

Recent progress in NNLO calculations

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Workshop on QCD Factorization
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NORTHWESTERN
UNIVERSITY



Outline

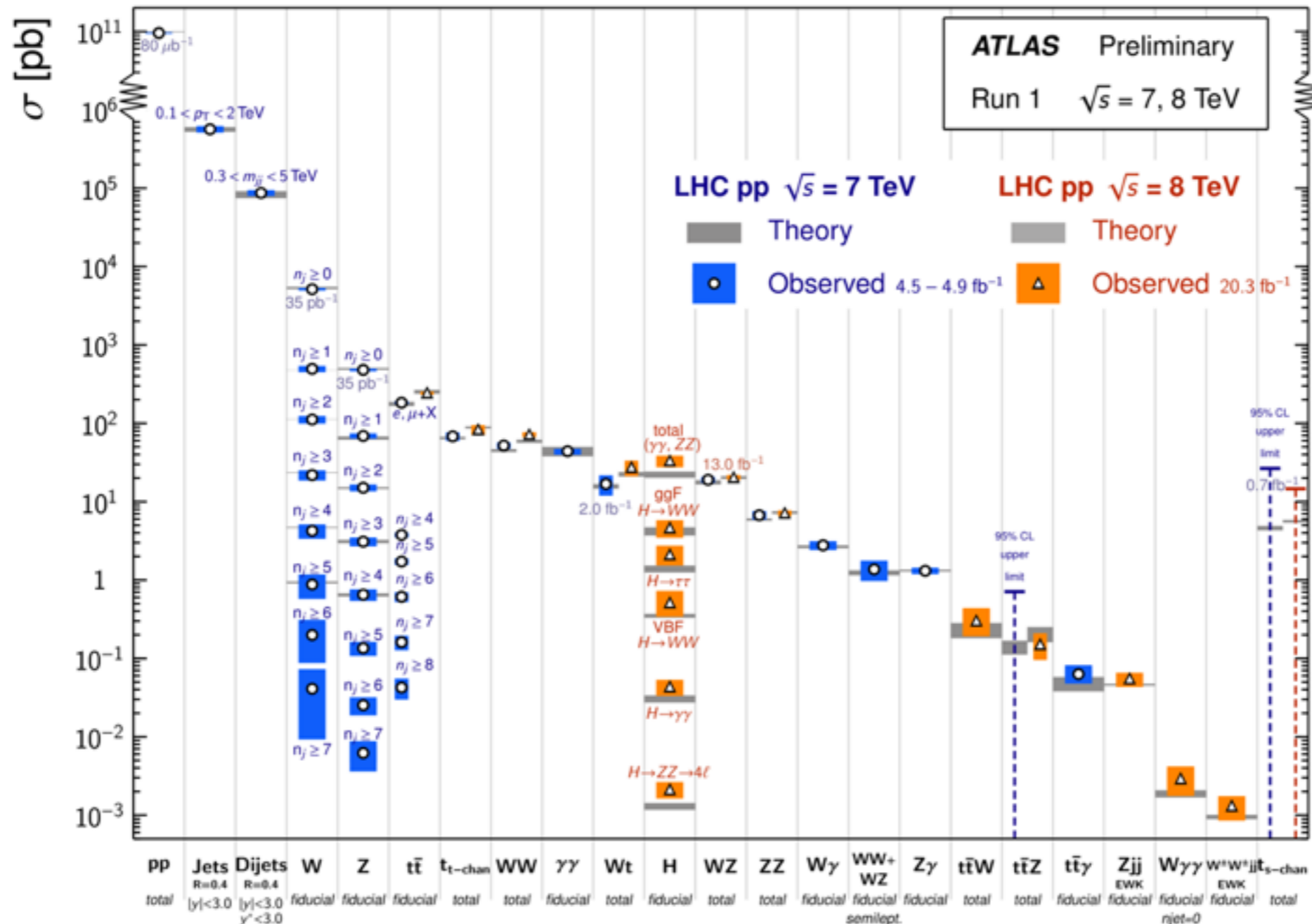
- Motivation and introduction
- A history of NNLO QCD
- N-jettiness as a subtraction scheme
- Jet phenomenology at NNLO: W +jet, H +jet, Z +jet at the LHC and at 100 TeV; comparison of exact versus approximate calculations

The LHC circa 2015

- Impressive agreement between SM theory and experiment. There is agreement on cross sections spanning many orders of magnitudes and containing numerous final states.

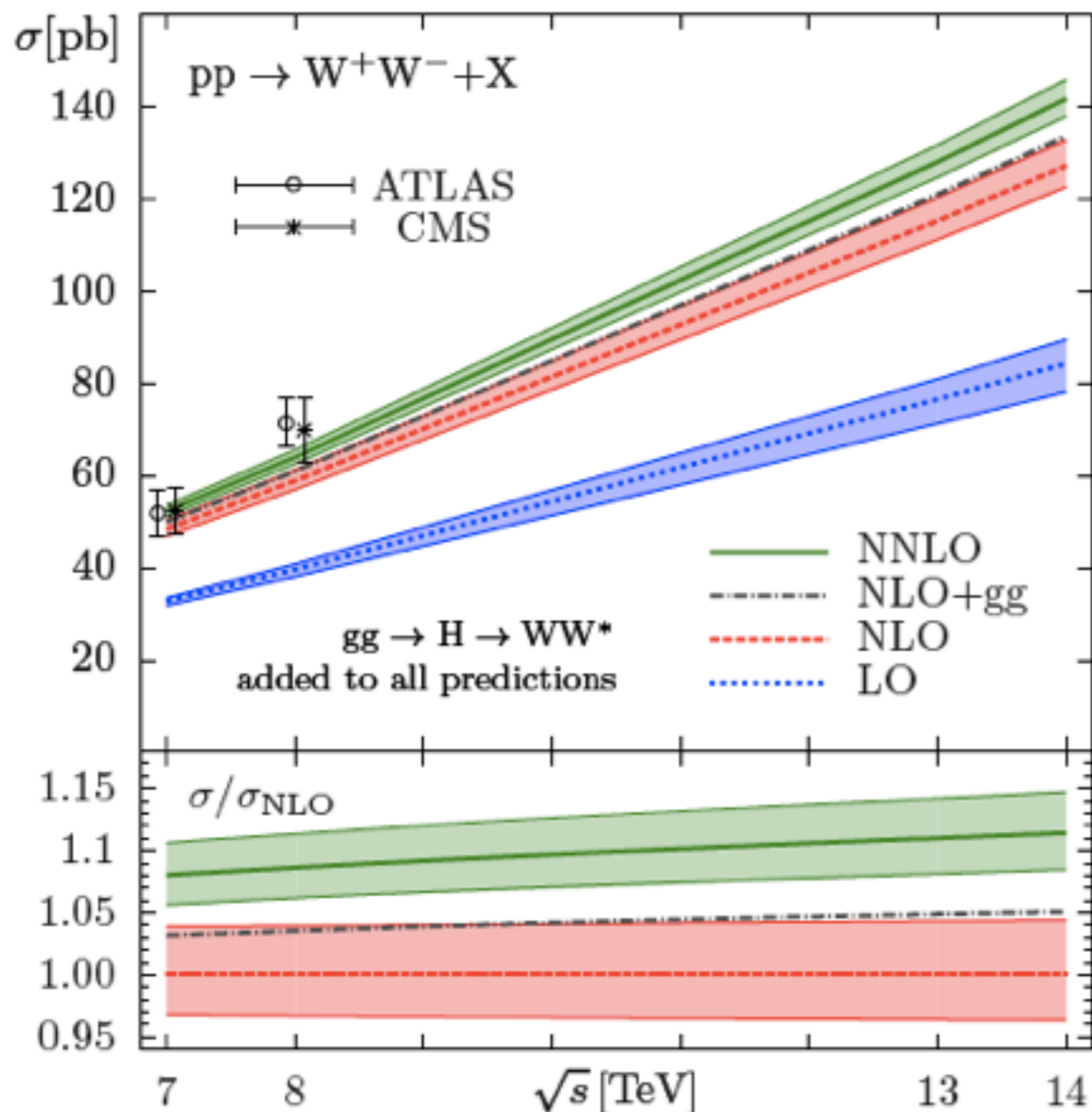
Standard Model Production Cross Section Measurements

Status: March 2015



The importance of higher orders

- Precision QCD has been a critical component of this success
- Example: until recently, WW cross section showed a $>2\sigma$ excess at both ATLAS and CMS, and for both 7 and 8 TeV. Could it be light charginos? (Curtin, Jaiswal, Meade | 206.6888)



- Sizable NNLO QCD corrections!

Gehrmann, Grazzini, Kallweit, Maierhofer, von Manteuffel, Pozzorini, Rathlev, Tancredi | 408.5243

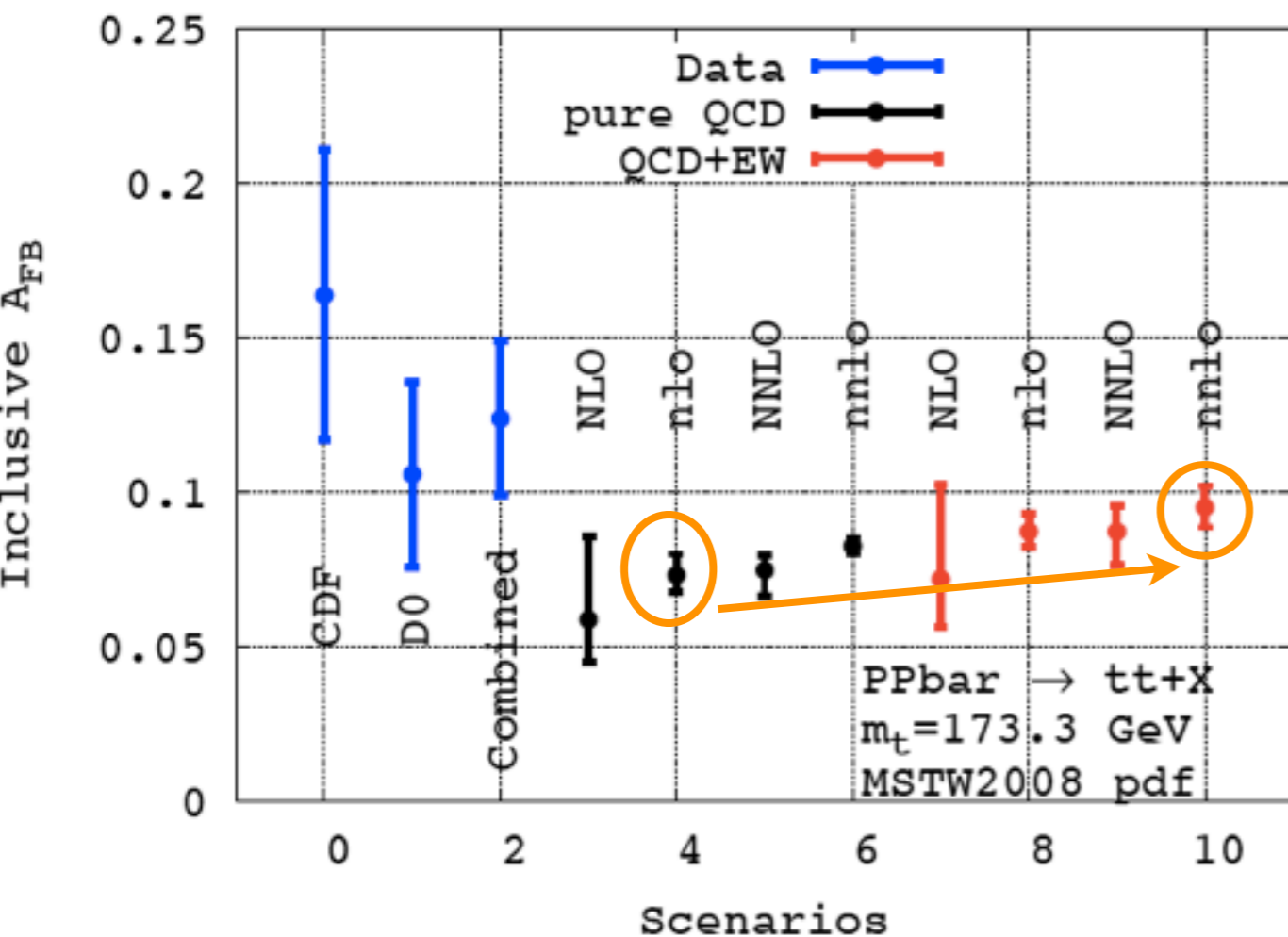
- Additional enhancement when extrapolating from 0-jet bin to full cross section

Curtin, Meade, Tien | 406.0848; Jaiswal, Okui, | 407.4537; Monni, Zanderighi | 410.4745

- Theory within 1σ agreement of ATLAS and CMS for both CM energies; proper interpretation not possible without NNLO QCD

The importance of higher orders

- For several years the $t\bar{t}$ asymmetry measured at Tevatron was several standard deviations away from the prediction; could it be a light axigluon?

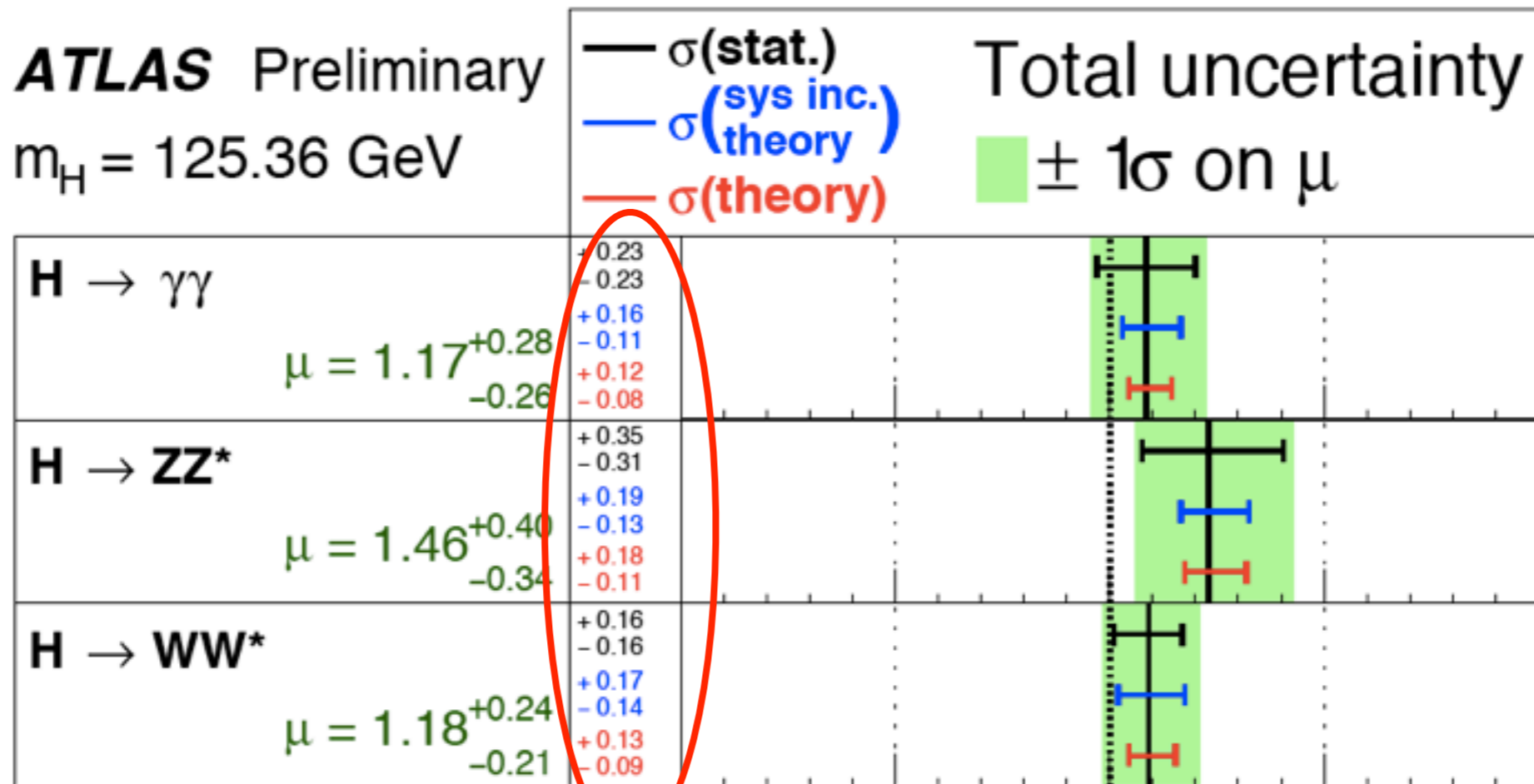


- Large NNLO QCD corrections! Not predicted by soft-gluon resummation, required a genuine NNLO prediction (Czakon, Fiedler, Mitov 1411.3007)
- Large electroweak corrections! QED generates an asymmetry via the same mechanism as QCD (Hollik, Pagani 1107.2606; Kuhn, Rodrigo 1109.6830)

