

# Effective Field Theory of Forward Scattering and Factorization Violation

(Part II)

In collaboration with Iain Stewart

Buffalo Factorization Violation Workshop 11/15

# Outline

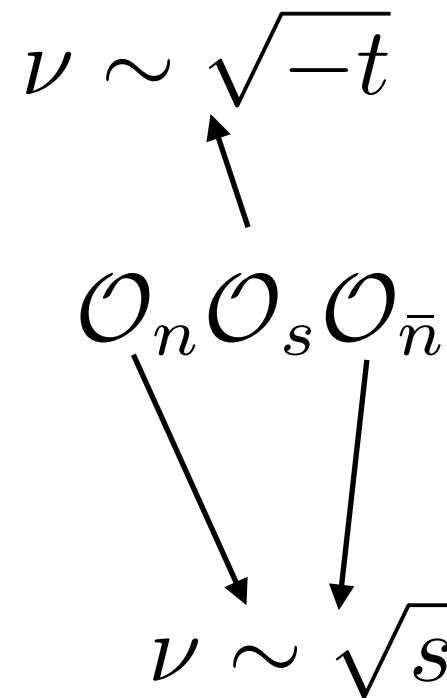
## Part II

- Reggeization from Rapidity renormalization
- Forward Scattering and BFKL
  - Forward Scattering Collinear and Soft Functions
  - BFKL equation from RGE
- Glauber Exponentiation, eikonal phase
- Glauber's in Hard Scattering
  - The Cheshire Glauber
  - One and Two Loop results
- Glauber Effects with Spectators
- Summary

# Reggeization

Follows from solving the RRG equation

Each sector has a natural rapidity scale



We have a choice to either run collinear ops from  $\nu \sim \sqrt{s}$  to  $\nu \sim \sqrt{-t}$  or run the soft function.

As opposed to RG anom. dim. we are integrating out rapidity shells, depends upon IR regulator. this dependence will drop out when we calculate something physics (e.g. BFKL)

There exists strong constraints on the rapidity anomalous dimensions due to the fact that there is **no hard matching coefficient**.

Consider running of single index operators

$$\nu \frac{d}{d\nu} \sum_{ij=q,g} \mathcal{O}_{ns}^{ij} = \nu \frac{d}{d\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) \frac{1}{\mathcal{P}_\perp^2} (\mathcal{O}_s^{qnA} + \mathcal{O}_s^{gnA}) = 0$$

$$\nu \frac{d}{d\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}), \quad \nu \frac{d}{d\nu} (\mathcal{O}_s^{qnA} + \mathcal{O}_s^{gnA}) = \gamma_{s\nu} (\mathcal{O}_s^{qnA} + \mathcal{O}_s^{gnA}).$$

mixing leads to universality

$$\gamma_{s_n\nu} \equiv \gamma_{s_n\nu}^{qq} + \gamma_{s_n\nu}^{gq} = \gamma_{s_n\nu}^{gg} + \gamma_{s_n\nu}^{qg}.$$

$$\gamma_{n\nu} \equiv \gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = \gamma_{n\nu}^{gg} + \gamma_{n\nu}^{qg},$$

$$\gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = -\gamma_{s_n\nu}^{qq} - \gamma_{s_n\nu}^{gq}, \quad \text{or } \gamma_{n\nu} = -\gamma_{s_n\nu}.$$



## Collinear mixing:

$$\vec{\mathcal{O}}_{\bar{n}}^{B\text{bare}} = \hat{V}_{\mathcal{O}_{\bar{n}}} \cdot \vec{\mathcal{O}}_{\bar{n}}^B(\nu, \mu), \quad \hat{V}_{\mathcal{O}_{\bar{n}}} = \begin{pmatrix} 1 + \delta V_{\bar{n}}^{qq} & \delta V_{\bar{n}}^{qg} \\ \delta V_{\bar{n}}^{gq} & 1 + \delta V_{\bar{n}}^{gg} \end{pmatrix}, \quad \vec{\mathcal{O}}_{\bar{n}}^B = \begin{pmatrix} \mathcal{O}_{\bar{n}}^{qB} \\ \mathcal{O}_{\bar{n}}^{gB} \end{pmatrix}.$$

$$\mathcal{O}_{\bar{n}}^{qB} = \bar{\chi}_{n,\omega} T^B \frac{\not{n}}{2} \chi_{n,\omega}, \quad \mathcal{O}_{\bar{n}}^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu,-\omega}^C \bar{n} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{n\perp,\omega}^{D\mu}.$$

$$\hat{\gamma}_{\mathcal{O}_n}^\nu = -\hat{V}_{\mathcal{O}_n}^{-1} \cdot \nu \frac{\partial}{\partial \nu} \hat{V}_{\mathcal{O}_n} = \begin{pmatrix} \gamma_{n\nu}^{qq} & \gamma_{n\nu}^{qg} \\ \gamma_{n\nu}^{gq} & \gamma_{n\nu}^{gg} \end{pmatrix}$$

