Reconstructed S -wave phase shift, PW-mixing ignored.
- Dashed: Reconstructed S -wave phase shift, PW-mixing disentangled.
- small p -wave: Level shift

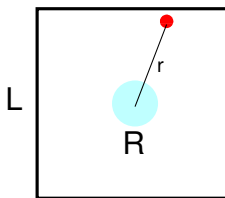
$$\Delta E \simeq -\frac{6\pi E_S \delta_P}{L^3 p \omega_1 \omega_2}$$

Rummukainen, Gottlieb, NPB (1995); Kim, Sachrajda, Sharpe, NPB (2005); Davoudi, Savage, PRD (2011), Z. Fu, PRD (2012); Leskovec, Prelovsek, PRD (2012); Dudek, Edwards, Thomas, PRD (2012); Hansen, Sharpe, PRD 86 (2012); Briceño, Davoudi, arXiv:1204.1110; Göckeler, Horsley, Lage, Meißner, Rakow, Rusetsky, Schierholz, Zanotti, PRD (2012)

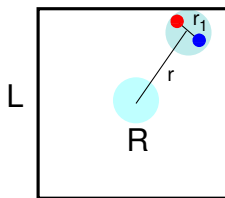
The need for an interpolation in energy ($K\pi$ scattering)



Three particles in a finite volume



2 particles



3 particles

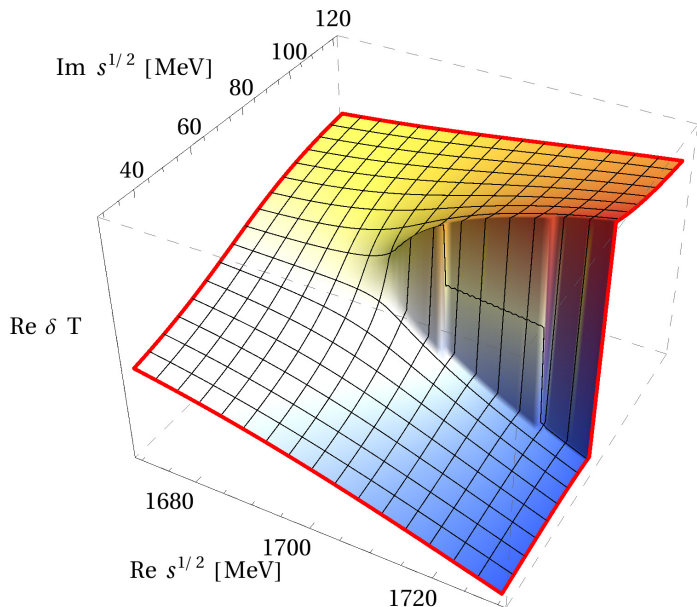
- In case of 2 particles: $r \gg R$, when particles are near the walls
- In case of 3 particles: it may happen that $r \gg R$, $r_1 \simeq R$, when the particles are near the walls

The problem with the disconnected contributions: is the finite-volume spectrum in the 3-particle case determined solely through the on-shell scattering matrix?

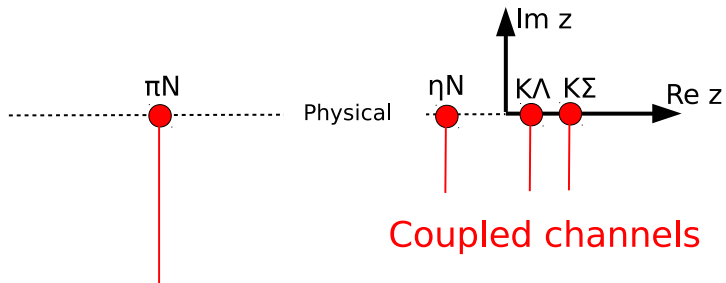
- Despite the presence of the disconnected contributions, the energy spectrum of the 3-particle system in a finite box is still determined by the on-shell scattering matrix elements in the infinite volume

Three particles: Threshold openings in the complex plane

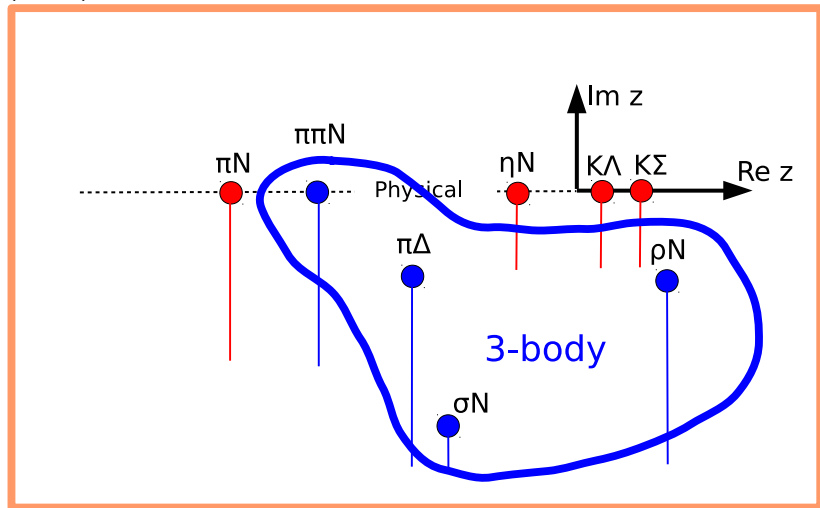
Existence shown model-independently in [S. Ceci, M.D., C. Hanhart et. al., PRC 84 (2011)]