- Within errors, the SM prediction now agrees with experiment

The future: precision Higgs physics

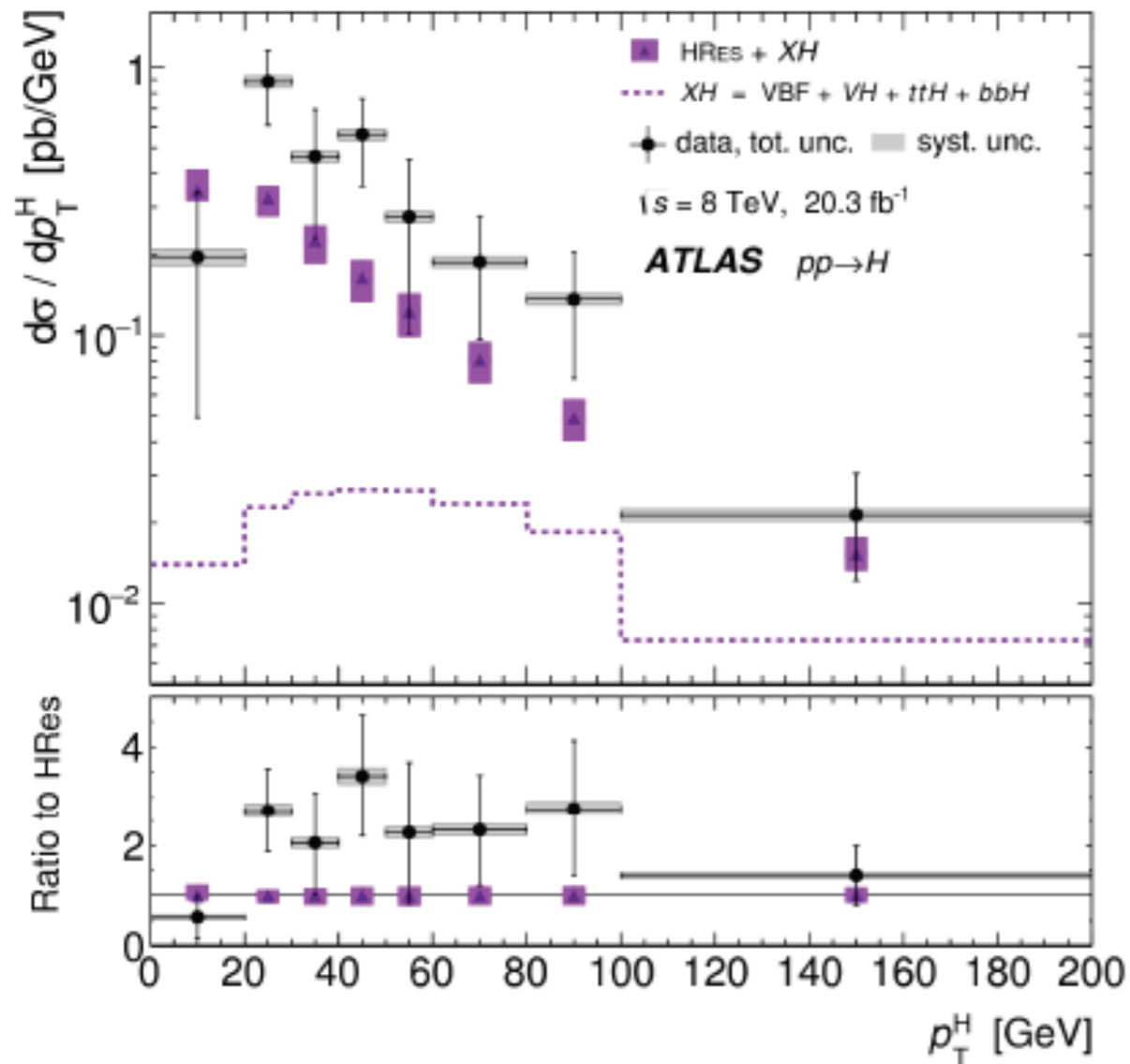
- There will be a continued need for further improvements in precision QCD as Run II data arrives.



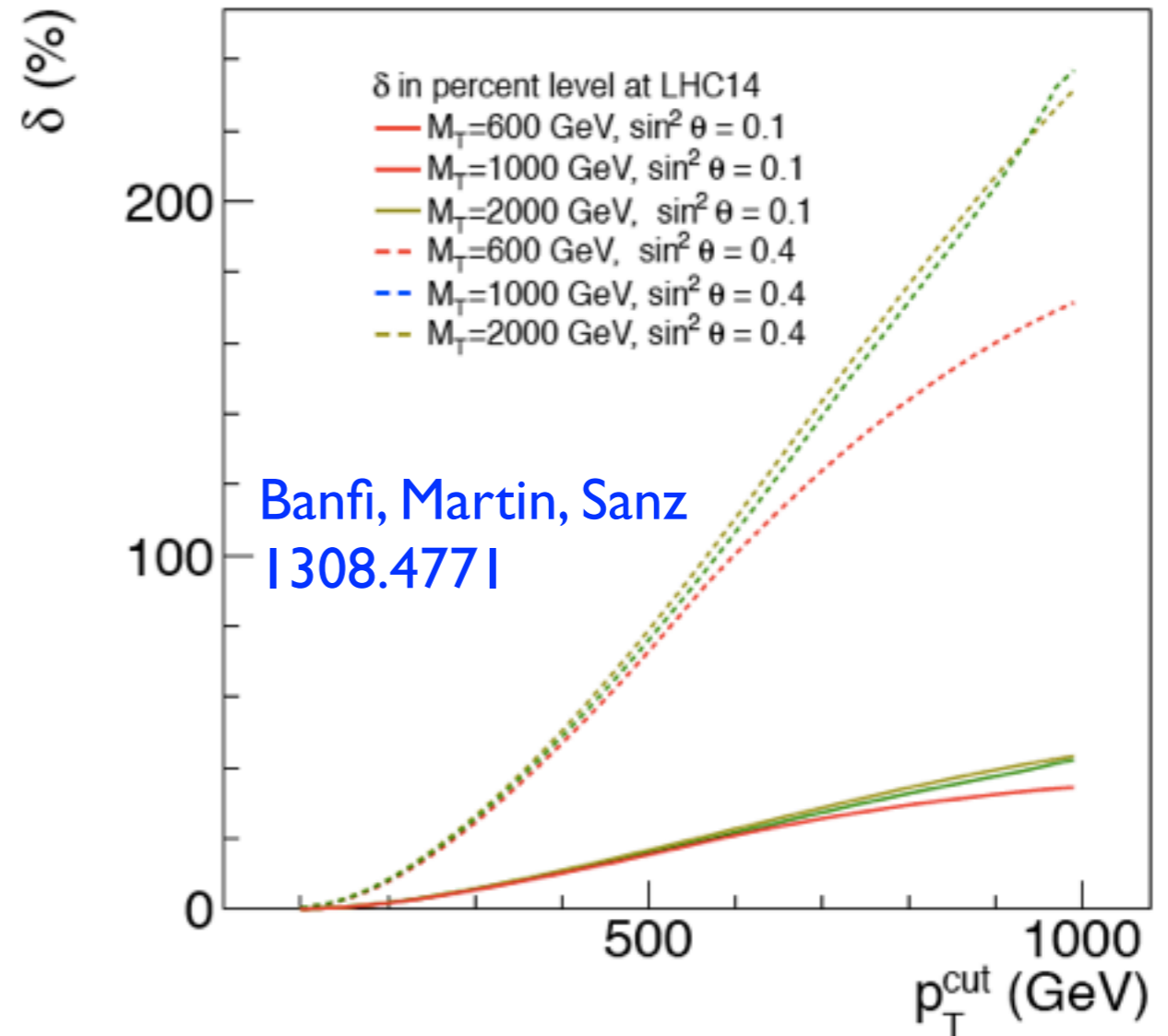
- The dominant component of the systematic error on the signal strength is theory.
- This will become a limiting factor in interpretation in Run II as statistical errors decrease.

Need better theory to sharpen our probes of BSM physics
in the Higgs sector!

Higher orders and jets



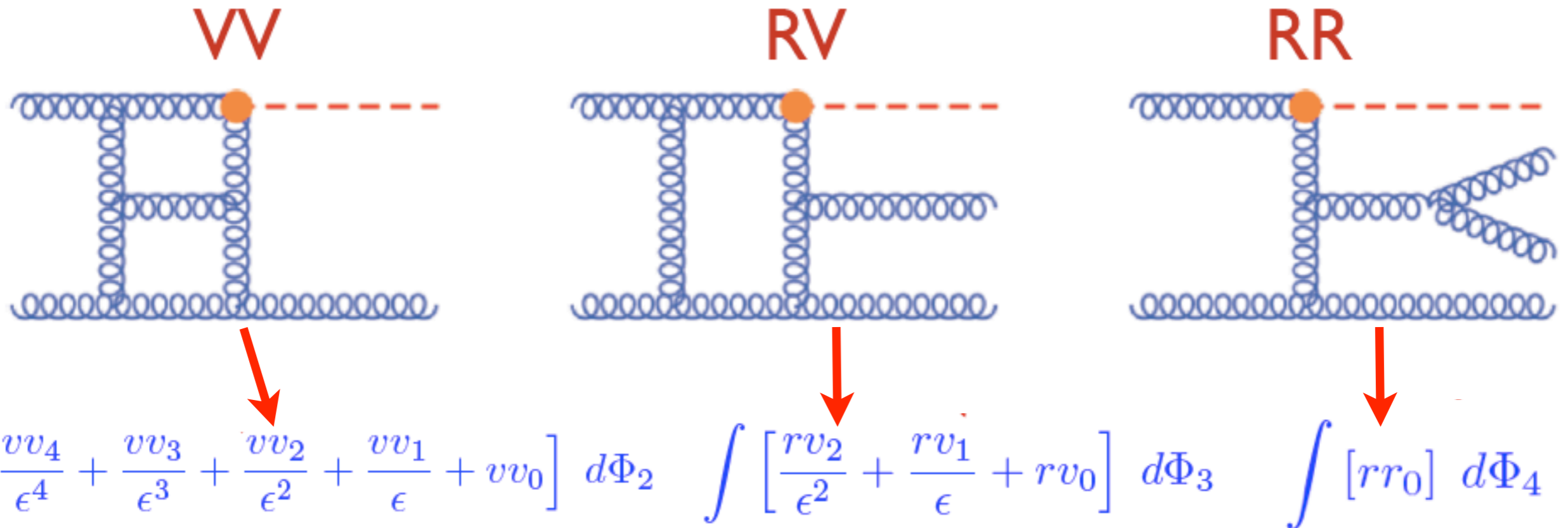
Top partners in H+j



- Probing new kinematical regions opened by recoil against high- p_T jets will be a major focus in Run II, as it could lead to discovery of BSM. Precision predictions for processes containing jets are needed to enable such searches

Fixed-order cross sections at NNLO

- Need the following ingredients for NNLO cross sections:

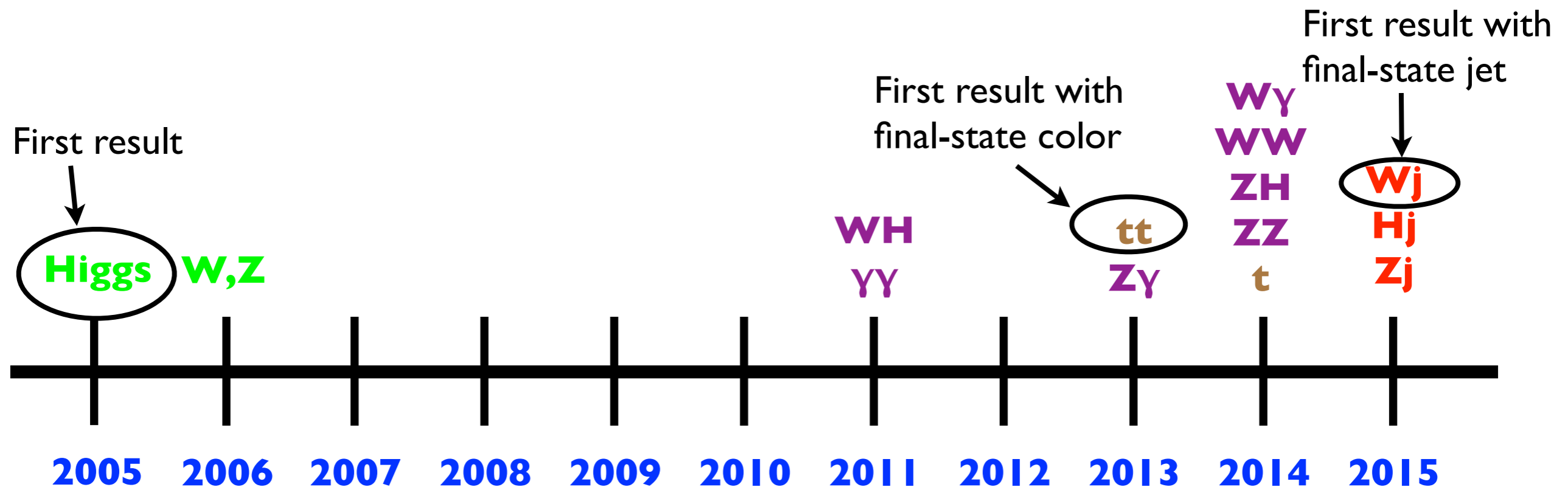


- In principle this is straightforward: draw all diagrams and calculate. In practice, it is complicated by the implicit poles in the real radiation corrections that only appear after integration over phase space.

- A generic, working procedure to extract infrared singularities in the presence of final-state jets was unknown for many years.

A timeline of NNLO hadron collider cross sections

- Complete NNLO hadron-collider cross sections with control over kinematics:



NNLO has become the standard for $2 \rightarrow 2$ scattering over the past 2-3 years

Techniques for NNLO

- Numerous proposed techniques for handling singularities at NNLO, which can be divided into two distinct categories:

(quasi-)Local: add and subtract counterterms that approximate real-emission matrix elements in all singular limits

Resummation-assisted: leverage knowledge of analytic resummation to remove double-real emission singularities.