## Soft mixing single index:

$$\vec{\mathcal{O}}_{s_n}^{A\text{bare}} = \hat{V}_{\mathcal{O}_{s_n}} \cdot \vec{\mathcal{O}}_{s_n}^A(\nu, \mu), \quad \hat{V}_{\mathcal{O}_{s_n}} = \begin{pmatrix} 1 + \delta V_{s_n}^{qq} & \delta V_{s_n}^{qg} \\ \delta V_{s_n}^{gq} & 1 + \delta V_{s_n}^{gg} \end{pmatrix}, \quad \vec{\mathcal{O}}_{s_n}^A = \begin{pmatrix} \mathcal{O}_s^{q_n A} \\ \mathcal{O}_s^{g_n A} \end{pmatrix},$$

$$\mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left( \bar{\psi}_S^n T^B \frac{\not{n}}{2} \psi_S^n \right),$$

$$\mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left( \frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} n \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right).$$

Soft mixing double index index:

$$\mathcal{O}_s^{AB} = 8\pi\alpha_s \sum_i C_i \mathcal{O}_i^{AB}$$

$$\vec{\mathcal{O}}_s^{AB\text{bare}} = \hat{V}_{\mathcal{O}_s} \cdot \vec{\mathcal{O}}_s^{AB}(\nu, \mu), \quad ($$

$$\hat{V}_{\mathcal{O}_s} = \begin{pmatrix} 1 + \delta V_s & 0 & 0 & 0 & 0 \\ \delta V_s^{Tqq} & & & & \\ \delta V_s^{Tgq} & & & & \\ \delta V_s^{Tgg} & \hat{V}_{\mathcal{O}_{s_n}} \otimes \hat{V}_{\mathcal{O}_{s_{\bar{n}}}} & & & \\ \delta V_s^{Tqg} & & & & \end{pmatrix},$$

$$\vec{\mathcal{O}}_s^{AB} = \begin{pmatrix} \mathcal{O}_s^{AB} \\ i \int d^4x T \mathcal{O}_s^{q_n A}(x) \mathcal{O}_s^{q_{\bar{n}} B}(0) \\ i \int d^4x T \mathcal{O}_s^{g_n A}(x) \mathcal{O}_s^{q_{\bar{n}} B}(0) \\ i \int d^4x T \mathcal{O}_s^{g_n A}(x) \mathcal{O}_s^{g_{\bar{n}} B}(0) \\ i \int d^4x T \mathcal{O}_s^{g_n A}(x) \mathcal{O}_s^{q_{\bar{n}} B}(0) \end{pmatrix}.$$

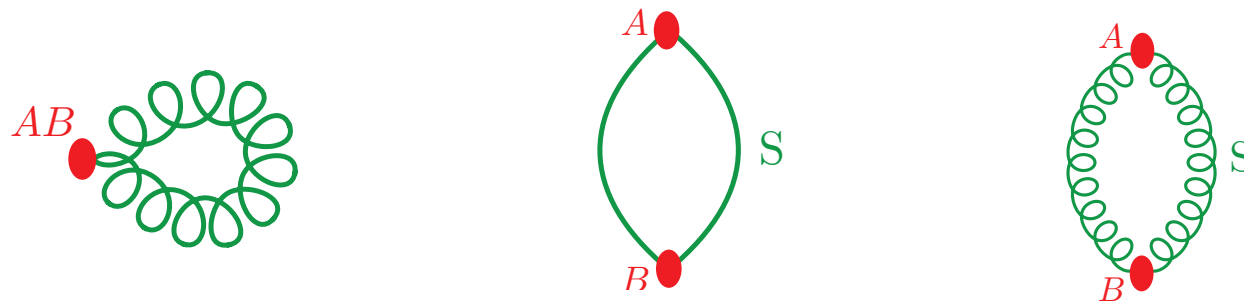
$$\hat{\gamma}_{\mathcal{O}_s}^\nu = -\hat{V}_{\mathcal{O}_s}^{-1} \cdot \nu \frac{\partial}{\partial \nu} \hat{V}_{\mathcal{O}_s}$$

$$\hat{\gamma}_{\mathcal{O}_s}^\nu = \begin{pmatrix} \gamma_{s\nu}^{\text{dir}} & 0 & 0 & 0 & 0 \\ \gamma_{s\nu}^{Tqq} & & & & \\ \gamma_{s\nu}^{Tgq} & & & & \\ \gamma_{s\nu}^{Tgg} & \hat{\gamma}_{\mathcal{O}_{s_n}}^\nu \otimes \hat{\gamma}_{\mathcal{O}_{s_{\bar{n}}}}^\nu & & & \\ \gamma_{s\nu}^{Tqg} & & & & \end{pmatrix}$$

