

# *Charting the Interaction Between Light Quarks*

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<http://www.phy.anl.gov/theory/staff/cdr.html>

# Universal Truths



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# PRIDE AND PREJUDICE

Starring  
Colin Firth  
and  
Jennifer Ehle



## Universal Truths



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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## Universal Truths

- Spectrum of excited states and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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- Spectrum of excited states and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
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- Running of quark mass entails that calculations at even modest  $Q^2$  require a Poincaré-covariant approach. **Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.**





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- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.



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- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. **Higgs mechanism is irrelevant to light-quarks.**
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. **Problem** because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.



# QCD's Challenges



[First](#)

[Contents](#)

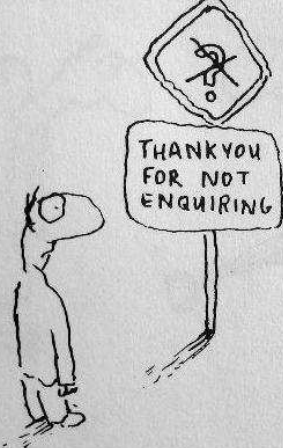
[Back](#)

[Conclusion](#)



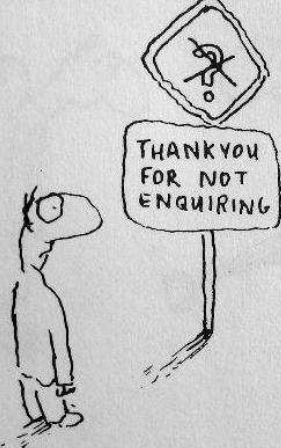
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- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.



## Understand Emergent Phenomena

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- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour  
arises from apparently simple rules



# *Dichotomy of Pion*

## *– Goldstone Mode and Bound state*

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[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)





# *Dichotomy of Pion*

## *– Goldstone Mode and Bound state*

- How does one make an **almost massless** particle  
..... from two **massive** constituent-quarks?





# Dichotomy of Pion

## – Goldstone Mode and Bound state

- How does one make an almost massless particle ..... from two massive constituent-quarks?
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Must exhibit  $m_\pi^2 \propto m_q$

Current Algebra ... 1968





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The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

- well-defined and valid chiral limit;
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**Highly Nontrivial**



# What's the Problem?



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# What's the Problem?

- Minimal requirements
  - detailed understanding of connection between **Current-quark** and **Constituent-quark** masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.



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- Differences!



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## Relativistic QFT!

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  - Here relativistic effects are crucial – *virtual particles*, quintessence of **Relativistic Quantum Field Theory** – must be included



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  - Interaction between quarks – the **Interquark “Potential”** – **unknown** throughout **> 98%** of a hadron's volume



# *Intranucleon Interaction*



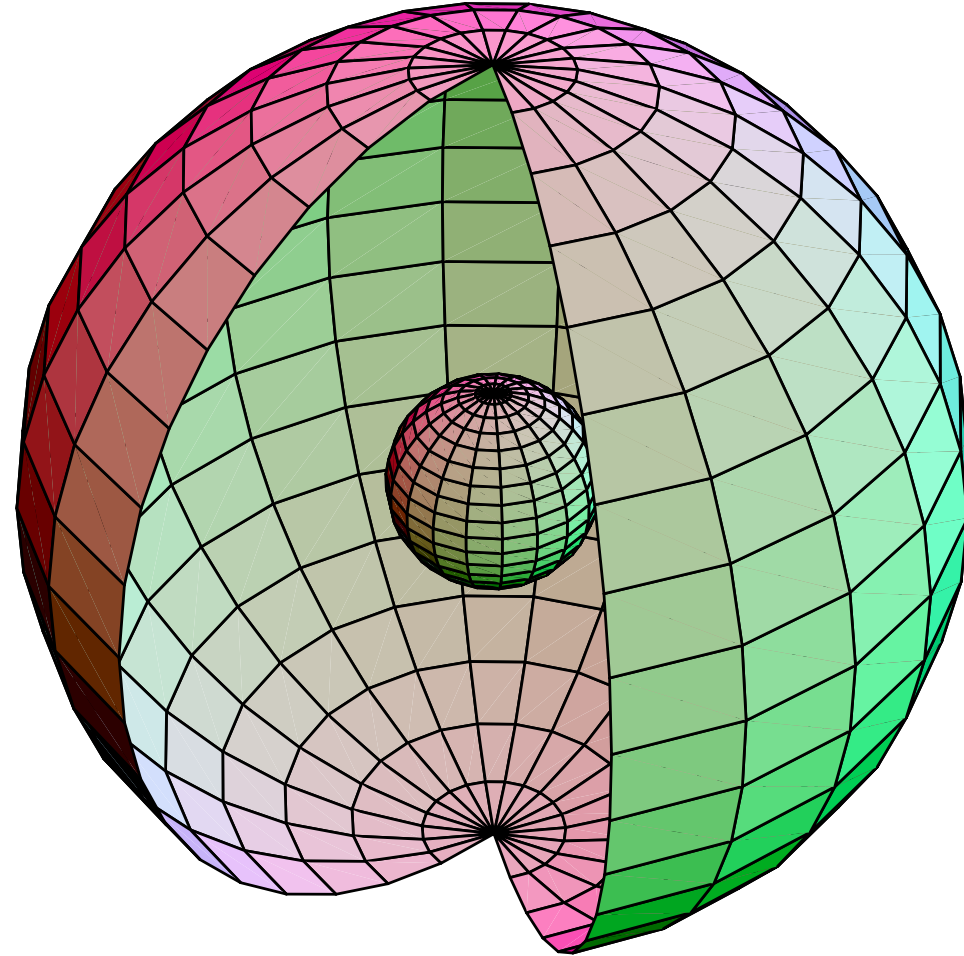
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# Intranucleon Interaction



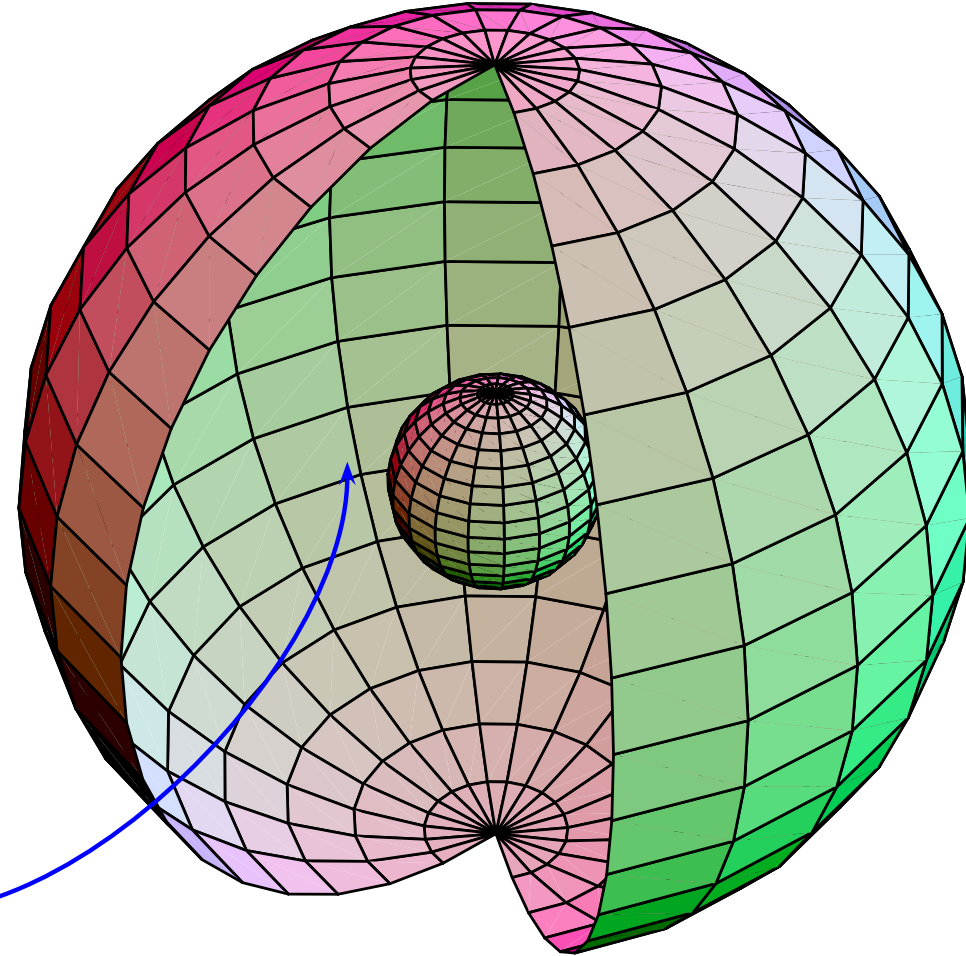
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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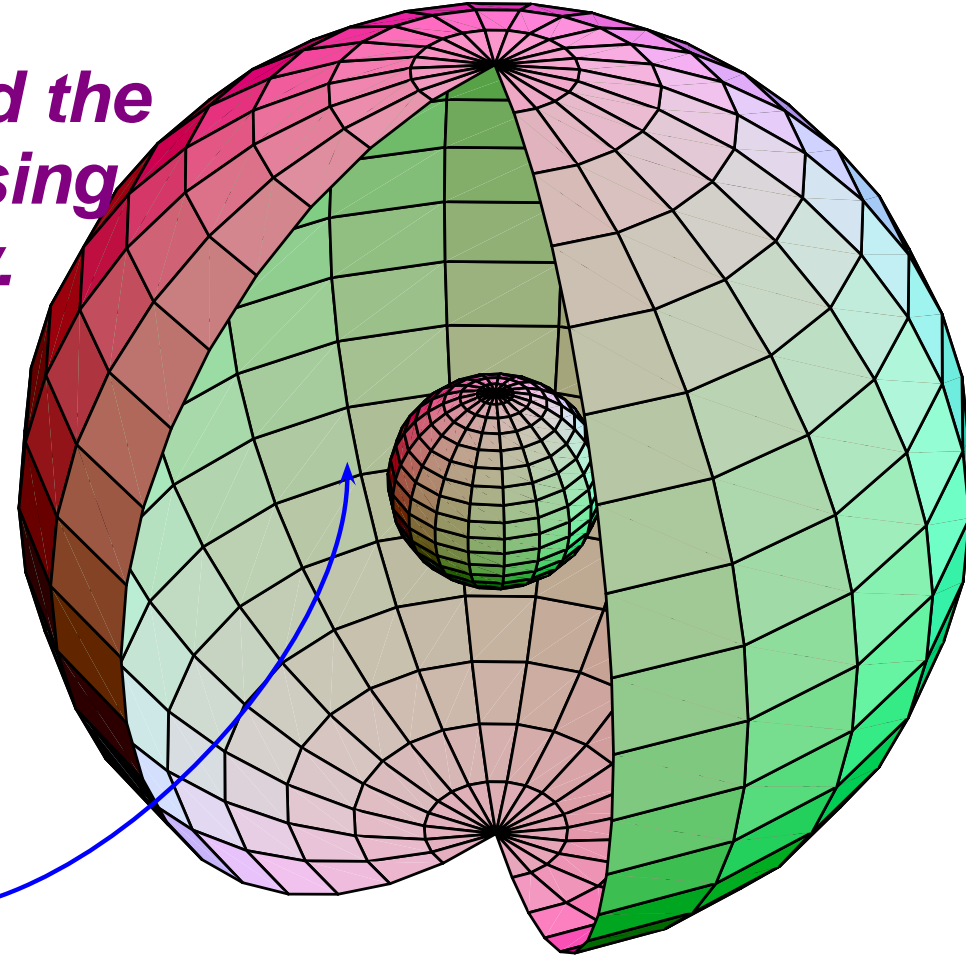


98% of the volume



# What is the Intranucleon Interaction?

*The question must be rigorously defined, and the answer mapped out using experiment and theory.*



**98% of the volume**



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First

Contents

Back

Conclusion

# Frontiers of Nuclear Science: A Long Range Plan (2007)



[First](#)

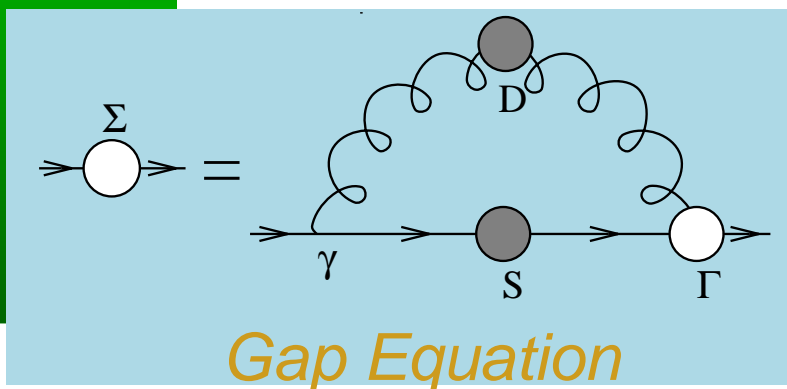
[Contents](#)

[Back](#)

[Conclusion](#)



# Frontiers of Nuclear Science: Theoretical Advances



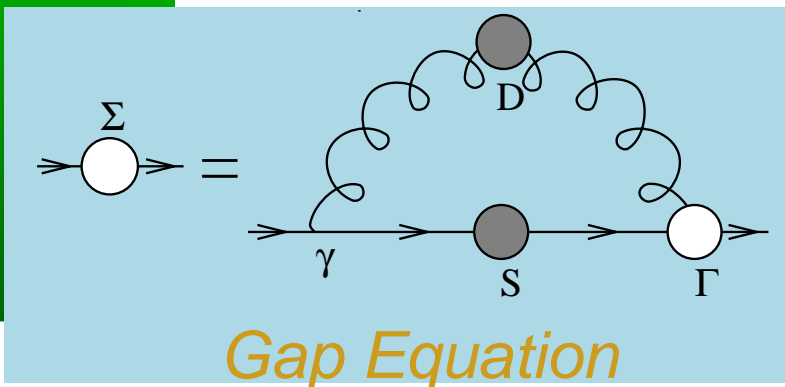
[First](#)

[Contents](#)

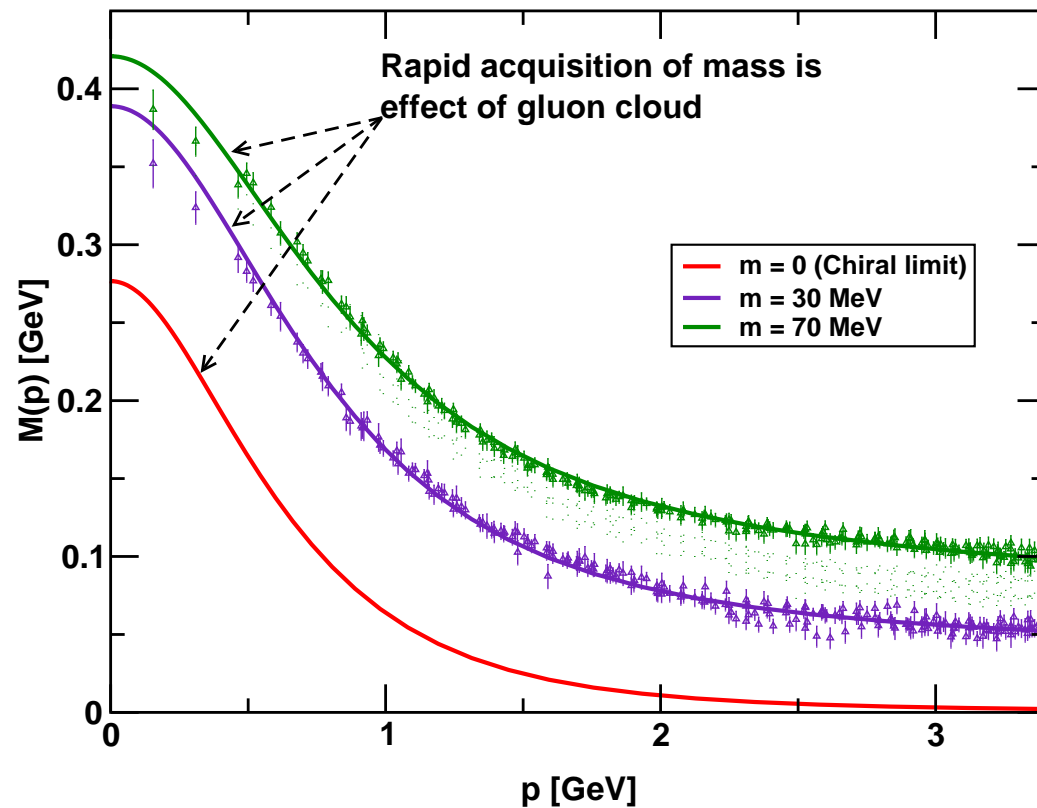
[Back](#)

[Conclusion](#)

# Frontiers of Nuclear Science: Theoretical Advances



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



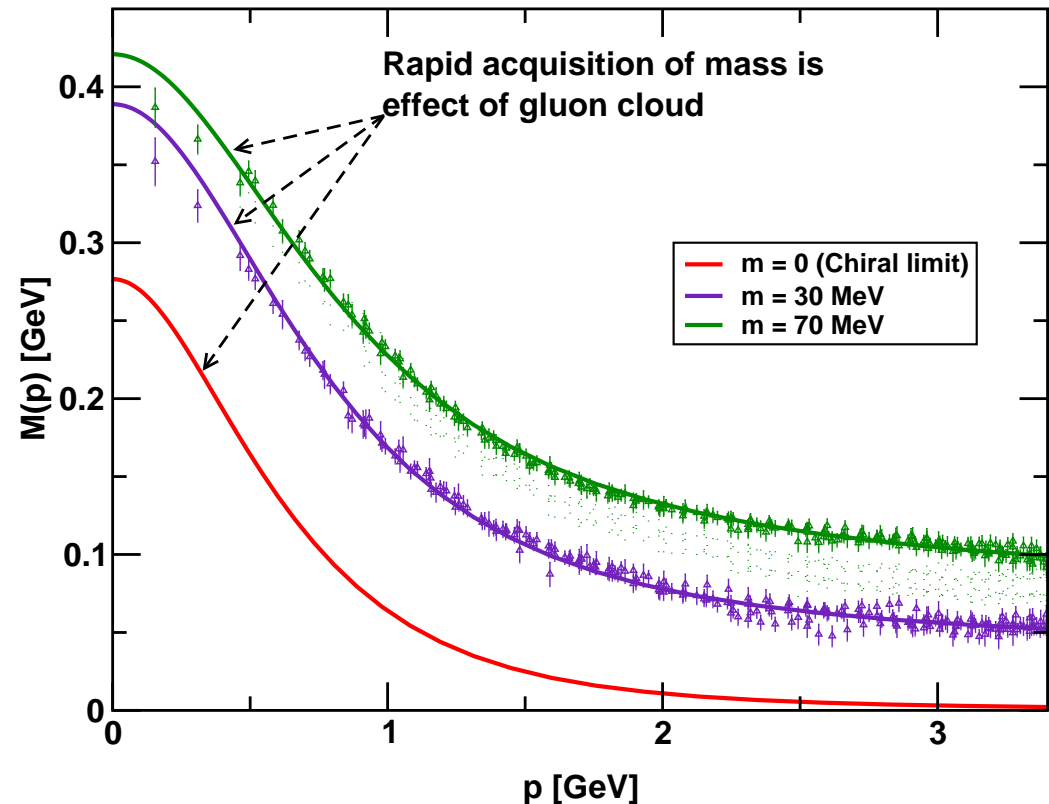
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# Frontiers of Nuclear Science: Theoretical Advances

## Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ( $m = 0$ , red curve) acquires a large constituent mass at low energies.

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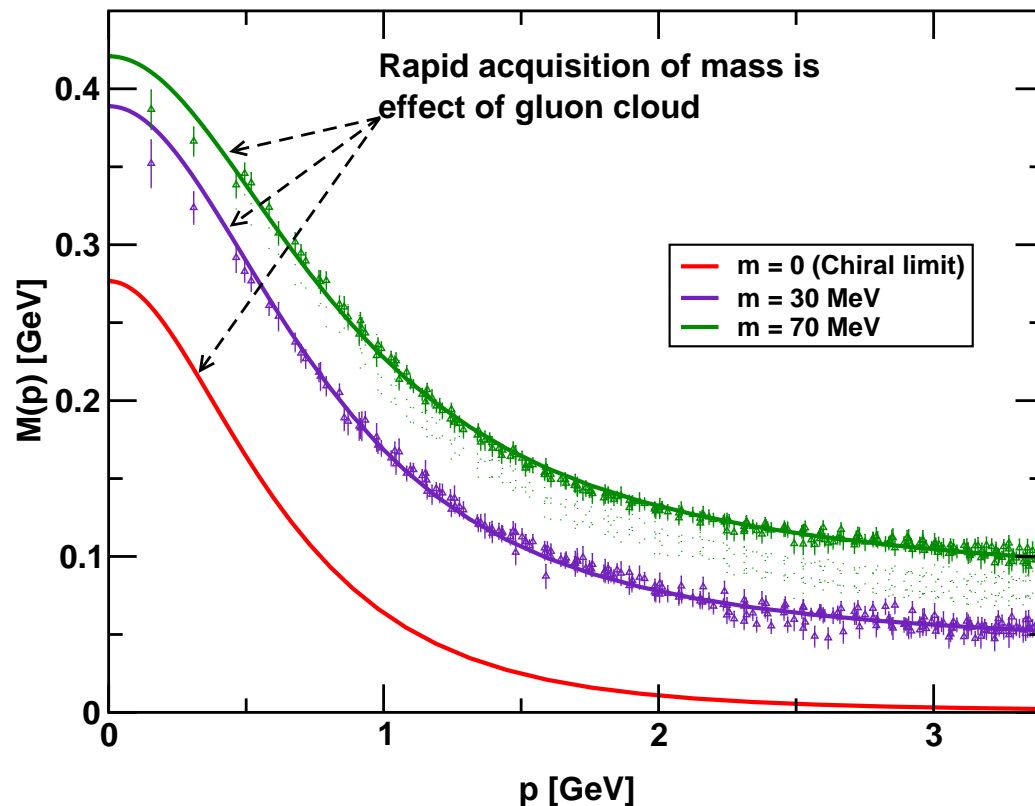


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[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



- Established understanding of two- and three-point functions



# Hadrons



- Established understanding of two- and three-point functions
- What about bound states?



- Without bound states, Comparison with experiment is **impossible**

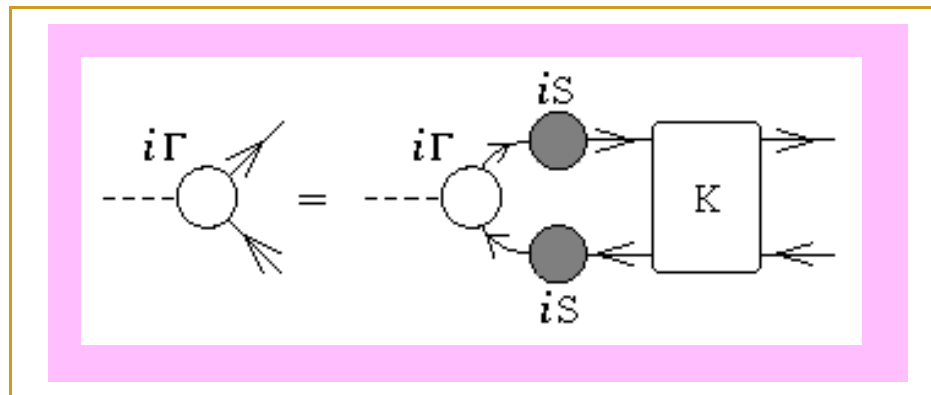




- Without bound states, Comparison with experiment is **impossible**
- They appear as pole contributions to  $n \geq 3$ -point colour-singlet Schwinger functions

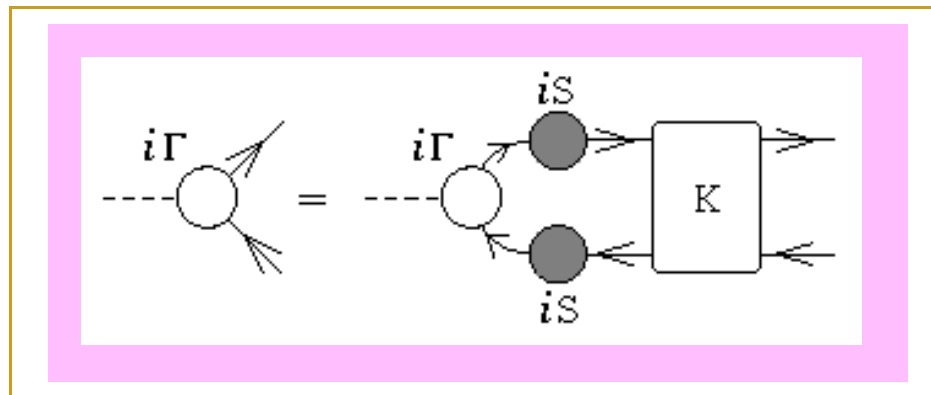


- Without bound states, Comparison with experiment is **impossible**
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel,  $K$ ?
- or What is the **long-range** potential in QCD?

# What is the light-quark Long-Range Potential?



# What is the light-quark Long-Range Potential?



Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD **is not related** in any simple way to the light-quark interaction.



# Bethe-Salpeter Kernel



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-) \\ - M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

## QFT Statement of Chiral Symmetry



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- **Nontrivial** constraint





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- Relation **must** be preserved by truncation
- **Failure**  $\Rightarrow$  Explicit Violation of QCD's Chiral Symmetry



# Persistent Challenge



[First](#)

[Contents](#)

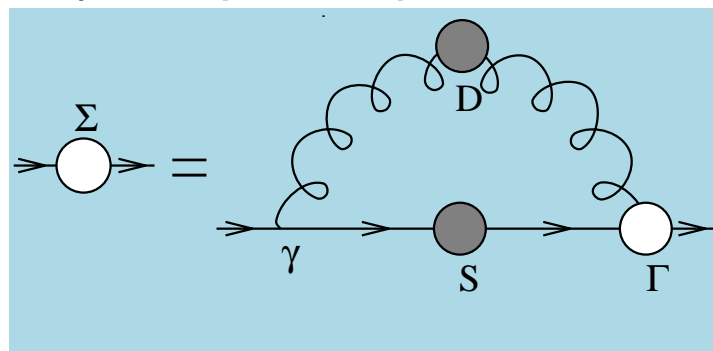
[Back](#)

[Conclusion](#)



# Persistent Challenge

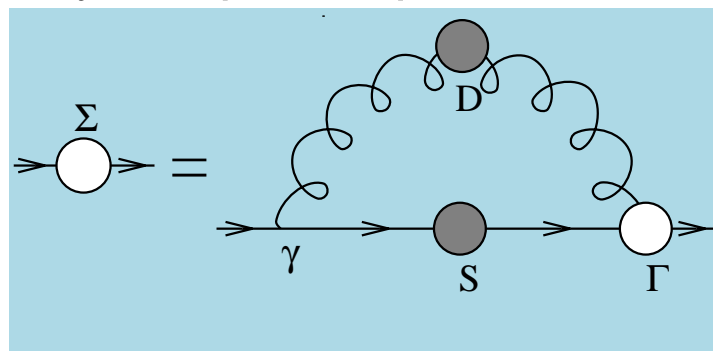
## ● Infinitely Many Coupled Equations





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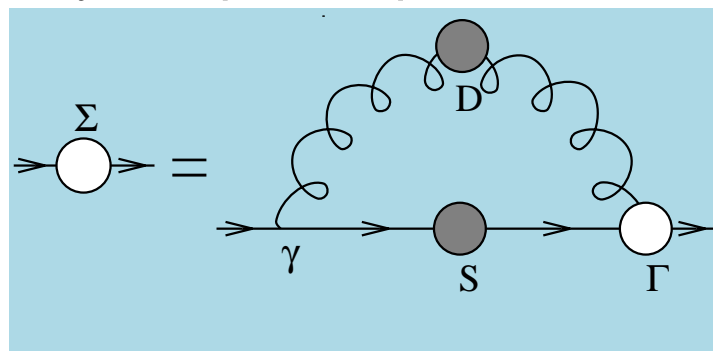
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- Coupling between equations **necessitates** truncation
  - Weak coupling expansion  $\Rightarrow$  Perturbation Theory

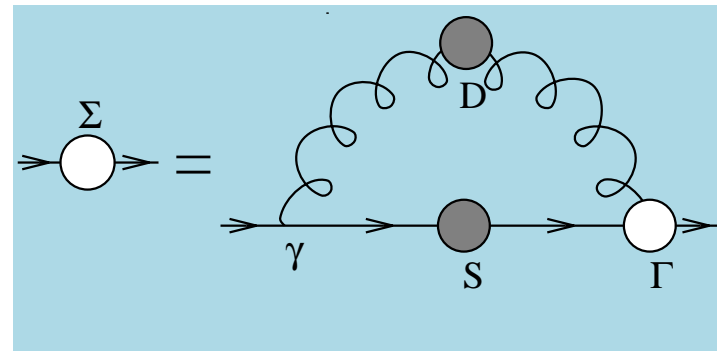






# Persistent Challenge

- Infinitely Many Coupled Equations



- Coupling between equations **necessitates** truncation
  - Weak coupling expansion  $\Rightarrow$  Perturbation Theory  
**Not useful** for the nonperturbative problems in which we're interested





# Persistent Challenge

- Infinitely Many Coupled Equations
  - There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- H.J. Munczek Phys. Rev. D **52** (1995) 4736  
*Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
- A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7  
*Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*





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# Radial Excitations & Chiral Symmetry



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(Maris, Roberts, Tandy  
nu-th/9707003 )

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$



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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Mass<sup>2</sup> of pseudoscalar hadron





# Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy  
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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

$$\mathcal{M}_H := \text{tr}_{\text{flavour}} \left[ M_{(\mu)} \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g.,  $T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5)$