$(z = E)$



$(z = E)$

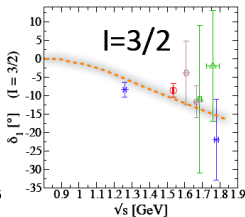
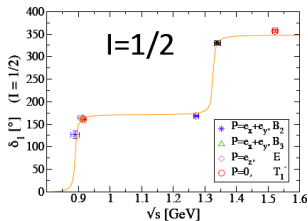


$K\pi$ scattering and K^* width in moving frames

Prelovsek, Leskovec, [Lang](#), Mohler,
this conf. and [arXiv: 1307.0736](#)

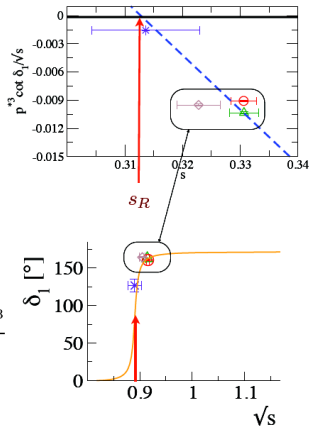
p-wave, coupled system of 5 $q\bar{q}$ and 3 $K\pi$ operators,
total momentum $P=(000),(001),(011)$

Representations B_2, B_3, E, T_1^-

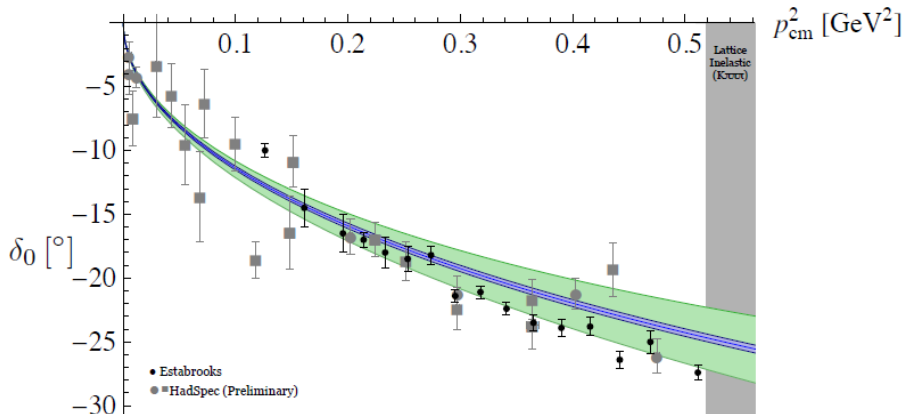


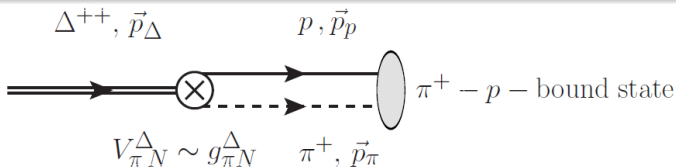
$$\frac{p^{*3}}{\sqrt{s}} \cot \delta_1(s) = \left[\sum_{K^*} \frac{g_{K^*}^2}{6\pi} \frac{1}{m_{K^*}^2 - s} \right]^{-1} \quad \Gamma[K^* \rightarrow K\pi] = \frac{g^2 p^{*3}}{6\pi s}$$

	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]	$m_{K^*(1410)}$ [GeV]	$g_{K^*(1410)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6	1.33 ± 0.02	input
exp	891.66 ± 0.26	5.72 ± 0.06	1.414 ± 0.0015	1.59 ± 0.03

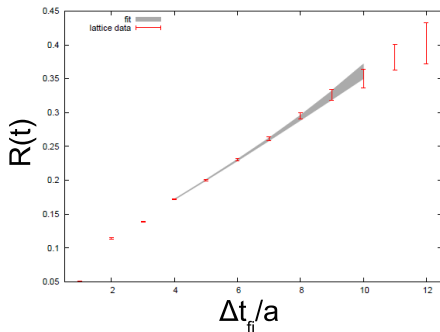


D. Wilson [HadSpec], Lattice 2013.





$$R(\Delta t_{fi}, \vec{Q}, \vec{q}) = \frac{C_\mu^{\Delta \rightarrow \pi N}(\Delta t_{fi}, \vec{Q}, \vec{q})}{\sqrt{C_\mu^\Delta(\Delta t_{fi}, \vec{Q}) C^{\pi N}(\Delta t_{fi}, \vec{Q}, \vec{q})}} \sim \langle \Delta | H | \pi N \rangle \Delta t_{fi}$$



$$\langle \Delta | H | \pi N \rangle \rightarrow g_{\pi N}^\Delta$$

$$g_{\pi N}^\Delta(\text{lat}) = 27.0 \pm 0.6 \pm 1.5$$

Rakow/QCDSF [Lattice 2013]:
 $\pi^+ p \rightarrow K^+ \Sigma^+$: $\sigma \sim 0.4$ (0.2) fm
 at SU(3) symmetrical point

