

Multiparton interactions in event generators

Stefan Prestel



Physics at the Falls

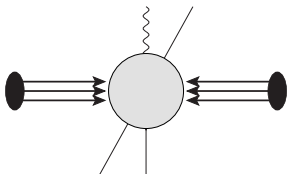
University of Buffalo, November 02, 2015

...or how much modelling do we need?

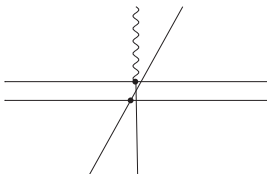


Single vs. double scattering

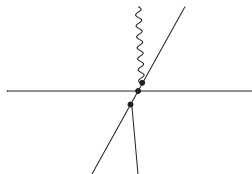
Detector event



Multiple scattering

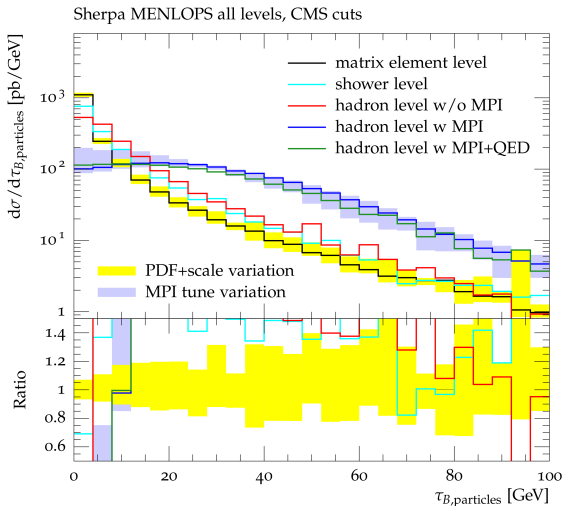


Perturbative scattering



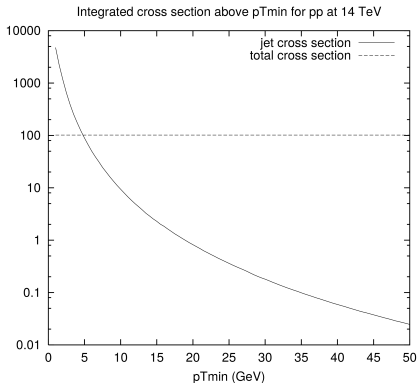
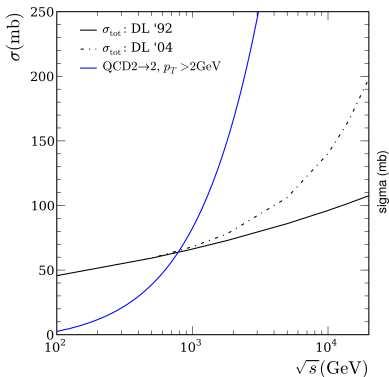
When colliding composite objects, many constituent scatterings "compete" for the collision energy – **and multiple scattering can look like single complicated scatterings! Which scattering is the most important?**

Double vs. multiparton scattering



Beam thrust in W +jets event, taken from Les Houches Proc. arXiv:1203.6803.
The difference (yellow vs. green) accumulates from many scatterings.

The dijet process



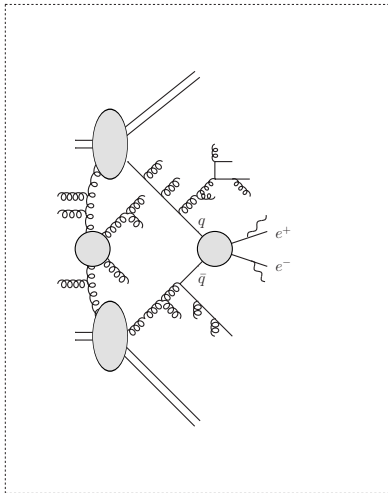
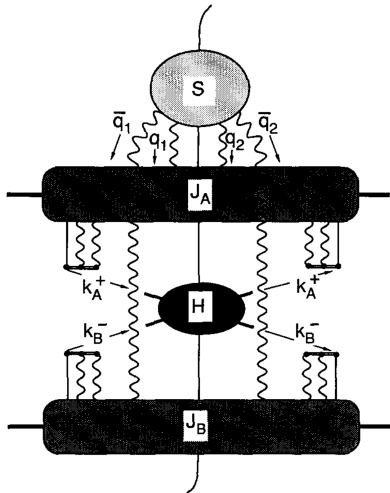
Perturbative cross section