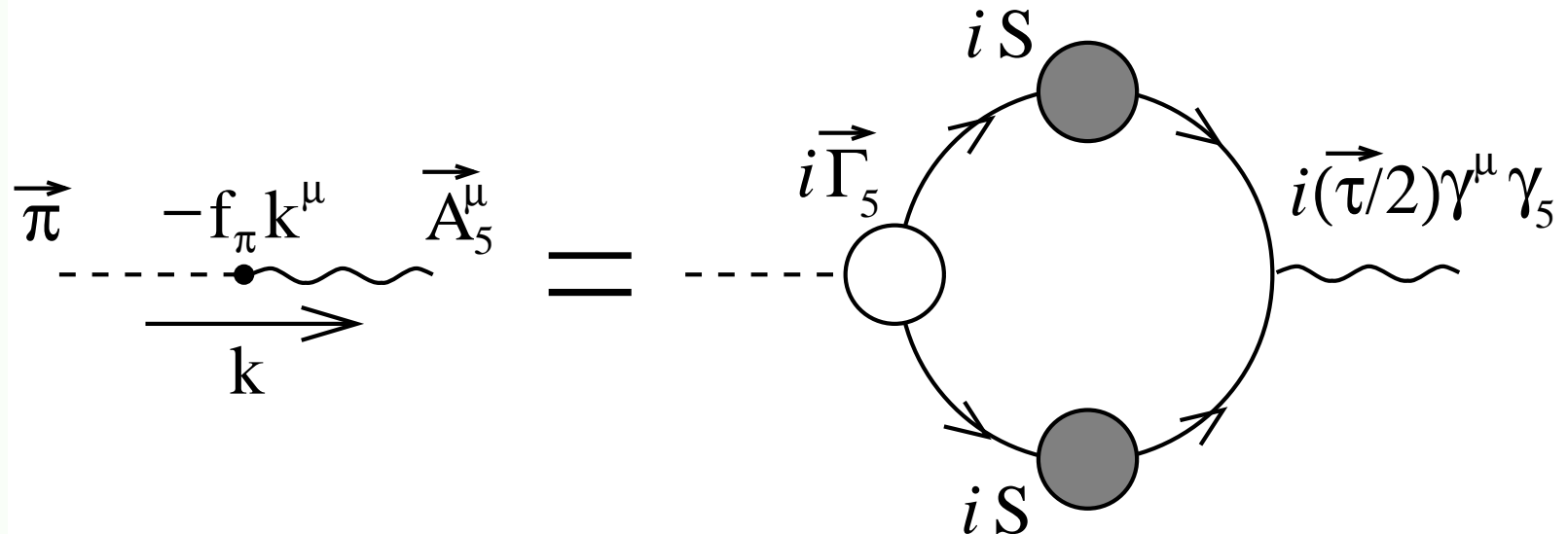
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$$f_H p_\mu = Z_2 \int_q^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu \mathcal{S}(q_+) \Gamma_H(q; P) \mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at  $x = 0$
- Pseudoscalar meson's leptonic decay constant



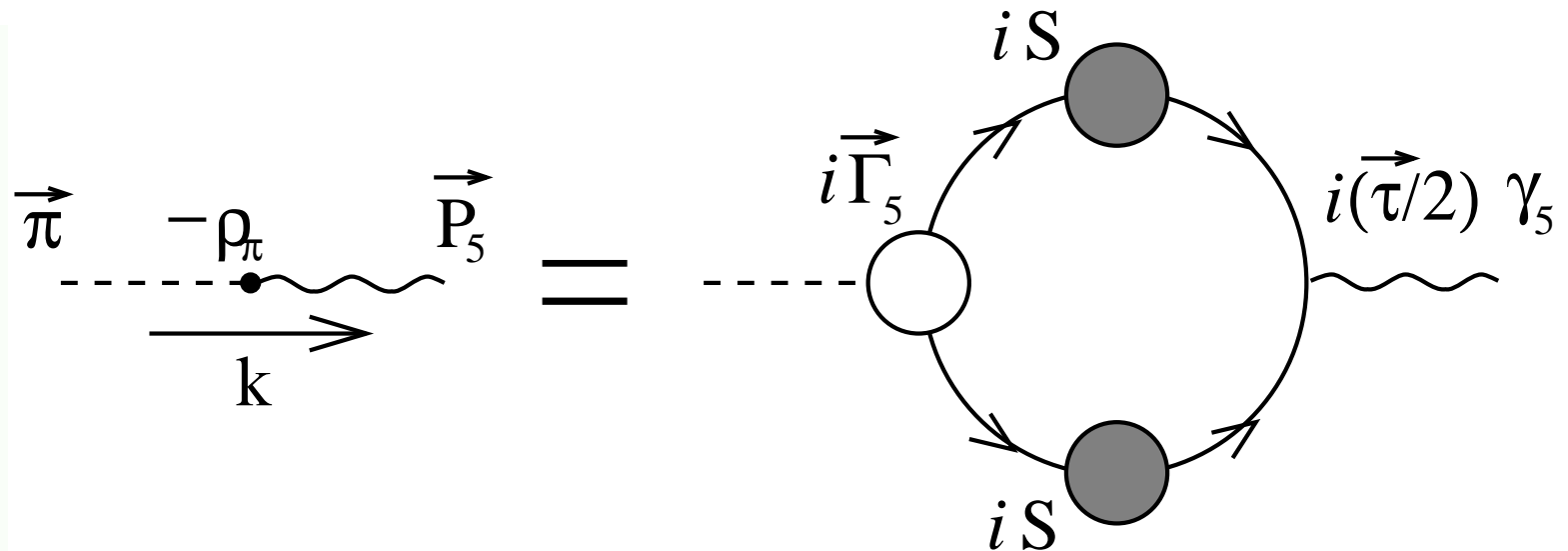
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- Light-quarks; i.e.,  $m_q \sim 0$

- $f_H \rightarrow f_H^0$  &  $\rho_\zeta^H \rightarrow \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$ , Independent of  $m_q$

Hence  $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \dots$  GMOR relation, a corollary



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- Heavy-quark + light-quark

$$\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}} \text{ and } \rho_\zeta^H \propto \sqrt{m_H}$$

Hence,  $m_H \propto m_q$

... QCD Proof of Potential Model result

Craig Roberts: Charting the interaction between light quarks

CLAS12 European Workshop ... 23 transparencies

- p. 13/40



# New Challenges



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# New Challenges

- **Next Steps** . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.



# New Challenges

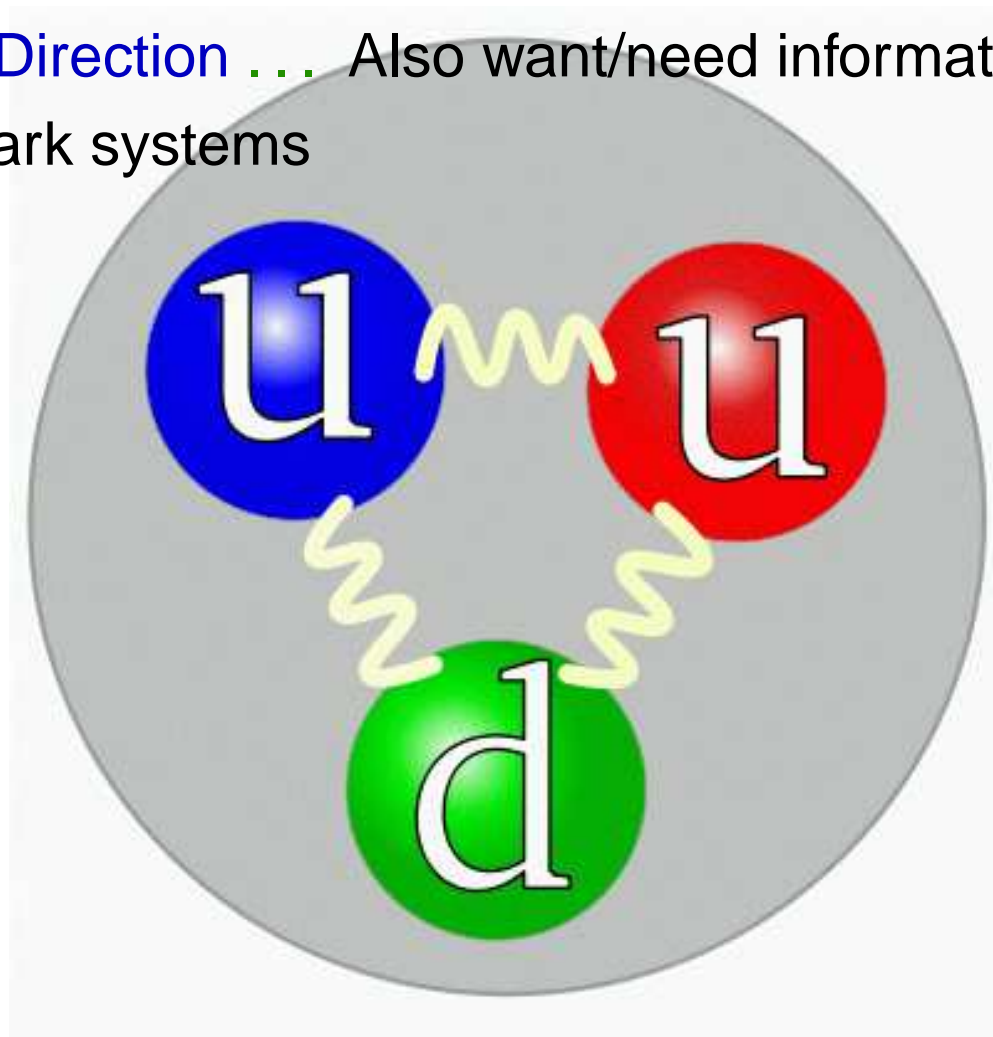
- **Next Steps** . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
- Move on to the problem of a **symmetry preserving** treatment of hybrids and exotics.





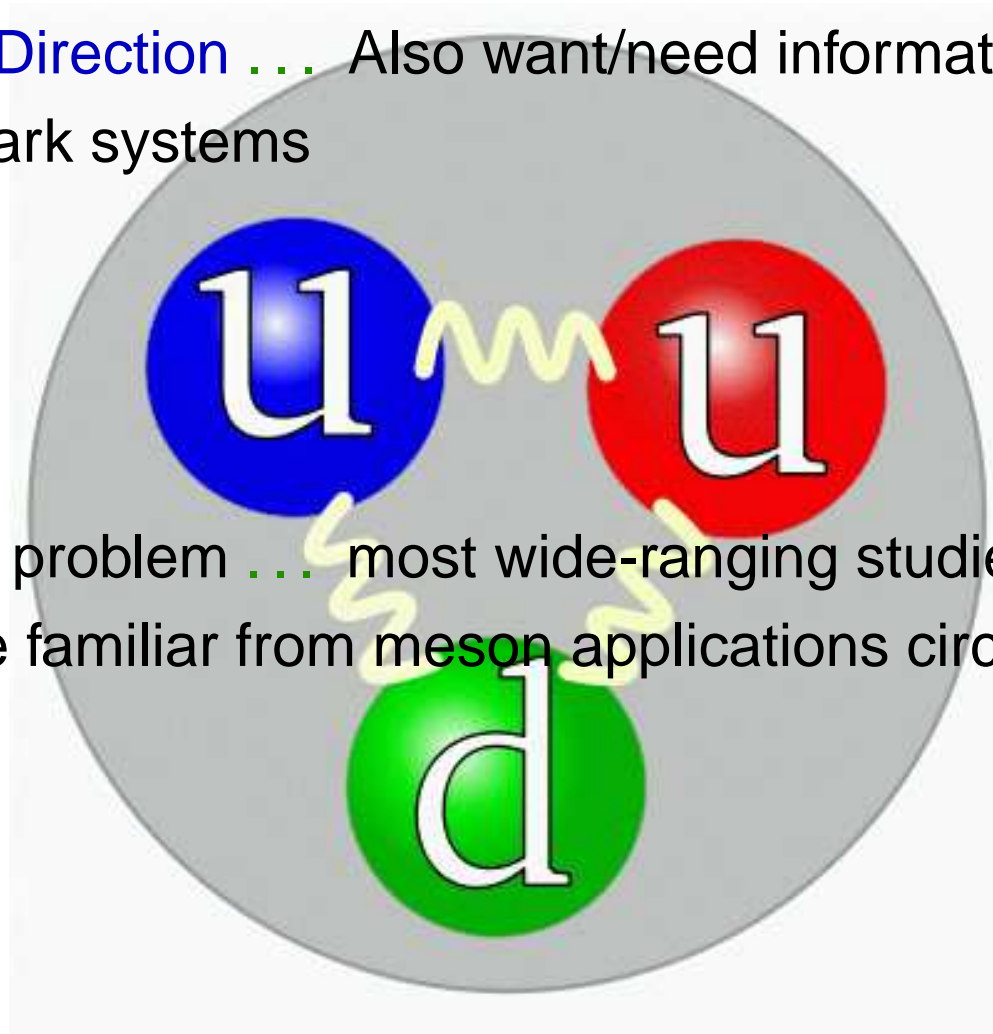
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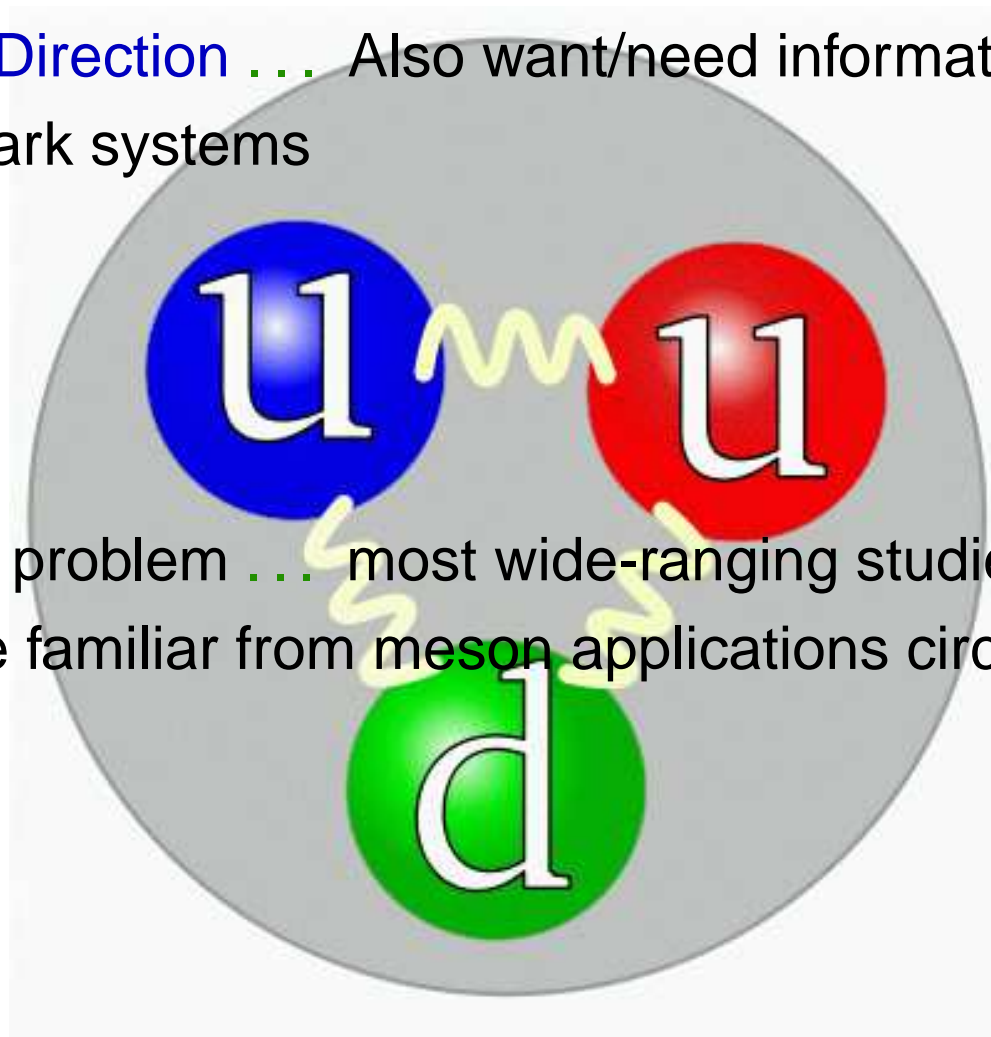


- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ~ 1995.



# New Challenges

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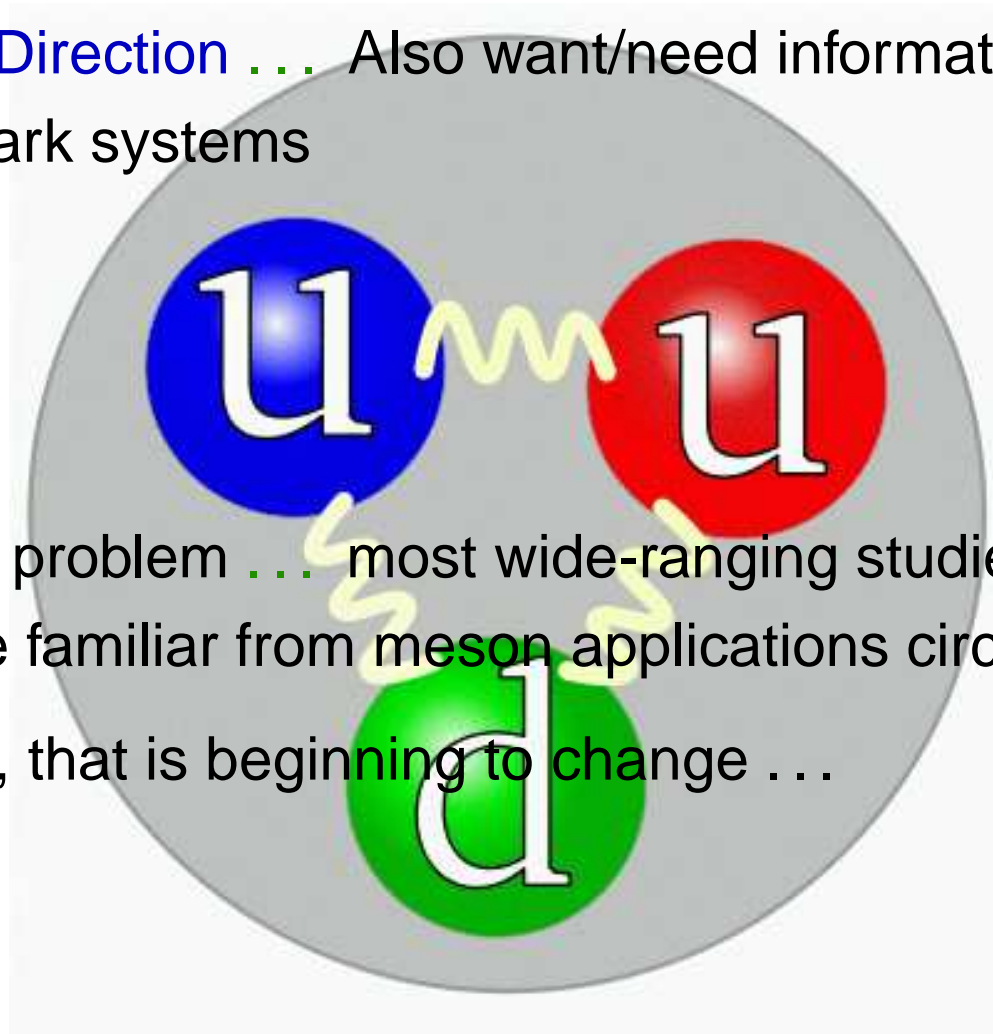
- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa  $\sim 1995$ .

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



# New Challenges

- **Another Direction . . .** Also want/need information about three-quark systems



- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa  $\sim 1995$ .
- However, that is beginning to change . . .



# *Unifying Study of Mesons and Baryons*



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function,  $M(p^2)$ , in study of baryons?



# Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function,  $M(p^2)$ , in study of baryons? Behaviour of  $M(p^2)$  is essentially a quantum field theoretical effect.



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- How does one incorporate dressed-quark mass function,  $M(p^2)$ , in study of baryons? Behaviour of  $M(p^2)$  is essentially a quantum field theoretical effect.
- In quantum field theory a nucleon appears as a pole in a six-point quark Green function.
  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour- $\bar{3}$  (antitriplet) channel



# *Faddeev equation*



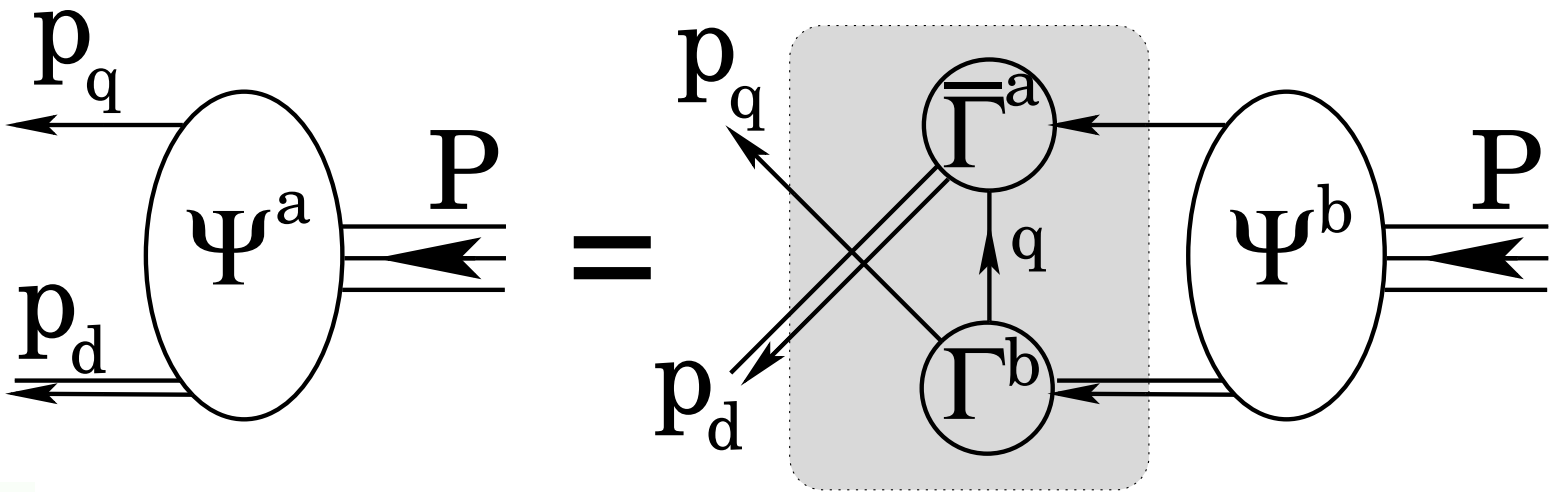
[First](#)

[Contents](#)

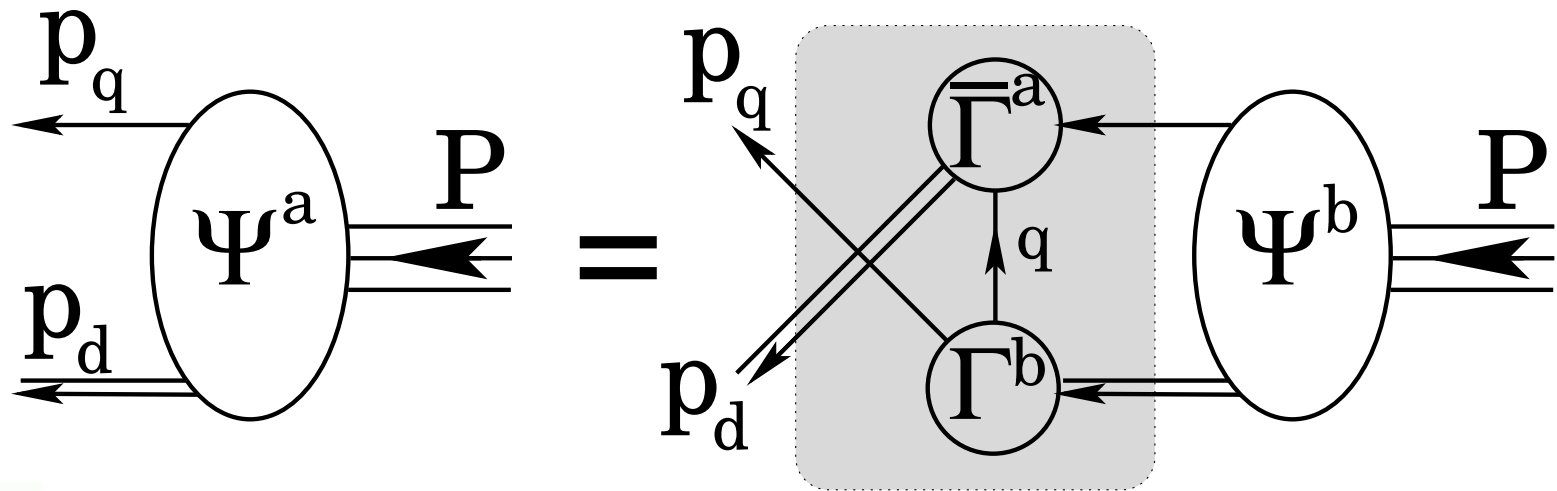
[Back](#)

[Conclusion](#)

# Faddeev equation



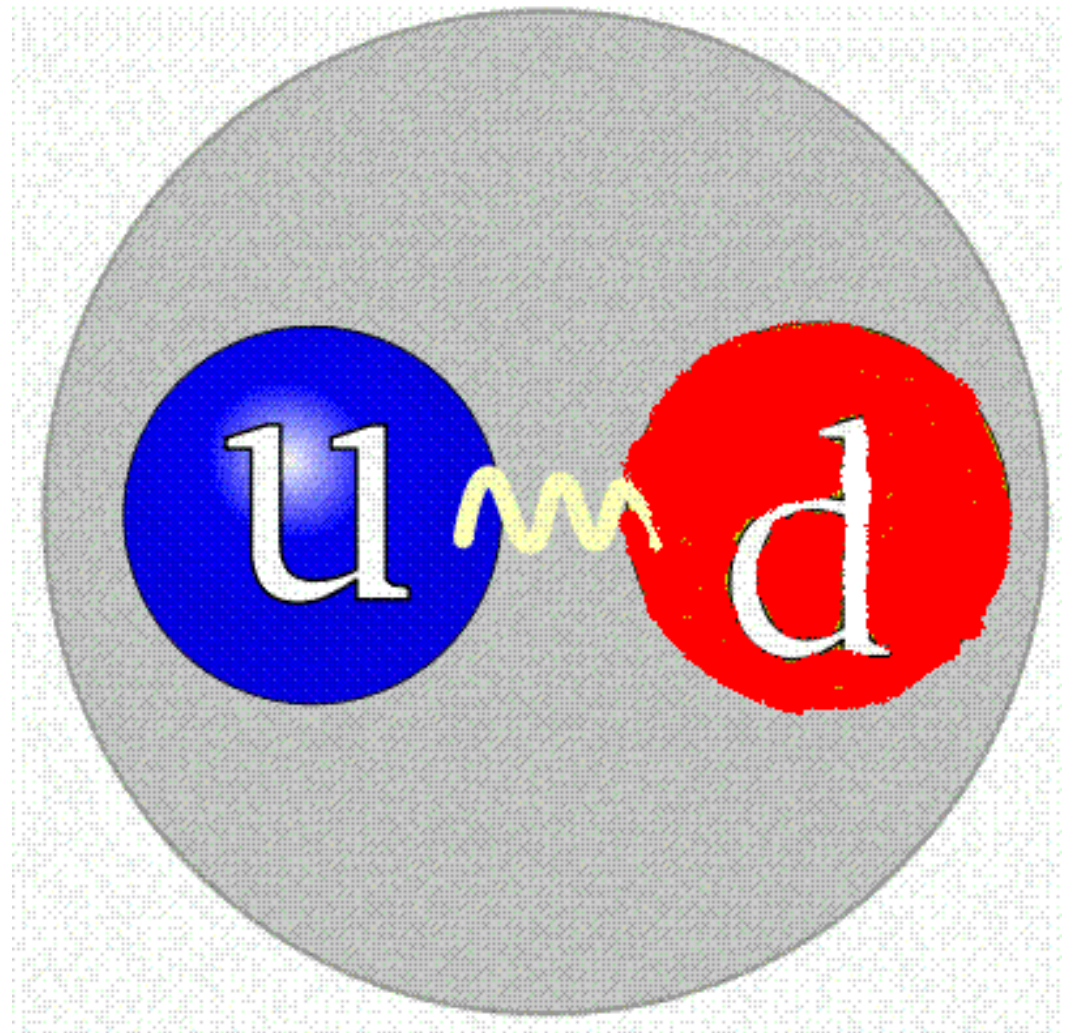
# Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (**Poincaré Covariant Faddeev Amplitude**) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame **Amplitude** has ... *s*-, *p*- & *d*-wave correlations



# Diquark correlations



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

## QUARK-QUARK

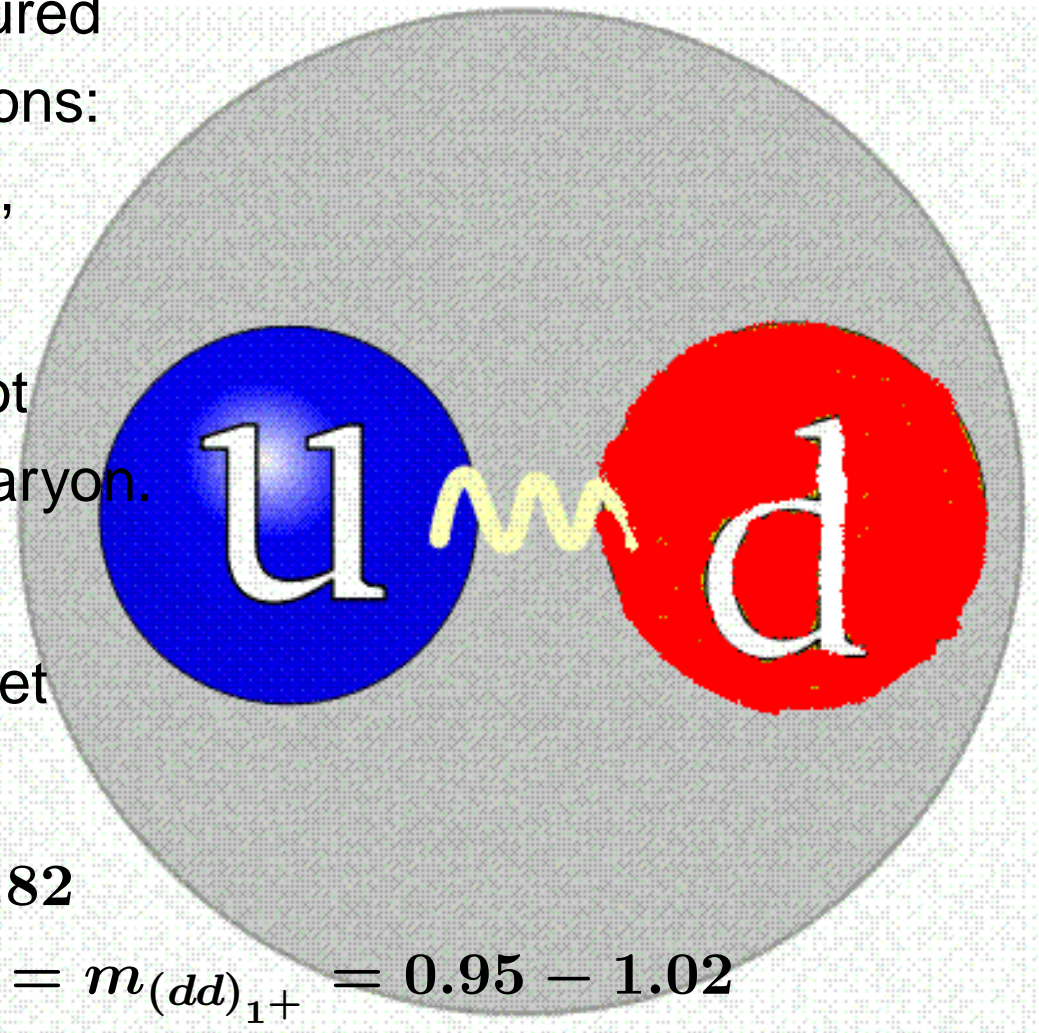
Craig Roberts: Charting the interaction between light quarks

