

Trident reaction kinematic fit with HPS HPS/INFN-GE

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Software/Analysis Tasks

- Momentum resolution/scale calibration using trident kinematic fit:

$$e(p) + W(P_i) \rightarrow e'(p') + e^+(p^+) + e^-(p^-) + [W(P_f)]$$

- standard HPS trigger;
 - e^+ and e^- have optimized acceptance;
 - e' carries out small momentum reducing acceptance to first SVT layers.
- Momentum/angular resolution/scale calibration using Moller scattered electrons:
$$e(p) + e^*(P_i) \rightarrow e'(p') + [e^{*'}(P_f)]$$
 - require a non-standard (one electron) trigger or study of accidentals;
 - $e^{*'}$ goes out of acceptance.

Trident Reaction Variables

- measured 5 variables \times 3 particles \implies 15 unknowns;
- recoil nucleus is not measured;
- one kinematic condition:

$$\tilde{M}_X^2 = (M_X^2 - M_W^2)/2 = 0$$

- 9 vertex conditions:

$$\vec{V}_{e'} = \vec{V}_{e^+} = \vec{V}_{e^-} = \vec{V}_{beam}$$

- one diffractive approximation condition (contradictory, used in MC generator):

$$\tilde{E}_X = E_X - M_W = T_{W'} = 0$$

- total 11 conditions to determine 15 unknowns:
 - 9 spacial for 9 unknowns;
 - 2 kinematic for 6 unknowns.

Kinematic Fit Conditions

Assuming reference point is in the known beam position:

- (in LDC system) 6 linear equations:

$$d_0^{e'} = d_0^{e^+} = d_0^{e^-} = 0$$

$$z_0^{e'} = z_0^{e^+} = z_0^{e^-} = 0$$

- 2 **non-linear** equations:

$$\tilde{M}_X^2(\phi_0^{e'}, \Omega^{e'}, \text{tg}\lambda^{e'}, \phi_0^{e^+}, \Omega^{e^+}, \text{tg}\lambda^{e^+}, \phi_0^{e^-}, \Omega^{e^-}, \text{tg}\lambda^{e^-}) = 0$$

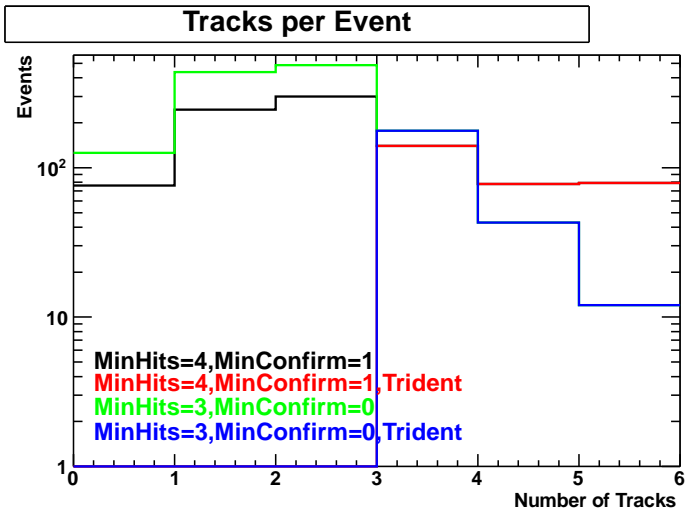
$$\tilde{E}_X(\phi_0^{e'}, \Omega^{e'}, \text{tg}\lambda^{e'}, \phi_0^{e^+}, \Omega^{e^+}, \text{tg}\lambda^{e^+}, \phi_0^{e^-}, \Omega^{e^-}, \text{tg}\lambda^{e^-}) = 0$$

Limitations:

- only 1 rigorous kinematic condition and it is non-linear,
- spacial conditions do not help to reconstruct momenta and angles.

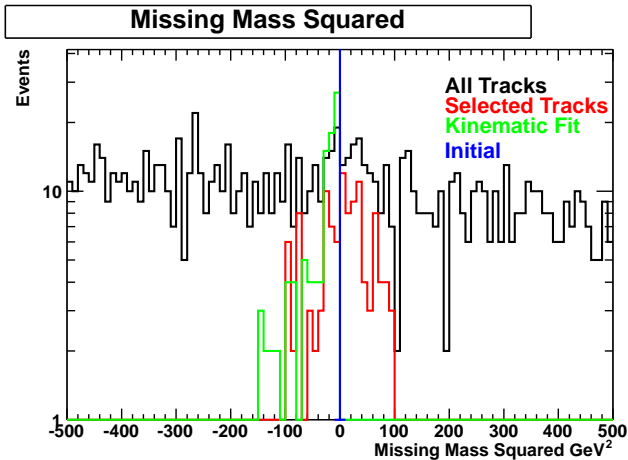
Event Selection

- need to enhance low-momentum acceptance,
- reduced minimal number of layers in Helical fit.