Local:

- **Sector decomposition:** Anastasiou, Melnikov, FP
- **Antennae subtraction:** Gehrmann, Gehrmann-de Ridder, Glover
- **Sector-improved residue subtraction:** Czakon; Boughezal, Melnikov, FP
- **Colorful subtraction:** Del Duca, Somogyi, Trocsanyi
- **Projection-to-Born:** Cacciari, Dreyer, Karlberg, Salam, Zanderighi

RG-

assisted:

- **q_T-subtraction:** Catani, Grazzini
- **N-jettiness subtraction:** Boughezal, Focke, Liu, FP; Gaunt, Stahlhofen, Tackmann, Walsh

Techniques for NNLO

- Numerous proposed techniques for handling singularities at NNLO, which can be divided into two distinct categories:

(quasi-)Local: add and subtract counterterms that approximate real-emission matrix elements in all singular limits

Resummation-assisted: leverage knowledge of analytic resummation to remove double-real emission singularities.

Resummation-assisted techniques are responsible for most of the progress and the first complete NNLO results for jet production processes that have appeared over the past few years

RG-

assisted:

- **q_T -subtraction**: Catani, Grazzini

- **N-jettiness subtraction**: Boughezal, Focke, Liu, FP; Gaunt, Stahlhofen, Tackmann, Walsh

Local subtraction schemes

$$\begin{aligned} d\sigma_{NNLO} = & \int_{d\Phi_{m+2}} (d\sigma_{NNLO}^R - d\sigma_{NNLO}^S) + \int_{d\Phi_{m+2}} d\sigma_{NNLO}^S \\ & + \int_{d\Phi_{m+1}} (d\sigma_{NNLO}^{V,1} - d\sigma_{NNLO}^{VS,1}) + \int_{d\Phi_{m+1}} d\sigma_{NNLO}^{VS,1} \\ & + \int_{d\Phi_m} d\sigma_{NNLO}^{V,2} \end{aligned}$$

- Construct local subtraction terms that reduce to the full matrix elements in the singular limits of phase space.
- For example, **sector-improved residue subtraction** uses the factorization of matrix elements into eikonal factors/splitting kernels times reduced matrix elements in the soft/collinear limits of phase space.
- The subtraction can be integrated analytically (antennae subtraction), or numerically (**sector-improved residue subtraction**) if the $1/\epsilon$ poles have been extracted.

q_T -subtraction

- For color-neutral final states, the transverse momentum completely determines the double-unresolved singular limit of the final state

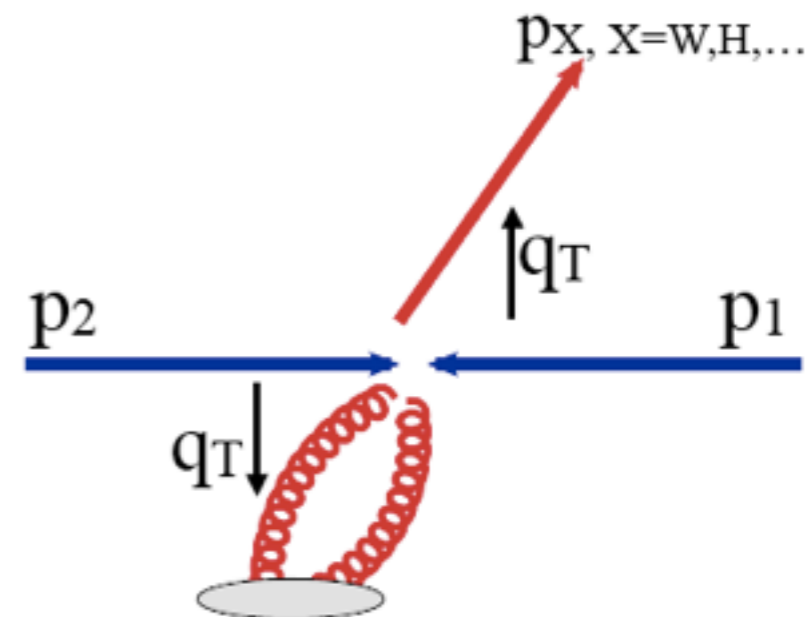
Catani, Grazzini (2007)

$$\sigma_{NNLO} = \int dq_T \frac{d\sigma}{dq_T} \theta(q_T^{\text{cut}} - q_T) + \int dq_T \frac{d\sigma}{dq_T} \theta(q_T - q_T^{\text{cut}})$$

Obtained from the Collins-Soper-Sterman factorization theorem for small- q_T

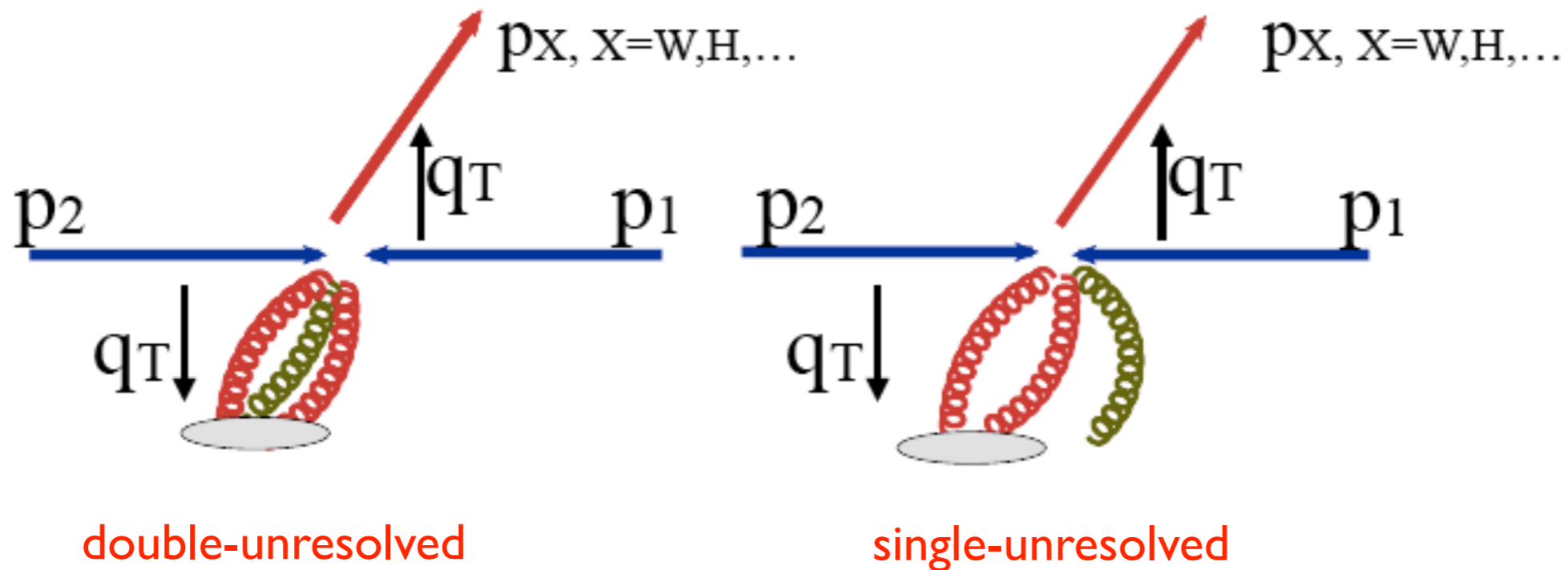
This is an NLO cross section for the color-neutral final state plus an additional jet

- Maximally reuses information from existing NLO calculations



q_T -subtraction

- Difficult to extend beyond color-neutral final states; q_T no longer separates double-unresolved limit from NLO singular limits



Need a different resolution parameter when final-state jets are present!

- Two requirements: must separate double-unresolved limits from single-unresolved and hard regions, and must be resumable

N-jettiness

- N-jettiness, \mathcal{T}_N , is an event-shape variable designed to veto final-state jets [Stewart, Tackmann, Waalewijn 0910.0467](#)

$$\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}.$$

N=number of final-state jets

Momenta of the two beams
and the final-state jets

Measure of the jet
hardness (we take $Q_i=2E_i$)

All final-state partons

$\mathcal{T}_N=0$: all radiation is either soft, or collinear to a beam/jet;
at NNLO, gives the double-unresolved limit

$\mathcal{T}_N>0$: at least one additional radiation is resolved; have N+1
final-state jets

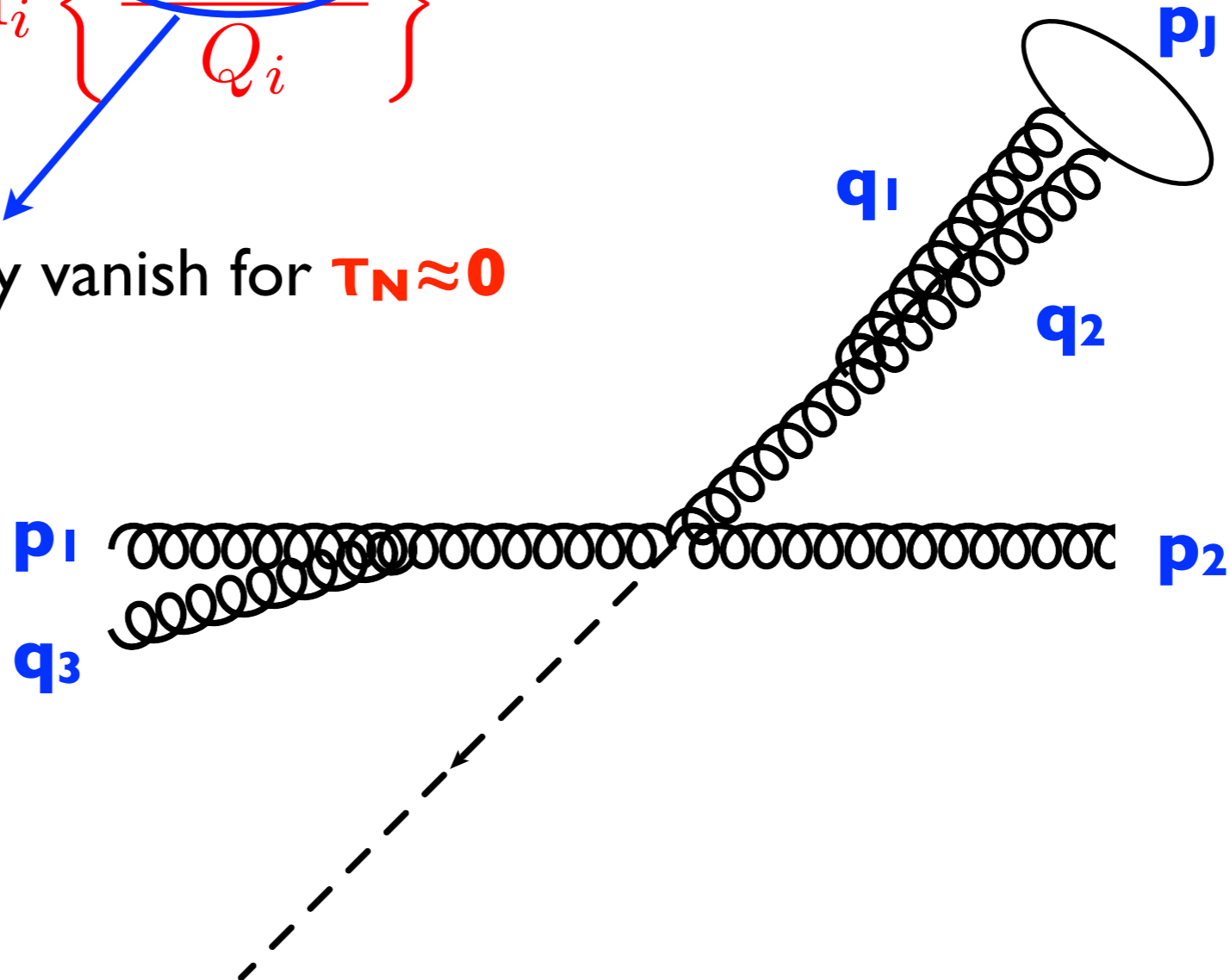
This is the resolution parameter we are looking for!

N-jettiness

$\tau_N \approx 0$: all radiation is either soft, or collinear to a beam/jet;
at NNLO, gives the double-unresolved limit

$$\tau_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}$$

Must all nearly vanish for $\tau_N \approx 0$

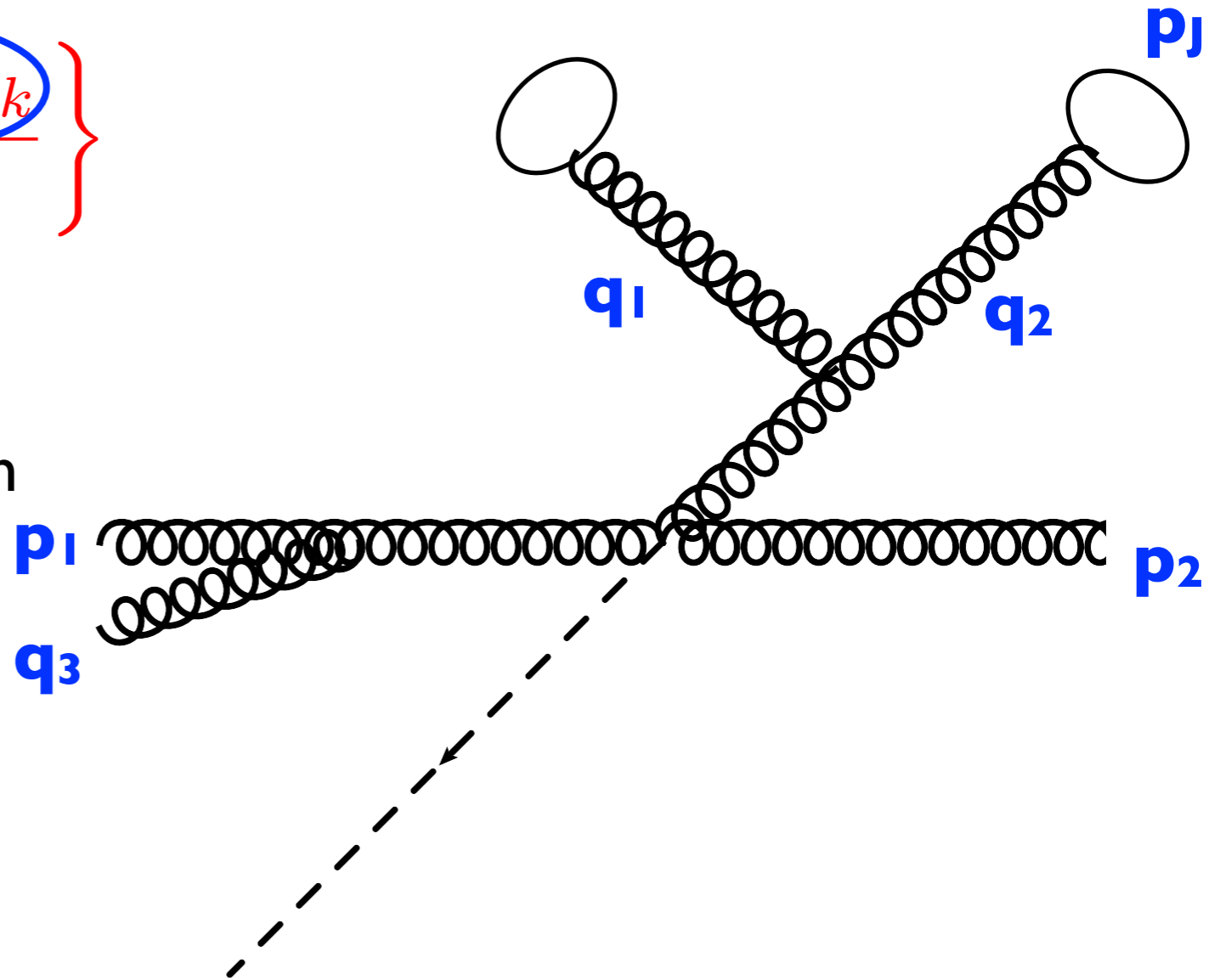


N-jettiness

$\tau_N > 0$: at least one additional radiation is resolved; have $N+1$ final-state jets

$$\tau_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}$$

Must have at least one dot product non-vanishing; clearly can obtain from an NLO calculation with $N+1$ jets