$$\sigma(pp \rightarrow jj + X) = \int_{p_{\perp\text{min}}}^{\frac{E_{\text{cm}}}{2}} dx_1 dx_2 f_1(x_1) f_2(x_2) \frac{d\hat{\sigma}}{dp_{\perp}} dp_{\perp} > \sigma(pp \rightarrow \text{anything}) \text{ for } \frac{p_{\perp\text{min}}}{E_{\text{cm}}} \rightarrow 0$$

as $f(x)$ not small (enough) for low $x \approx \frac{p_{\perp\text{min}}}{E_{\text{cm}}}$ to suppress $\frac{p_{\perp\text{min}}}{E_{\text{cm}}} \rightarrow 0$ divergence!

Hints from factorisation

Still consistent with perturbative QCD: PDFs are the *inclusive* probability to find parton at x , with all other interactions above $x \approx \frac{p_{\perp, \min}}{E_{cm}}$ integrated out!



Multiple interactions (indirect evidence)

Collider observables can “see” these additional interactions.

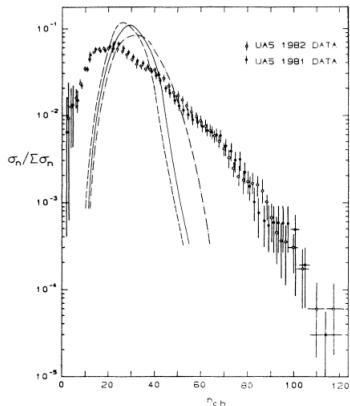


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low p_T only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

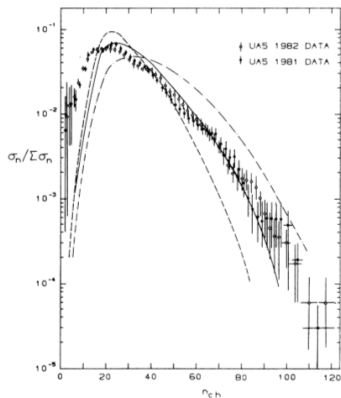
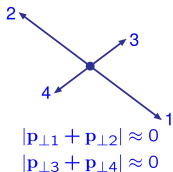


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T\min}=2.0$ GeV; solid line, $p_{T\min}=1.6$ GeV; dashed-dotted line, $p_{T\min}=1.2$ GeV.

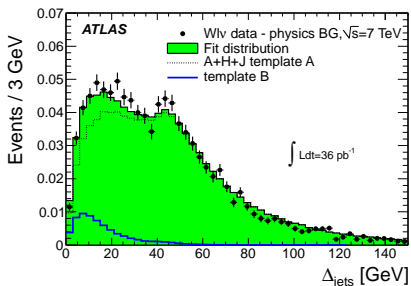
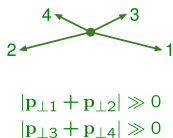
Multiple interactions (“direct evidence”)

Double parton scattering has typical kinematics...