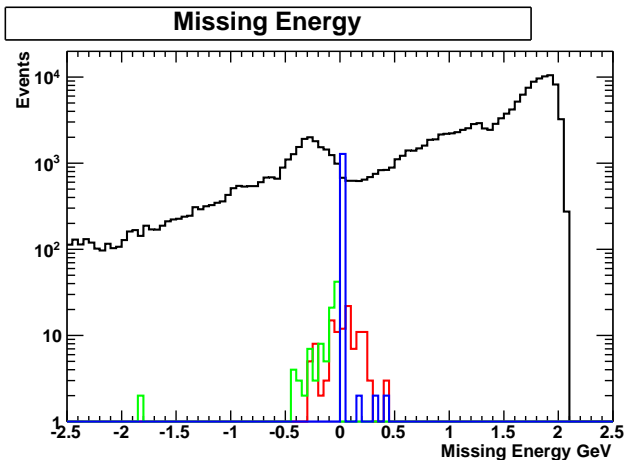
Missing Mass Distribution

- MS enhanced initial resolution $\sigma_{M_x^2}^{MC} \simeq 3 \times 10^{-3} \text{ GeV}^2$,
- reconstructed $\sigma_{M_x^2}^{trk} \simeq 3 \times 10^1 \text{ GeV}^2$,
- kinematic fit $\sigma_{M_x^2}^{kfit} \simeq 1 \times 10^1 \text{ GeV}^2 \sim \sigma_{M_x^2}^{trk}/3$.



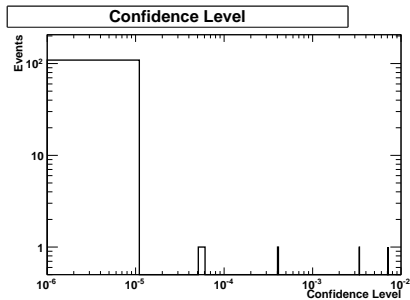
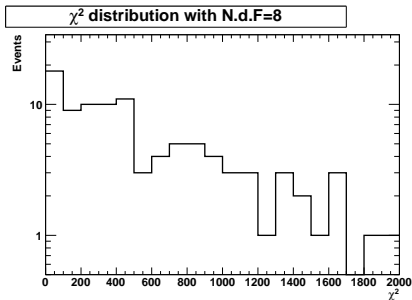
Missing Energy Distribution

- MS enhanced initial resolution $\sigma_{E_x}^{MC} = 6 \times 10^{-6} \text{ GeV}$,
- reconstructed $\sigma_{E_x}^{trk} = 0.1 \text{ GeV}$,
- kinematic fit $\sigma_{E_x}^{kfit} = 0.03 \text{ GeV} \sim \sigma_{E_x}^{trk} / 3$.



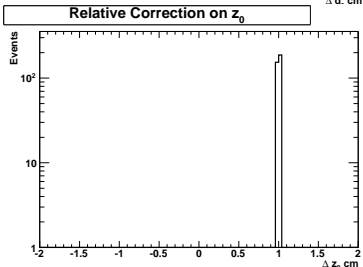
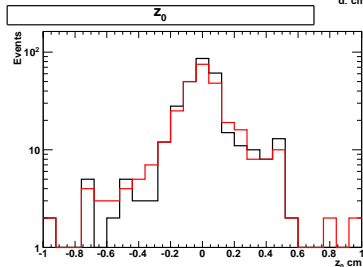
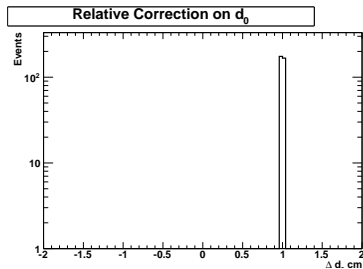
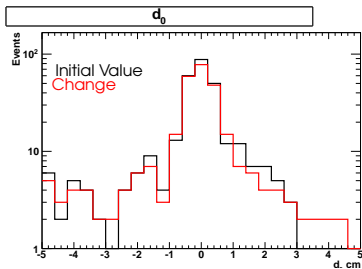
Fit Quality

- χ^2 values are larger than expected for N.d.F.=8,
- Confidence Level is not acceptable $\ll 0.05$,
- possible reasons:
 - underestimated covariance matrix,
 - highly non-linear constraints.