N-jettiness subtraction

- N-jettiness can be applied to obtain exact NNLO cross sections
[Boughezal, Focke, Liu, FP 1504.02131](#)
- Introduce τ_N^{cut} that separates the $\tau_N=0$ doubly-unresolved limit of phase space from the single-unresolved and hard regions

$$\begin{aligned}\sigma_{NNLO} &= \int d\Phi_N |\mathcal{M}_N|^2 + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^{\leq} \\ &+ \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^{\leq} + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^{\geq} \\ &+ \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^{\geq} \\ &\equiv \sigma_{NNLO}(\tau_N < \tau_N^{\text{cut}}) + \sigma_{NNLO}(\tau_N > \tau_N^{\text{cut}})\end{aligned}$$

$$\theta_N^{\leq} = \theta(\tau_N^{\text{cut}} - \tau_N) \quad \text{and} \quad \theta_N^{\geq} = \theta(\tau_N - \tau_N^{\text{cut}})$$

N-jettiness subtraction

- For $\tau_N > \tau_N^{\text{cut}}$, at least one of the two additional radiations that appear at NNLO is resolved; this region of phase space contains the NLO correction to the N+1 jet process. Can be obtained from any NLO program.
- For $\tau_N < \tau_N^{\text{cut}}$, both additional radiations are unresolved. A factorization theorem giving the all-orders result for small N-jettiness was derived [Stewart, Tackmann, Waalewijn 0910.0467](#)

$$\sigma(\tau_N < \tau_N^{\text{cut}}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n \right] + \dots$$

describes hard radiation; in dim-reg, coincides with the 2-loop virtual corrections

describes radiation collinear to an initial-state beam

describes soft radiation

describes radiation collinear to a final-state jet

- The ellipses denote power corrections that become negligible for small τ_N^{cut}

Ingredients for the factorization theorem

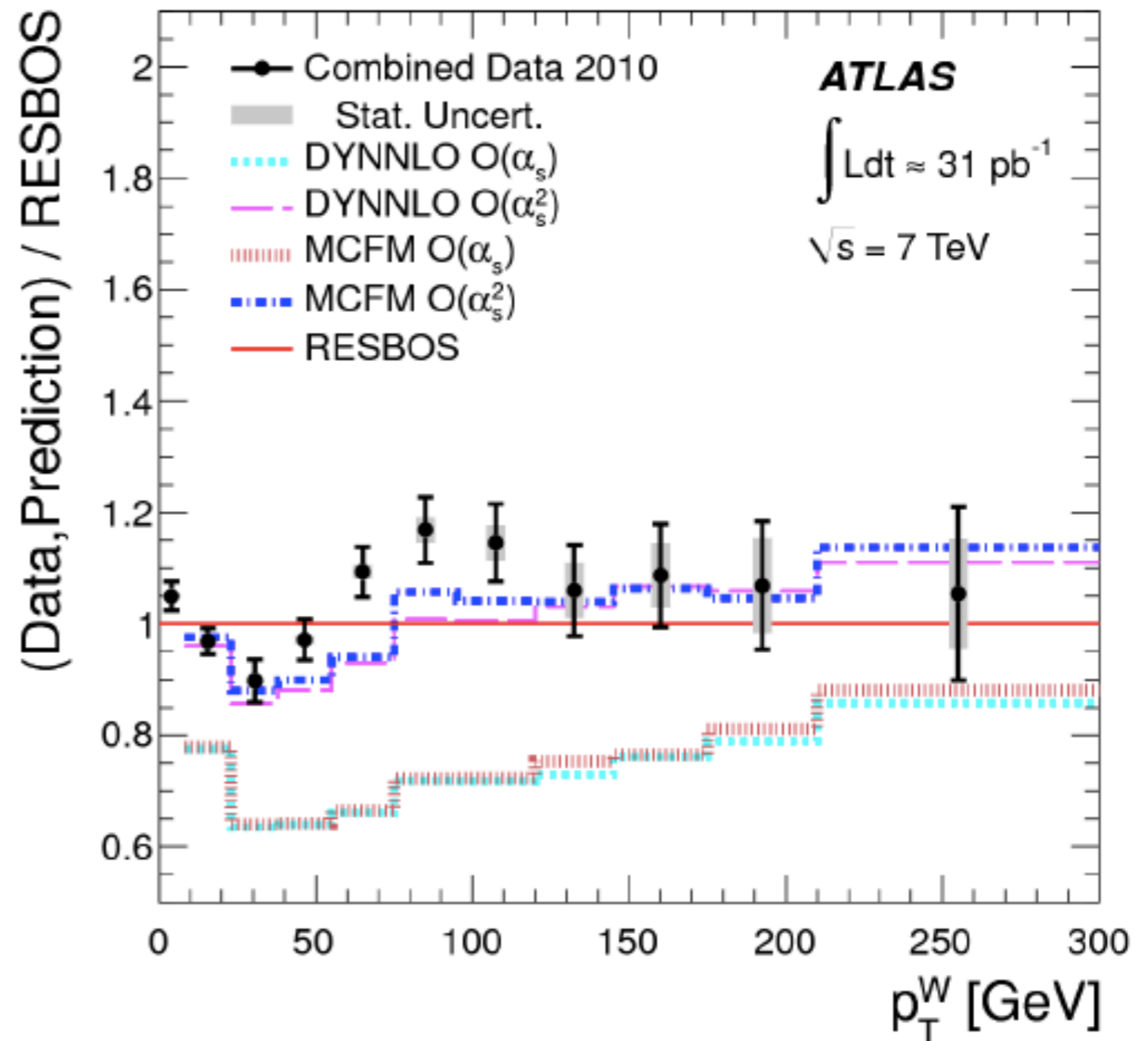
$$\sigma(\tau_N < \tau_N^{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n \right] + \dots$$

- Expand this formula to $\mathcal{O}(\alpha_s^2)$, and turn off all resummation, to get the NNLO cross section below the cut. Need each of these separate functions to NNLO.
- The beam and jet functions depend only on the flavor of the parton (quark, gluon); the soft function depends only on the parton flavors and the external hard directions; the hard function is the only process-dependent piece.
 - **H@NNLO**: for W/H+j, Gehrmann, Tancredi (2012); Gehrmann, Jaquier, Glover, Koukoutsakis (2012)
 - **B@NNLO**: Gaunt, Stahlhofen, Tackmann (2014)
 - **S@NNLO**: Boughezal, Liu, FP (2015)
 - **J@NNLO**: Becher, Neubert (2006); Becher, Bell (2011)

Within the past two years all ingredients have become available to apply this idea to $2 \rightarrow 2$ scattering at the LHC!

W+jet@NNLO

- W+jet and the associated W- p_T spectrum are important benchmark processes in the SM
- Can be used to determine the high-x gluon PDF, at ATLAS/CMS or LHCb. In the latter case, the NLO theoretical error is a limiting factor [Farry, Gauld 1505.01399](#)

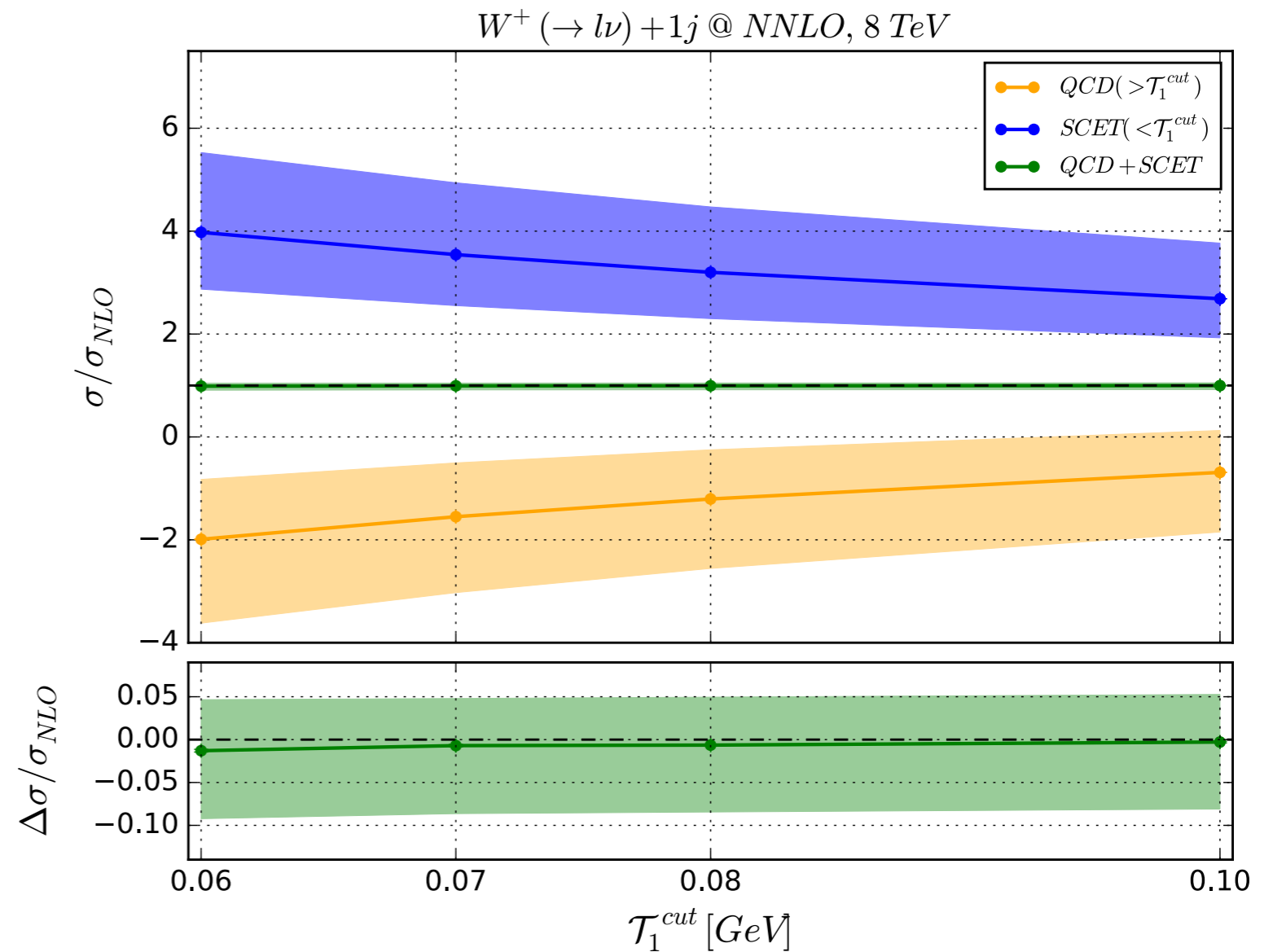


W+jet@NNLO: validation

- A powerful check of the N-jettiness subtraction formalism is the independence of the final result from $\mathcal{T}_N^{\text{cut}}$.
- The above-cut and below cut contributions separately depend on $\ln^n(\mathcal{T}_N^{\text{cut}})$, where n ranges from 1 to 4 at NNLO. This dependence must cancel when the two regions are summed.

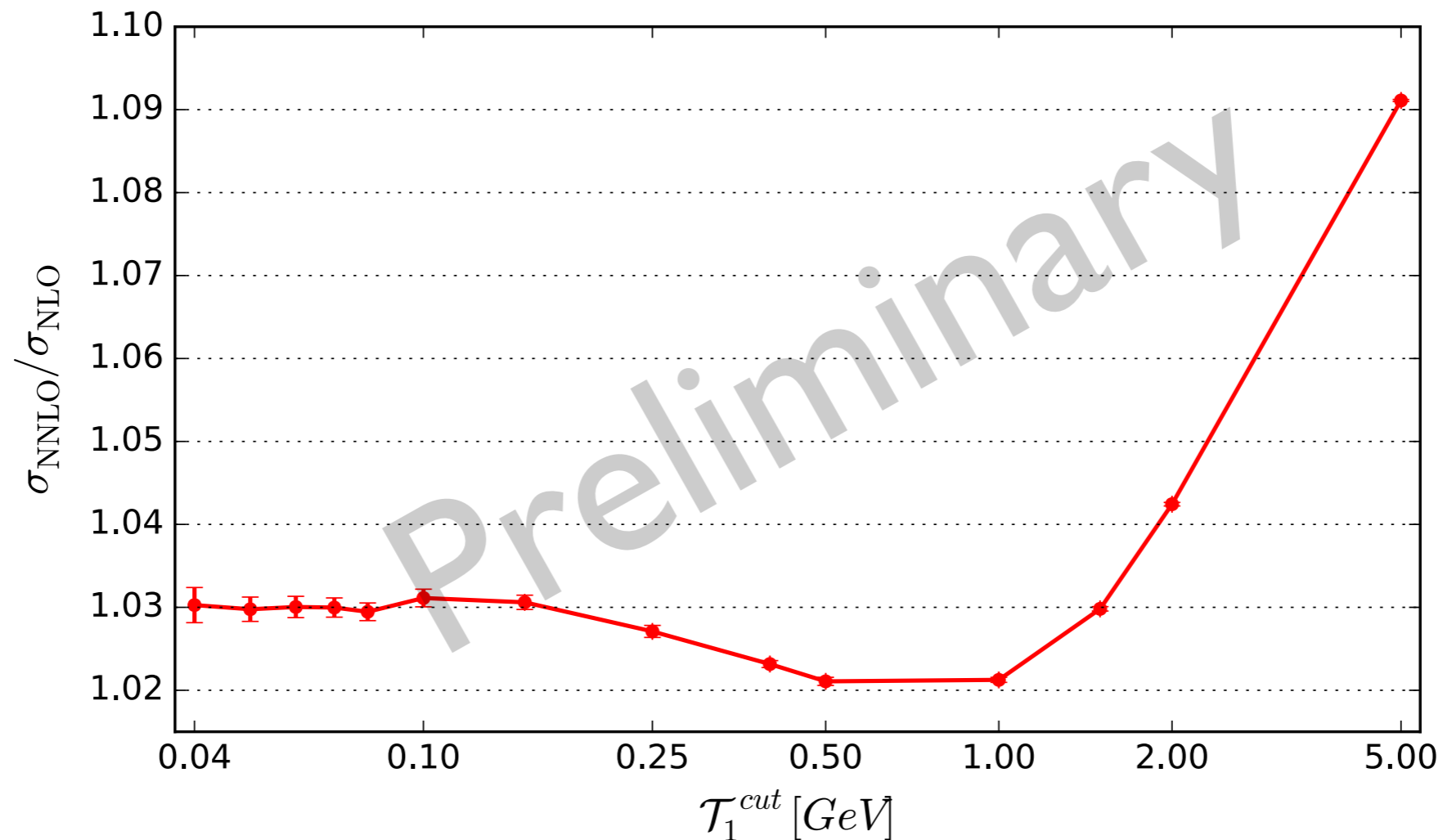
Boughezal, Focke, Liu FP 1504.02131

- CMS cuts: $p_{Tj} > 30$ GeV, $|\eta_j| < 2.4$
- CT10 PDFs, $\mu_0 = M_W$, vary by factor of 2 to estimate error
- NLO prediction using N-jettiness agrees with known results.
- Sum of above-cut and below-cut contributions stable to better than 0.1% of σ_{total}