Double Parton Scattering



Double BremsStrahlung



...that's seen in the data (ATLAS New J.Phys. 15 (2013) 033038)

Multiple scatterings and the inelastic (non-diffractive) cross section

A scattering above $p_{\perp min}$ is accompanied by an average $\langle n(p_{\perp min}) \rangle$ interactions, so that

$$d\sigma^{inc}(p_{\perp min}, E_{cm}) = \langle n(p_{\perp min}) \rangle \cdot \sigma^{ND}(p_{\perp min}, E_{cm})$$

where

$$\begin{aligned}\sigma^{ND} &\approx (\sigma^{tot} - \sigma^{el}) - \int (d\sigma_{SD} - d\sigma_{DD} - d\sigma_{CD}) \\ \sigma^{ND} &< \sigma(pp \rightarrow \text{anything})\end{aligned}$$

is the inelastic non-diffractive cross-section (defined implicitly by total, elastic cross section and diffractive parametrisation).

Now the average number of scatterings is unconstrained, but the (single scattering) cross section remains small enough.

Multiparton interactions from a “consistency condition”

Question: Can't we just overlay many scatterings to approximate the result? Just like we do for Pile-Up?

Answer: No!

Consistency condition: For large p_{\perp} , model must preserve the perturbative hard scattering cross section, otherwise factorisation of inclusive cross section violated!

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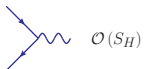
Consistency condition: For large p_{\perp} , model must preserve the perturbative hard scattering cross section, otherwise factorisation of inclusive cross section violated!

Solution: Subtract what you add!

For every additional scattering, we need "virtual corrections"

(This should sound familiar from parton shower unitarity)

”Virtual corrections” for second scattering




”Virtual corrections” for second scattering

$$\left[\begin{array}{c} \text{Diagram 1} \\ \mathcal{O}(S_H) \end{array} + \int \begin{array}{c} \text{Diagram 2} \\ \mathcal{O}(S_H S_{2 \rightarrow 2}) \end{array} \right]$$


The image shows a mathematical expression enclosed in large square brackets. On the left side of the brackets is a blue Feynman diagram consisting of two incoming lines meeting at a vertex, with a wavy line extending from that vertex. Below this diagram is the label $\mathcal{O}(S_H)$. To the right of the first term is a plus sign, followed by an integral symbol \int . Inside the integral is a green Feynman diagram of a bubble loop. The bubble is a horizontal oval with two external lines on the left and two on the right. The word "MPP" is written vertically inside the bubble. Below this diagram is the label $\mathcal{O}(S_H S_{2 \rightarrow 2})$. The entire expression is enclosed in large square brackets.

"Virtual corrections" for second scattering




$$\left[\mathcal{O}(S_H) - \mathcal{O}(S_H) \int \text{MFT} + \int \text{MFT} \mathcal{O}(S_H S_{2 \rightarrow 2}) \right]$$

"Virtual corrections" for second scattering


$$\left[\mathcal{O}(S_H) - \mathcal{O}(S_H) \int \text{MFP} + \int \text{MFP} \mathcal{O}(S_H S_{2 \rightarrow 2}) \right]$$

Remember PS unitarity:
First terms in a "no-scattering" factor.

"Virtual corrections" for second scattering




$$\left[\mathcal{O}(S_H) - \mathcal{O}(S_H) \int \text{MPI} + \int \text{MPI} \mathcal{O}(S_H S_{2 \rightarrow 2}) \right]$$

Remember PS unitarity:
First terms in a "no-scattering" factor.



$$\left[\Pi^{\text{MPI}}(E_{cm}, p_{\perp \min}) \mathcal{O}(S_H) + \int \text{MPI} \Pi^{\text{MPI}}(E_{cm}, p_{\perp 1}) \mathcal{O}(S_H S_{2 \rightarrow 2}) \right]$$


”Virtual corrections” for multiple scatterings



$$\left[\mathcal{O}(S_H) - \mathcal{O}(S_H) \int \text{MFT} + \int \text{MFT} \otimes \left\{ \mathcal{O}(S_H S_{2 \rightarrow 2}) + \int \text{MFT} \mathcal{O}(S_H S_{2 \rightarrow 2} S'_{2 \rightarrow 2}) \right\} \right]$$