CLAS12 European Workshop ... 23 transparencies

- p. 17/40

# Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue-red, blue-green, green-red
- Confined ... Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD
$$m_{[ud]_{0+}} = 0.74 - 0.82$$
$$m_{(uu)_{1+}} = m_{(ud)_{1+}} = m_{(dd)_{1+}} = 0.95 - 1.02$$



Argonne  
NATIONAL  
LABORATORY



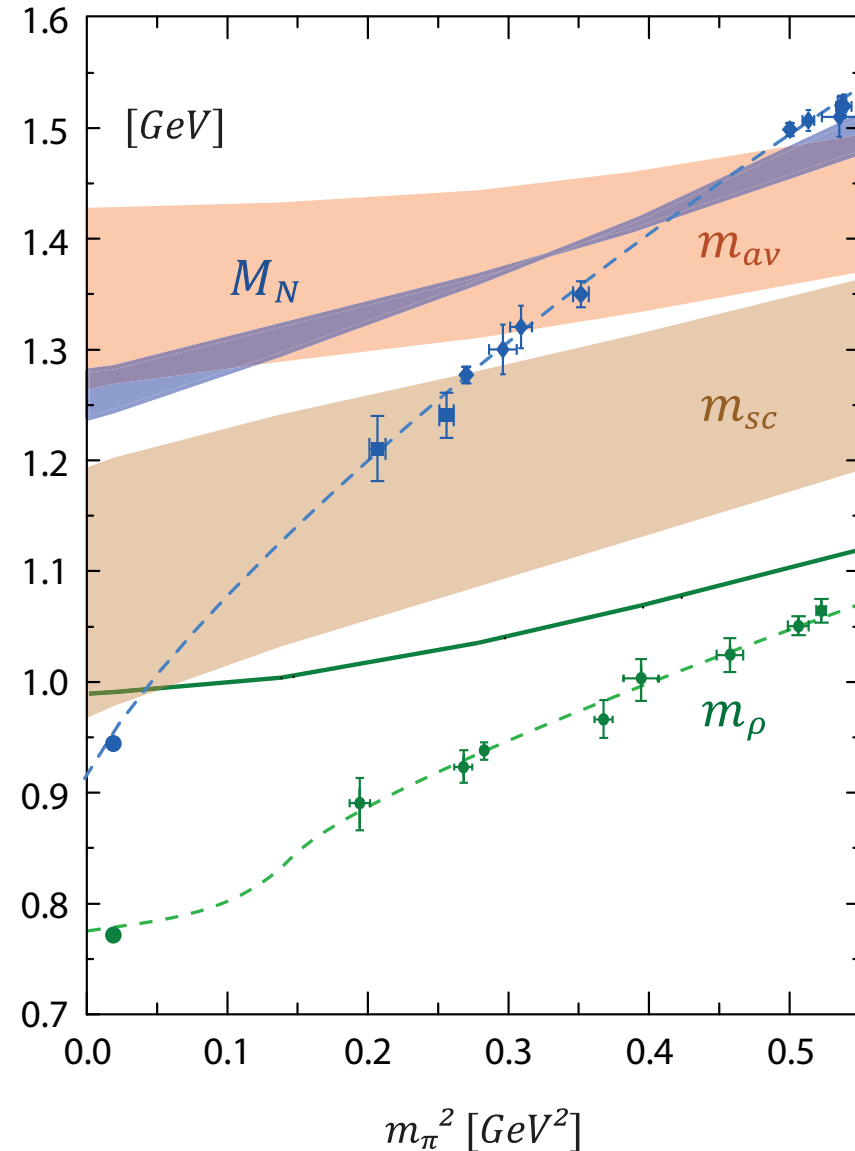
Eichmann *et al.*

– arXiv:0802.1948 [nucl-th]

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# Ab-initio study of mesons & nucleons

- Bethe-Salpeter & Faddeev equations built from same RG-improved rainbow-ladder interaction



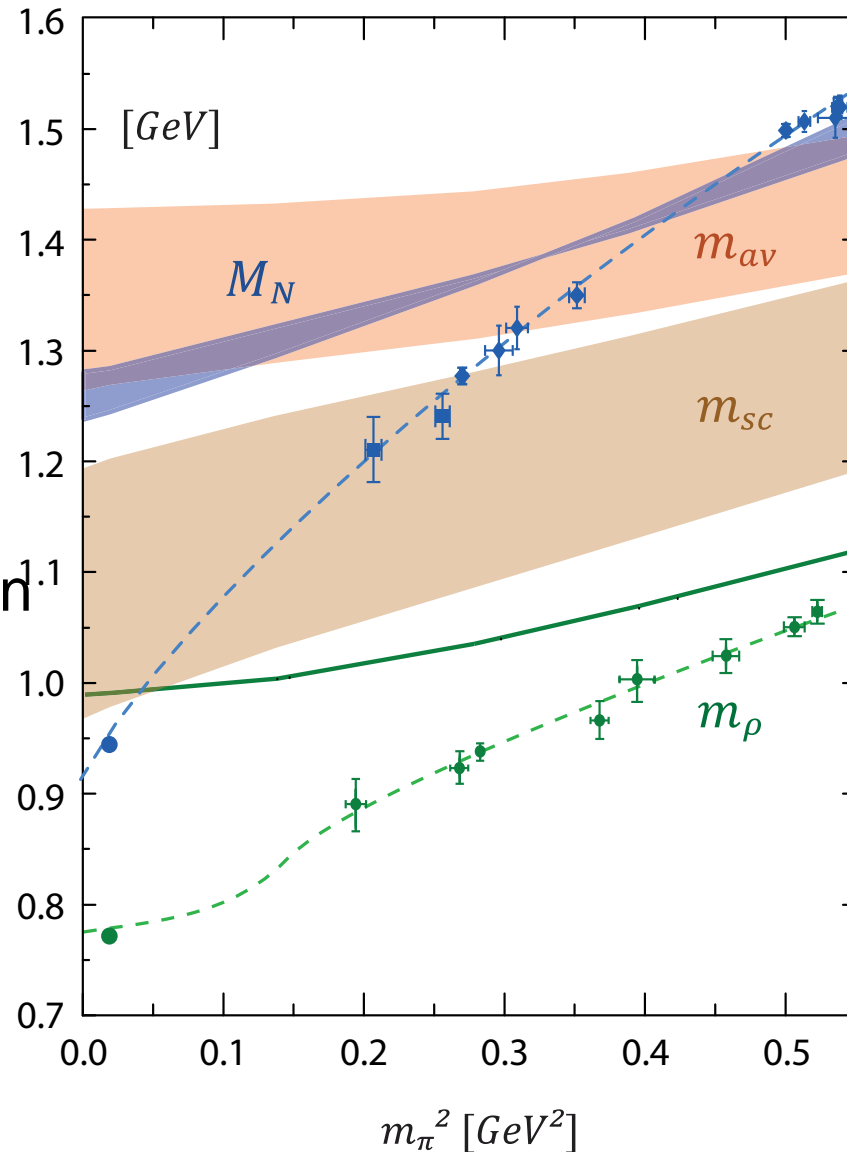
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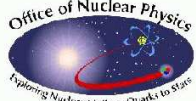
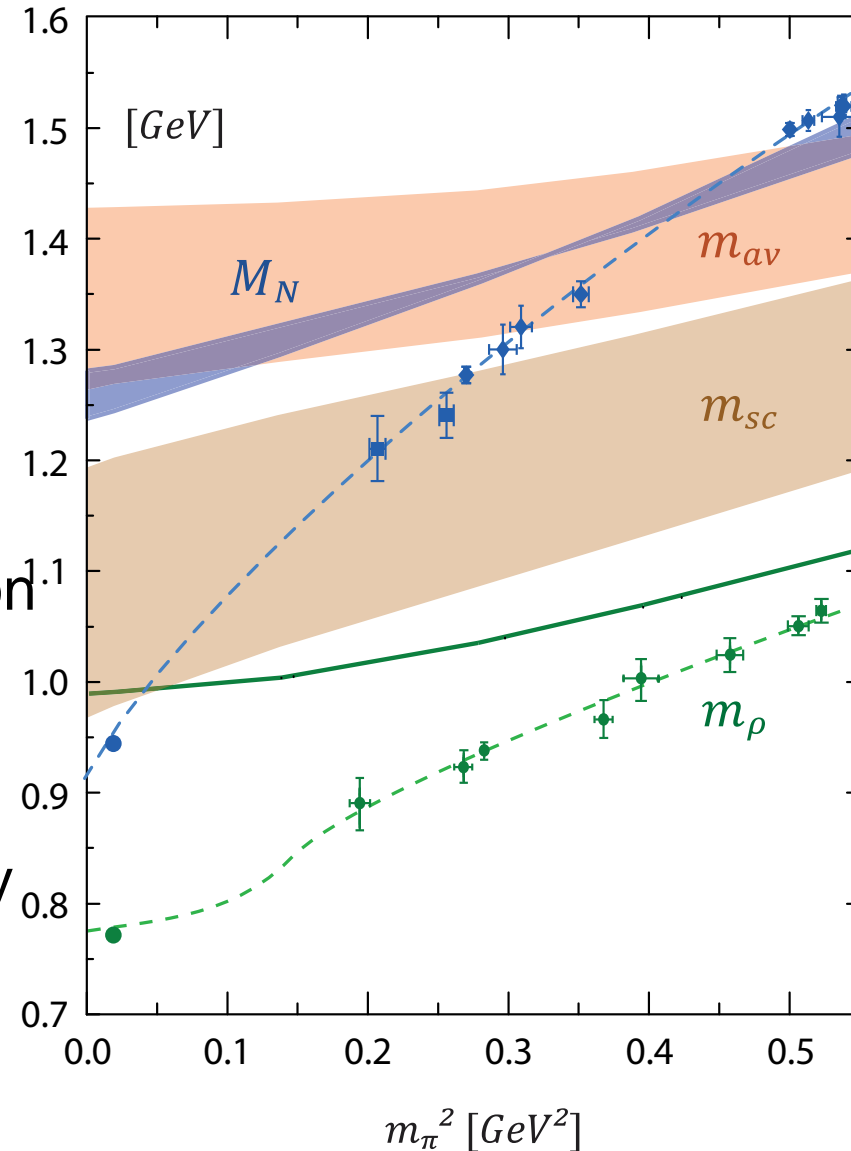


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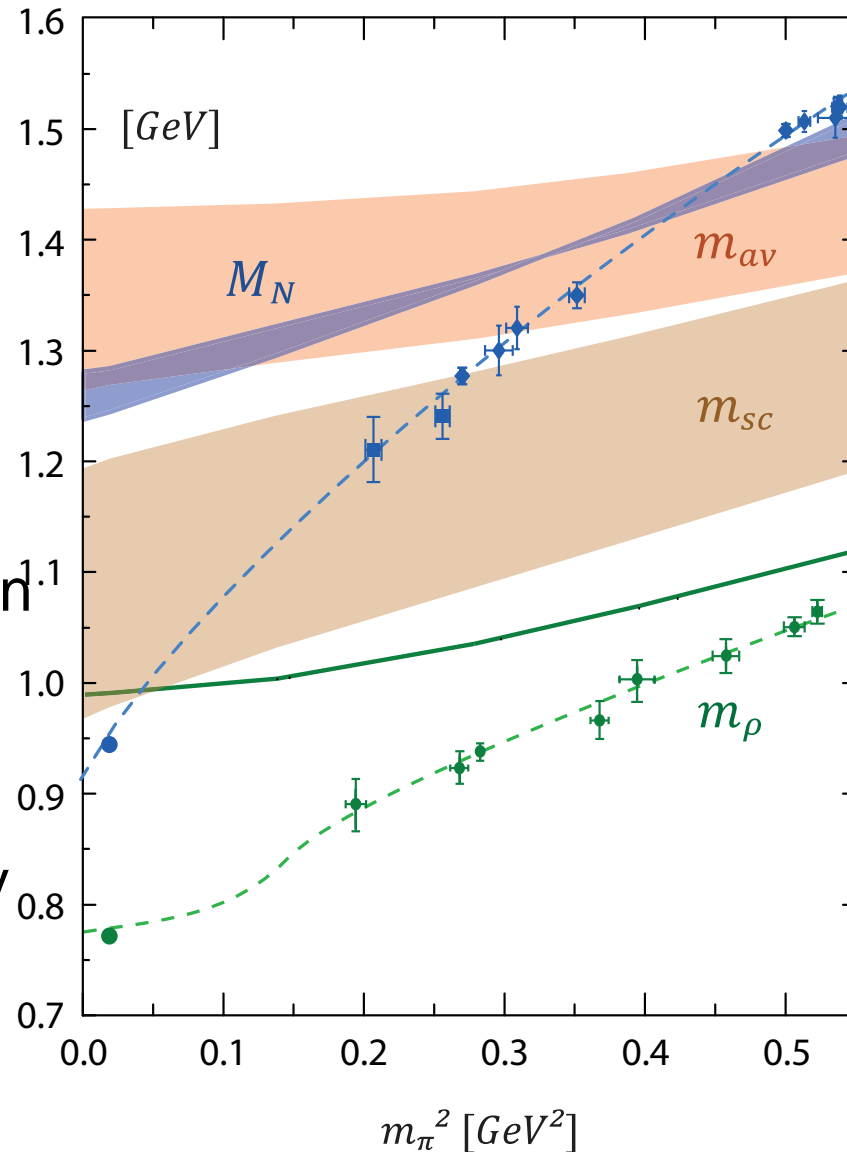
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● *Systematically improvable*



# Nucleon-Photon Vertex



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

M. Oettel, M. Pichowsky  
and L. von Smekal, nu-th/9909082

6 terms ...

# ***Nucleon-Photon Vertex***

constructed systematically ... current conserved automatically  
for on-shell nucleons described by Faddeev Amplitude



[First](#)

[Contents](#)

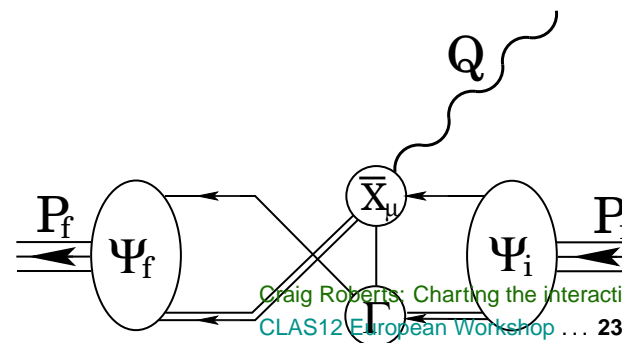
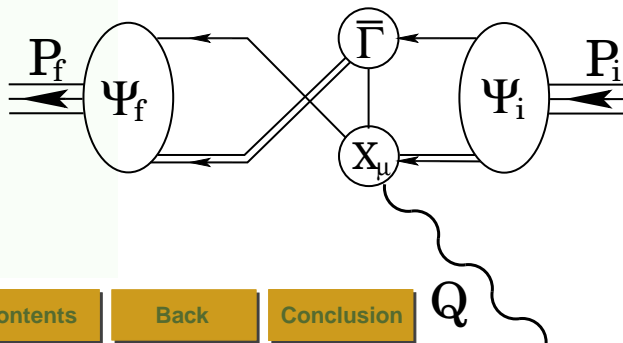
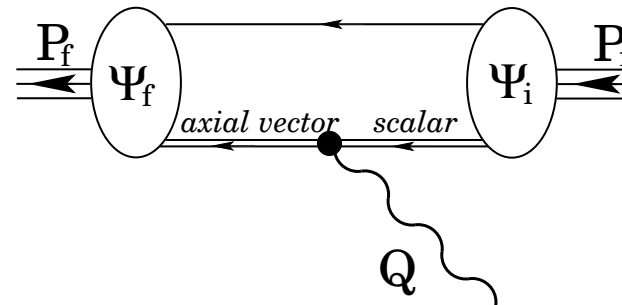
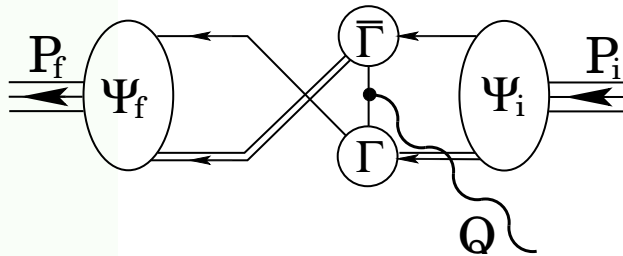
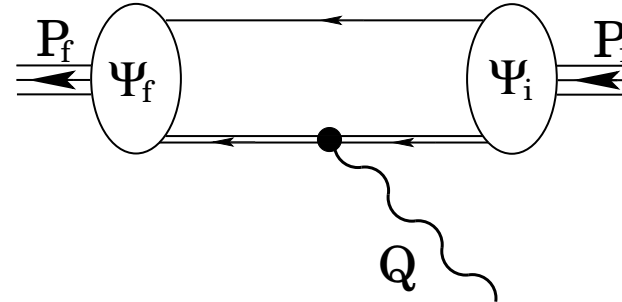
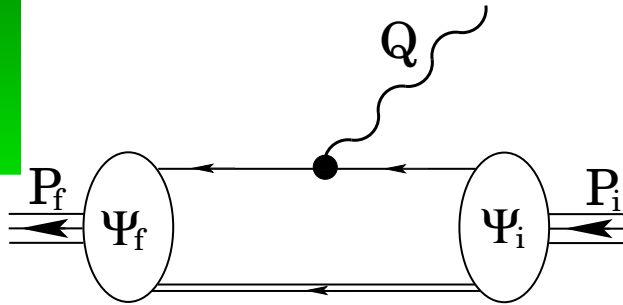
[Back](#)

[Conclusion](#)

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# *DSE-based Faddeev Equation*



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)





Cloët *et al.*

- arXiv:0710.2059 [nucl-th]
- arXiv:0710.5746 [nucl-th]
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# ***DSE-based Faddeev Equation***



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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# *DSE-based Faddeev Equation*

- Faddeev equation input – algebraic parametrisations of DSE results, constrained by  $\pi$  and  $K$  observables



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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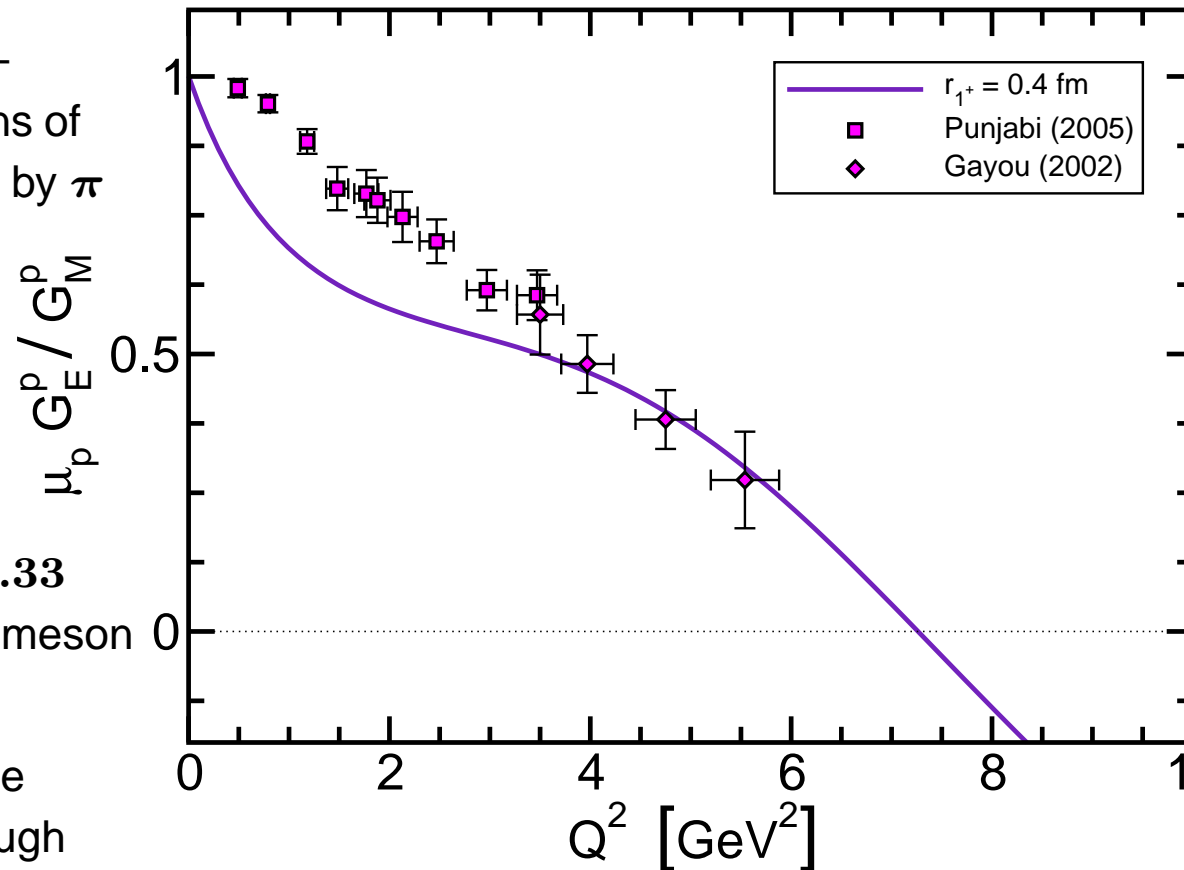
- Faddeev equation input – algebraic parametrisations of DSE results, constrained by  $\pi$  and  $K$  observables
- Two parameters
  - $M_{0+} = 0.8$  GeV,
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  - chosen to give  $M_N = 1.18$ ,  $M_\Delta = 1.33$
  - allow for pseudoscalar meson contributions



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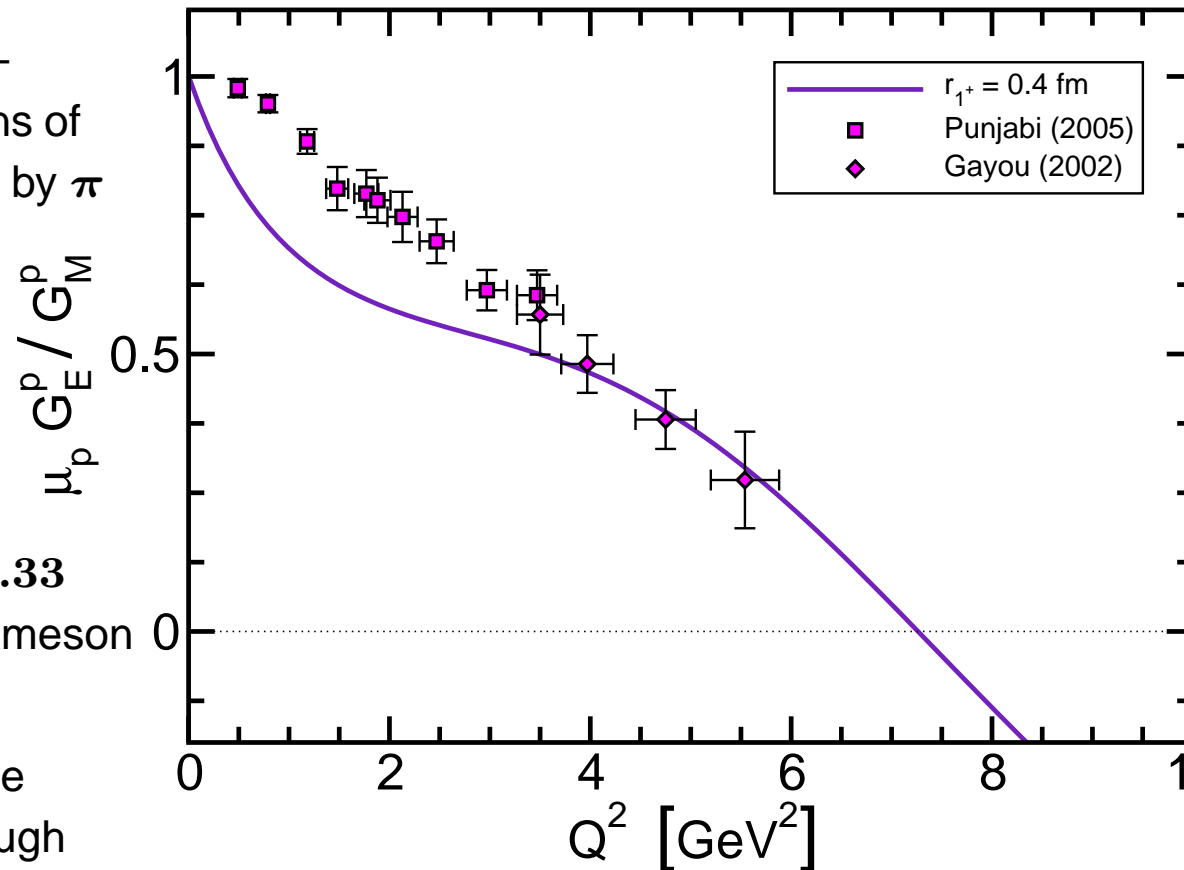
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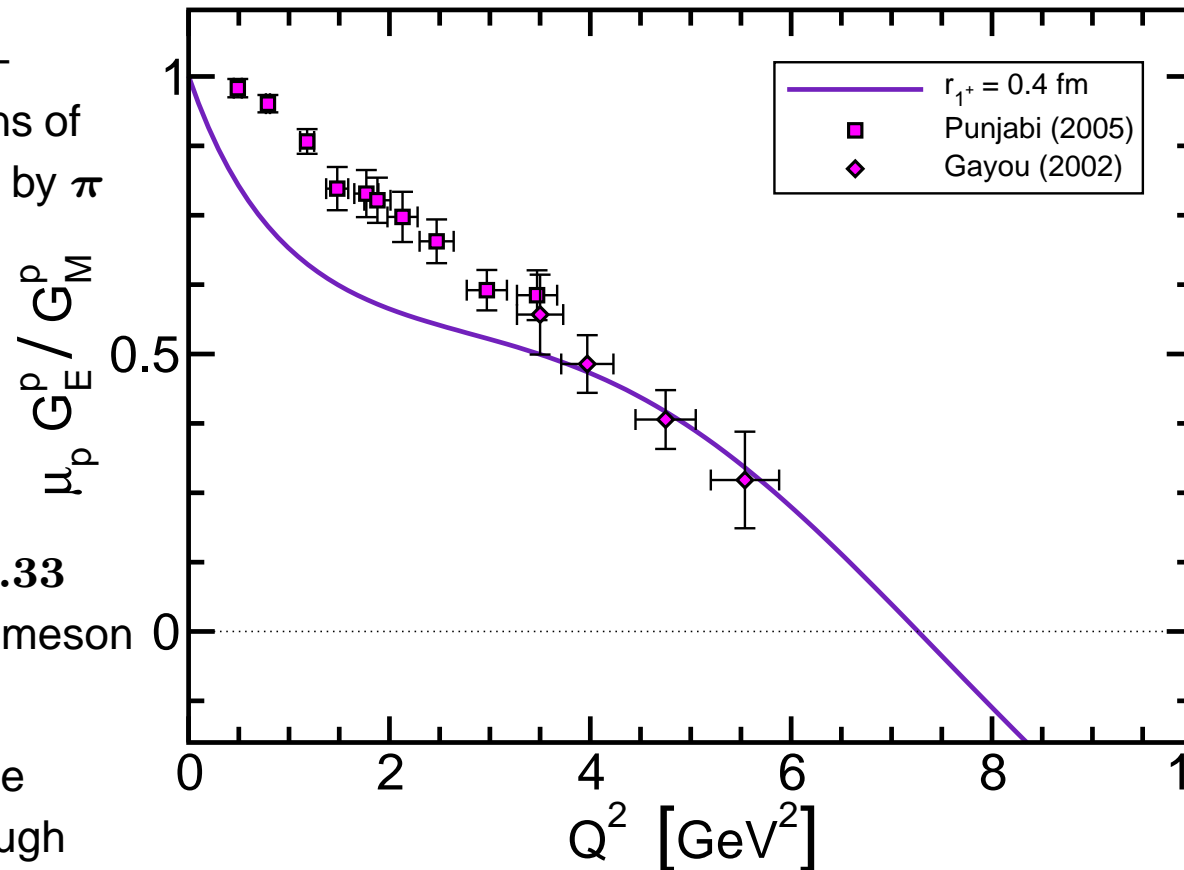
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- Sensitivity to details of the current – expressed through diquark radius
- On  $Q^2 \lesssim 4$  GeV $^2$  result lies below experiment. This can be attributed to omission of pseudoscalar-meson-cloud contributions
- Always a zero but position depends on details of current





# ab initio

## *Faddeev Equation*

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[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