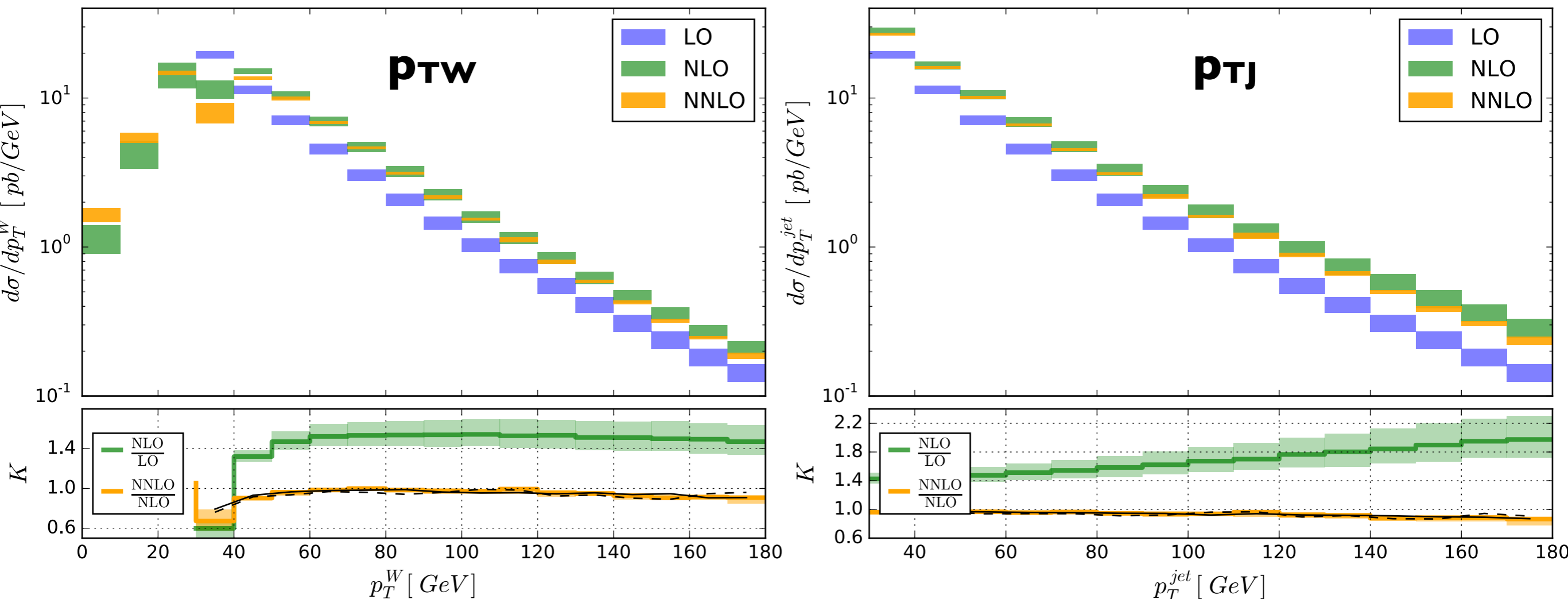
Power corrections in $Z+j$ @NNLO

- Here is a more detailed study of the τ^{cut} dependence for $Z+j$ at NNLO



- Result becomes independent of τ^{cut} below 0.2 GeV for standard fiducial cuts; can achieve numerical control of the cross section to the per-mille level, more than sufficient for phenomenology

W+jet@NNLO

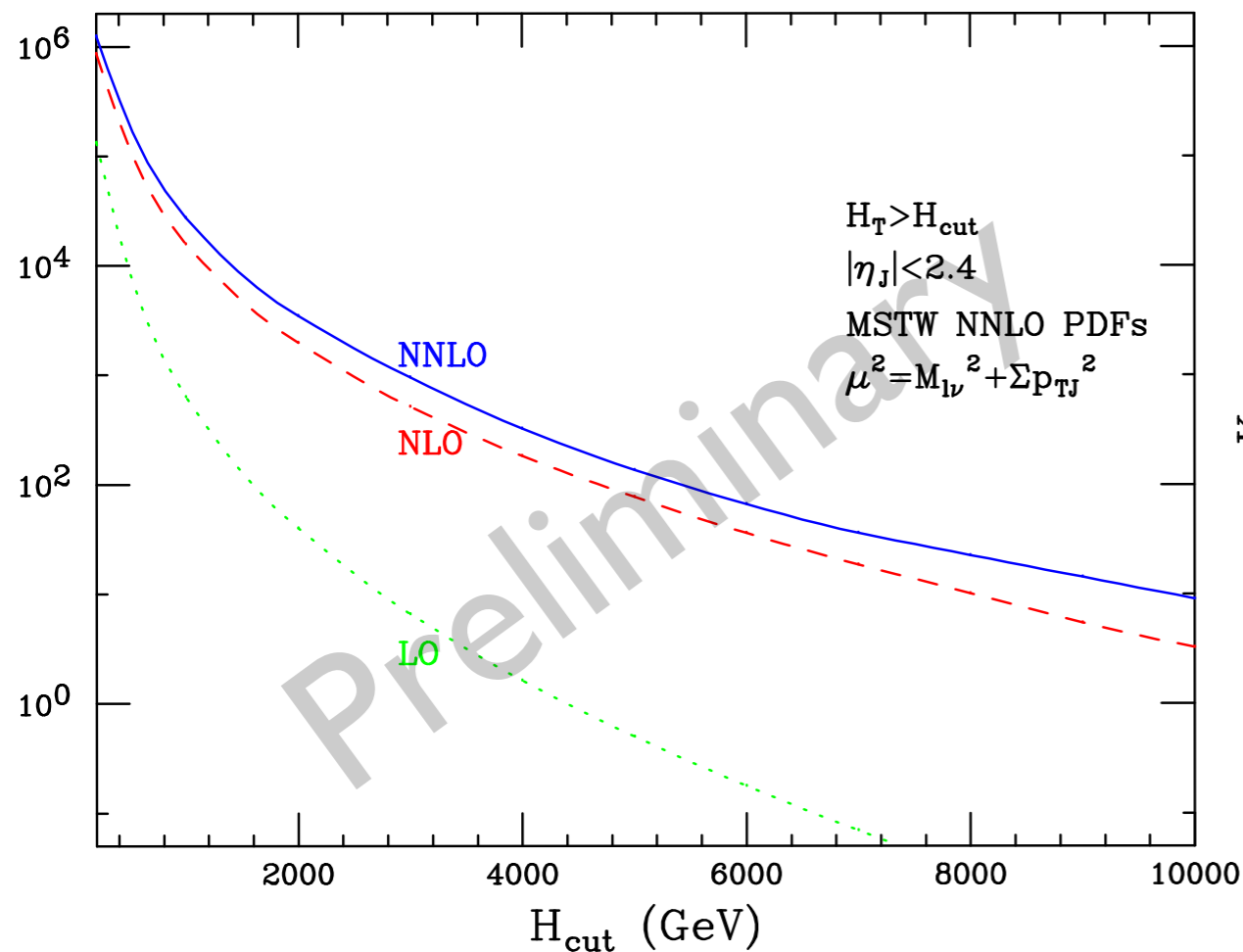


- Tiny NNLO corrections; at the -1% level for the central scale choice
- For transverse momenta less than 200 GeV, ratio of NNLO over NLO cross section is flat (with the exception of the Sudakov shoulder in the $p_{T,W}$ distribution caused by the $p_{T,J} > 30$ GeV cut)

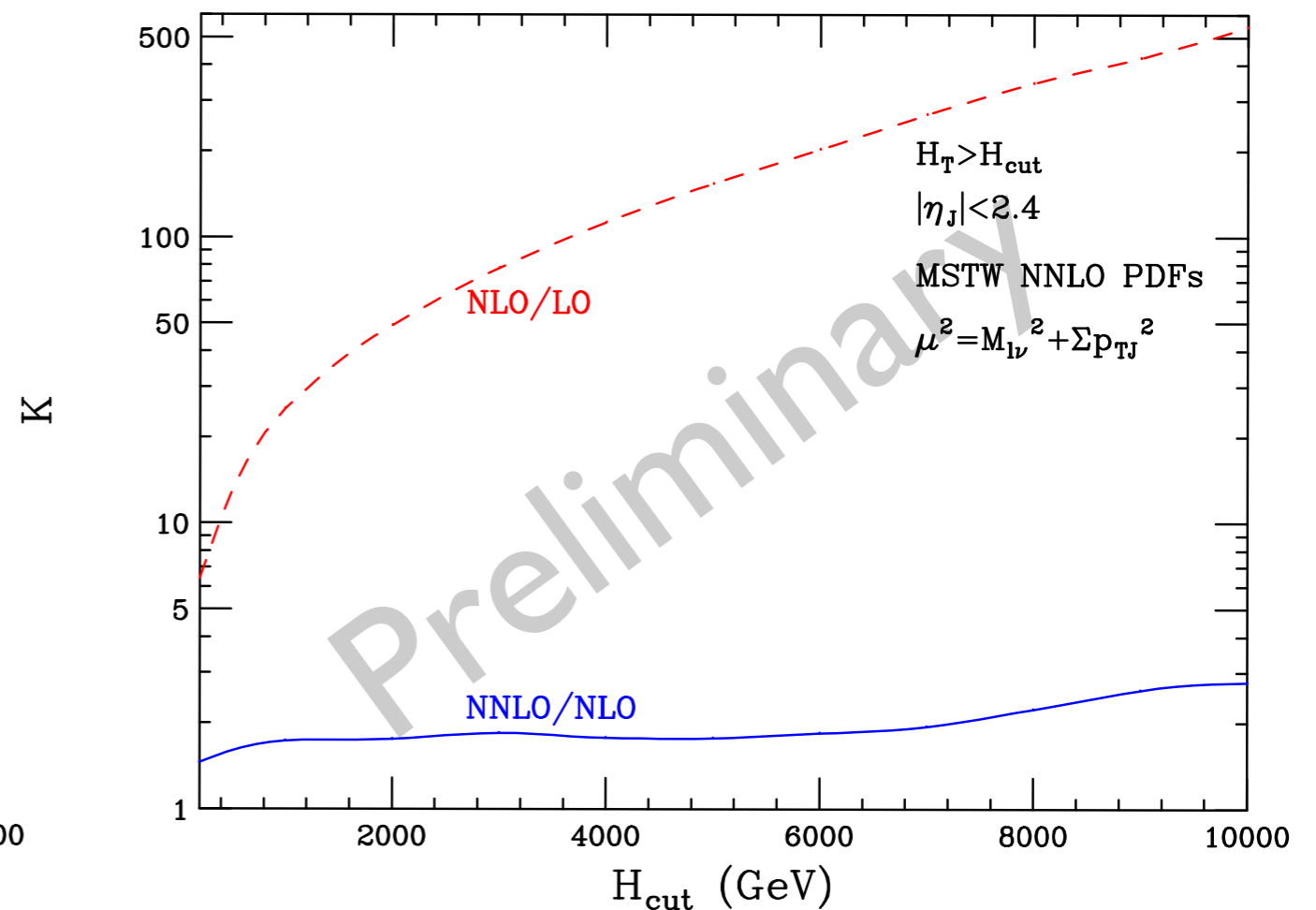
W+jet at 100 TeV

- Recent interest in studying a possible future 100 TeV pp collider; how do the NNLO corrections to W+jet look at such an energy? Define $H_T = \sum p_{Tj}$, a measure of the total jet activity.

Wj @ 100 TeV

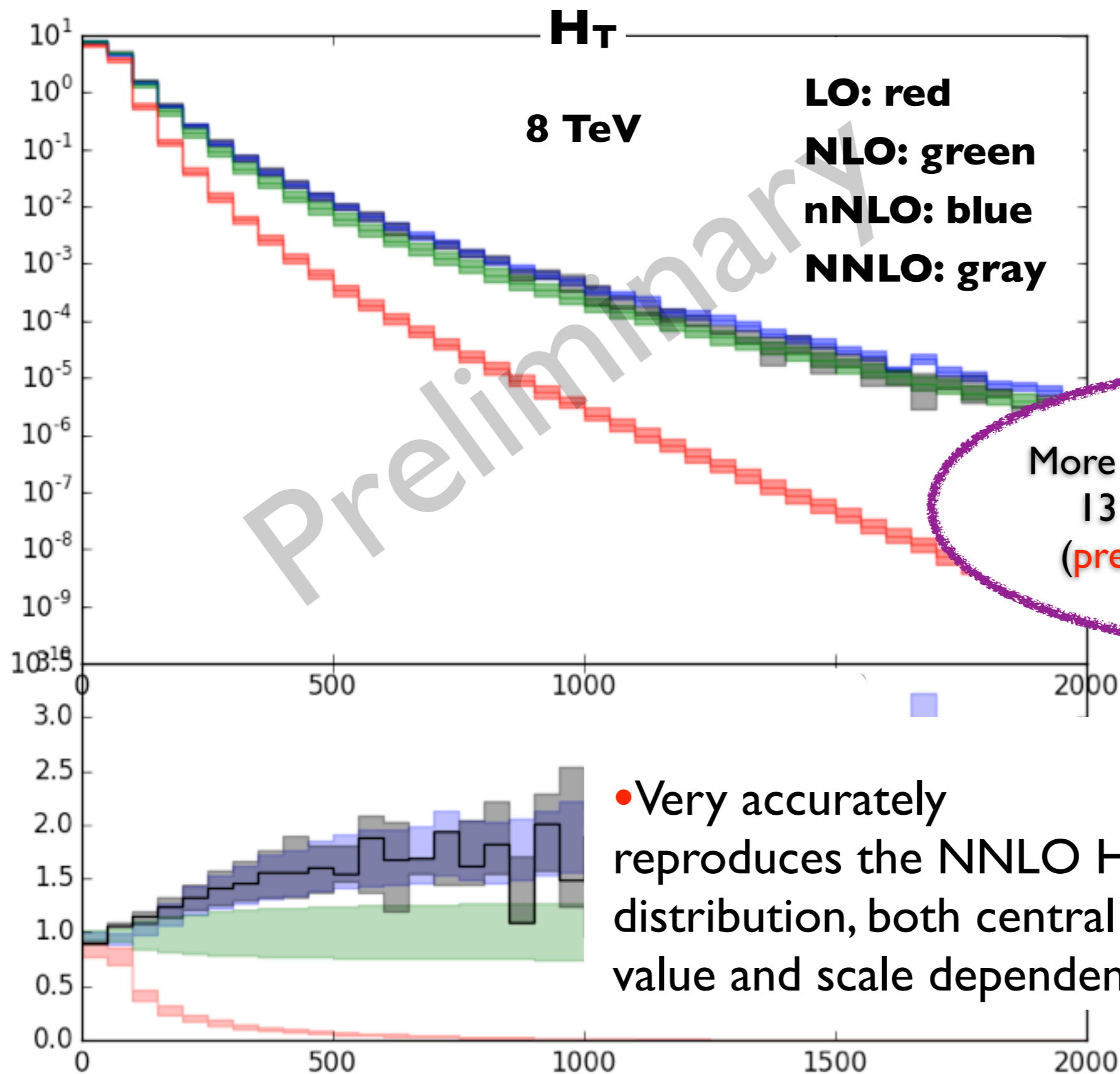


Wj @ 100 TeV



- **Large NNLO corrections** that increase with H_T . Similar large corrections to the p_{Tj} distribution; p_{TW} does not receive similar large NNLO corrections.
- This result shows that beyond NNLO will be needed for accurate 100 TeV pheno.