The diagram illustrates the structure of virtual corrections for multiple scatterings. It features a vertex on the left with two incoming blue lines and one outgoing wavy line. This vertex is enclosed in a large square bracket. To the right of the vertex, the expression is: $\mathcal{O}(S_H) - \mathcal{O}(S_H) \int \text{MFT} + \int \text{MFT}$. The MFT diagrams are represented as ovals with four external green lines. This is followed by a tensor product symbol \otimes and a large curly bracket containing $\mathcal{O}(S_H S_{2 \rightarrow 2}) + \int \text{MFT} \mathcal{O}(S_H S_{2 \rightarrow 2} S'_{2 \rightarrow 2})$. The $S_{2 \rightarrow 2}$ and $S'_{2 \rightarrow 2}$ terms are represented by diagrams with four external green lines.

”Virtual corrections” for multiple scatterings



$$\left[\mathcal{O}(S_H) - \mathcal{O}(S_H) \int \text{MPI} + \int \text{MPI} \right] \otimes \left\{ \mathcal{O}(S_H S_{2 \rightarrow 2}) - \mathcal{O}(S_H S_{2 \rightarrow 2}) \int \text{MPI} + \int \text{MPI} \mathcal{O}(S_H S_{2 \rightarrow 2} S'_{2 \rightarrow 2}) \right\}$$

The diagram illustrates the structure of virtual corrections for multiple scatterings. It consists of two main parts enclosed in large square brackets. The first part is a sum of three terms: a tree-level operator $\mathcal{O}(S_H)$, a loop correction term $-\mathcal{O}(S_H) \int \text{MPI}$ (where MPI is a grey oval with two green lines), and a self-energy correction term $+\int \text{MPI}$. The second part is a tensor product \otimes with a set of curly braces containing three terms: a tree-level operator $\mathcal{O}(S_H S_{2 \rightarrow 2})$, a loop correction term $-\mathcal{O}(S_H S_{2 \rightarrow 2}) \int \text{MPI}$, and a self-energy correction term $+\int \text{MPI} \mathcal{O}(S_H S_{2 \rightarrow 2} S'_{2 \rightarrow 2})$. The MPI diagrams are represented by grey ovals with two green lines extending from them.

"Virtual corrections" for multiple scatterings

Remember PS unitarity:
First terms in a "no-scattering" factor.

$$\left[\mathcal{O}(S_H) - \mathcal{O}(S_H) \int \text{MPI} + \int \text{MPI} \right] \otimes \left\{ \mathcal{O}(S_H S_{2 \rightarrow 2}) - \mathcal{O}(S_H S_{2 \rightarrow 2}) \int \text{MPI} + \int \text{MPI} \mathcal{O}(S_H S_{2 \rightarrow 2} S'_{2 \rightarrow 2}) \right\}$$

"Virtual corrections" for multiple scatterings

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$$\left[\mathcal{O}(S_H) - \mathcal{O}(S_H) \int \text{MPI} + \int \text{MPI} \right] \otimes \left\{ \mathcal{O}(S_H S_{2 \to 2}) - \mathcal{O}(S_H S_{2 \to 2}) \int \text{MPI} + \int \text{MPI} \mathcal{O}(S_H S_{2 \to 2} S'_{2 \to 2}) \right\}$$

$$\left[\Pi^{\text{MPI}}(E_{cm}, p_{\perp min}) \mathcal{O}(S_H) + \int \text{MPI} \right] \otimes \left\{ \Pi^{\text{MPI}}(p_{\perp 1}, p_{\perp min}) \mathcal{O}(S_H S_{2 \to 2}) + \int \text{MPI} \Pi^{\text{MPI}}(p_{\perp 1}, p_{\perp 2}) \mathcal{O}(S_H S_{2 \to 2} S'_{2 \to 2}) \right\}$$

If you have a hammer...

...everything looks like a parton shower. Assume

$\delta p_{\perp} \langle n(p_{\perp}) \rangle \equiv$ Probability for scattering with $p_{\perp} \in [p_{\perp min}, p_{\perp min} + \delta p_{\perp}]$.