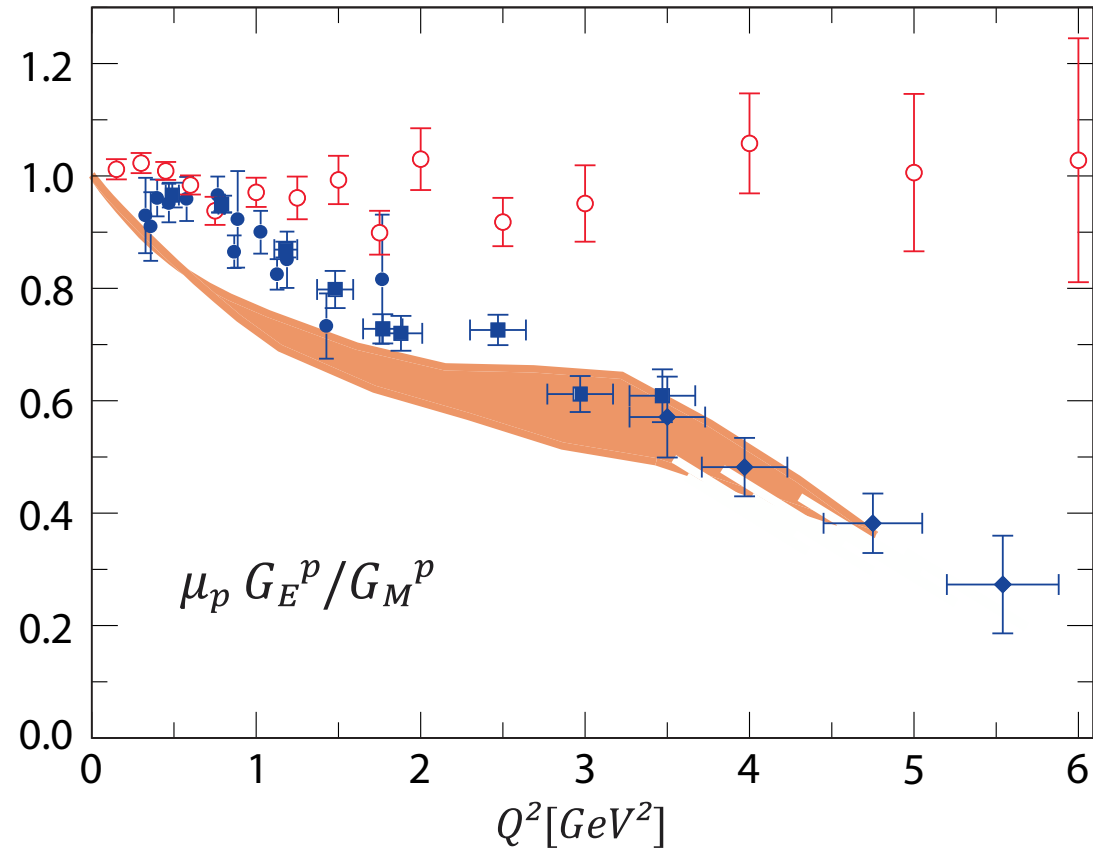


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ab initio

# Faddeev Equation



First

Contents

Back

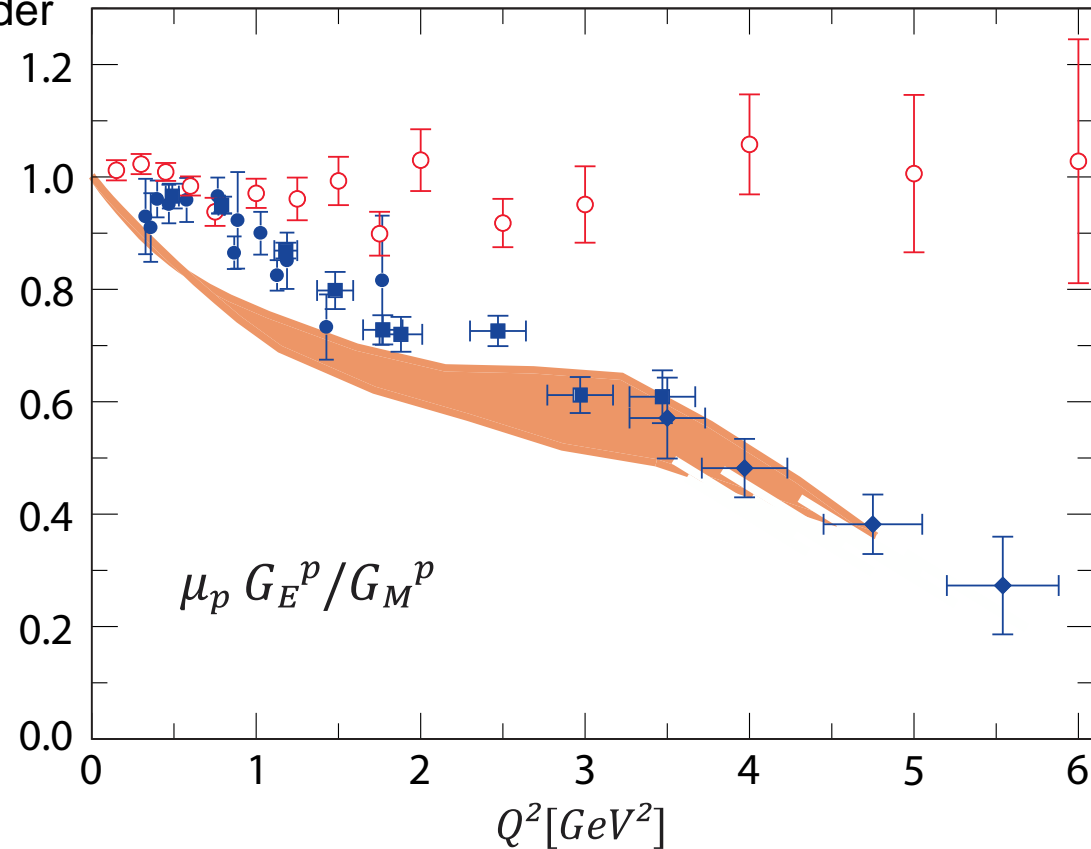
Conclusion



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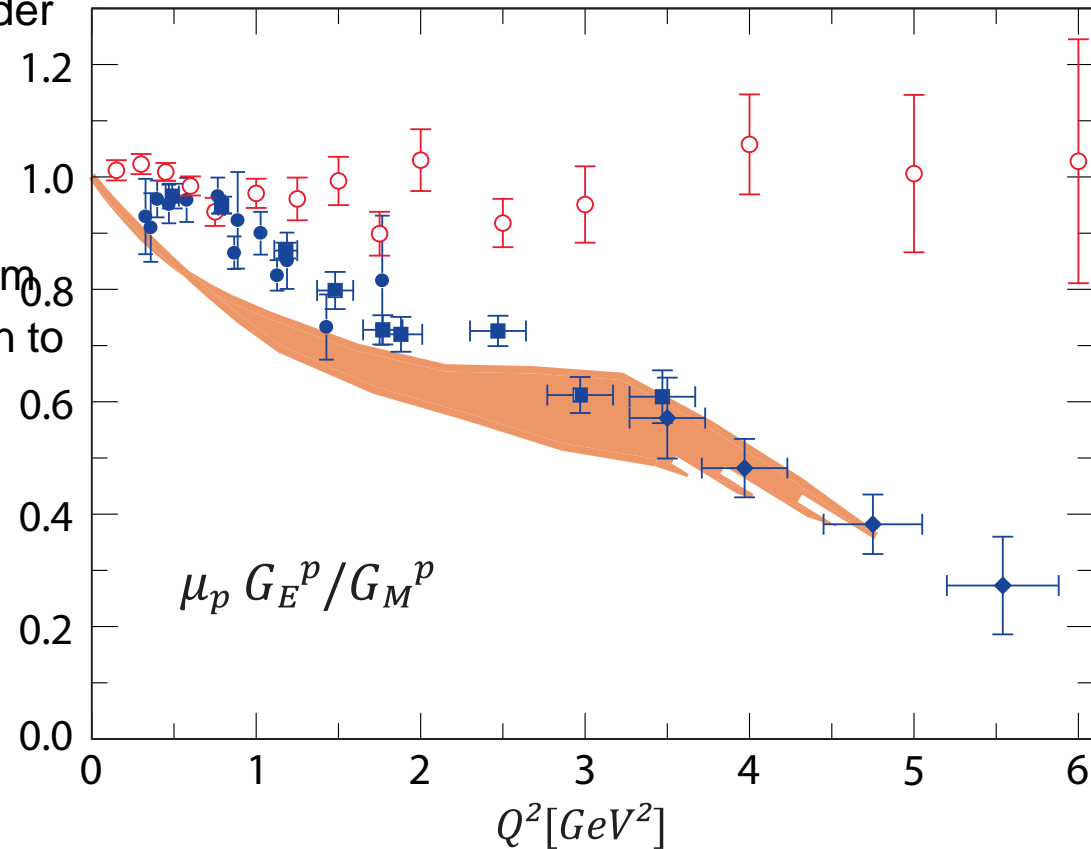
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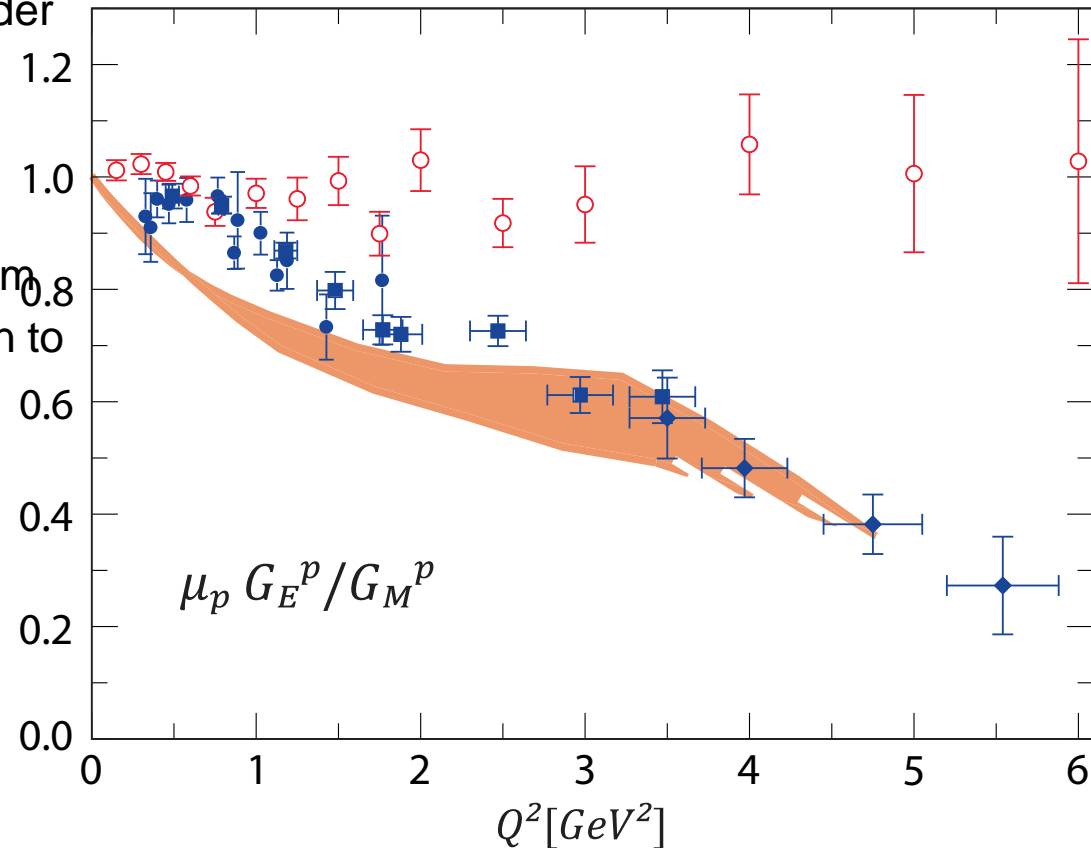


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- Calculation unifies  $\pi$ ,  $\rho$  and nucleon properties – keystone is behaviour of dressed-quark mass function and hence veracious description of QCD's Goldstone mode



# Ratio of Neutron Pauli & Dirac Form Factors

$$\frac{\hat{Q}^2}{(\ln \hat{Q}^2 / \hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

$$\hat{\Lambda} = \Lambda / M_N = 0.44$$

Ensures proton ratio  
constant for  $\hat{Q}^2 \geq 4$

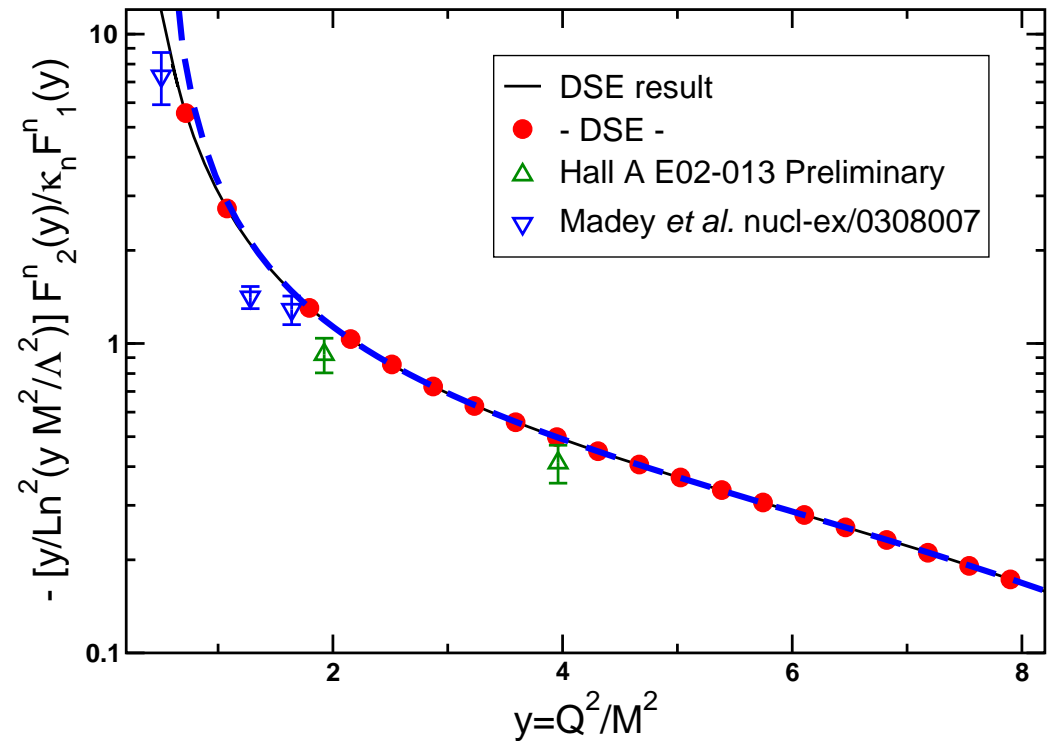


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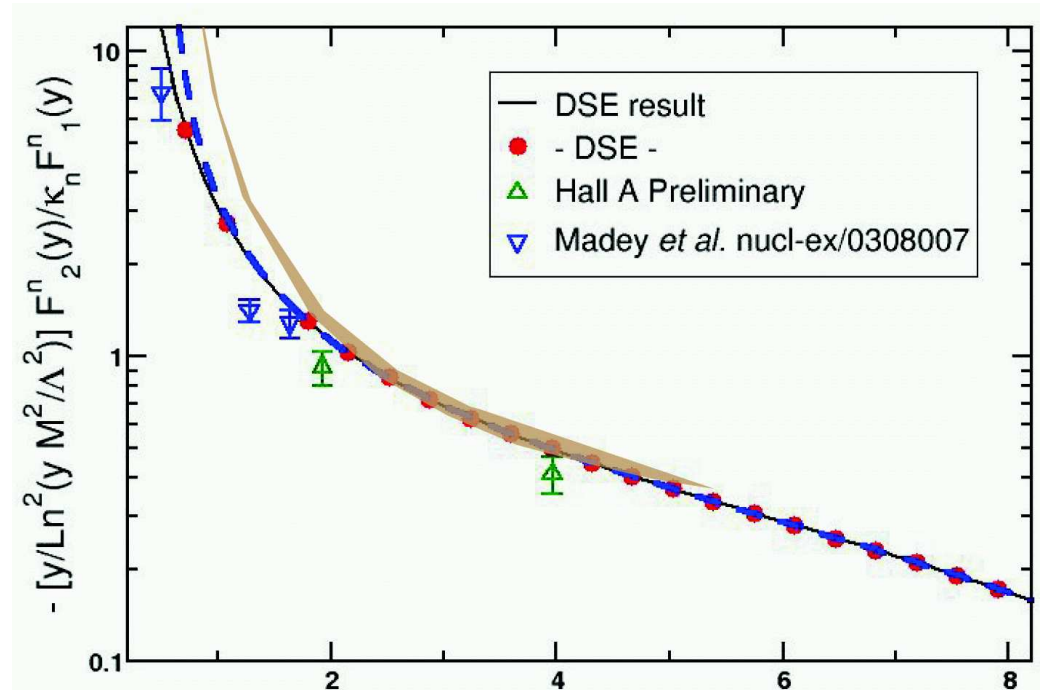
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**Brown band**  
– *ab initio* RL result





[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# *Pion Cloud*

## *F2 – neutron*

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[First](#)

[Contents](#)

[Back](#)

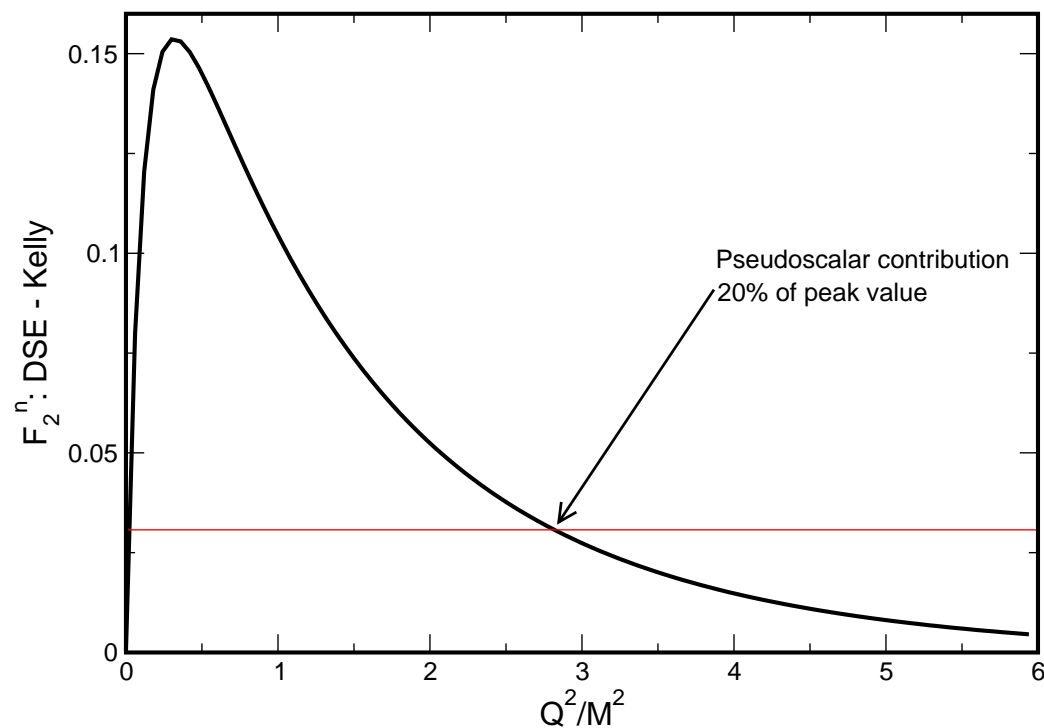
[Conclusion](#)



# Pion Cloud

## F2 – neutron

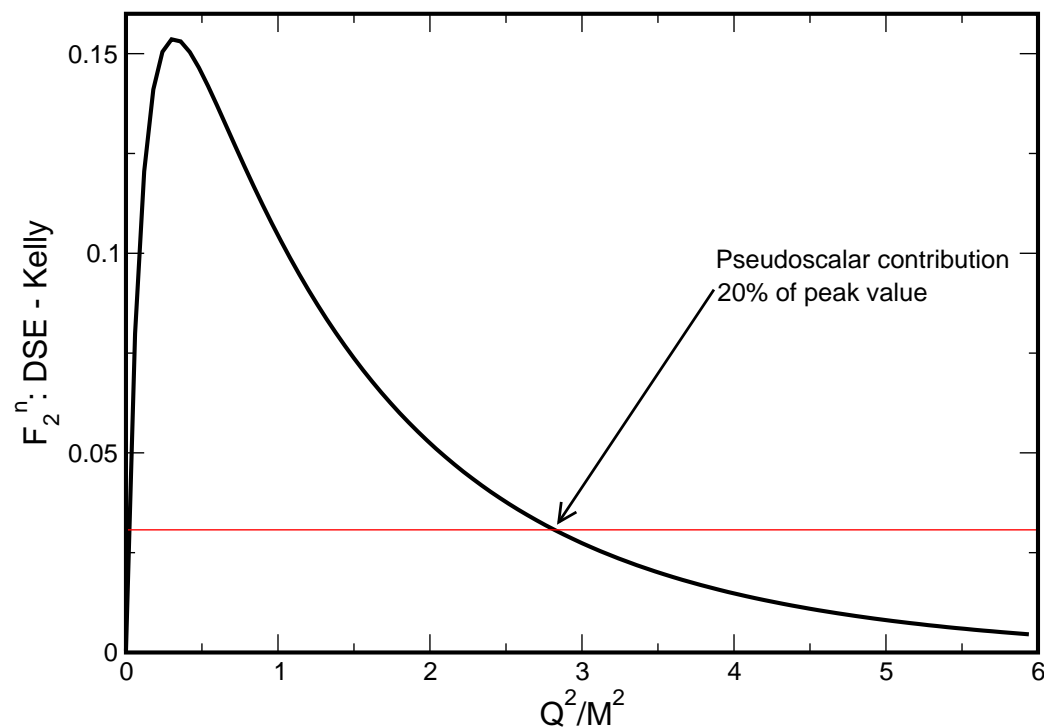
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# Pion Cloud

## F2 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core
- Pseudoscalar meson cloud (and related effects) significant for  $Q^2 \lesssim 3 - 4 M_N^2$





# Epilogue



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



# Epilogue



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



● DCSB exists in QCD.

# Epilogue



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



# Epilogue

● DCSB exists in QCD.

- It is manifest in dressed propagators and vertices



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



# Epilogue

## ● DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It predicts, amongst other things, that
  - light current-quarks become heavy constituent-quarks:  $4 \rightarrow 400 \text{ MeV}$
  - pseudoscalar mesons are unnaturally light:  $m_\rho = 770$  cf.  $m_\pi = 140 \text{ MeV}$
  - pseudoscalar mesons couple unnaturally strongly to light-quarks:  $g_{\pi\bar{q}q} \approx 4.3$
  - pseudoscalar mesons couple unnaturally strongly to the lightest baryons

$$g_{\pi\bar{N}N} \approx 12.8 \approx 3g_{\pi\bar{q}q}$$





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$$g_{\pi\bar{N}N} \approx 12.8 \approx 3g_{\pi\bar{q}q}$$
- **It impacts dramatically upon observables.**







# Epilogue

- Dyson-Schwinger Equations
  - Poincaré covariant unification of meson and baryon observables





- Dyson-Schwinger Equations
  - Poincaré covariant unification of meson and baryon observables
  - All global and pointwise corollaries of DCSB are manifested naturally without fine-tuning





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  - Poincaré covariant unification of meson and baryon observables
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  - Excited states:
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    - Baryons are within practical reach





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  - Ab-initio study of  $N \rightarrow \Delta$  transition underway





nothing!

# Epilogue

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    - Baryons are within practical reach
  - Ab-initio study of  $N \rightarrow \Delta$  transition underway
- Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks



# Contents

1. Universal Truths
2. QCD's Challenges
3. Dichotomy of the Pion
4. Dressed-Quark Propagator
5. Frontiers of Nuclear Science
6. Hadrons
7. Confinement
8. Bethe-Salpeter Kernel
9. Persistent Challenge
10. Radial Excitations
11. Radial Excitations & Lattice-QCD
12. Pion FF
13. Calculated Pion FF
14. All Pion Form Factors
15. Nucleon Challenge
16. Unifying Meson & Nucleon
17. Faddeev equation
18. Diquark correlations
19. Ab-initio study of mesons & nucleons
20.  $r_\pi f_\pi$
21. Nucleon-Photon Vertex
22. DSE-based Faddeev Equation
23. Ratio of Neutron FFs
24. Pion Cloud



# Dyson-Schwinger Equations



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# *Dyson-Schwinger Equations*

## *Dressed-Quark Propagator*

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[First](#)

[Contents](#)

[Back](#)

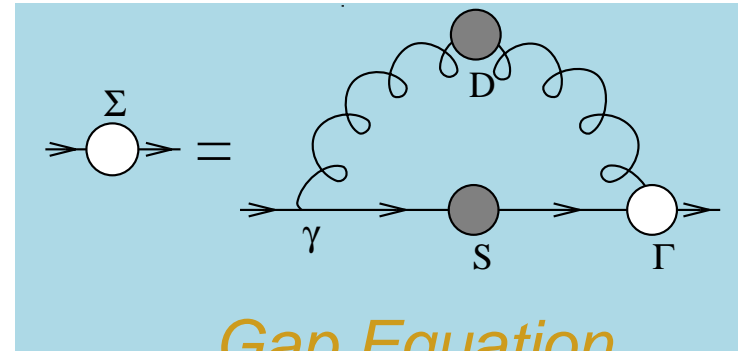
[Conclusion](#)



# Dyson-Schwinger Equations

## Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



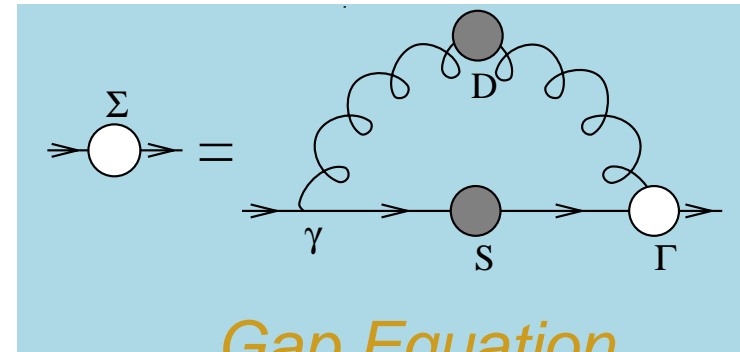
*Gap Equation*



# Dyson-Schwinger Equations

## Dressed-Quark Propagator

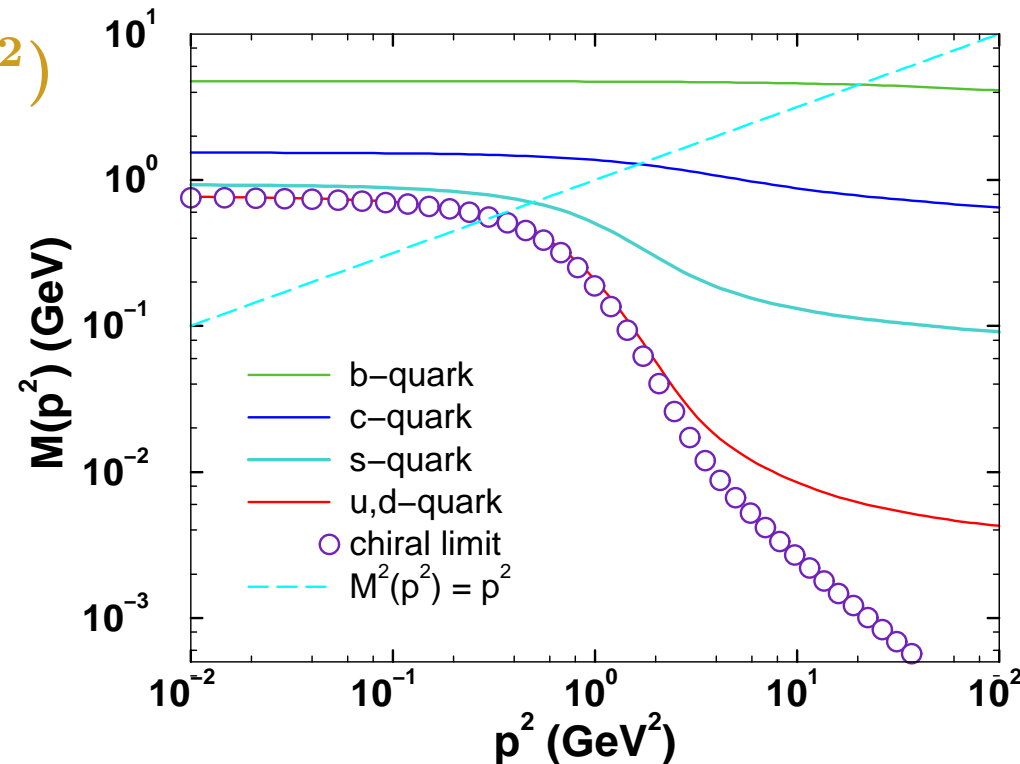
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Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

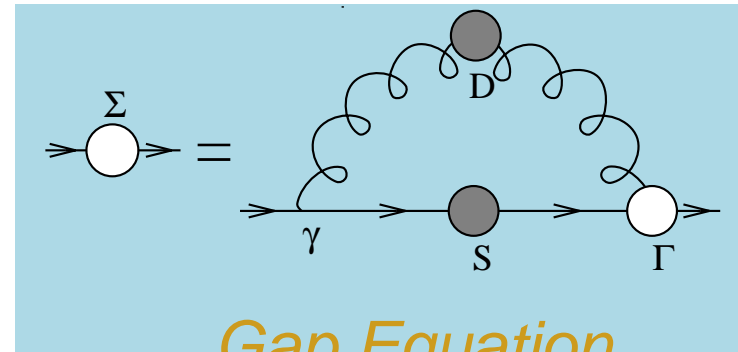
⇒ IR Enhancement of  $M(p^2)$



# Dyson-Schwinger Equations

## Dressed-Quark Propagator

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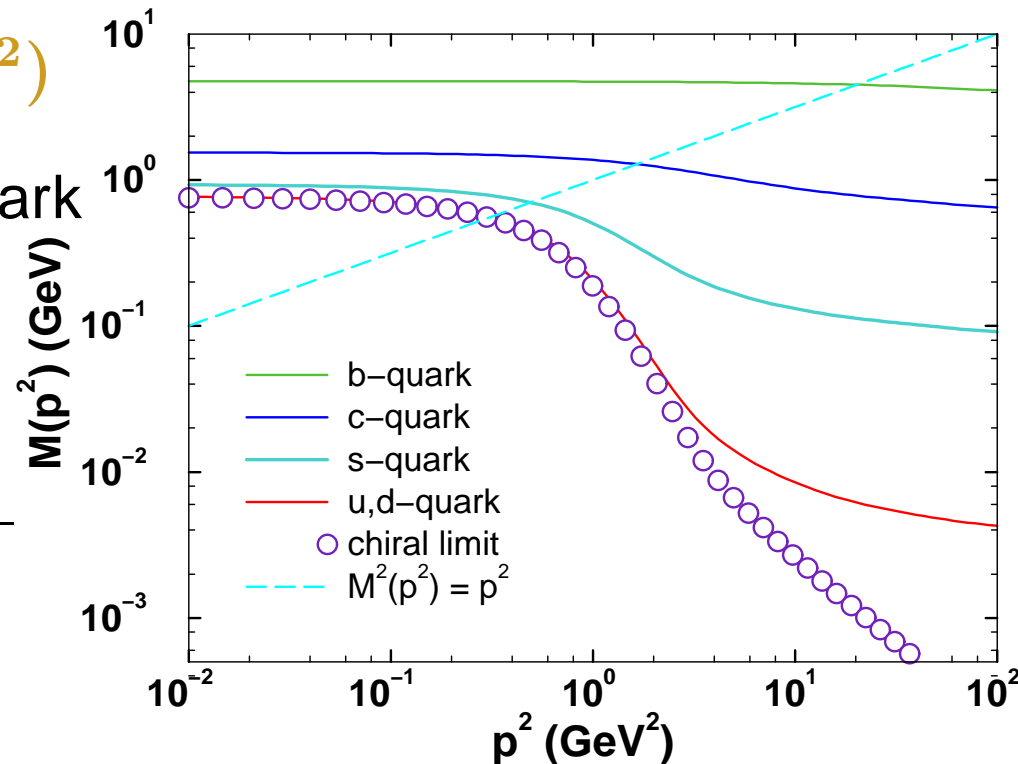
- Gap Equation's Kernel Enhanced on **IR domain**