W+jet@NNLO: LoopSim



H+jet@NNLO: validation

- As discussed previously, the production of the Higgs at high- p_T will provide an important probe of BSM physics in Run II
- Need improvement on two fronts:

$O(\alpha_s)$ corrections to $1/m_t$ suppressed operators

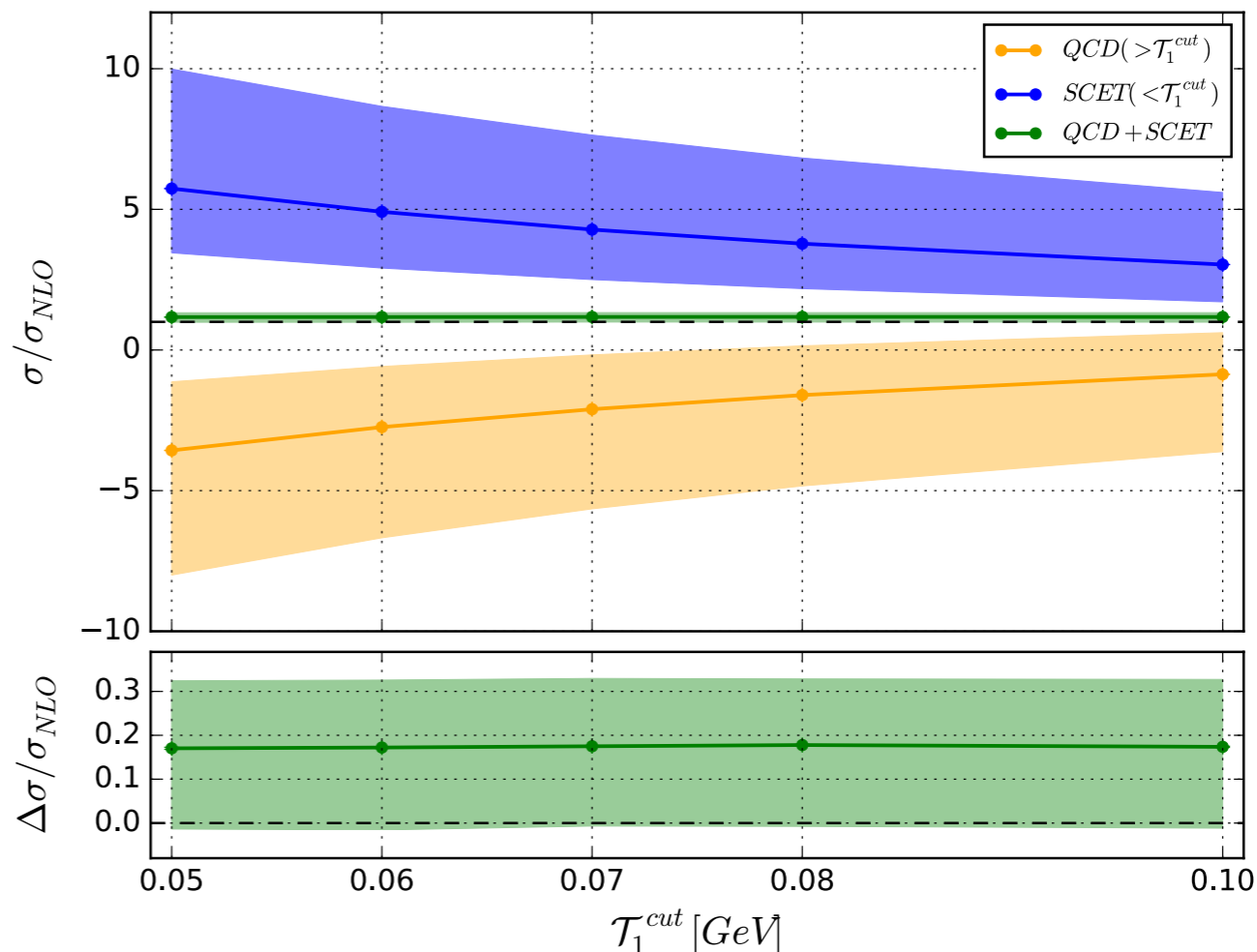
$O(\alpha_s^2)$ correction in the $m_t \rightarrow \infty$ limit

Our focus

Boughezal, Focke, Giele, Liu, FP 1505.03893

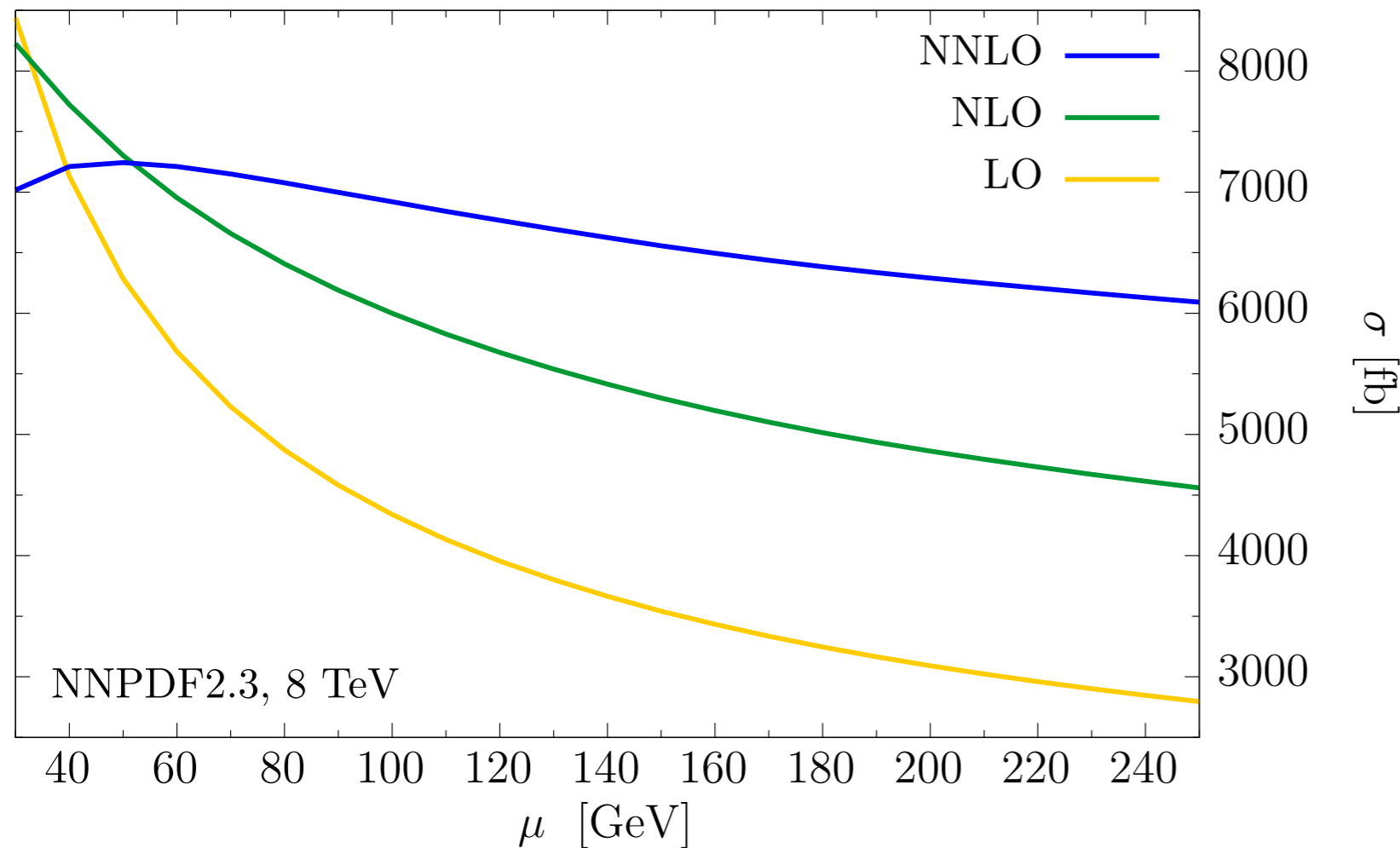
Harlander, Neumann, Ozeren 1206.0157;
Dawson, Lewis, Zeng 1409.6299

- $p_{Tj} > 30$ GeV, $|\eta_j| < 2.4$
- NNPDF PDFs, $\mu_0 = M_H$, vary by factor of 2 to estimate error
- Perfect stability with respect to varying τ^{cut}



H+jet@NNLO: validation

- An additional check is possible in this case. The dominant qg and gg scattering channels were also computed using the sector-improved residue subtraction technique [Boughezal, Caola, Melnikov, FP, Schulze 1504.07922](#)

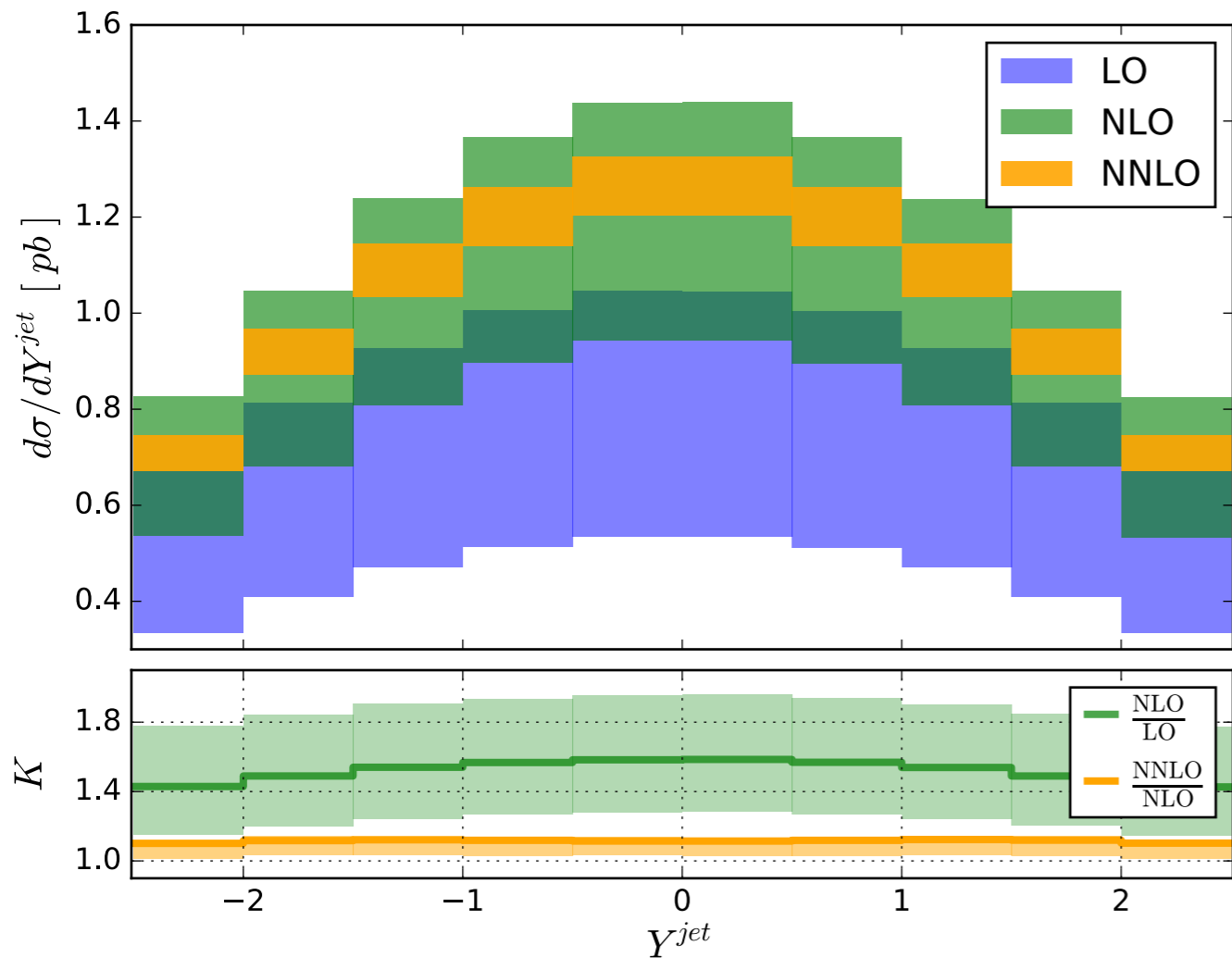


- Agreement between the two calculations at the per-mille level
- Effect of missing qq channels in the sector-improved calculation at the 1-2% level
- Reduced scale dependence at NNLO; preference for smaller scales

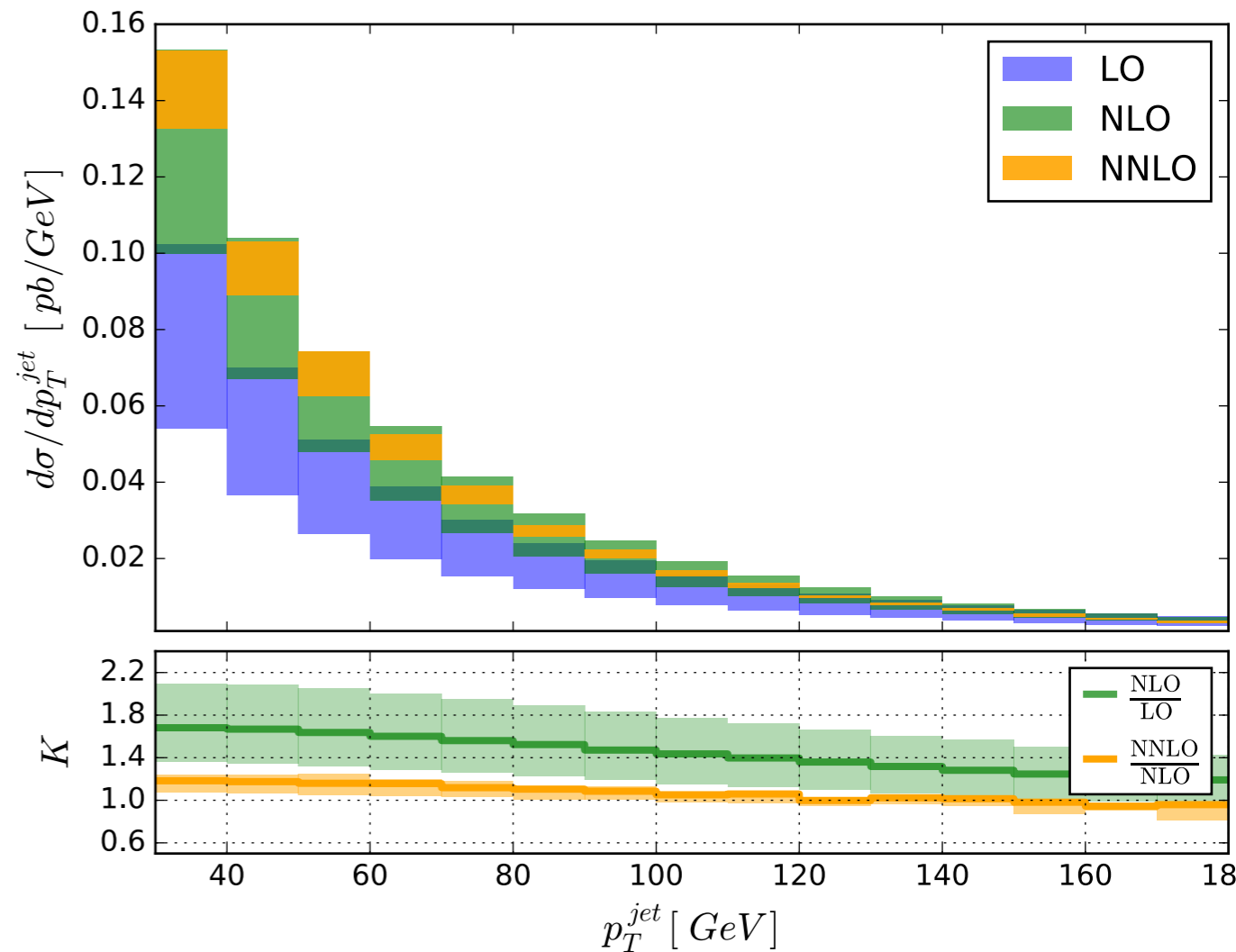
Important, unique validation of NNLO calculational technology!