Then the probability of no scattering is

$$1 - \delta p_{\perp} \langle n(p_{\perp}) \rangle$$

or, if δp_{\perp} is divided into m parts, and the scattering probabilities are independent

$$[1 - \delta p_{\perp} / m \langle n(p_{\perp}) \rangle]^m \xrightarrow{m \rightarrow \infty} \exp \left(- \int_{p_{\perp min}}^{p_{\perp min} + \delta p_{\perp}} dp_{\perp} \langle n(p_{\perp}) \rangle \right) \equiv \Pi^{\text{MPI}}(p_{\perp min} + \delta p_{\perp}, p_{\perp min})$$

We can define a no-additional scattering probability which contains "all-order virtual corrections" – just like a parton shower Sudakov factor.

\implies Can recycle the PS algorithm to produce additional scatterings.

Scattering probabilities

The probability of a hardest scattering at $p_{\perp 1}$ is

$$\Pi^{\text{MPI}}(p_{\perp 0}, p_{\perp 1}) \langle n(p_{\perp 1}) \rangle$$

The probability of having a second hardest scattering at $p_{\perp 2} < p_{\perp 1}$ is

$$\int_{p_{\perp 2}}^{p_{\perp 0}} dp_{\perp 1} \Pi^{\text{MPI}}(p_{\perp 0}, p_{\perp 1}) \langle n(p_{\perp 1}) \rangle \Pi^{\text{MPI}}(p_{\perp 1}, p_{\perp 2}) \langle n(p_{\perp 2}) \rangle$$

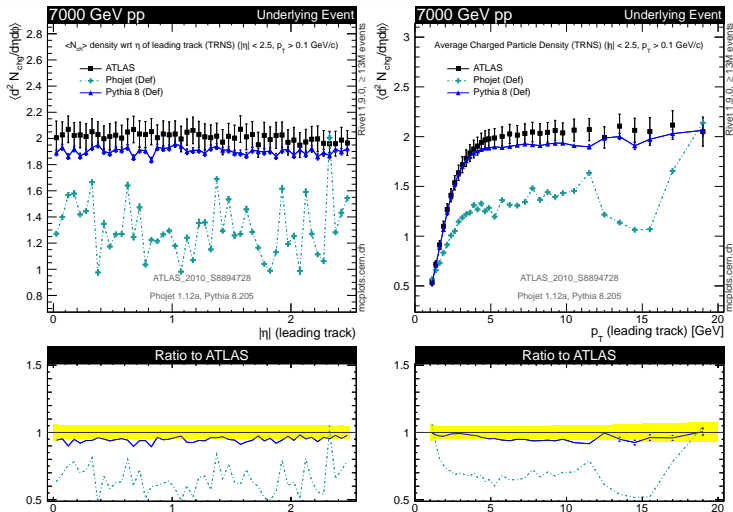
...and so on. The probability for any partonic scattering is thus σ^{inc} , i.e. the perturbative result. The double parton scattering cross section is:

$$\langle \mathcal{O} \rangle = \int d\Phi_H \int_{p_{\perp \text{min}}}^{p_{\perp 0}} d\Phi_{2 \rightarrow 2} d\hat{\sigma}_H(\Phi_H) d\hat{\sigma}_{2 \rightarrow 2}(\Phi_{2 \rightarrow 2}) \mathcal{O}(\Phi_H \Phi_{2 \rightarrow 2})$$

$$\underbrace{\frac{1}{\sigma^{\text{ND}}} \Pi^{\text{MPI}}(p_{\perp 0}, p_{\perp}) f_0^+(x_0^+, \mu_{F,0}^2) f_0^-(x_0^-, \mu_{F,0}^2) \frac{f_1^+(\frac{x_1^+}{1-x_0^+}, p_{\perp})}{1-x_0^+} \frac{f_1^-(\frac{x_1^-}{1-x_0^-}, p_{\perp})}{1-x_0^-}}$$

Our simplest DPD model. Note rescaled PDF to enforce flavour sum rule.