⇒ **IR** Enhancement of  $M(p^2)$

● Euclidean Constituent-Quark

Mass:  $M_f^E: p^2 = M(p^2)^2$

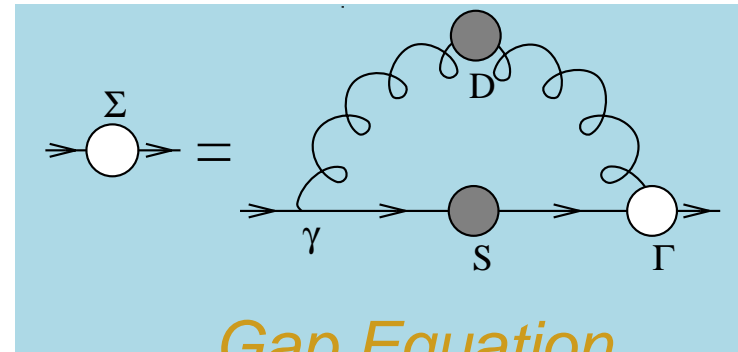
flavour	$u/d$	$s$	$c$	$b$
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	$\sim 10$	$\sim 1.5$	$\sim 1.1$



# Dyson-Schwinger Equations

## Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

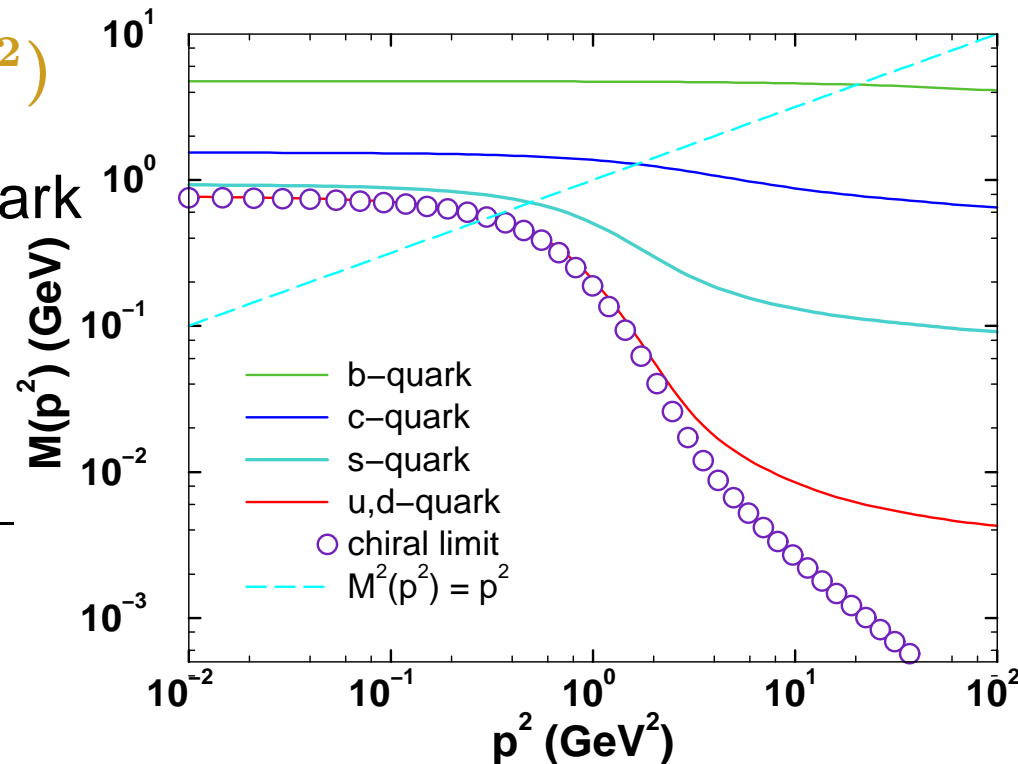
- Gap Equation's Kernel Enhanced on IR domain

⇒ IR Enhancement of  $M(p^2)$

Euclidean Constituent-Quark

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Predictions confirmed in numerical simulations of lattice-QCD



# Confinement



[First](#)

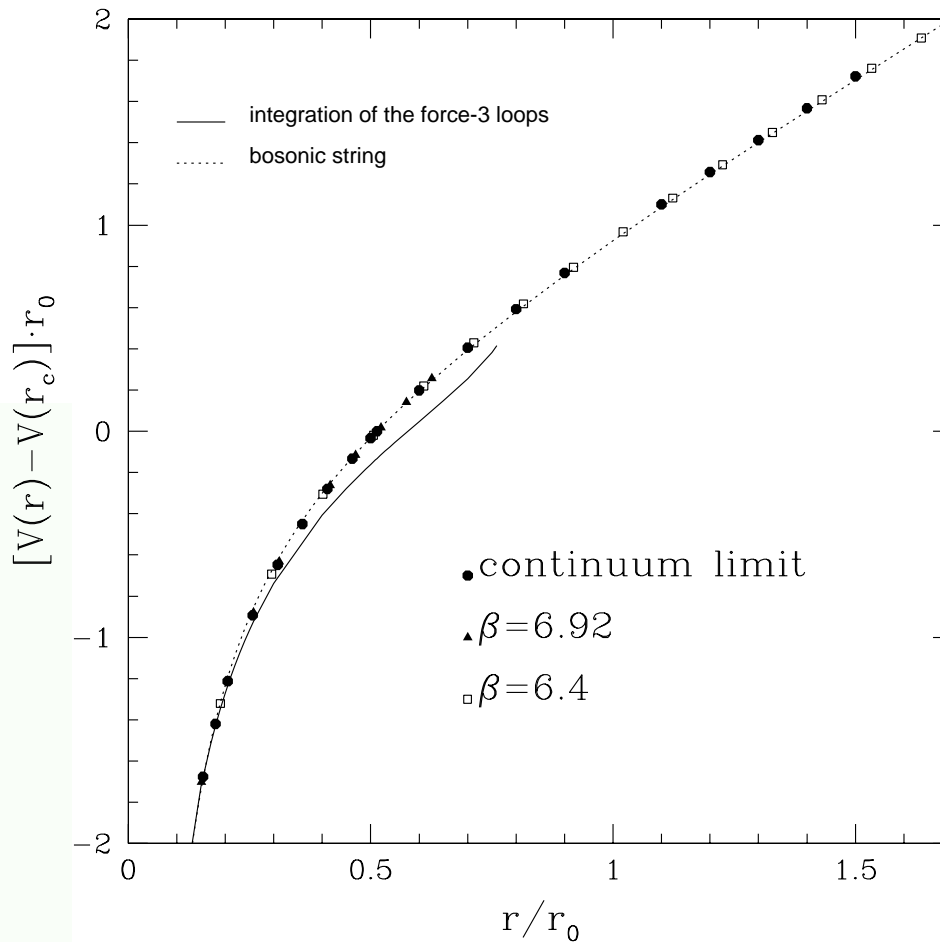
[Contents](#)

[Back](#)

[Conclusion](#)

# Confinement

## ● Infinitely Heavy Quarks ... Picture in Quantum Mechanics



$$V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r}$$

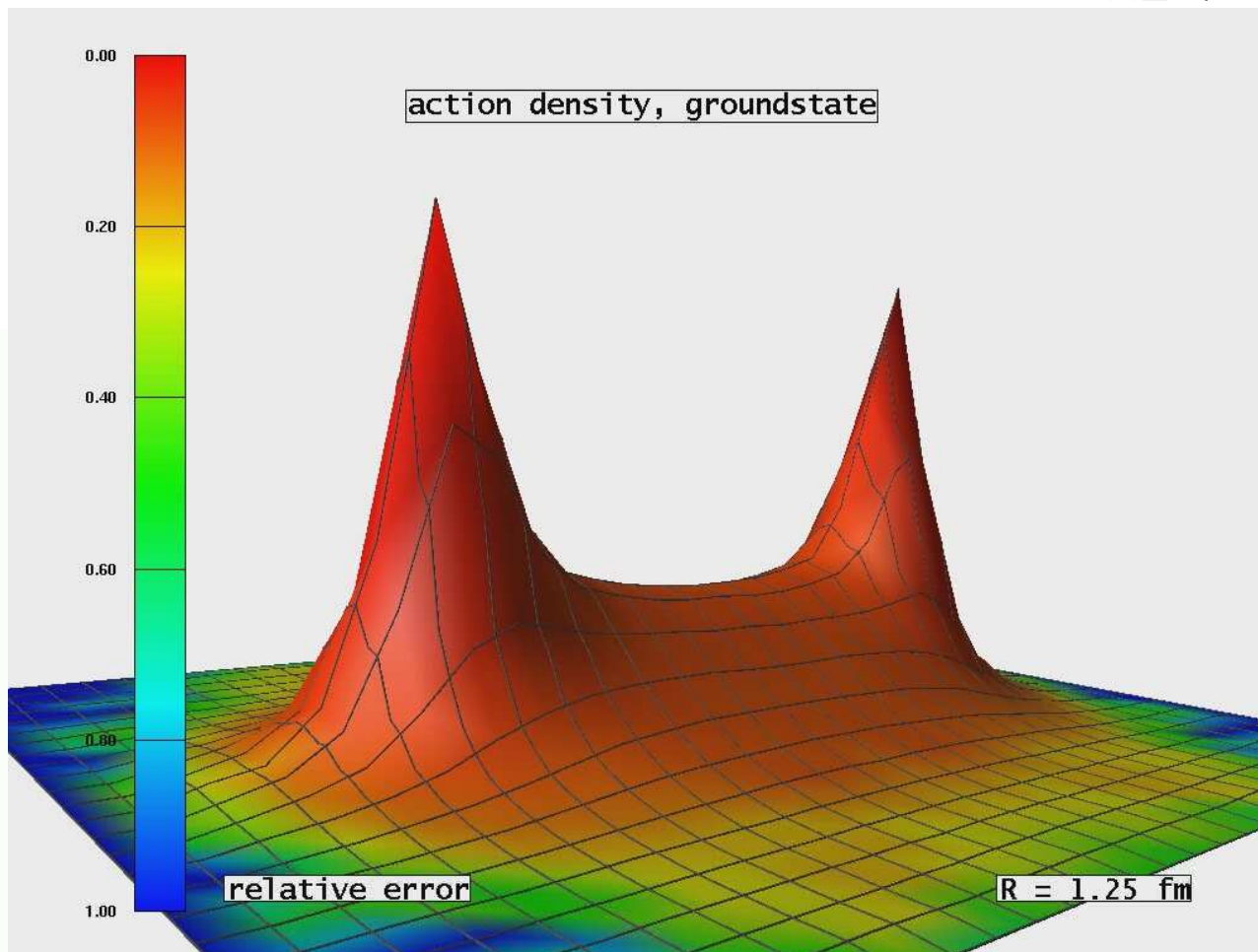
$$\sigma \sim 470 \text{ MeV}$$

Necco & Sommer  
he-lq/0108008



# Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force =  $F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$



Bali, *et al.*  
he-lq/0512018



# Confinement

- What happens in the real world; namely, in the presence of light-quarks?

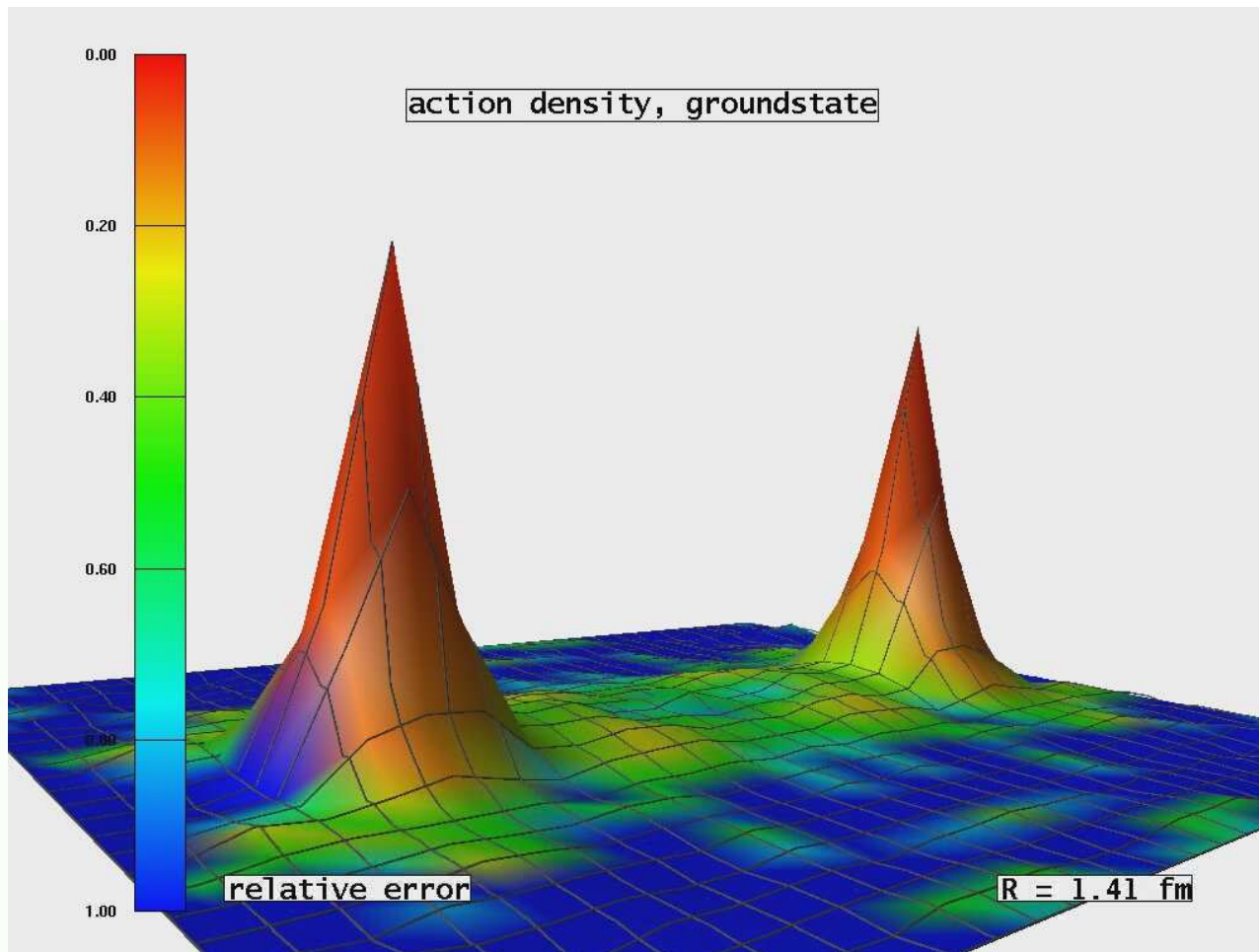




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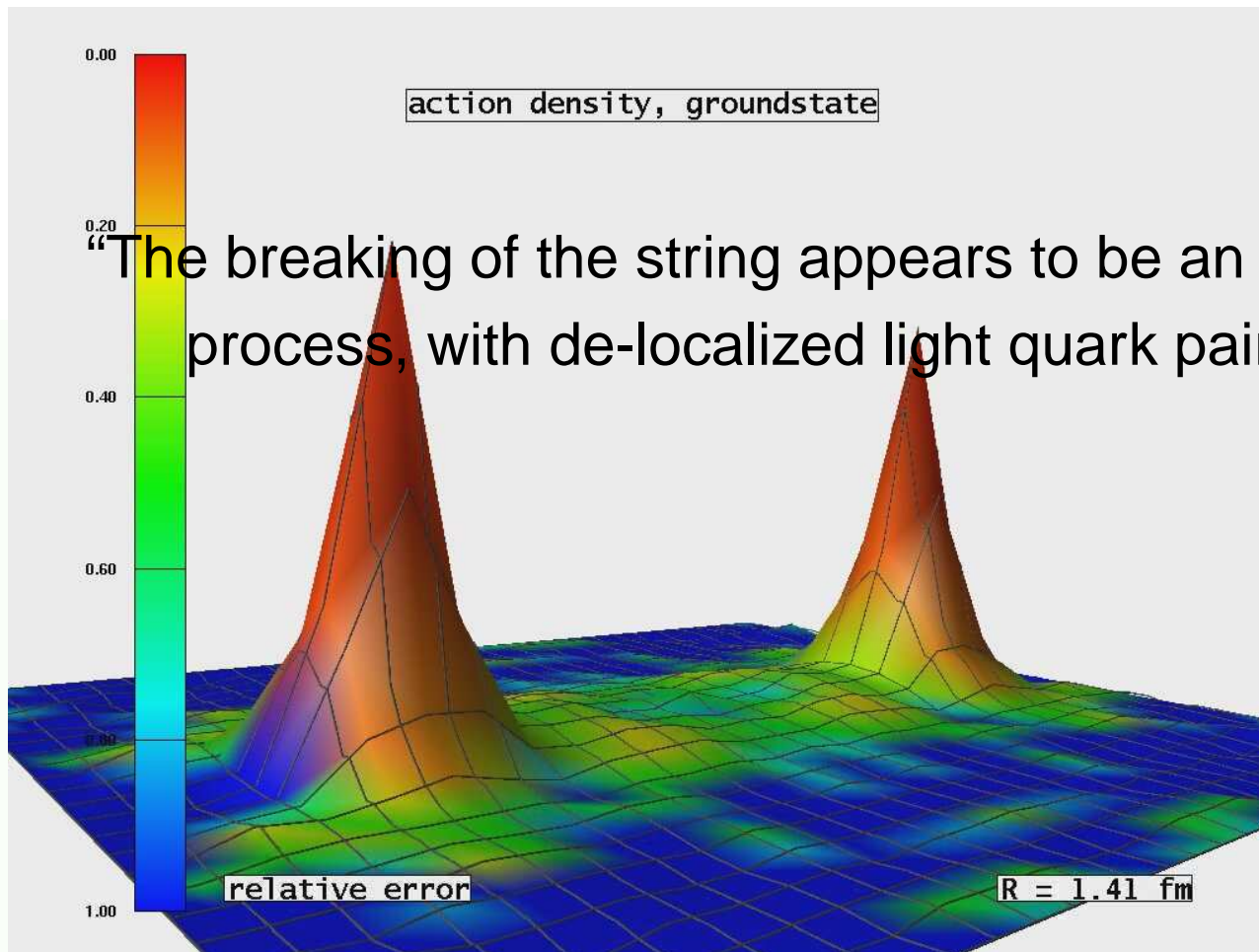
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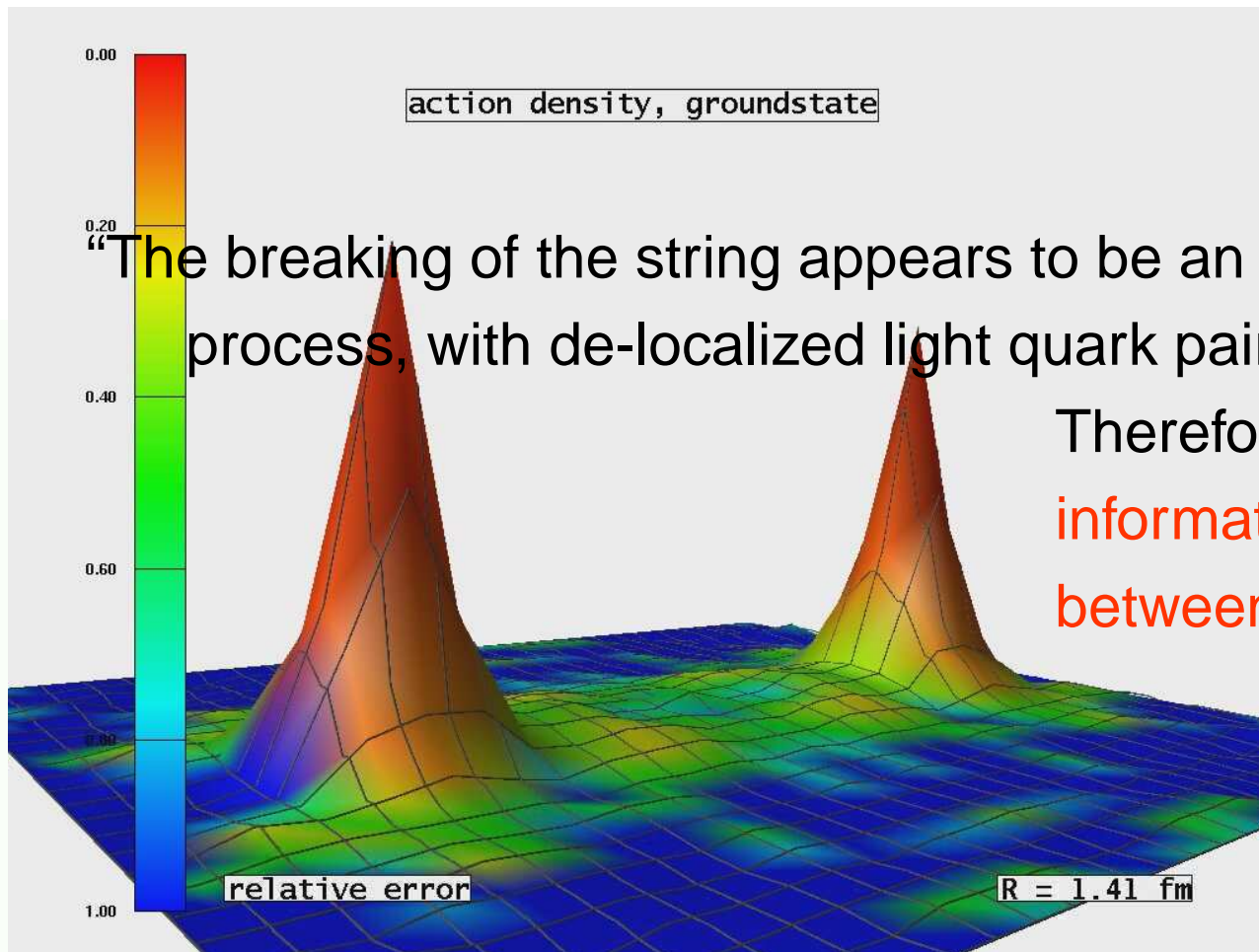


“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”



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“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

Therefore ... **No**  
**information on *potential***  
**between light-quarks.**



# Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts  
nu-th/0406030

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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ALL pseudoscalar mesons **except  $\pi(140)$**  in **chiral limit**
- **Dynamical Chiral Symmetry Breaking**  
– Goldstone’s Theorem –  
impacts upon **every pseudoscalar meson**





# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032

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[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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 $\Rightarrow f_{\pi_1} < 8.4 \text{ MeV}$   
*Diehl & Hiller*  
*he-ph/0105194*



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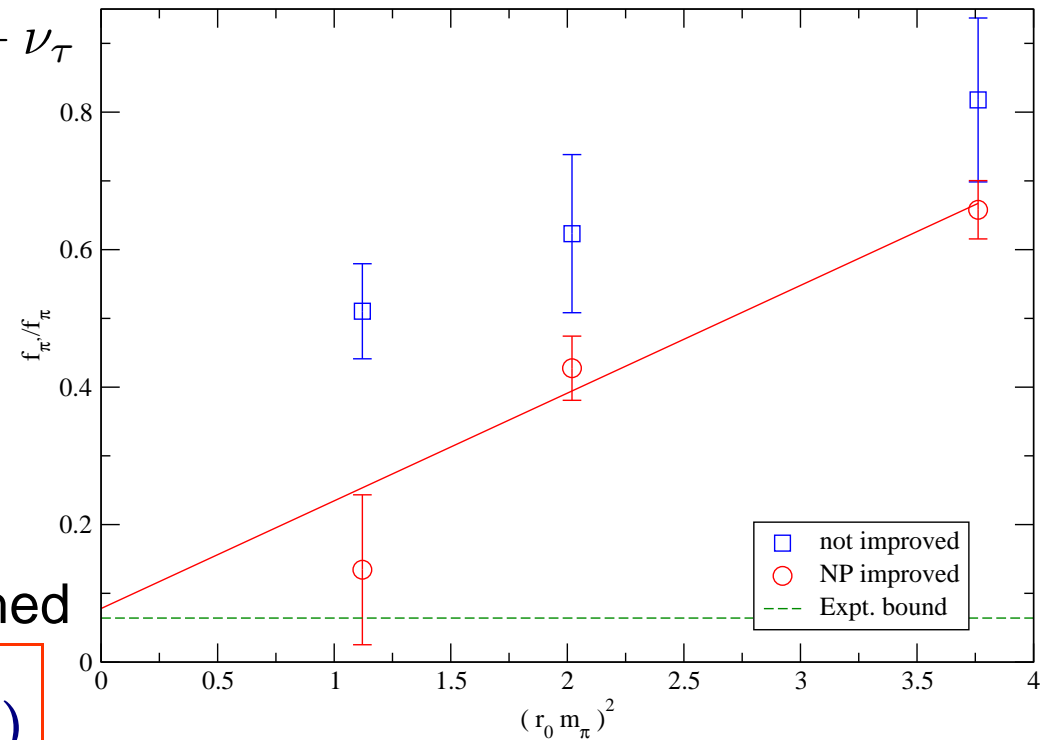
- Lattice-QCD check:

$$16^3 \times 32,$$

$$a \sim 0.1 \text{ fm},$$

two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$$



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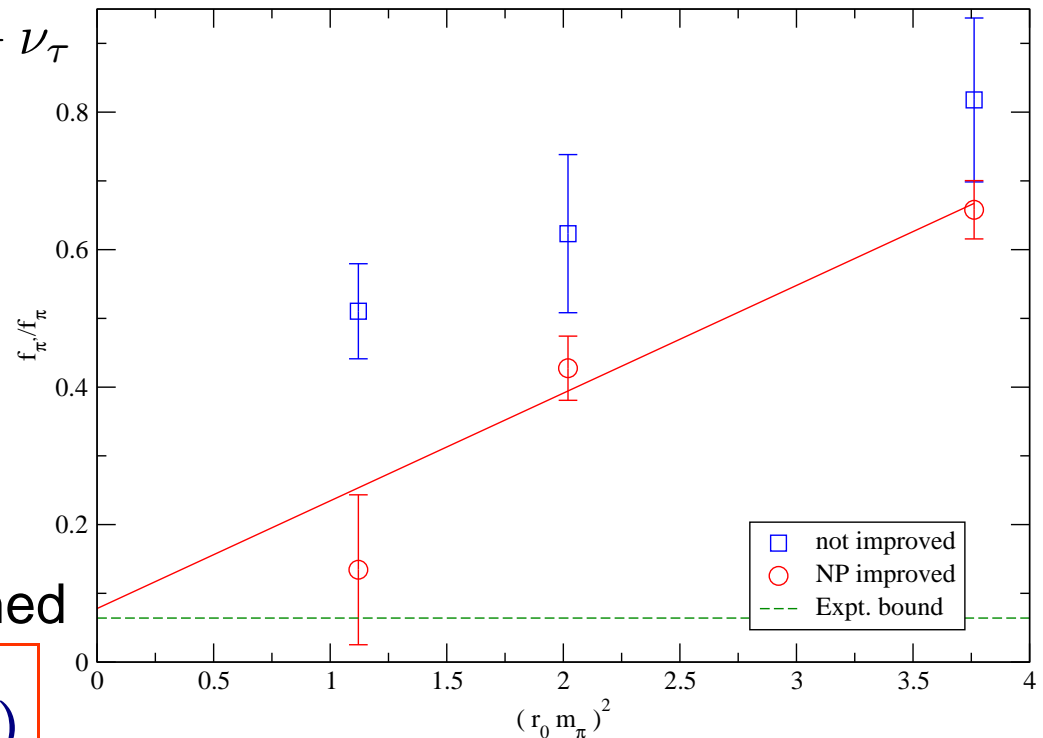
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- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)



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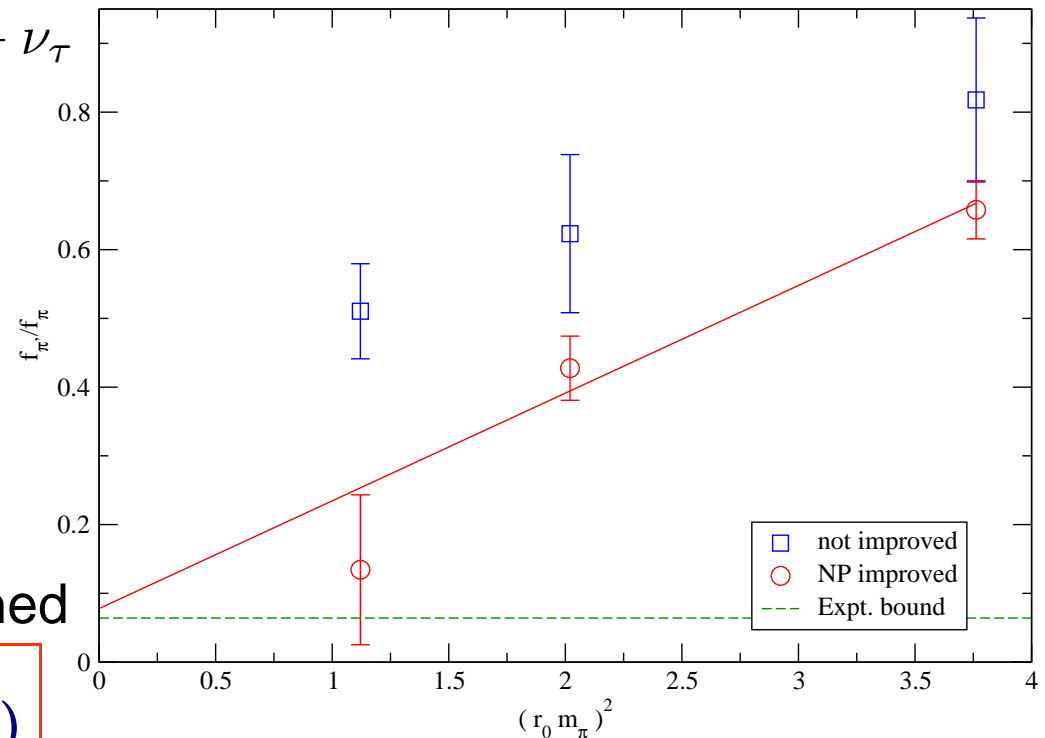
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- The suppression of  $f_{\pi_1}$  is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.