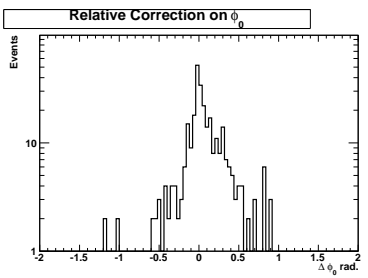
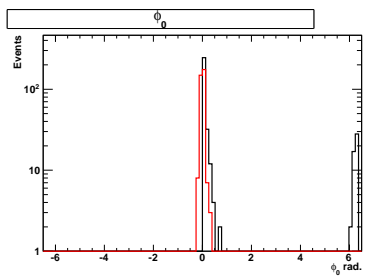
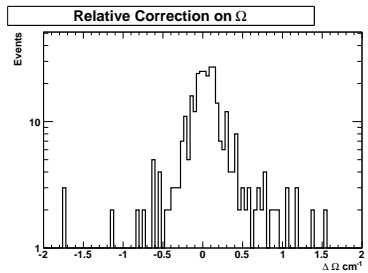
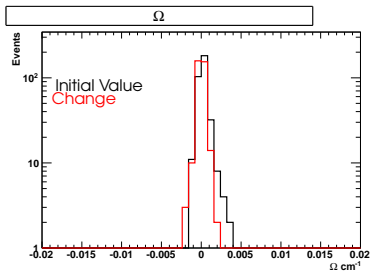
Fitted Position Sensitive Variables

- Correction of the order of 100% to all variables.



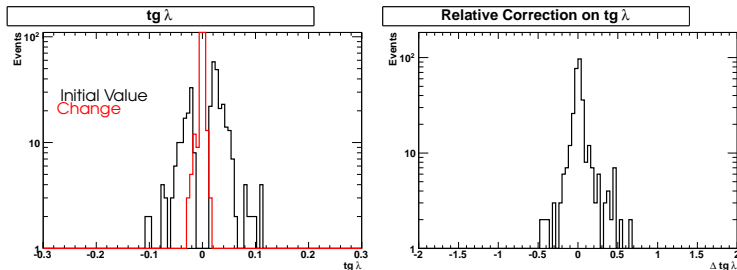
Fitted Kinematic Variables

- Correction of the order of 20%.



Fitted Kinematic Variables Cont.

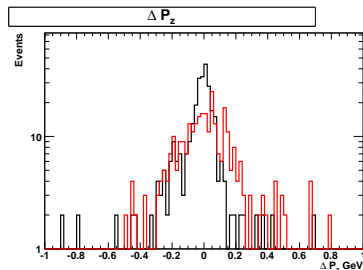
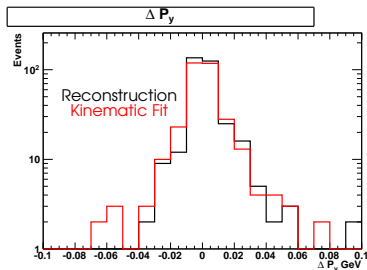
- Correction of the order of 10%.



- Qualitatively corrections have good behavior:
 - vertex correction just moves all tracks to beam spot,
 - kinematic corrections are smaller, in particular along \vec{B} .

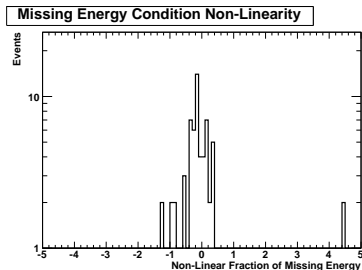
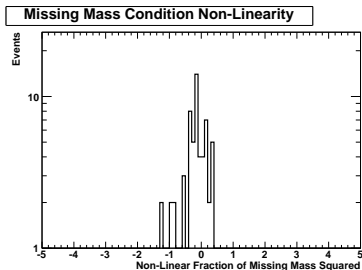
Momentum Resolution Improvements

- X,Y components lost 20% of resolution,
- Z component resolution worsened by a factor of 3.
 - System is too undetermined (2 equations for 6 unknowns)?
 - Bug of implementation?
 - Linear approximation fails?