The pedestal, or how entangled are MPI?



Activity uniform in rapidity. More additional particles for harder core scatterings (trigger bias!) \Rightarrow Not captured by model where all scatterings have same probability. \Rightarrow Model matter overlap, i.e. introduce impact parameter dependence.

The pedestal, or *how entangled are MPI?*

Redefine the naive scattering as

$$\langle n(p_{\perp}) \rangle \rightarrow \langle n(p_{\perp}) \rangle \cdot \mathcal{A}(b) / \langle \mathcal{A} \rangle$$

The probability for any partonic scattering is still σ^{inc} , but the double-parton cross section is now

$$\langle \mathcal{O} \rangle = \int d^2b \int_{p_{\perp min}}^{p_{\perp 0}} d\Phi_H \int d\Phi_{2 \rightarrow 2} d\hat{\sigma}_H(\Phi_H) d\hat{\sigma}_{2 \rightarrow 2}(\Phi_{2 \rightarrow 2}) \mathcal{O}(\Phi_H \Phi_{2 \rightarrow 2})$$

$$\frac{1}{\sigma^{ND}} \frac{\mathcal{A}(b)}{\langle \mathcal{A} \rangle} \Pi^{MPI}(b, p_{\perp 0}, p_{\perp}) f_0^+(x_0^+, \mu_{F,0}^2) f_0^-(x_0^-, \mu_{F,0}^2) \frac{f_1^+(\frac{x_1^+}{1-x_0^+}, p_{\perp})}{1-x_0^+} \frac{f_1^-(\frac{x_1^-}{1-x_0^-}, p_{\perp})}{1-x_0^-}$$

with

$$\Pi^{MPI}(b, p_{\perp 0}, p_{\perp}) \exp \left(- \frac{\mathcal{A}(b)}{\langle \mathcal{A} \rangle} \int_{p_{\perp}}^{p_{\perp 0}} d\Phi_{2 \rightarrow 2} \frac{d\hat{\sigma}_{2 \rightarrow 2}(\Phi_{2 \rightarrow 2})}{\sigma^{ND}} \right)$$

Evolution of proton scattering (w/ PYTHIA bias)

- MPI models infer a model for double-parton distributions. DPDs should evolve (to LL as double DGLAP would?)
⇒ Additional scatterings should evolve by parton showering.
- Scatterings are correlated by momentum conservation. Thus, scatterings should evolve together as one big system.
- Formalism of p_{\perp} -ordered showers and MPI is analogous.

⇒ Combine MPI and PS into a single, interleaved evolution equation

Evol [*in state*] ($p_{\perp 0}, p_{\perp c}$)

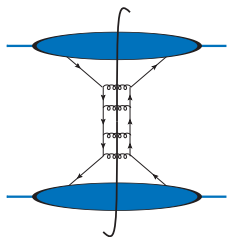
$$= \mathcal{O}(\text{in state}) \exp \left[- \int_{p_{\perp c}}^{p_{\perp 0}} (d\mathcal{P}^{\text{MPI}} + d\mathcal{P}^{\text{FSR}} + d\mathcal{P}^{\text{ISR}}) \right]$$

$$+ \int_{p_{\perp c}}^{p_{\perp 0}} (d\mathcal{P}^{\text{MPI}} + d\mathcal{P}^{\text{FSR}} + d\mathcal{P}^{\text{ISR}}) \exp \left[- \int_{p_{\perp 1}}^{p_{\perp 0}} (d\mathcal{P}^{\text{MPI}} + d\mathcal{P}^{\text{FSR}} + d\mathcal{P}^{\text{ISR}}) \right]$$

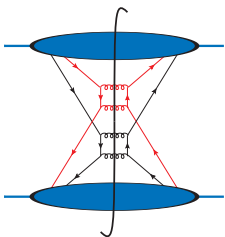
Evol [*new state*] ($p_{\perp 1}, p_{\perp c}$)