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# Pion Form Factor

Procedure Now Straightforward



[First](#)

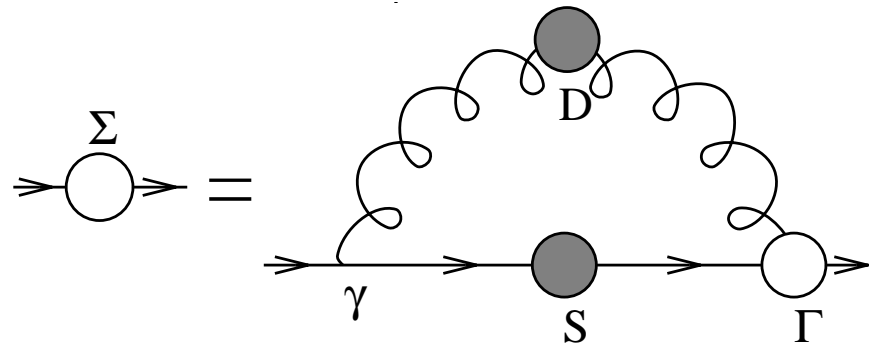
[Contents](#)

[Back](#)

[Conclusion](#)

# Pion Form Factor

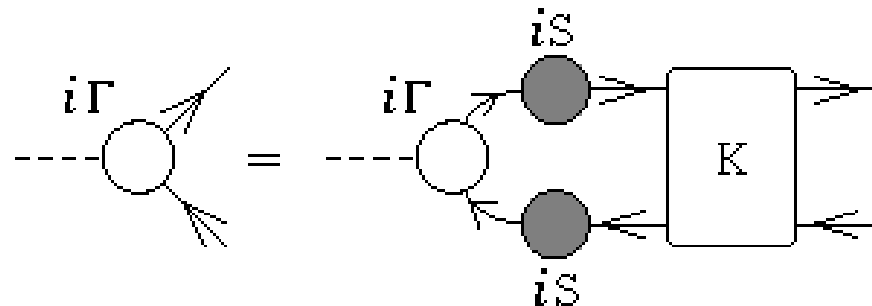
- Solve Gap Equation
  - ⇒ Dressed-Quark Propagator,  $S(p)$





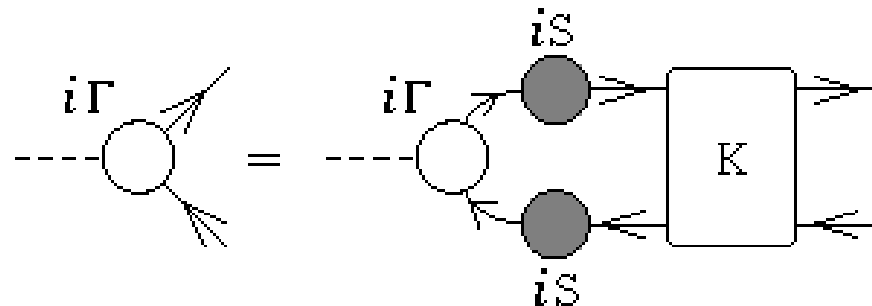
# Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel,  $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude,  $\Gamma_\pi$



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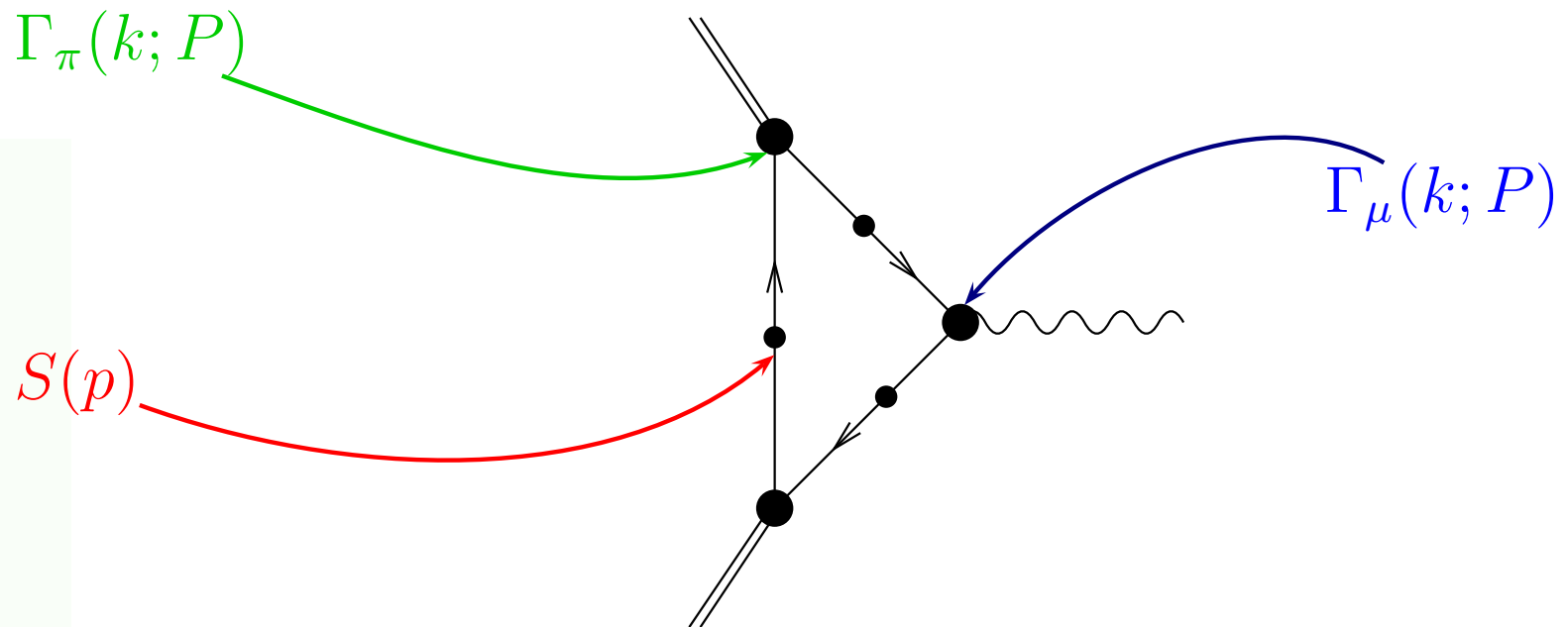


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex,  $\Gamma_\mu$



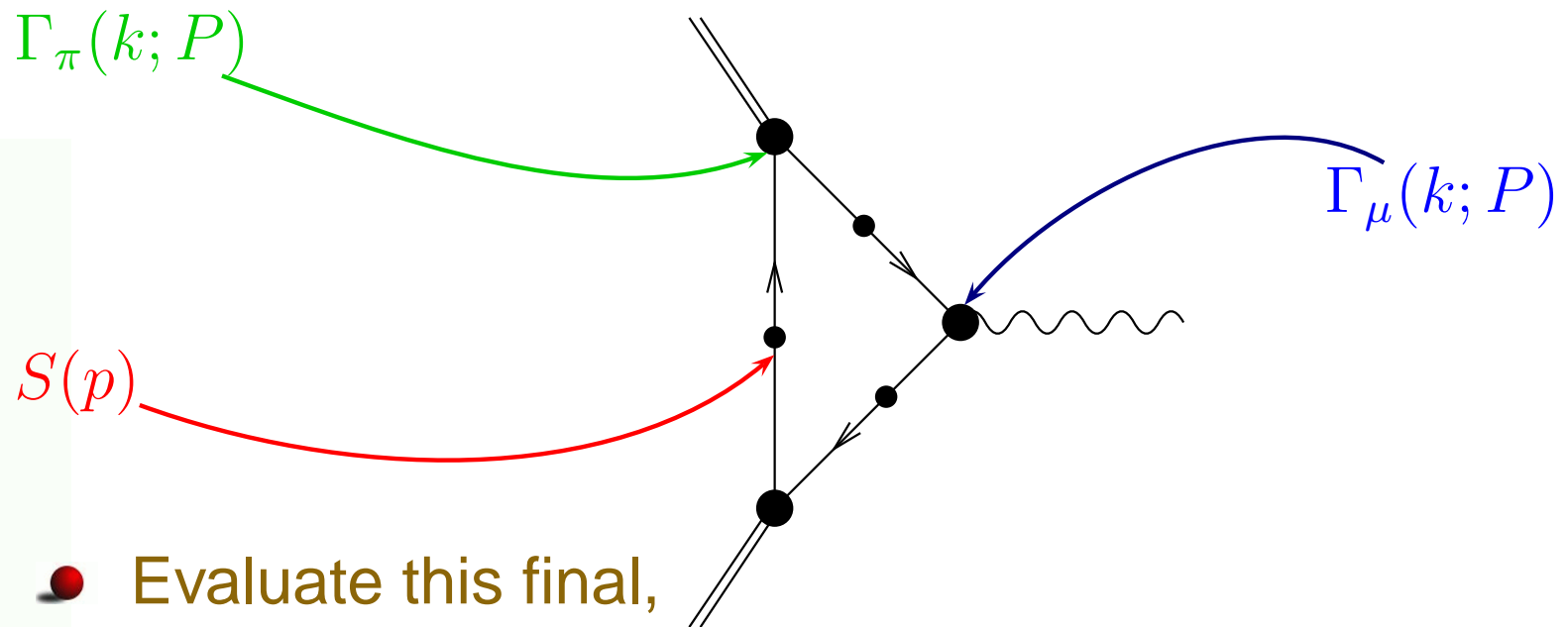
# Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



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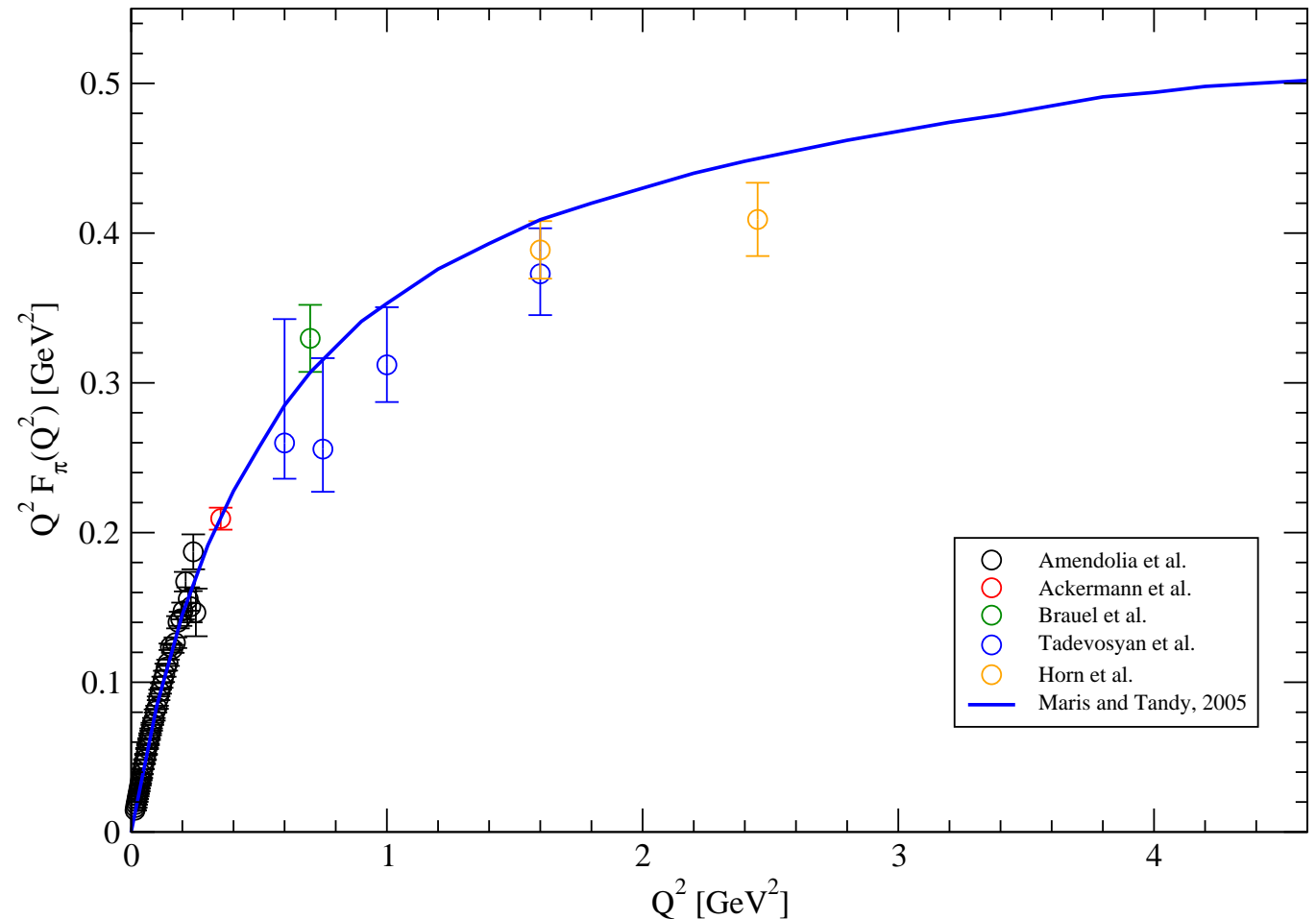


- Evaluate this final, three-dimensional integral



# Calculated Pion Form Factor

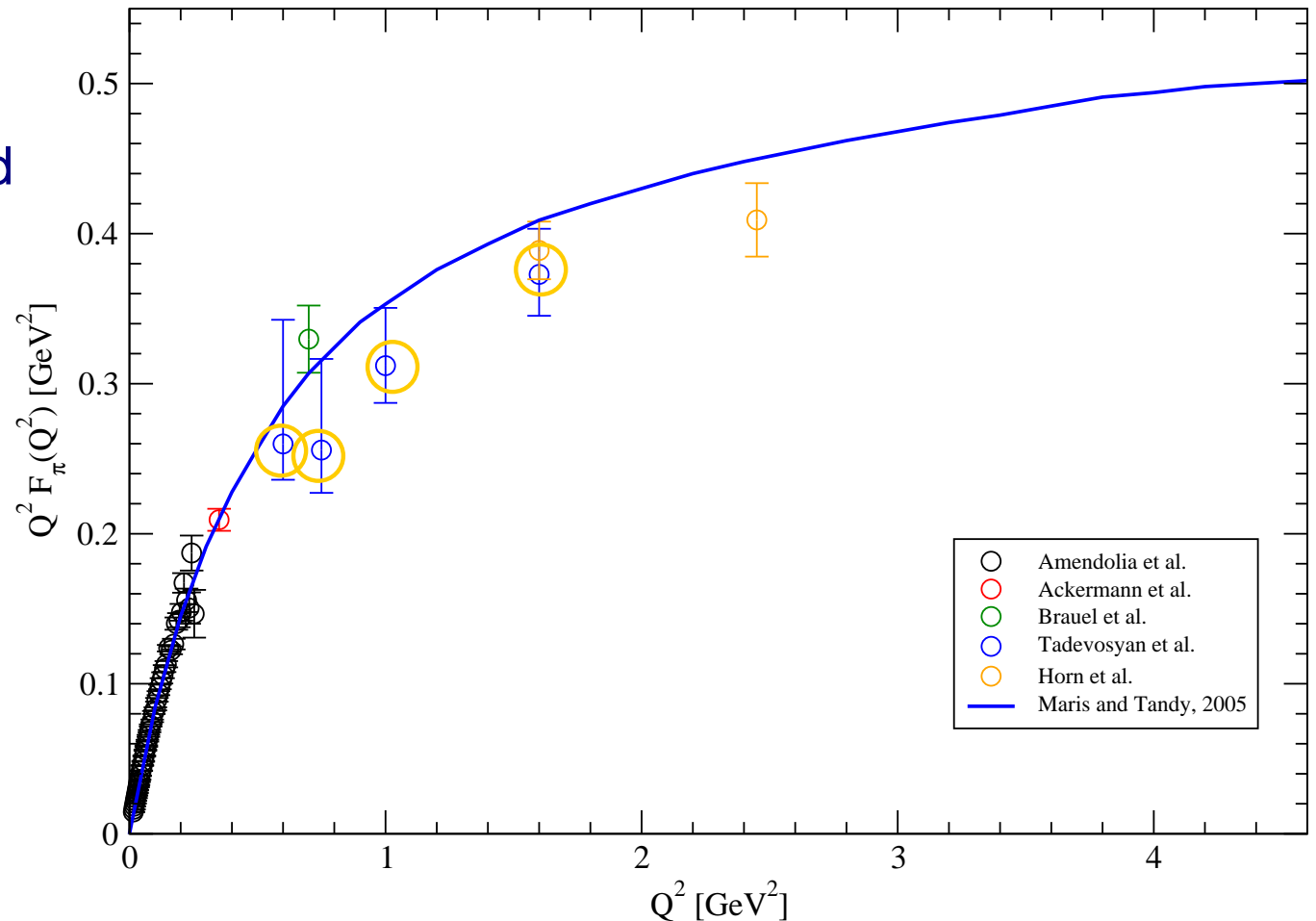
Calculation first published in 1999; No Parameters Varied  
Numerical method improved in 2005



# Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied  
Numerical method improved in 2005

Data published  
in 2001.  
Subsequently  
revised





# Timelike Pion Form Factor



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



*Ab initio* calculation into  
timelike region. Deeper than  
ground-state  $\rho$ -meson pole

# Timelike Pion Form Factor



[First](#)

[Contents](#)

[Back](#)

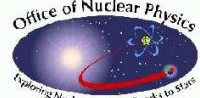
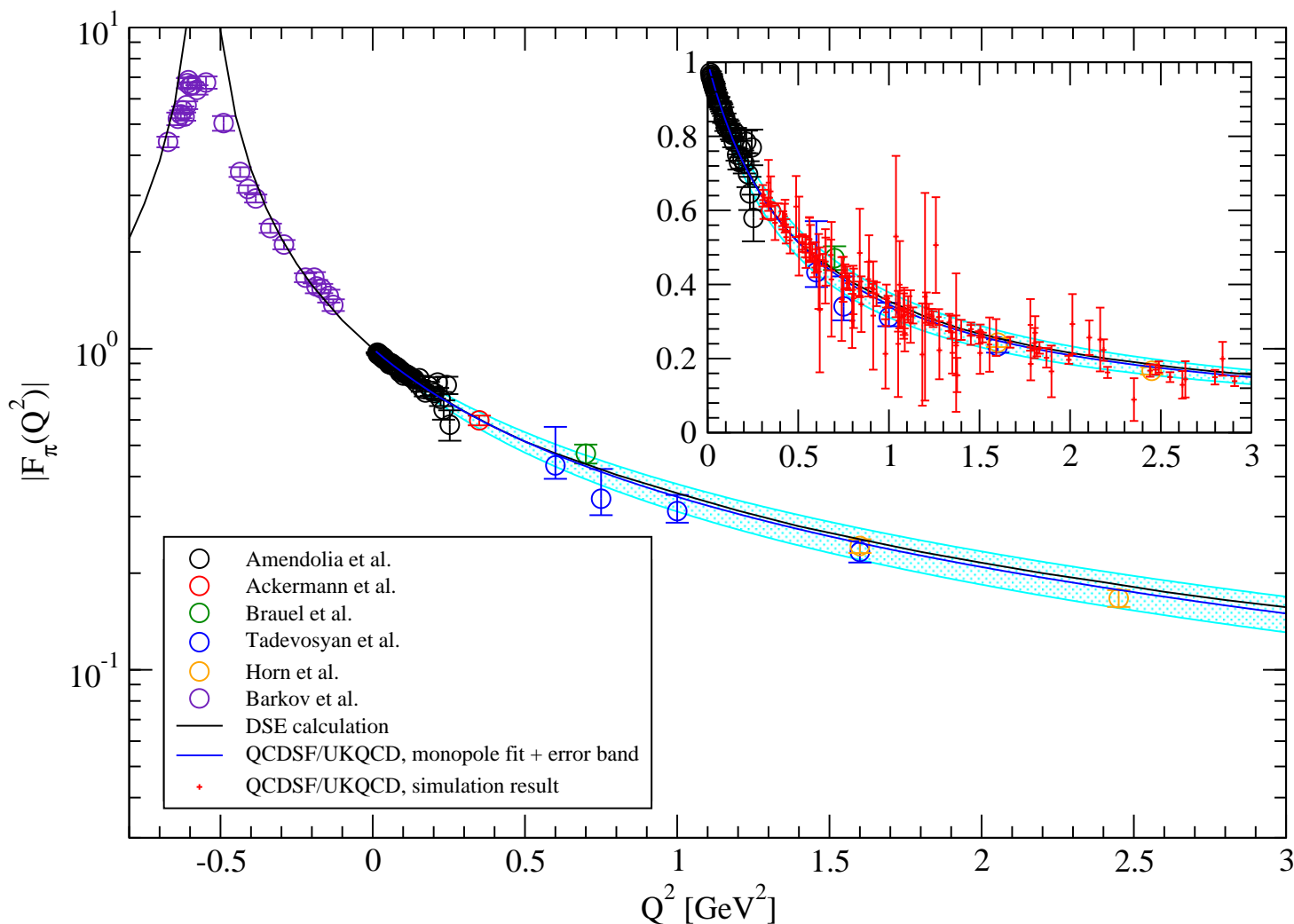
[Conclusion](#)





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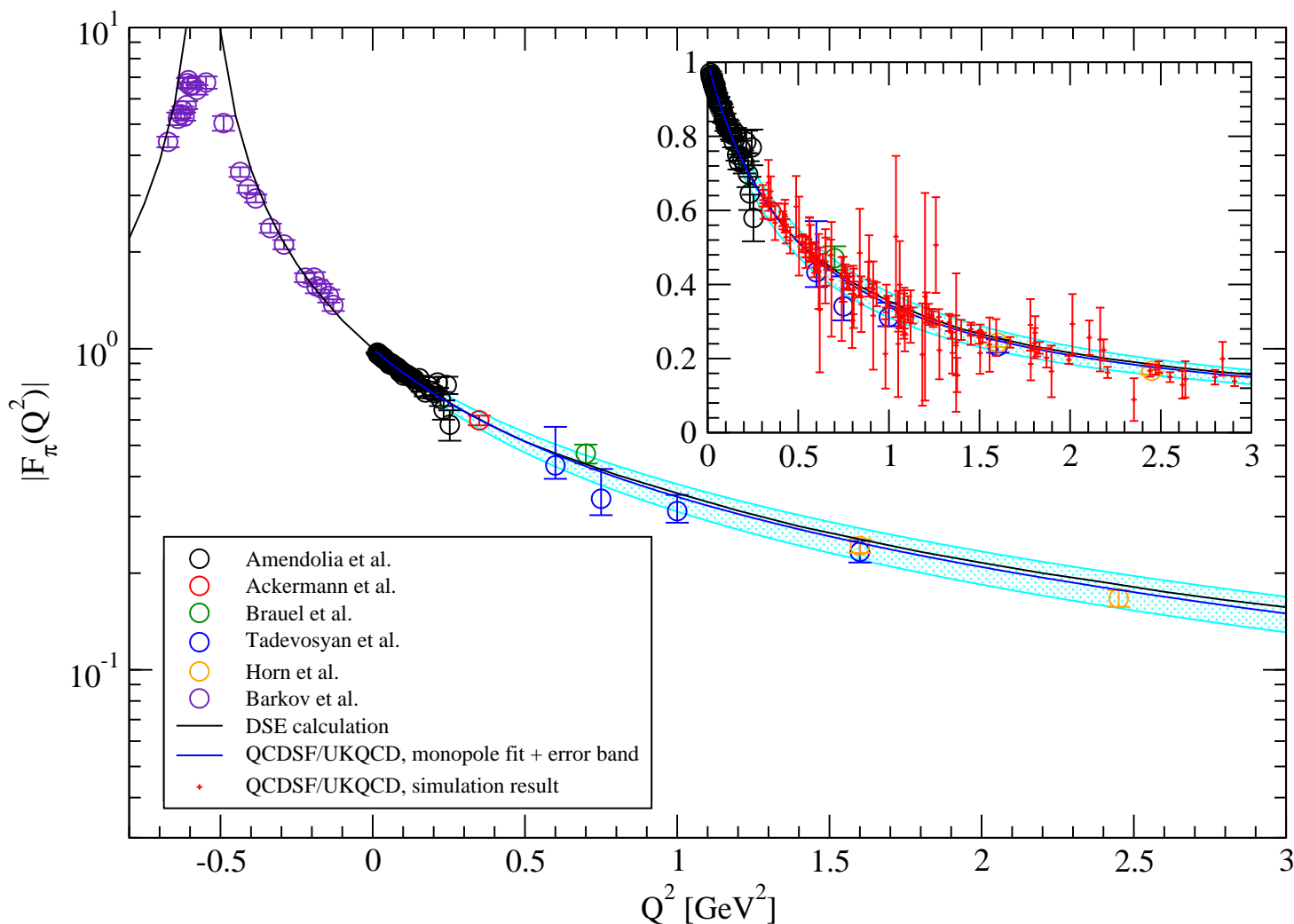
Argonne  
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LABORATORY



*Ab initio* calculation into timelike region. Deeper than ground-state  $\rho$ -meson pole

# Timelike Pion Form Factor

$\rho$ -meson not put in “by hand” – generated dynamically as a bound-state of dressed-quark and dressed-antiquark



# *Dimensionless product: $r_\pi f_\pi$*



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



# Dimensionless product: $r_\pi f_\pi$



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



# *Dimensionless product: $r_\pi f_\pi$*

- Improved rainbow-ladder interaction





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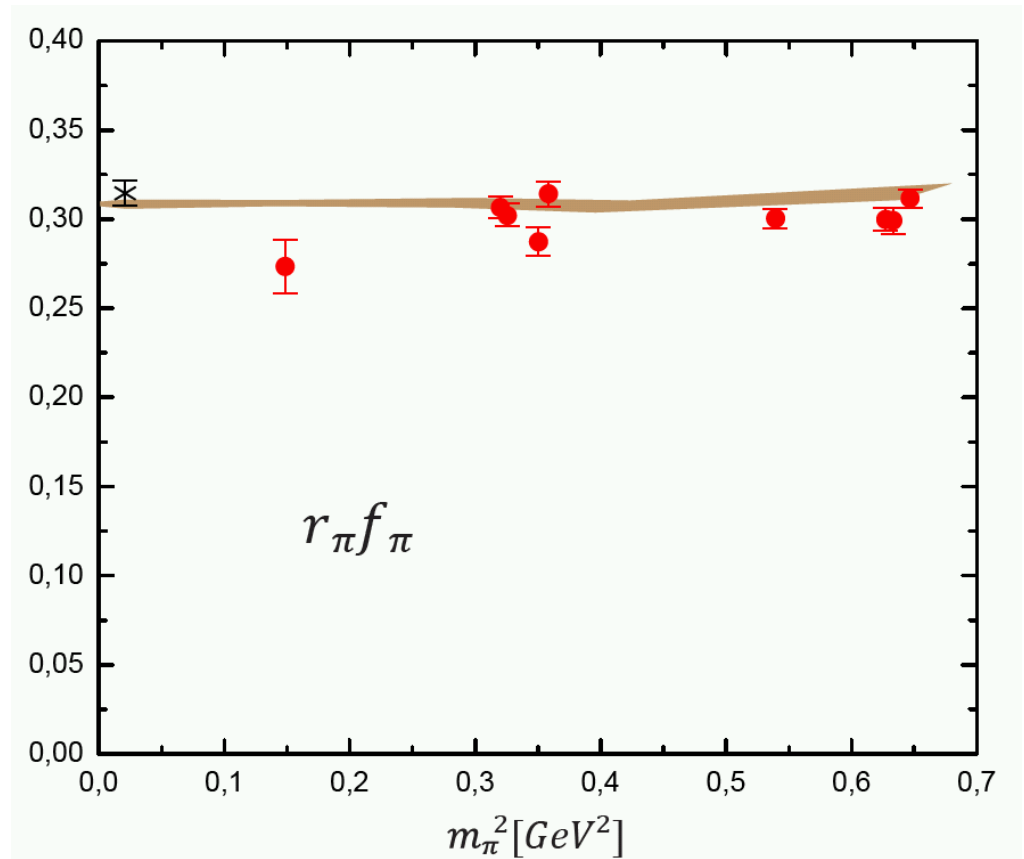
- Improved rainbow-ladder interaction
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● DSE prediction





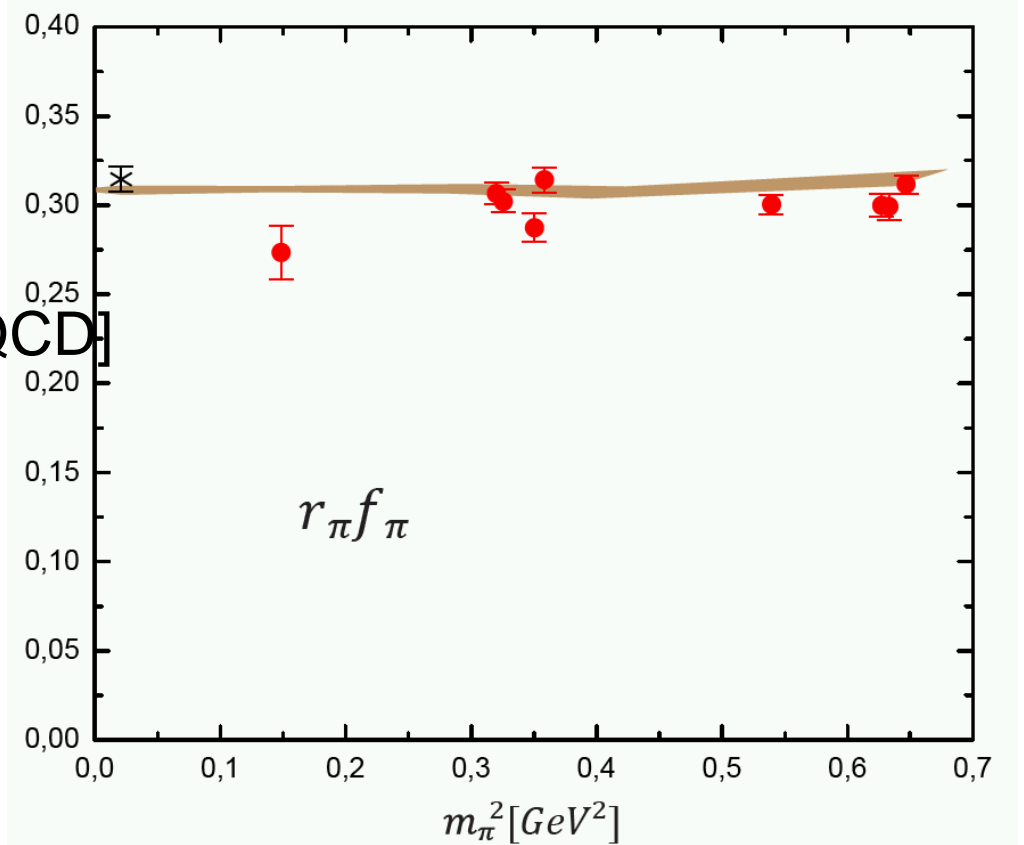
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● Lattice results