H+jet@NNLO

Y_J



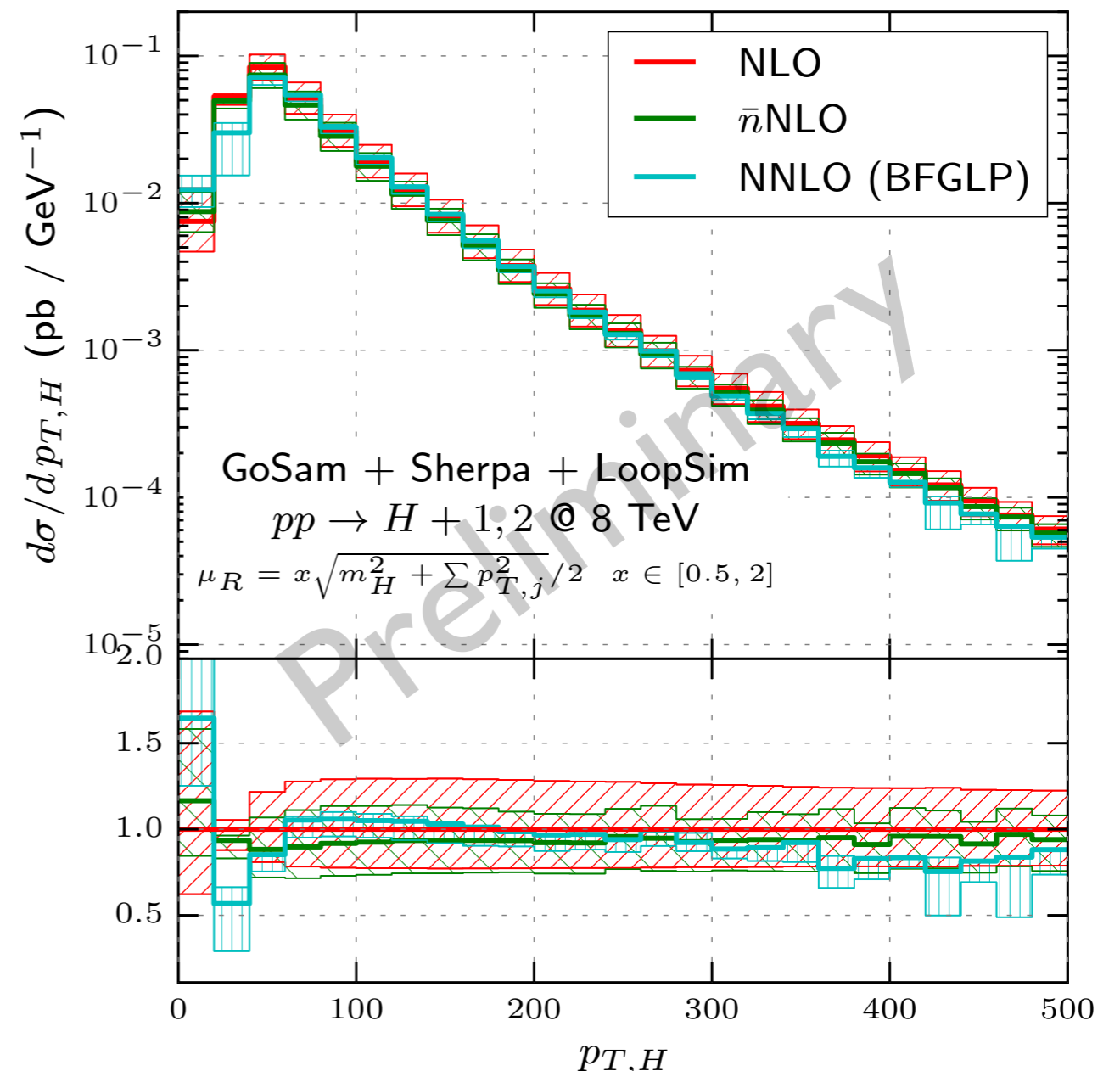
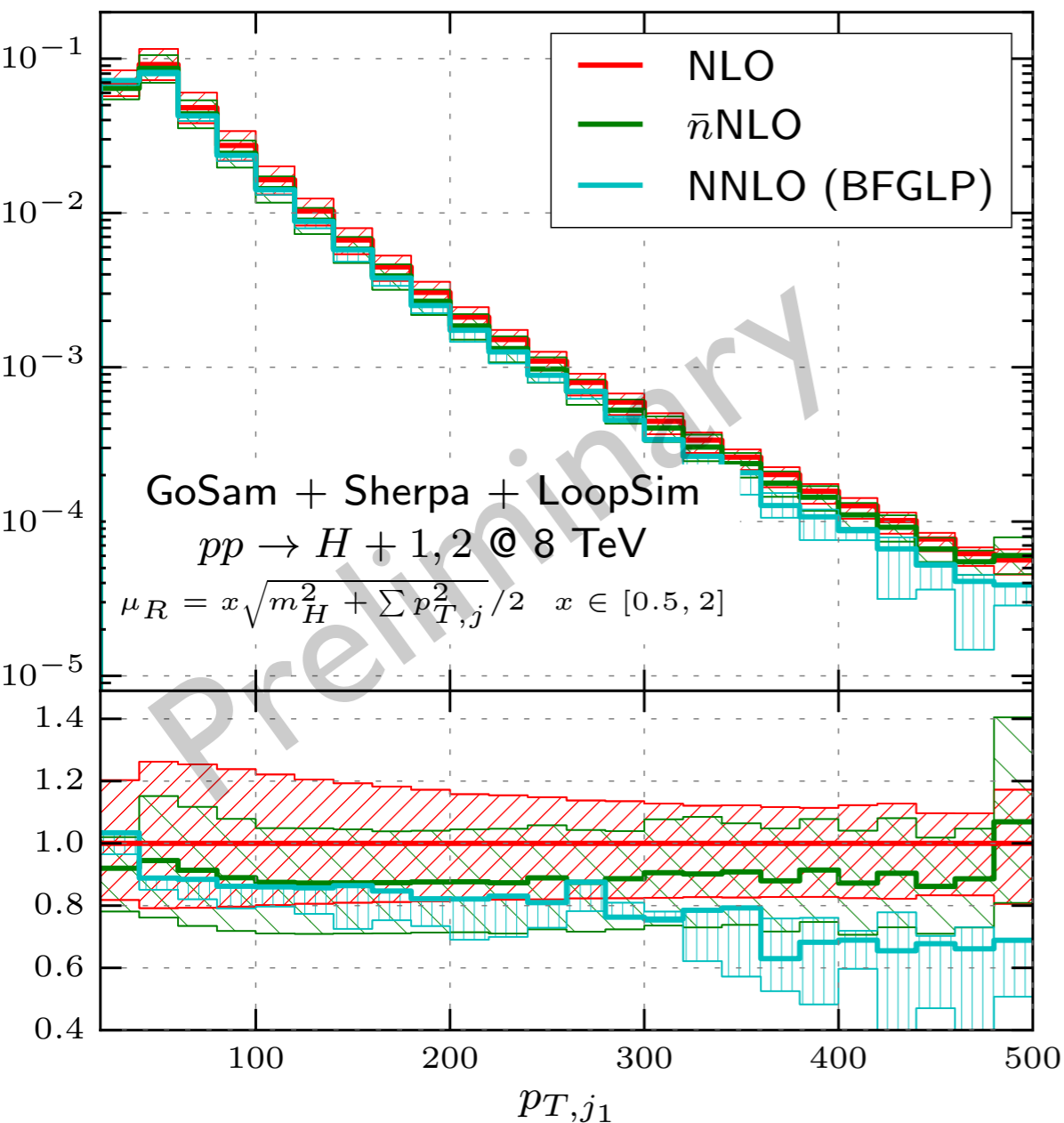
p_{TJ}



- NNLO corrections are flat as a function of jet rapidity
- However, the NNLO/NLO ratio for the p_{TJ} distribution varies by $\sim 20\%$ as p_{TJ} is varied from 30 GeV to 180 GeV

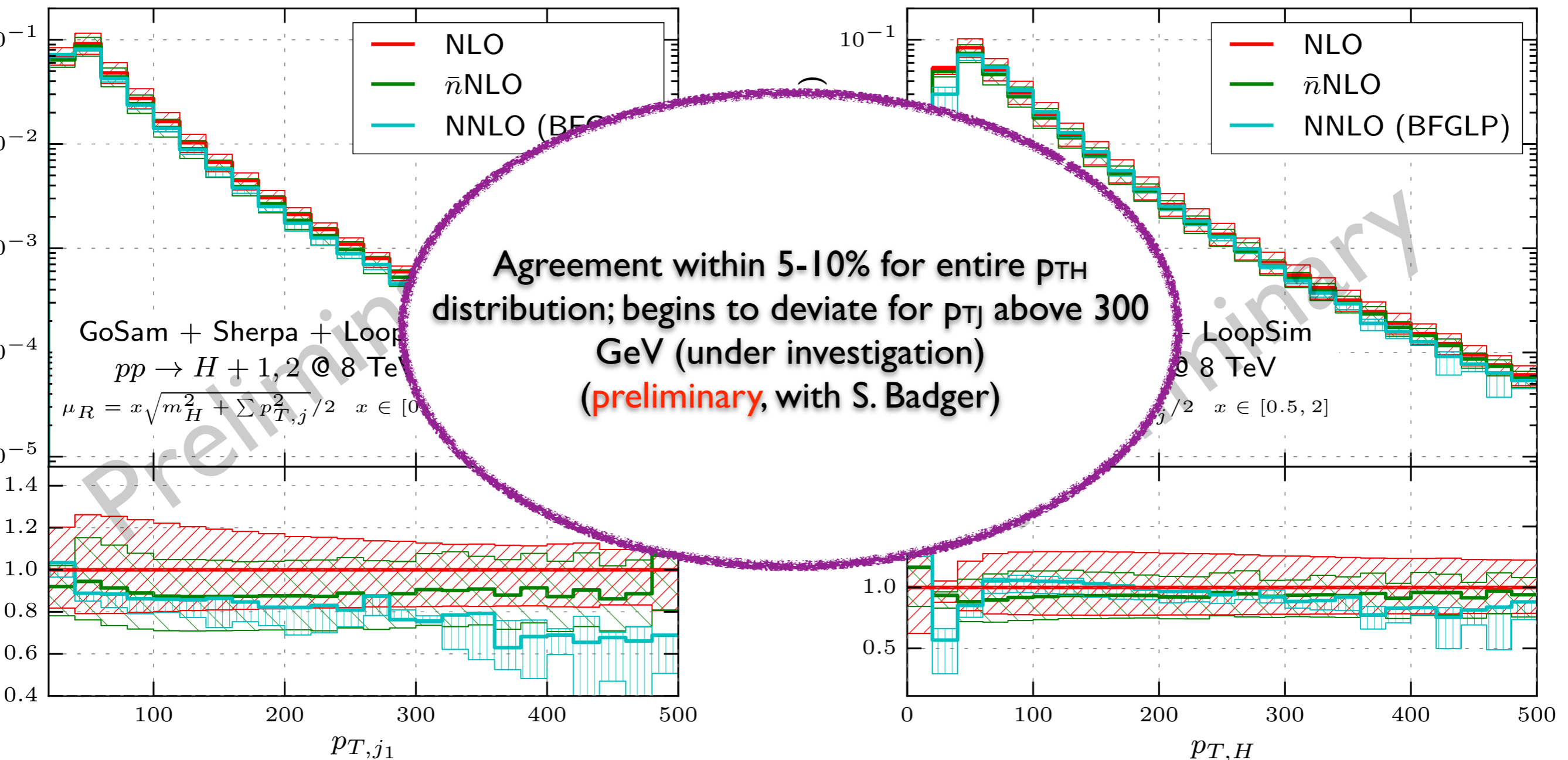
H+jet@NNLO

- Such full NNLO calculations provide the opportunity of testing various approximate techniques, such as LoopSim