# At the Level of Single Glauber (1-loop)

$$\nu \frac{d}{d\nu} \sum_{ij=q,g} \left( O_{ns\bar{n}}^{ij}(0) + \sum_{kk'=q,g} i T \int d^4x O_{ns}^{ik}(x) O_{\bar{n}s}^{jk'}(0) \right) = 0.$$

only contributions at one loop



$$\gamma_{s\nu} \equiv \gamma_{s\nu}^{\text{dir}} + \gamma_{s\nu}^{Tqq} + \gamma_{s\nu}^{Tgg} = -\gamma_{n\nu} - \gamma_{\bar{n}\nu}.$$

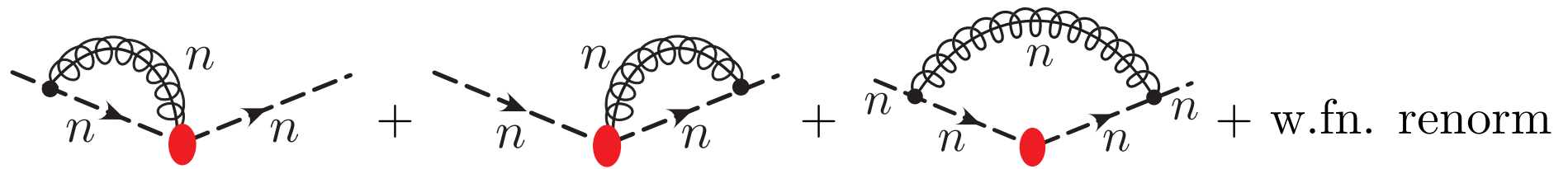
$$\gamma_{s\nu} = -\frac{\alpha_s(\mu)C_A}{\pi} \ln\left(\frac{-t}{m^2}\right).$$

$$\gamma_{s\nu}^{\text{dir}} = \frac{\alpha_s C_A}{4\pi} \left[ -4h(\epsilon, \mu^2/m^2) + \frac{4}{\epsilon} \right] = -\frac{\alpha_s(\mu)C_A}{\pi} \ln\left(\frac{\mu^2}{m^2}\right),$$

$$\gamma_{s\nu}^{Tqq} = 0,$$

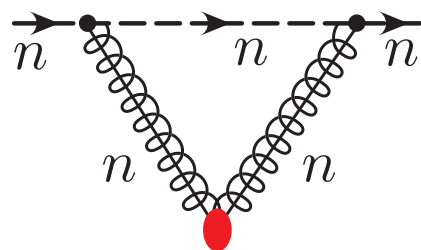
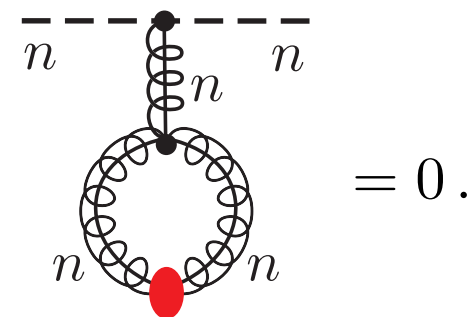
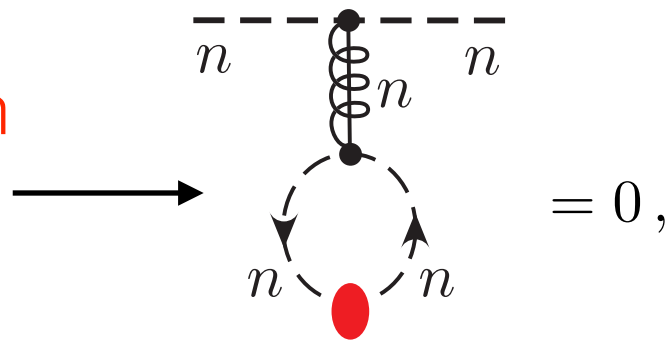
$$\gamma_{s\nu}^{Tgg} = \frac{\alpha_s C_A}{4\pi} \left[ -4g(\epsilon, \mu^2/(-t)) - \frac{4}{\epsilon} \right] = -\frac{\alpha_s(\mu)C_A}{\pi} \ln\left(\frac{-t}{\mu^2}\right).$$