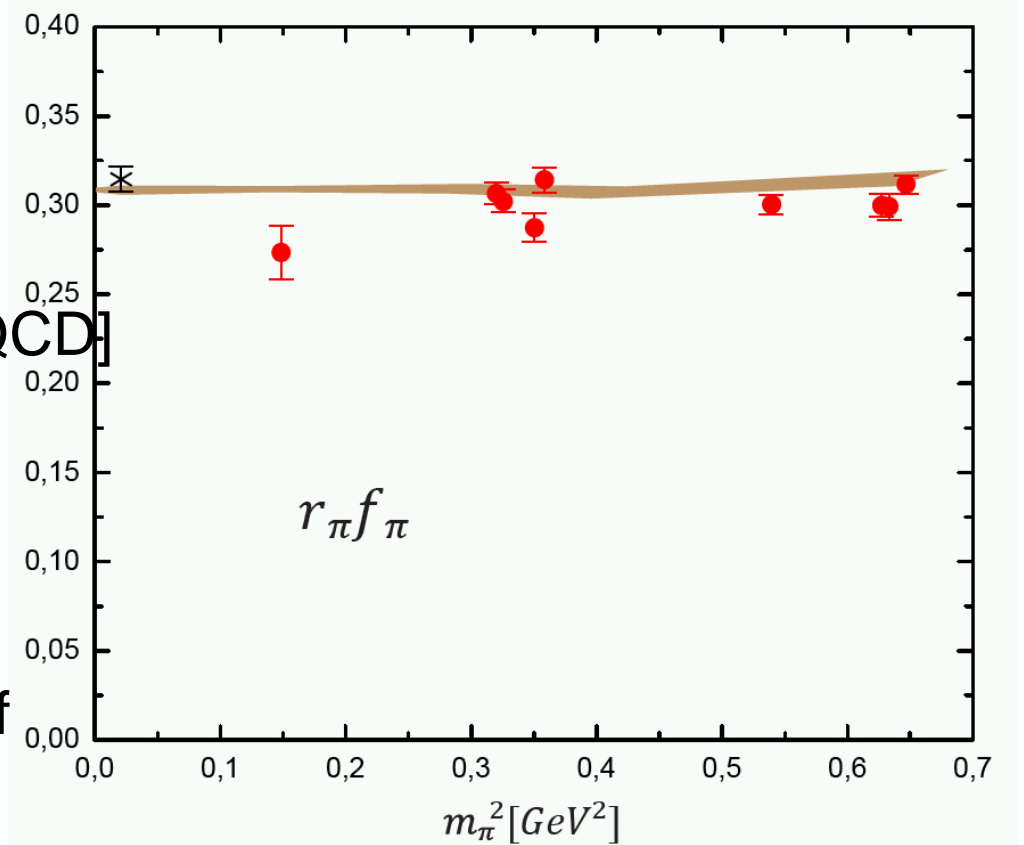
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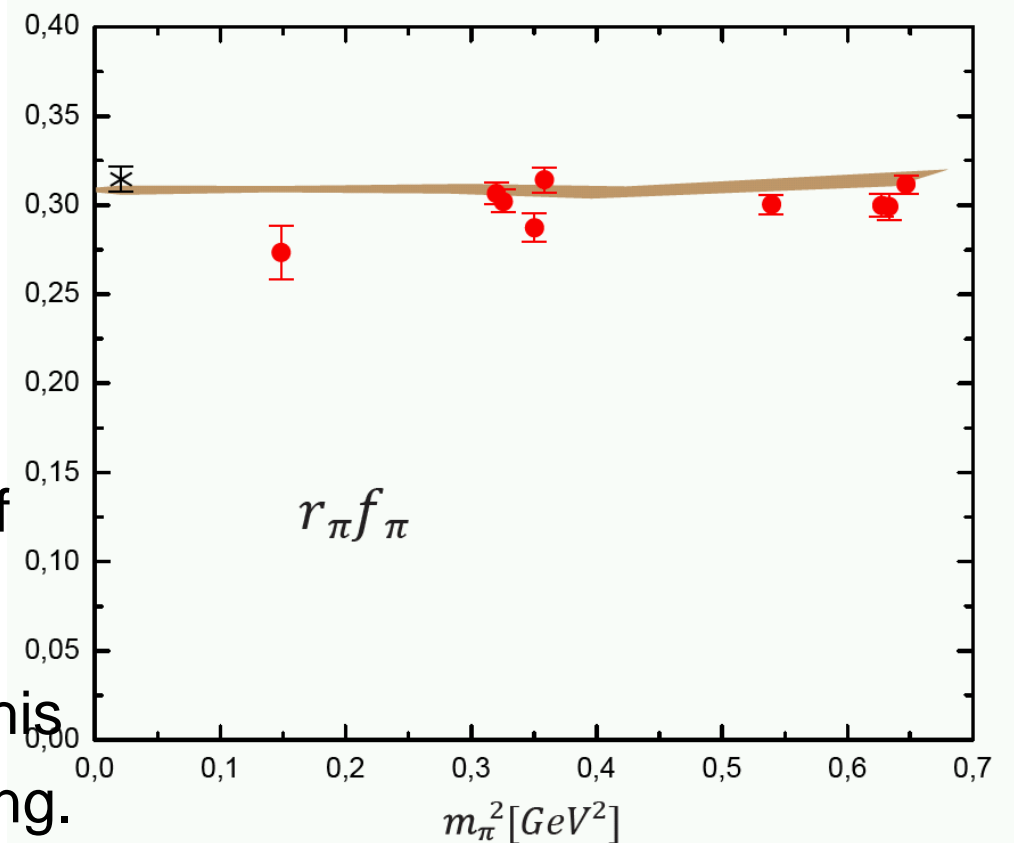
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obtains independent of  
current-quark mass.
- We have understood this  
Implications far-reaching.



# Pion Form Factors



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

- There is a sense in which it is easy to fabricate a model that can *reproduce* the elastic electromagnetic pion form factor



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## *Infidelity without simultaneity*

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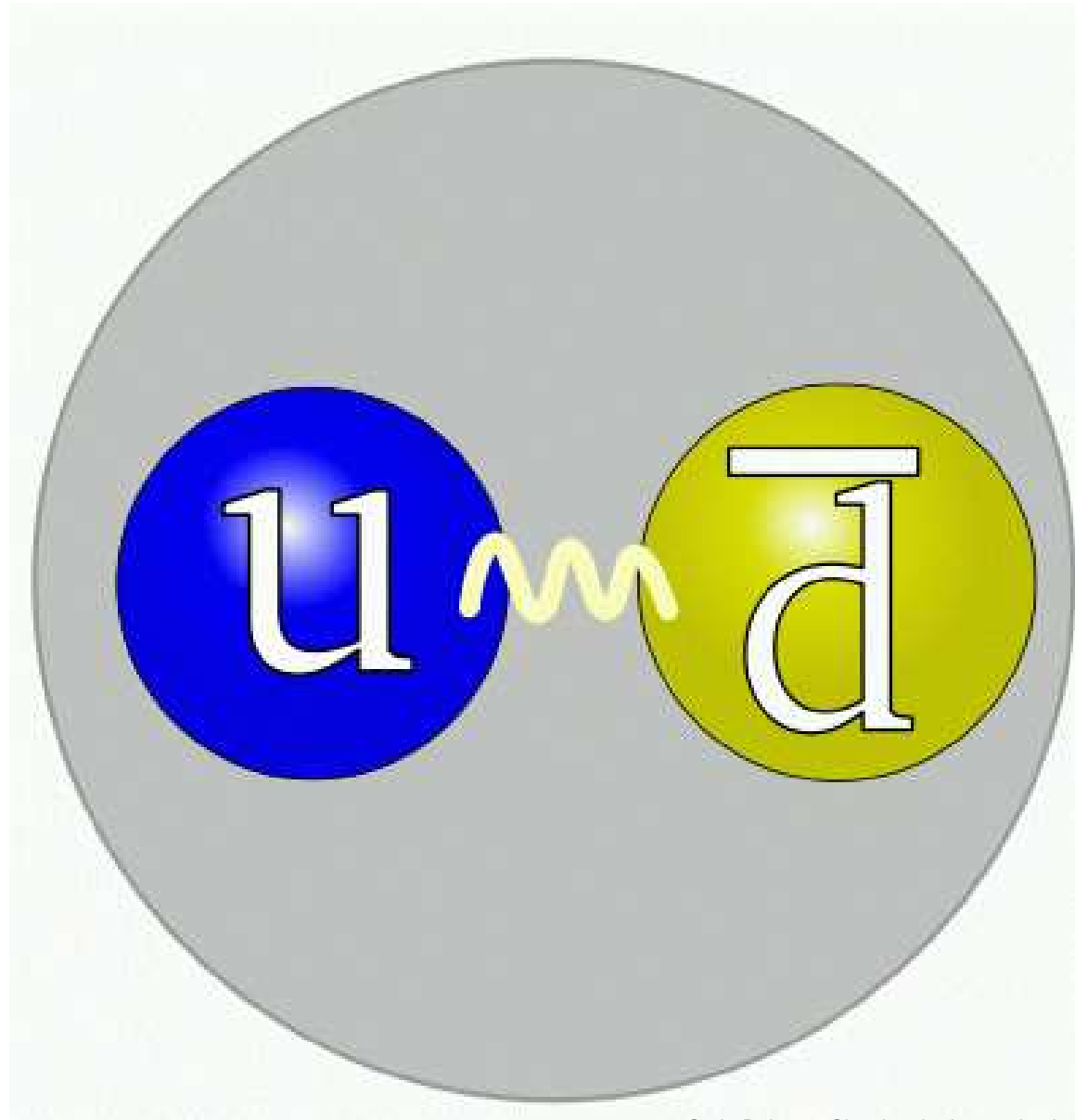
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- The latter is connected with the **Abelian anomaly** – therefore **fundamentally connected** with **chiral symmetry and its dynamical breaking** – no mere model can successfully describe this without fine tuning
- Must similarly require prediction of  $\gamma^* \pi \rightarrow \pi\pi$  and all other anomalous processes



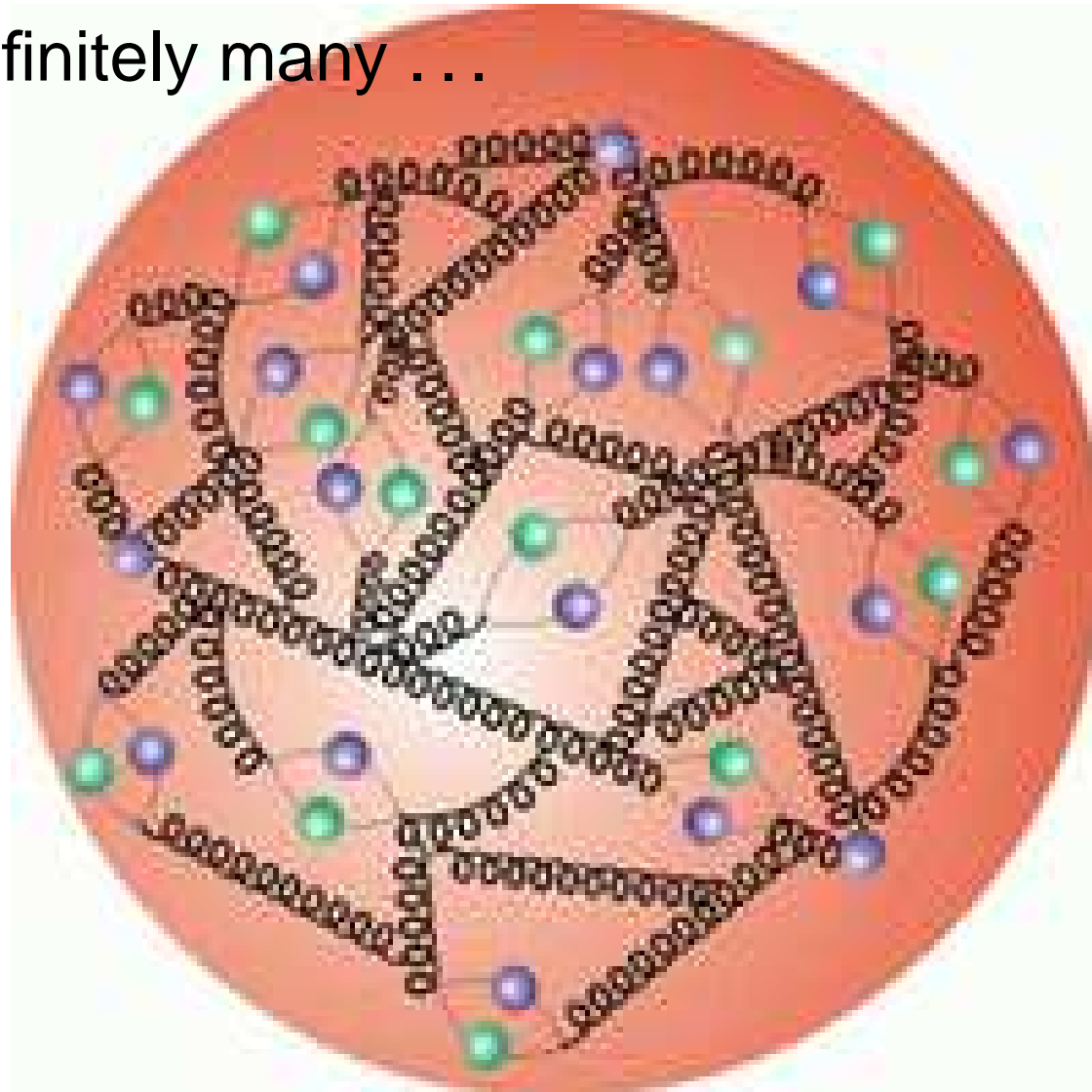


# Answer for the pion



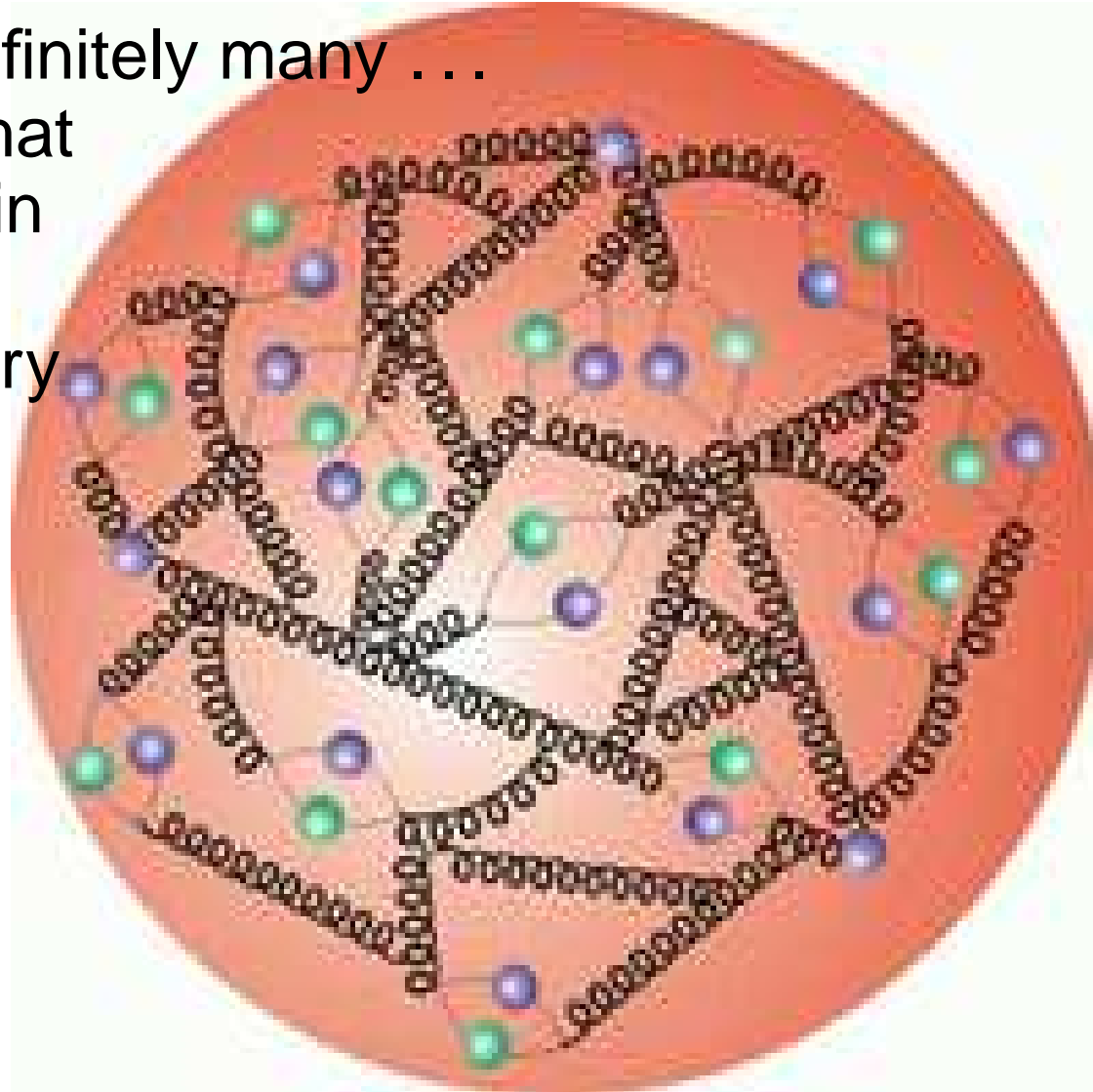
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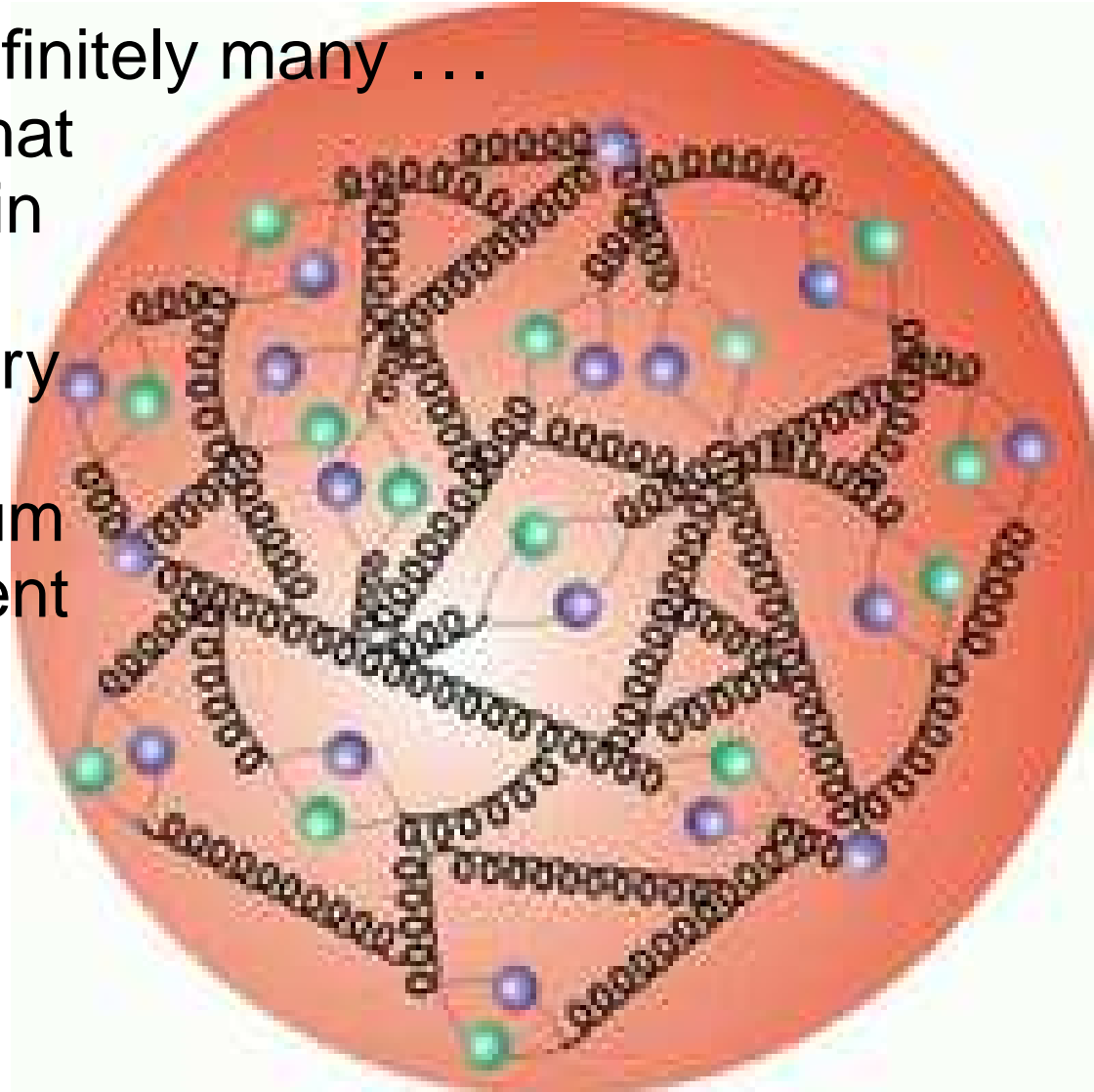


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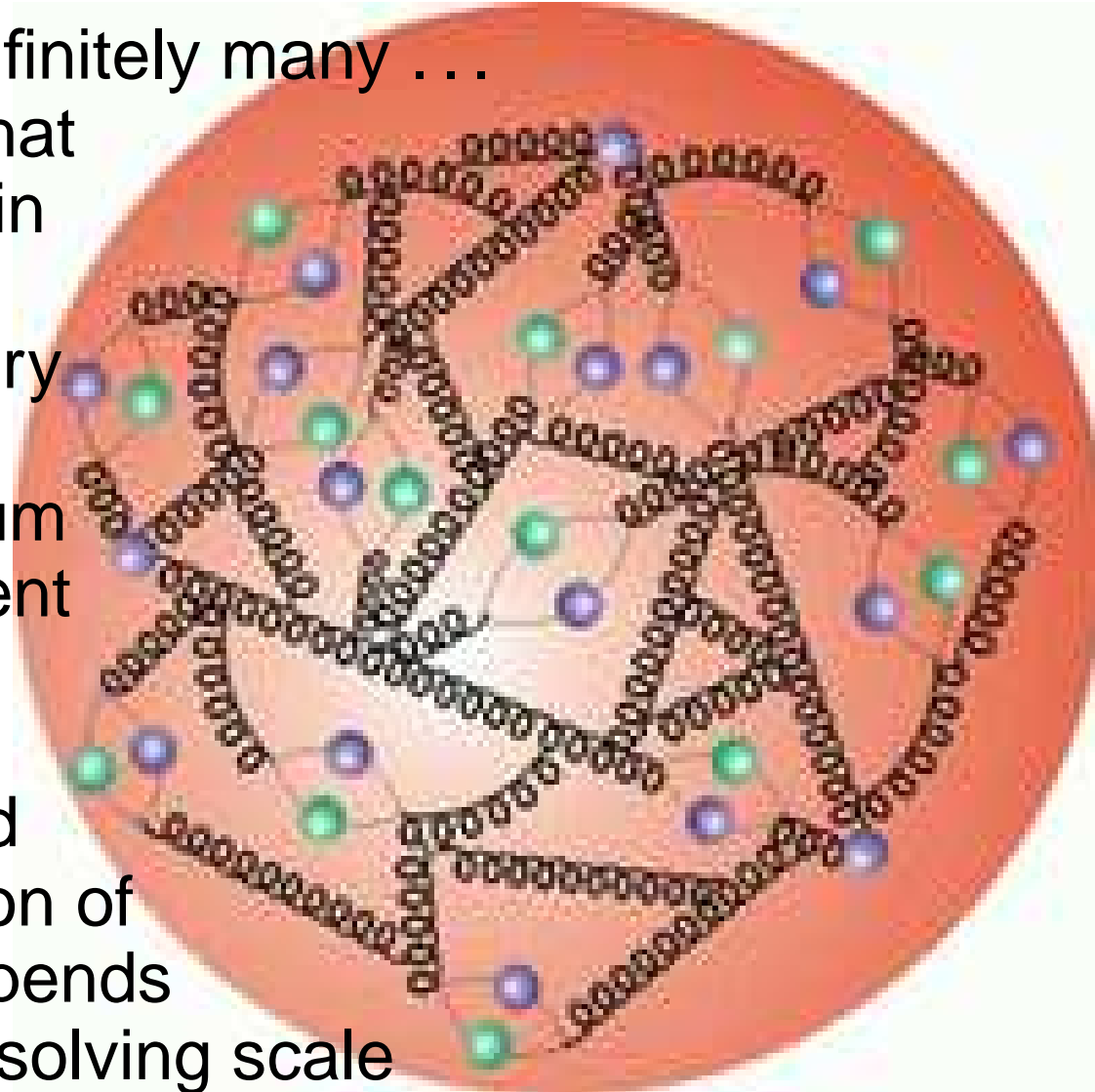
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momentum-dependent dressing

...

perceived distribution of mass depends on the resolving scale





# *Ab-initio study of mesons & nucleons*

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[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Eichmann *et al.*

- arXiv:0802.1948 [nucl-th]
- arXiv:0810.1222 [nucl-th]

# *Ab-initio study of mesons & nucleons*



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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# *Ab-initio study of mesons & nucleons*

- Leading-order truncation of DSEs – rainbow-ladder



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



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- Symmetry preserving and systematic approach can elucidate and account for these effects
  - Use this knowledge to constrain interaction in infrared
  - Interaction in ultraviolet predicted by perturbative expansion of DSEs

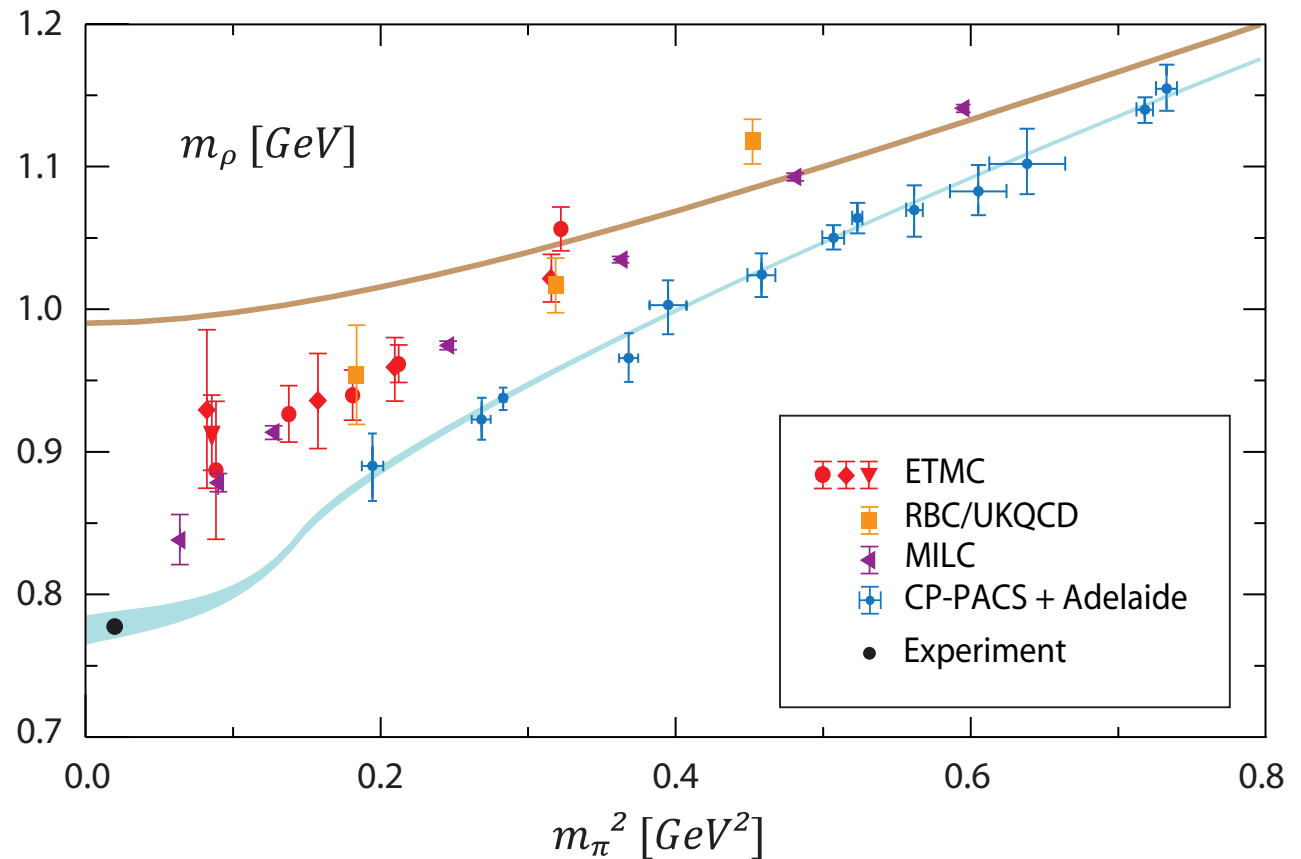


Eichmann *et al.*

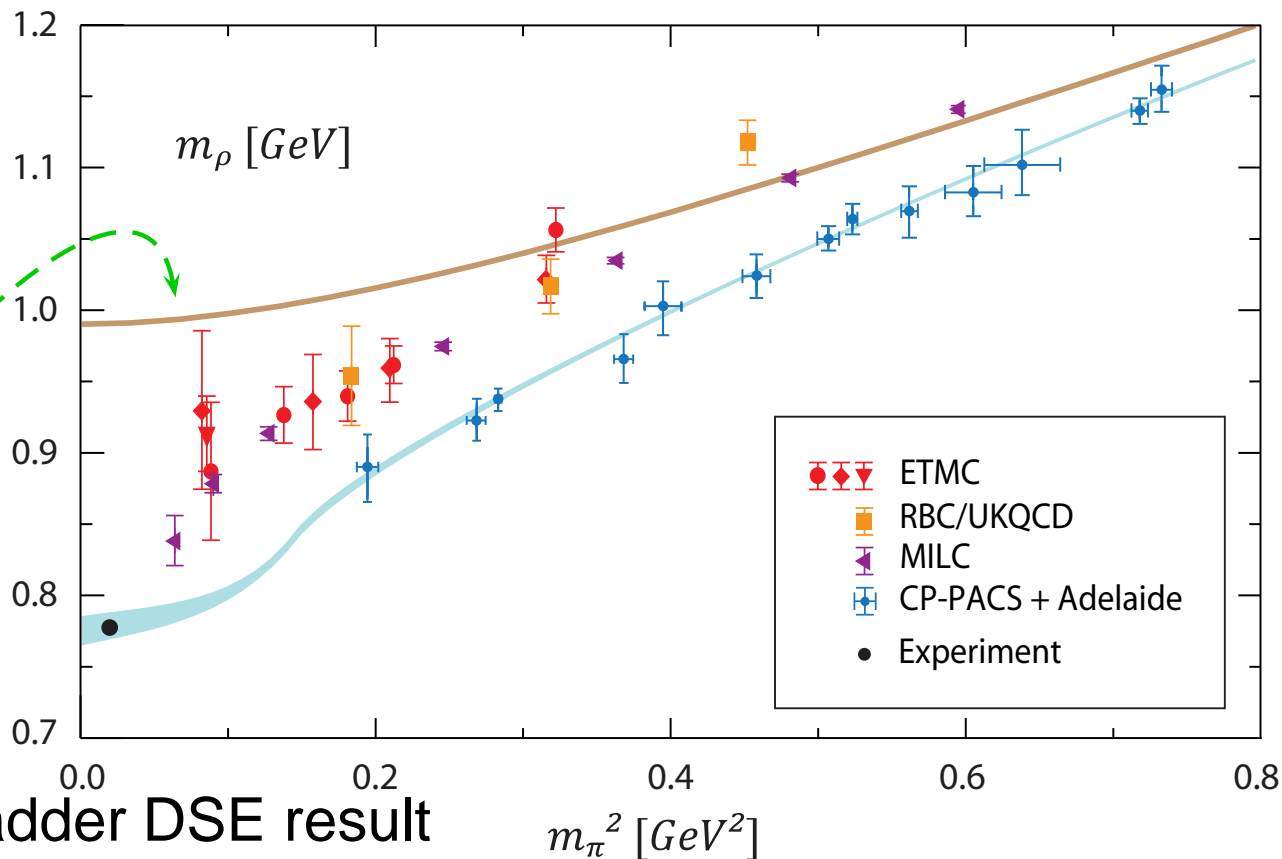
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# Ab-initio study of mesons & nucleons