H+jet@NNLO

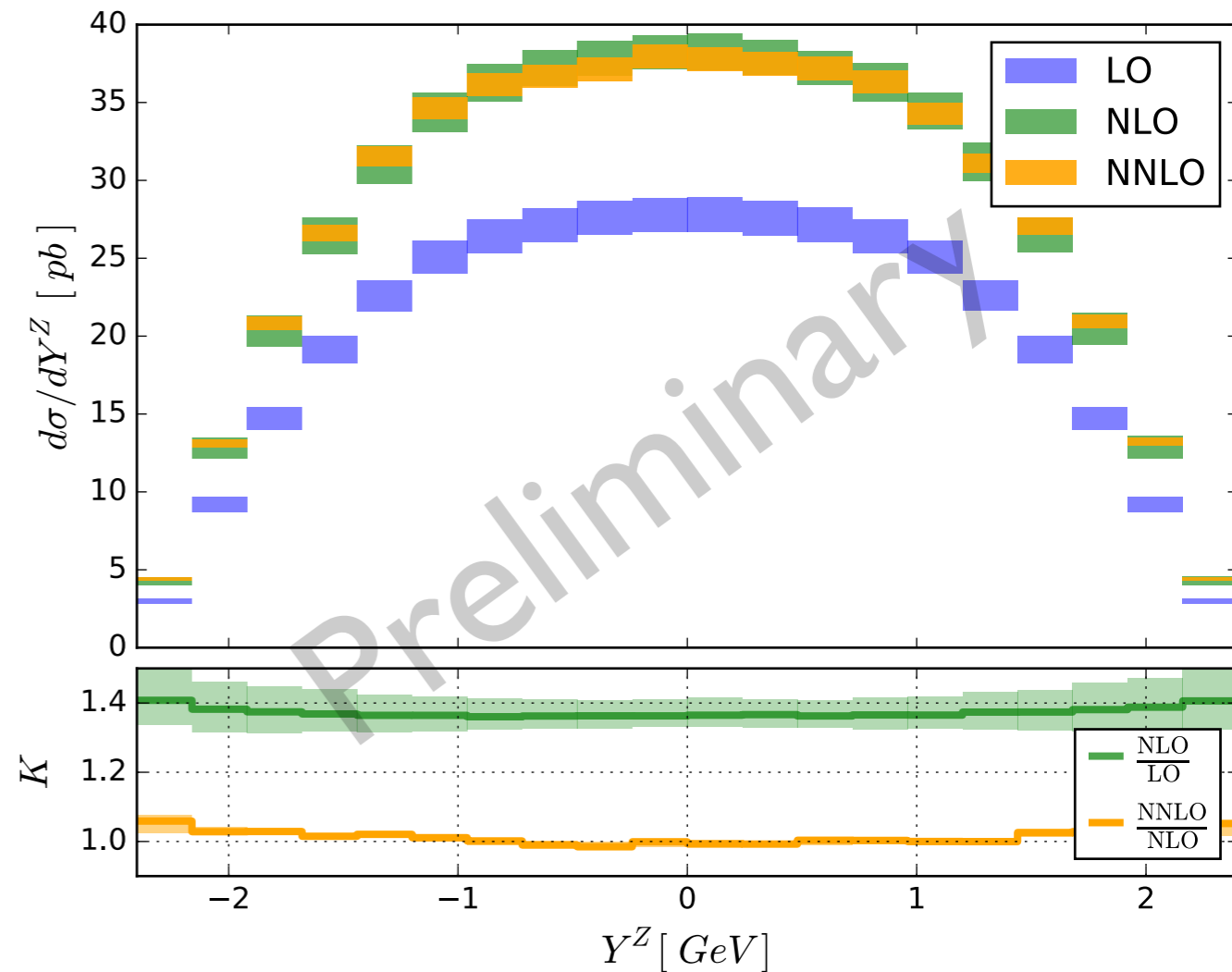
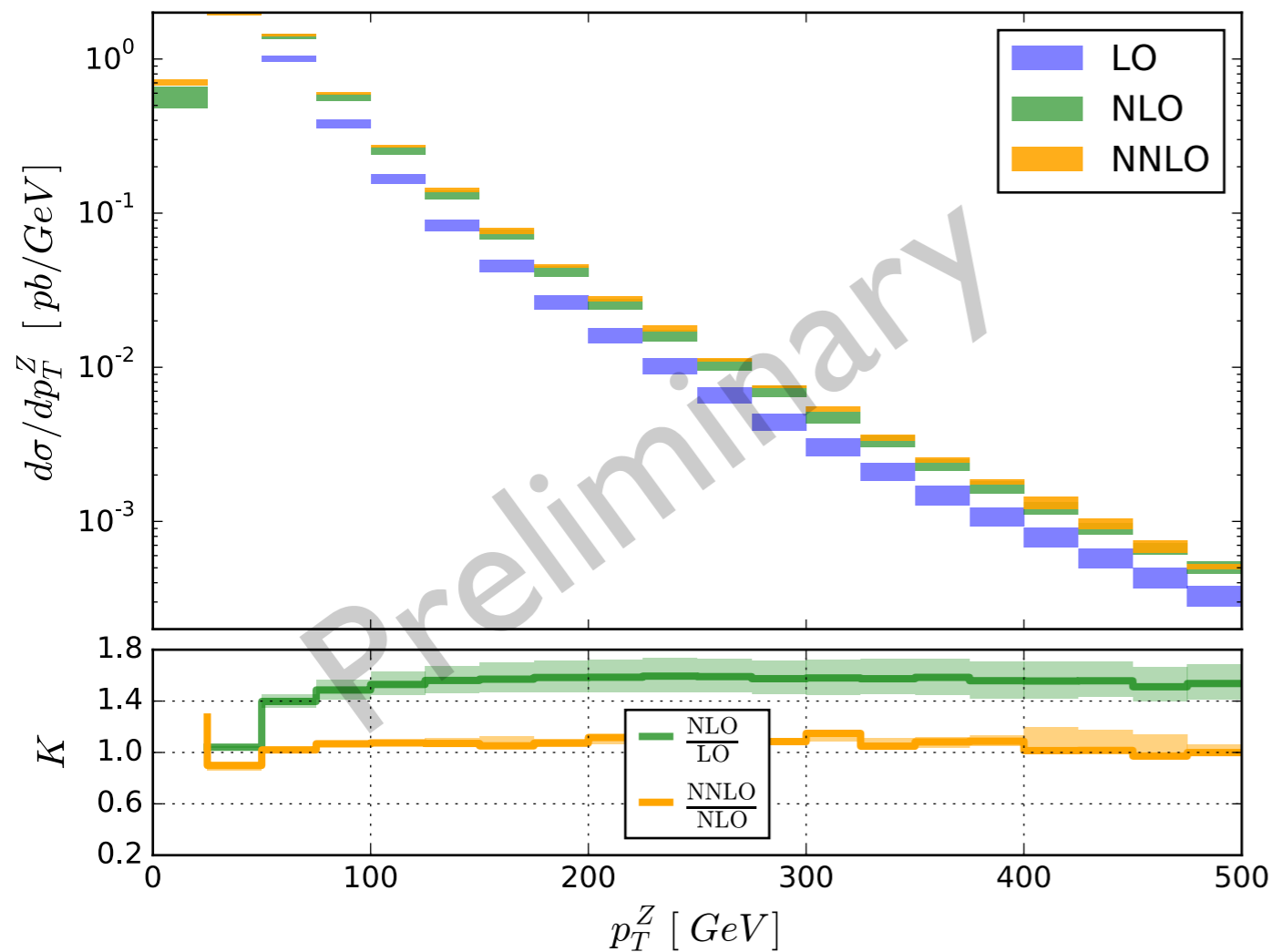
- Such full NNLO calculations provide the opportunity of testing various approximate techniques, such as LoopSim.



Z+jet@NNLO

- Z+jet is another important SM benchmark; needed for gluon PDF determination; detector calibration; background to dark matter searches in the mono-jet channel

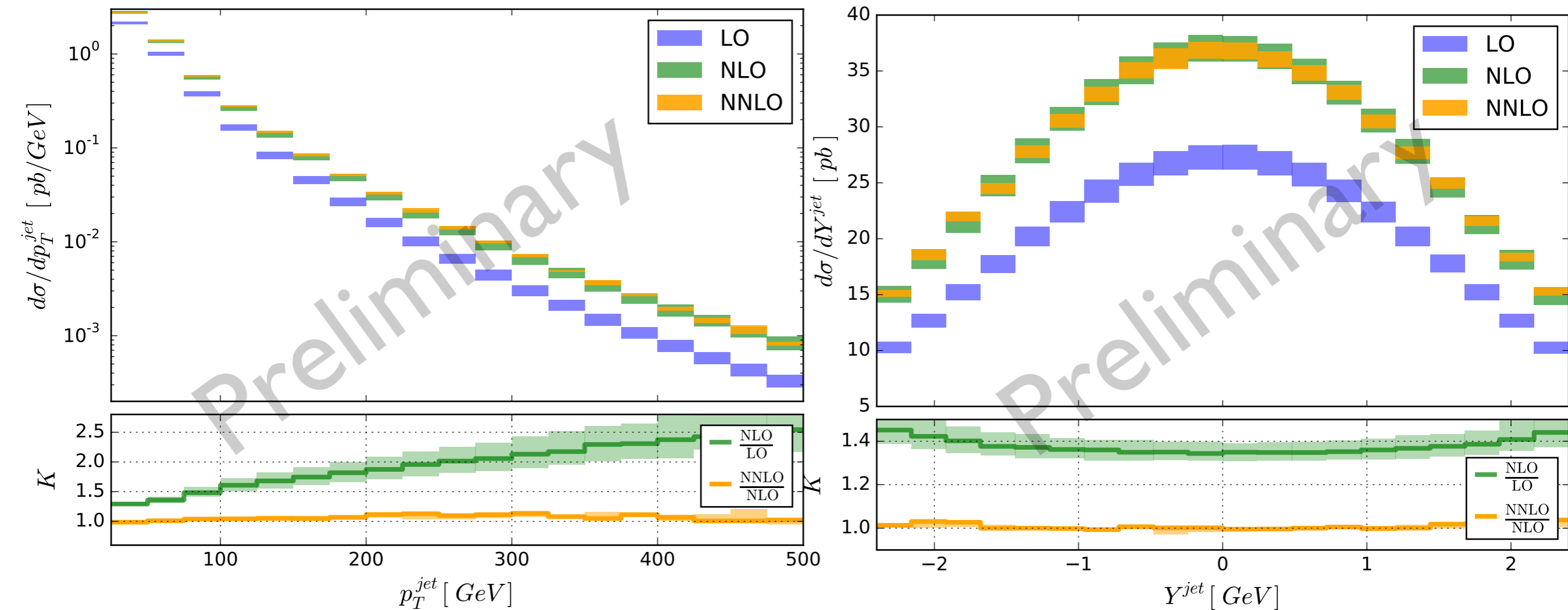
Boughezal, Campbell, Ellis, Focke, Giele, Liu, FP to appear



- Z-boson p_T and Y receive small NNLO corrections, with a flat K-factor

Z+jet@NNLO

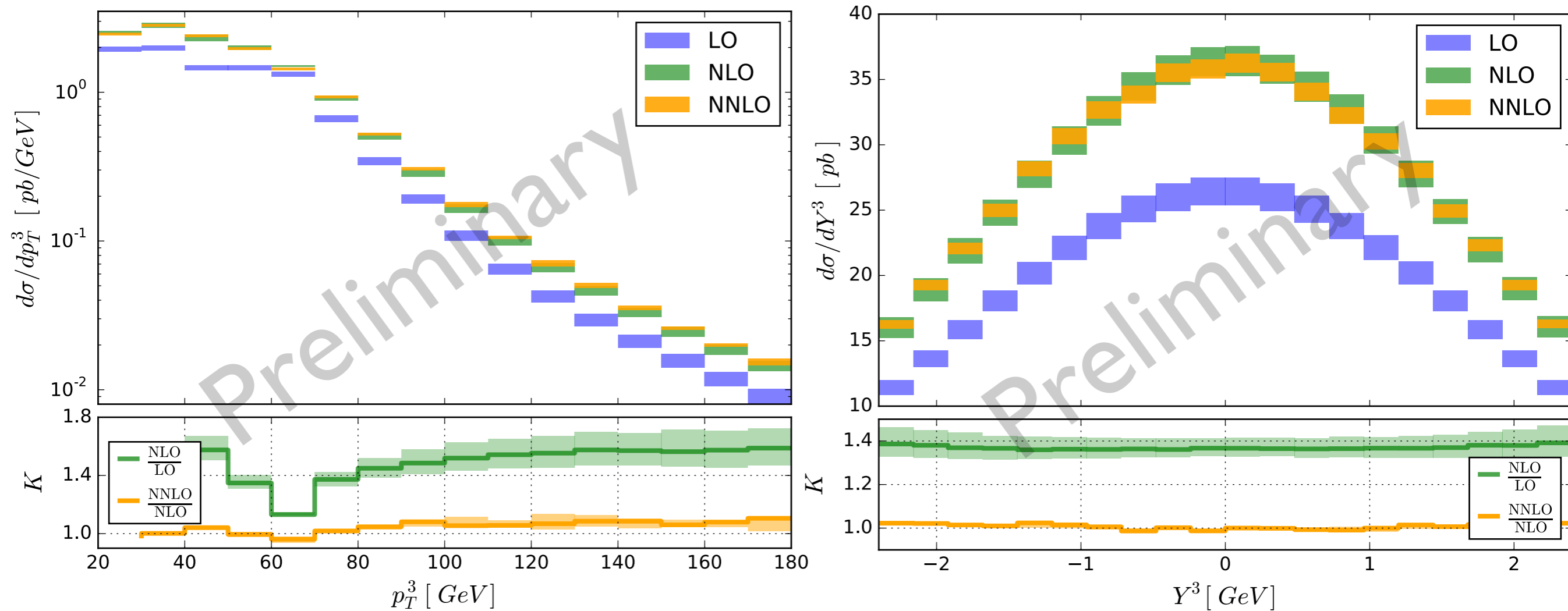
- Z+jet is another important SM benchmark; needed for gluon PDF determination; detector calibration; background to dark matter searches in the mono-jet channel



- NNLO jet distribution corrections also small, even though NLO corrections are large and very dependent on kinematics

Z+jet@NNLO

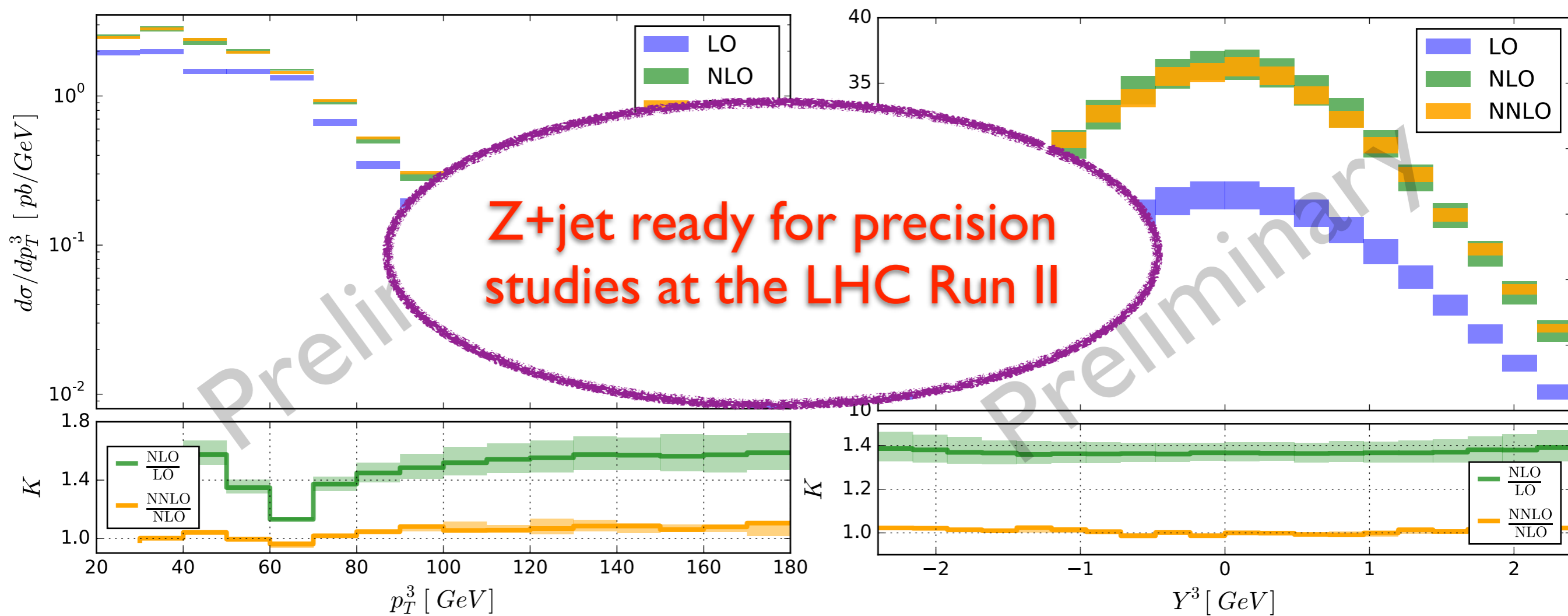
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- Wiggles in the lepton p_T distribution caused by the invariant mass cut used by ATLAS/CMS; leads to a well-known Jacobian peak at $M_{ll}/2$

Z+jet@NNLO

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Conclusions

- We have entered the era of percent-level jet phenomenology
- The N-jettiness subtraction scheme has solved the problem of NNLO corrections to jet production processes
- NNLO corrections to V +jet are at the percent level; ready for precision measurements at the LHC Run II
- Applications also outside of HEP, for example to a future EIC experimental program