Subdividing MPI into double and single feed terms

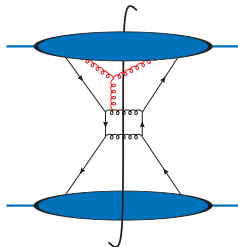
PYTHIA 8 further includes rescattering corrections (lower left). This is truly non-factorising (final parton is put back into proton)



$2 \rightarrow 4$ scattering



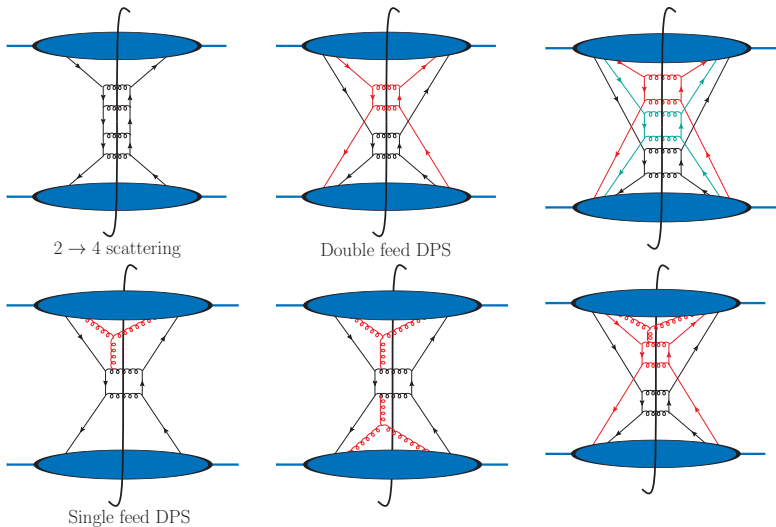
Double feed DPS



Single feed DPS

Subdividing MPI into double and single feed terms

PYTHIA 8 further includes rescattering corrections (lower left). This is truly non-factorising (final parton is put back into proton)



Parameters of Multiparton Interaction models

⇒ Perturbative MPI *model* keeping the inc. cross section. Unknowns:

- σ^{ND} depends on σ^{tot} , σ^{elast} and diffractive model. The parameters of these thus influence MPI.
- Most MPI very soft, but σ^{inc} is still divergent for $p_{\perp min} \rightarrow 0$, i.e. needs extra regulator parameter $p_{\perp 0}$. Regulator should be larger if E_{cm} becomes larger (to not violate the total cross section), i.e. get parameters for energy scaling of $p_{\perp 0}$
- Cut $p_{\perp c}$ deciding when to convert to hadrons.
- Parameters for matter profile $\mathcal{A}(b)$.

MPI models in general-purpose event generators:

HERWIG: Pick interactions prior to running according to Poissonian,

SHERPA: MPI after hard process evolution in a p_{\perp} -ordered sequence.

PYTHIA: MPI + ISR + FSR combined in one single p_{\perp} sequence.

Summary

- Prediction incomplete before assessing/including non-pert. effects.
- $2 \text{ partons} \rightarrow 2 \text{ partons}$ cross section naively exceeds total cross section.
- Factorisation hints this may be due to additional scatterings.
- Multiparton interaction models attempt to describe hadron-collider data by "resolving" these additional scatterings.
- Models fulfill some consistency conditions, e.g. should yield the inclusive cross section when additional scattering are integrated out.
- Models come with a handful of parameters.

References

Almost the whole talk is based on Sjöstrand, van Zijl (PRD 36 (1987) 2019).

The current PYTHIA MPI model is interleaved (Skands, Sjöstrand JHEP 0403 (2004) 053) and contains rescattering (Corke, Sjöstrand JHEP 1001 (2010) 035).

The HERWIG++ model is documented in Bähr, Gieseke, Seymour (JHEP 0807 (2008) 076) and the SHERPA model is in hep-ph/0601012

.
Another fun approach (combining MPI and minimum bias) is DIPSY (JHEP 0712 (2007) 012)