# Ab-initio study of mesons & nucleons



Rainbow-Ladder DSE result

one parameter for IR – “confinement radius”

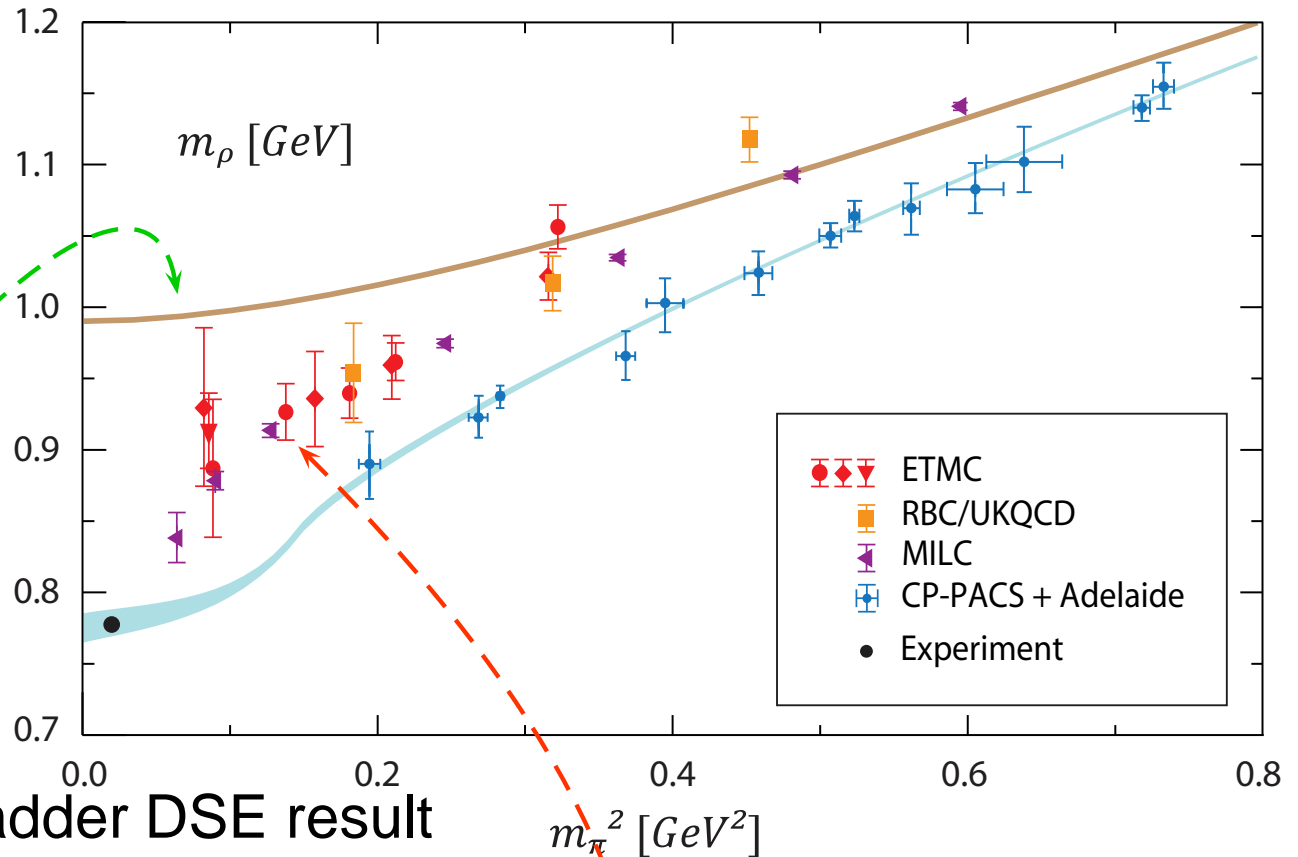
Results insensitive to value on material domain



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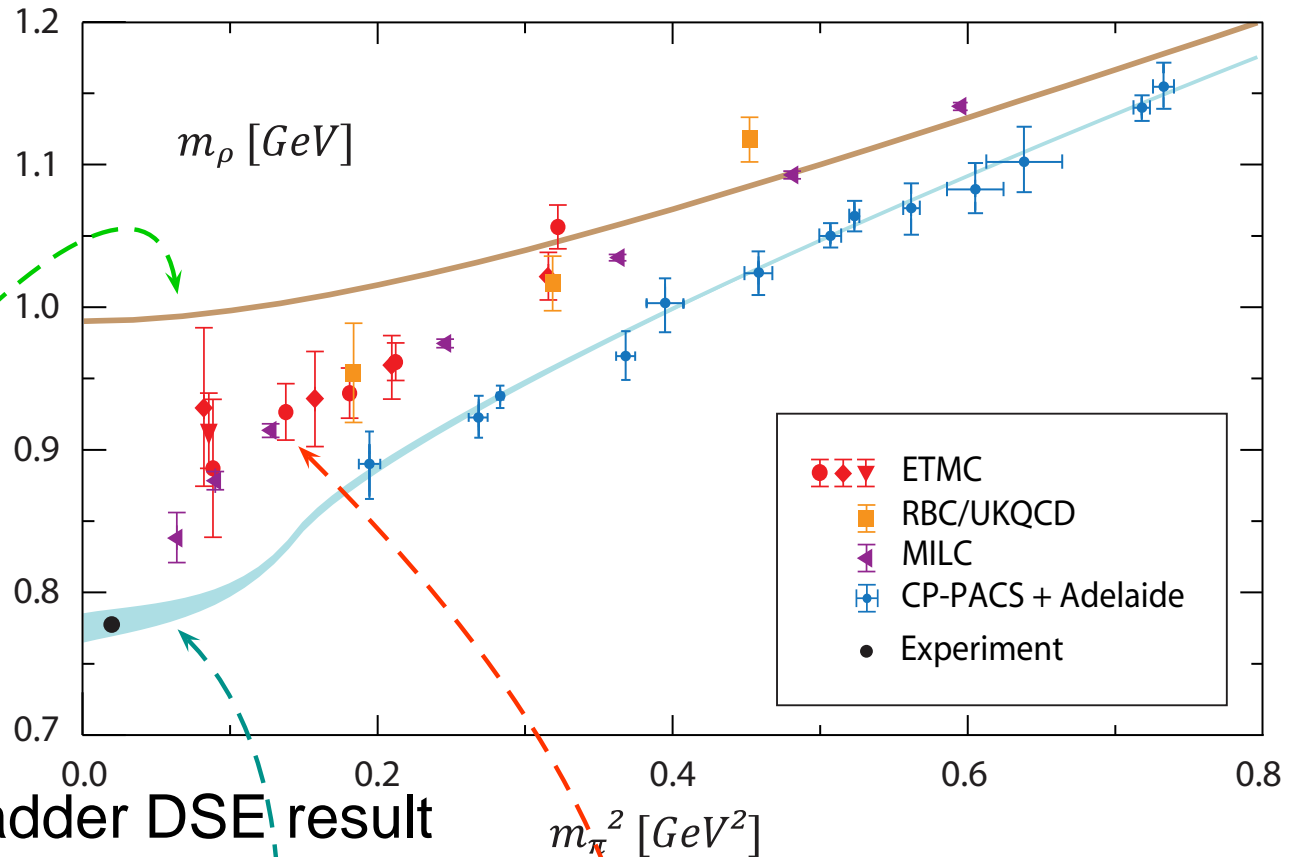
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● Rainbow-Ladder DSE result

one parameter for IR + “confinement radius”

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– arXiv:0802.1948 [nucl-th]

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- Precisely the same interaction



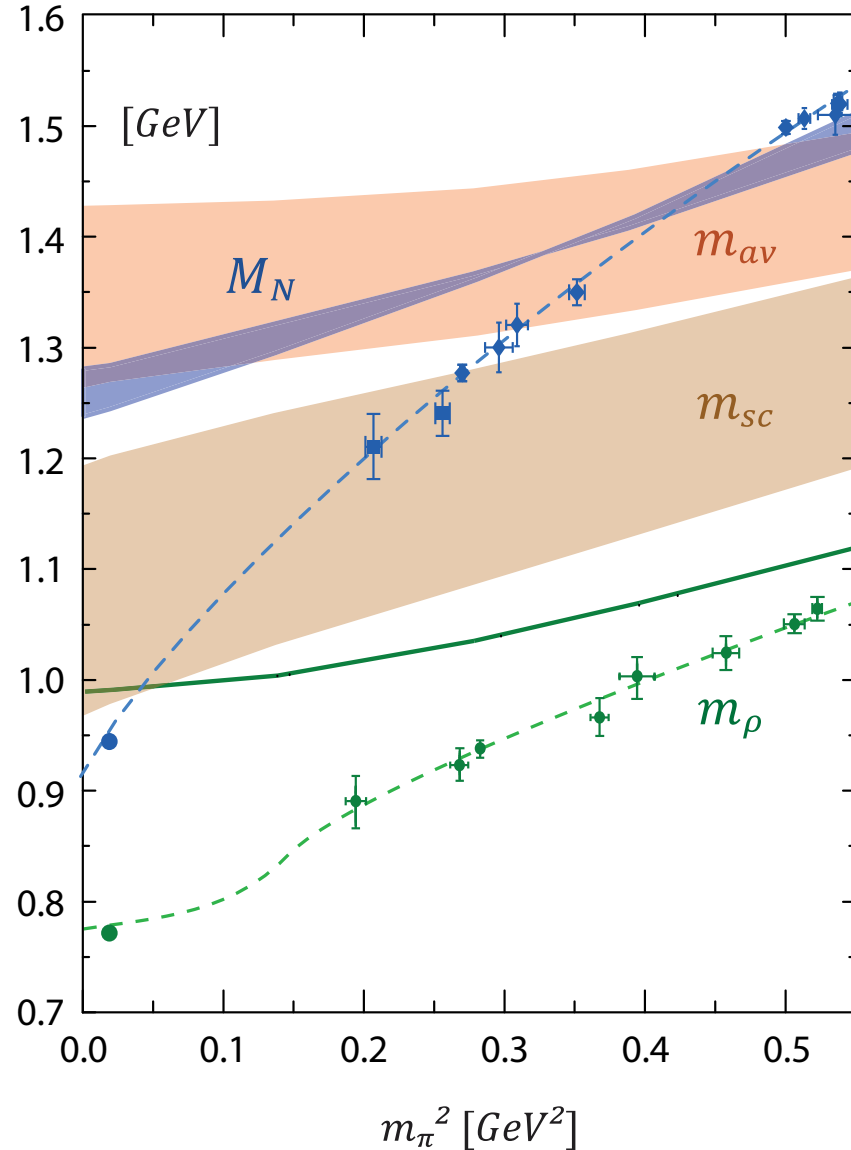
Eichmann *et al.*

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# Ab-initio study of mesons & nucleons

- Precisely the same interaction
- Same  $\rho$ -meson curve



First

Contents

Back

Conclusion

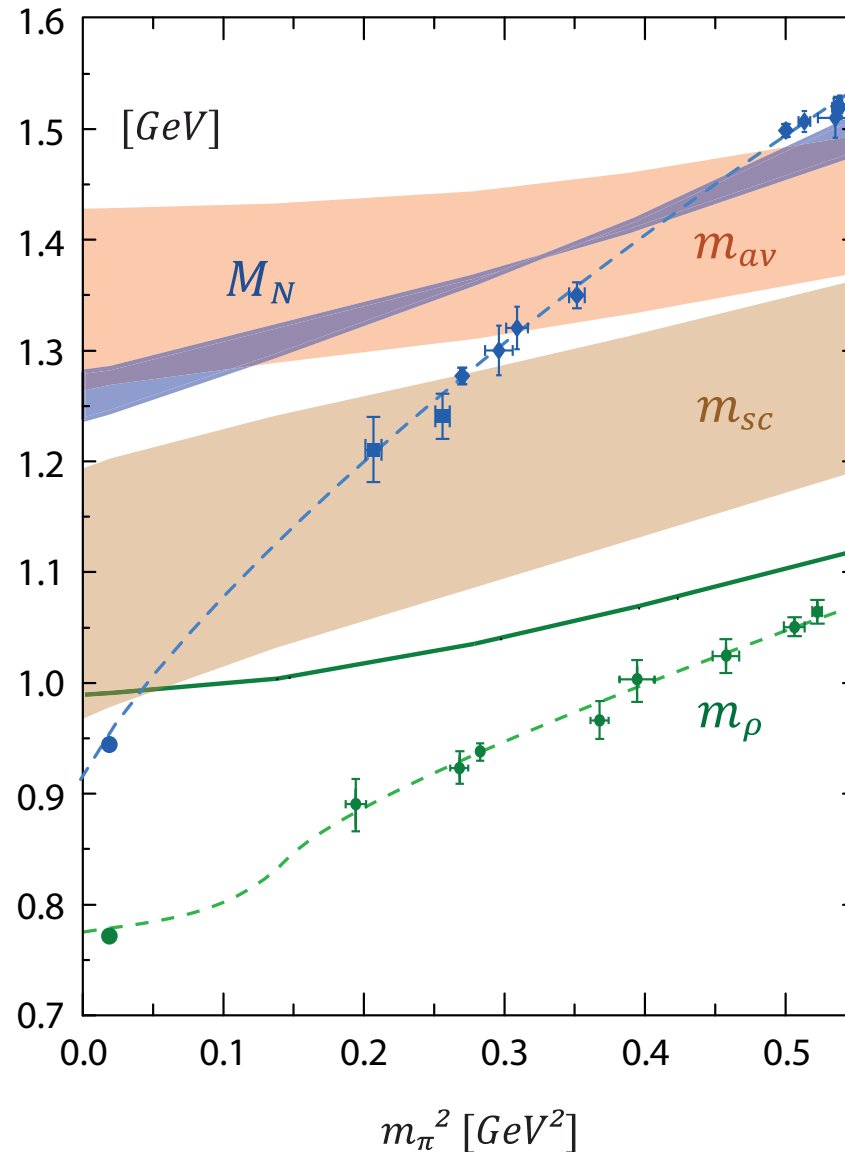
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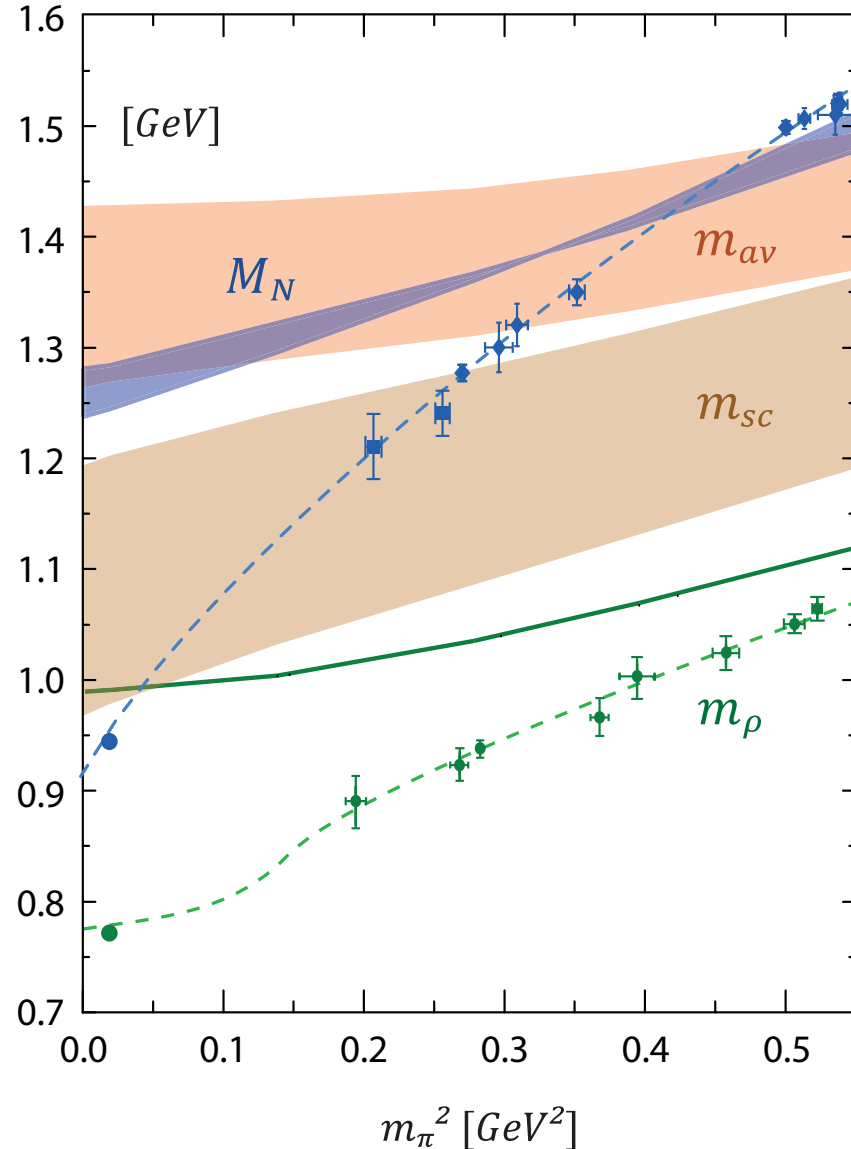
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- But ...  $[m_{av} - m_{sc}], m_\rho$  &  $M_N$  ... are *independent* of that parameter



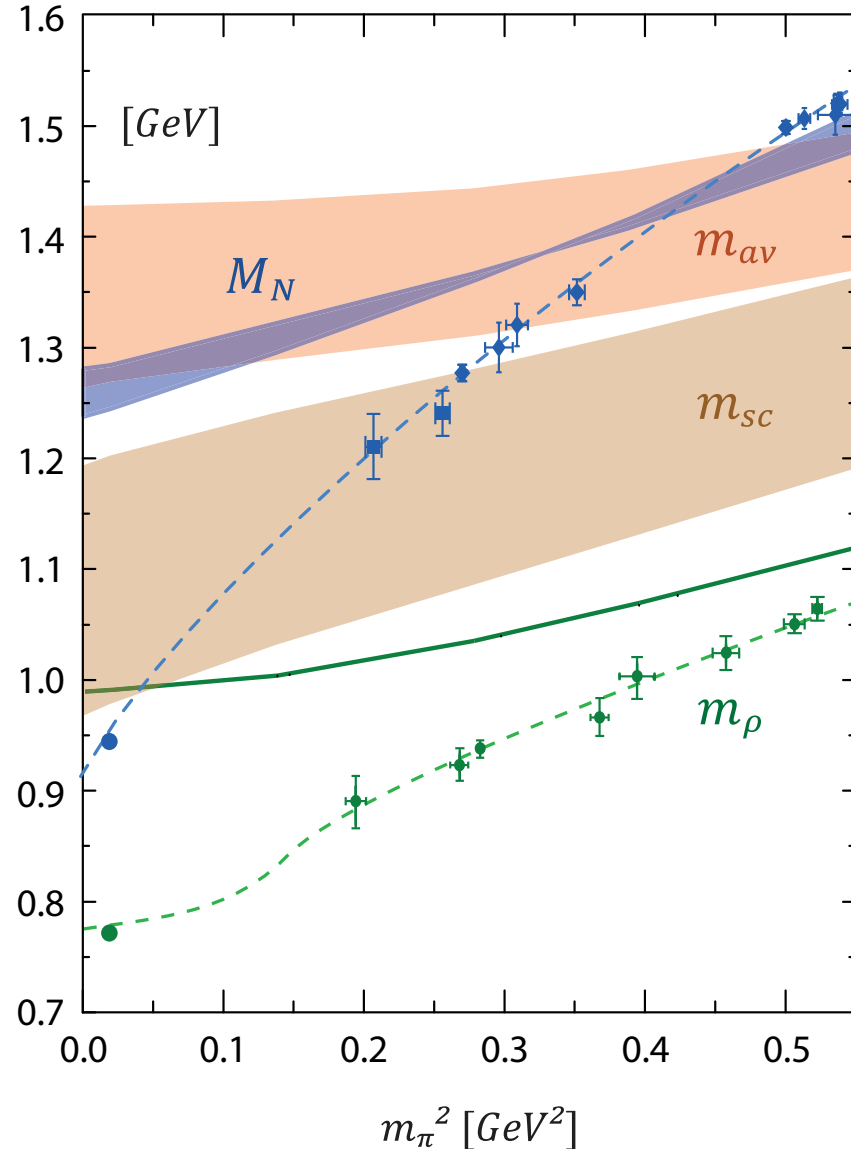
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# Ab-initio study of mesons & nucleons

- Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode



First

Contents

Back

Conclusion

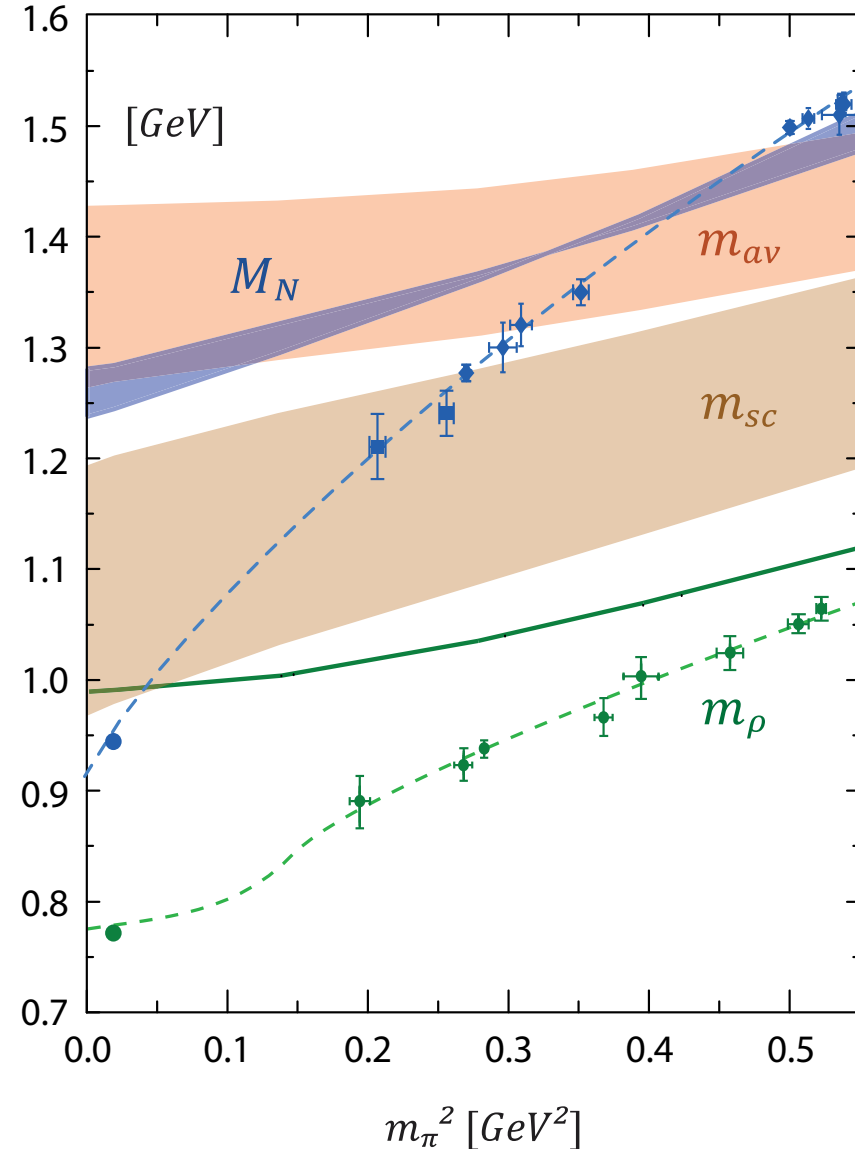
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# Ab-initio study of mesons & nucleons

- Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode
- DSE and lattice agree on heavy-quark domain



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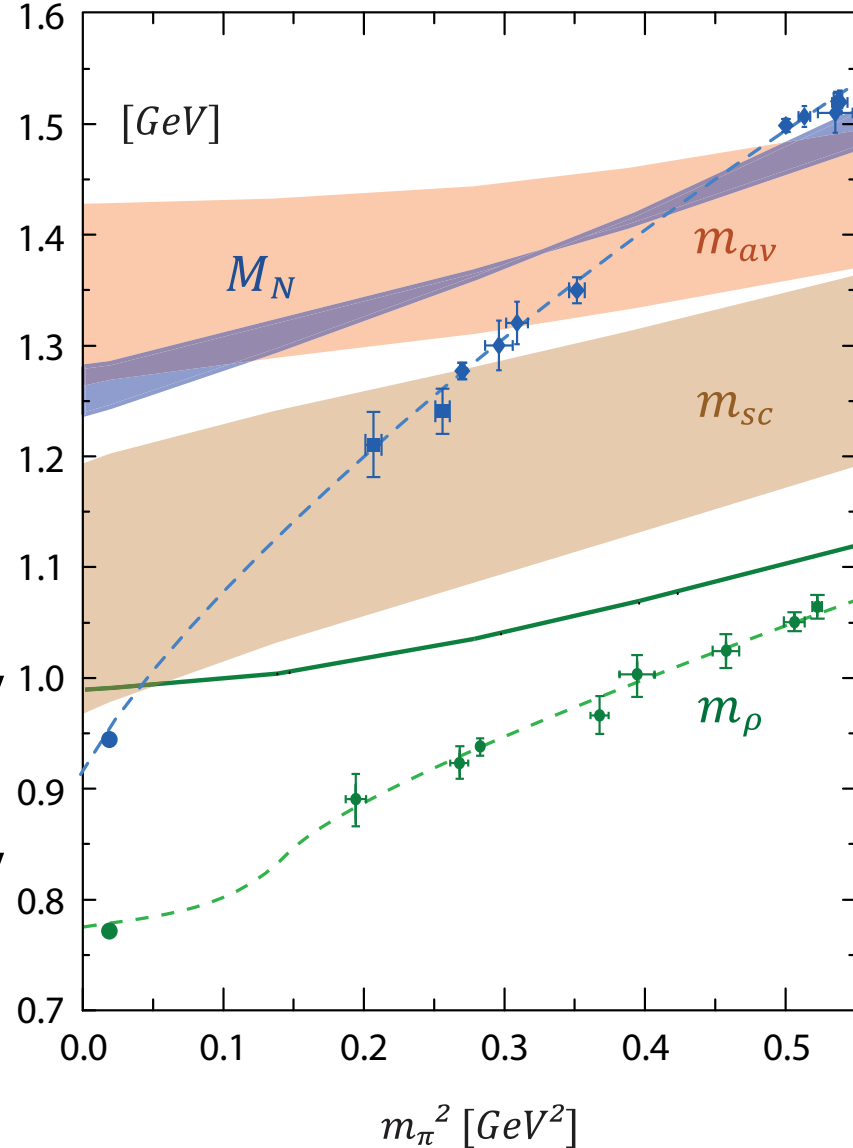
Contents

Back

Conclusion

# Ab-initio study of mesons & nucleons

- Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode
- DSE and lattice agree on heavy-quark domain
- Prediction: at physical  $m_\pi^2$ ,  
 $M_N^{\text{quark-core}} = 1.26(2) \text{ GeV}$   
 cf. FRR+lattice-QCD,  
 $M_N^{\text{quark-core}} = 1.27(2) \text{ GeV}$   
 $\Rightarrow$  subleading corrections,  
 including  $0^-$ -meson loops,  
 $\delta M_N = -320 \text{ MeV}$ ,



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