Non-Linearity

- In missing mass condition the linear term $< 70\%$,
- in missing energy condition the linear term $< 70\%$.
 - Non-linear contribution on the correction interval is suppressed by a factor of ~ 2 ,
 - but in absolute value it gives $3 \div 6\%$ (assuming linear relation to measured variables),
 - this can be compared to resolution of 5% .



Conclusions

- Trident reaction is not closed, hence allowing to reconstruct strictly only 1 variable out of 6;
- diffractive assumption adds 1 (potentially 4) more variable, but at cost of a slight contradiction;
- still 4 variables remain unconstrained;
- fit quality is very bad, probably due to high non-linearity,
- momentum resolution was not improved.

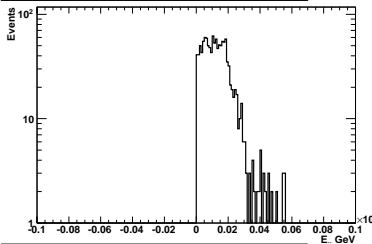
ToDo:

- try to fit with complete diffractive assumption $P_X = 0$.

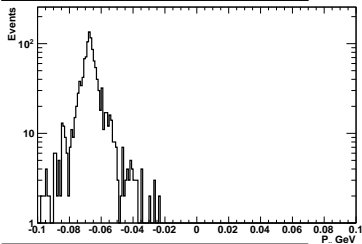
Initial State Problem

- initial W has $P_x = 67$ MeV and $P_z = 1.5$ MeV.

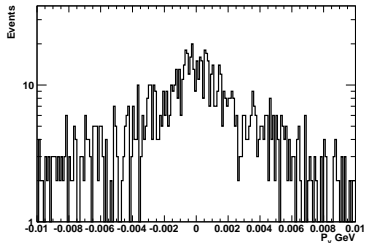
Initial Missing Energy GeV



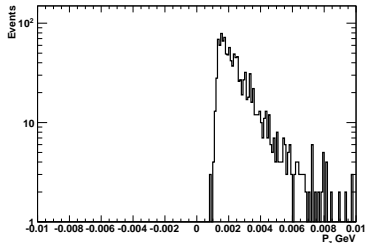
Initial Missing Momentum GeV



Initial Missing Momentum GeV



Initial Missing Momentum GeV



Diffractive Assumption

- In elastic scattering on a heavy target (W) one may neglect energy transfer:

$$T_{W'} = \frac{-t}{2M_W} = o\left(\frac{1}{M_W}\right) \rightarrow 0$$

- however, this assumption implies also:

$$t \rightarrow 0 \text{ and therefore } \vec{P}_{W'} \rightarrow 0$$

- but in practice the 3-momentum transfer is not suppressed:

$$\vec{P}_{W'} \simeq \sqrt{2M_W T_{W'}} = \sqrt{-t}$$

Moller Scattering

- Moller scattering:

$$e(k) + e^*(p) \rightarrow e'(k_1) + [e^{*'}(k_2)]$$

$$k(E_b, 0, 0, E_b) + p(m, 0, 0, 0) = k_1(E_1, \vec{P}_1) + k_2(E_2, \vec{P}_2)$$

- expressing $k_{1,2}^2$, dividing on $2E_b$ and neglecting $o(m^2)$ terms:

$$(1 - \cos \theta_{2,1}) = \frac{m}{E_b} \left[\frac{E_b}{E_{2,1}} - 1 \right]$$

- using energy conservation $E_2 = E_b - E_1$, for θ_2 one gets:

$$(1 - \cos \theta_2) = \frac{m}{E_b} \left[\frac{1}{1 - \frac{E_1}{E_b}} - 1 \right]$$

Moller Scattering Cont.

- using E_1 extracted from the first relation:

$$E_1 = \frac{E_b}{1 + \frac{E_b}{m}(1 - \cos \theta_1)}$$

- one obtains:

$$(1 - \cos \theta_2) = \frac{m^2}{E_b^2} \frac{1}{(1 - \cos \theta_1)}$$

- expanding both for small angles ($E_b = 2.2$ GeV):

$$\theta_2 = \left(\frac{m}{E_b} \right)^2 \frac{1}{\theta_1} \sim \frac{5.4 \times 10^{-8}}{\theta_1}$$

Moller Scattering Cont.

- if electron $e'(k_1)$ is detected in HPS:

$$\theta_1 > 10 \text{ mrad}$$

- the other electron of Moller scattering ($e^{*'}(k_2)$) goes at polar angle:

$$\theta_2 < 5.4 \mu\text{rad}$$

- it cannot be measured by HPS detector.