# Simpler to run collinear functions as there are no TOP's involved



$$= \mathcal{S}^{nq} \frac{\alpha_s C_A}{4\pi} \left[ w^2 \frac{2h\left(\epsilon, \frac{\mu^2}{m^2}\right)}{\eta} + w^2 \frac{2}{\epsilon} \ln\left(\frac{\nu}{\bar{n} \cdot p}\right) + \frac{3}{2\epsilon} \right] = \mathcal{S}^{nq} \delta V_n^{qq},$$

after zero-bin subtraction



$$= \mathcal{S}^{nq} \frac{\alpha_s C_A}{4\pi} \left[ w^2 \frac{2g\left(\epsilon, \frac{\mu^2}{-t}\right)}{\eta} - w^2 \frac{2}{\epsilon} \ln\left(\frac{\nu}{\bar{n} \cdot p}\right) - \frac{3}{2\epsilon} \right] = \mathcal{S}^{nq} \delta V_n^{gq},$$

$$\gamma_{n\nu}^{qq} = -\frac{\alpha_s C_A}{4\pi} \left[ -2h(\epsilon, \mu^2/m^2) + \frac{2}{\epsilon} \right] = \frac{\alpha_s(\mu) C_A}{2\pi} \ln \left( \frac{\mu^2}{m^2} \right),$$

$$\gamma_{n\nu}^{gq} = -\frac{\alpha_s C_A}{4\pi} \left[ -2g(\epsilon, \mu^2/(-t)) - \frac{2}{\epsilon} \right] = \frac{\alpha_s(\mu) C_A}{2\pi} \ln \left( \frac{-t}{\mu^2} \right),$$

$$\gamma_{n\nu}^{gg} = -\frac{\alpha_s C_A}{4\pi} \left[ -2g(\epsilon, \mu^2/(-t)) - 2h(\epsilon, \mu^2/m^2) \right] = \frac{\alpha_s(\mu) C_A}{2\pi} \ln \left( \frac{-t}{m^2} \right),$$

$$\gamma_{n\nu}^{qg} = 0.$$

$$\gamma_{n\mu}^{qq} = \frac{\alpha_s(\mu) C_A}{2\pi} \left[ 2 \ln \left( \frac{\nu}{\bar{n} \cdot p} \right) + \frac{3}{2} \right],$$

$$\gamma_{n\mu}^{gq} = -\frac{\alpha_s(\mu) C_A}{2\pi} \left[ 2 \ln \left( \frac{\nu}{\bar{n} \cdot p} \right) + \frac{3}{2} \right],$$

$$\gamma_{n\mu}^{gg} = -\frac{2\alpha_s(\mu) n_F T_F}{3\pi},$$

$$\gamma_{n\mu}^{qg} = \frac{2\alpha_s(\mu) n_F T_F}{3\pi}.$$

Obey Consistency Conditions

$$\gamma_{n\mu}^{qq} + \gamma_{n\mu}^{gq} = \gamma_{n\mu}^{gg} + \gamma_{n\mu}^{qg} = 0.$$

$$\gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = \gamma_{n\nu}^{gg} + \gamma_{n\nu}^{qg}$$

To sum logs all  
we need is

$$\gamma_{n\nu} = \frac{\alpha_s C_A}{2\pi} \ln \left( \frac{-t}{m^2} \right).$$

$$\frac{d}{d \log \nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})$$

$$(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{-t}) = \left( \frac{s}{-t} \right)^{-\gamma_{n\nu}/2} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{s}).$$

$$\begin{aligned} & (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{-t}) \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{AB}(\nu = \sqrt{-t}) \frac{1}{\mathcal{P}_\perp^2} (\mathcal{O}_{\bar{n}}^{qB} + \mathcal{O}_{\bar{n}}^{gB})(\nu = \sqrt{-t}) \\ &= \left( \frac{s}{-t} \right)^{-\gamma_{n\nu}} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{s}) \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{AB}(\nu = \sqrt{-t}) \frac{1}{\mathcal{P}_\perp^2} (\mathcal{O}_{\bar{n}}^{qB} + \mathcal{O}_{\bar{n}}^{gB})(\nu = \sqrt{s}), \end{aligned}$$

# Summing Logs in IR safe quantities

Consider observables for which the soft radiation is not measured. For this class the RRG equation will reduce to the BKFL equation

Consider the total cross section for hadron scattering try to capture s dependence

$$\sigma_{pp} = \int \frac{d\sigma}{dt} dt \sim \int dt \sum_{X_s, X_n, X_{\bar{n}}} \langle p_n \bar{p}_n | X_s, X_n, X_{\bar{n}} \rangle O(t(\mathcal{P}_\perp^n, \mathcal{P}_\perp^{\bar{n}})) \langle X_s, X_n, X_{\bar{n}} | p_n \bar{p}_n \rangle,$$

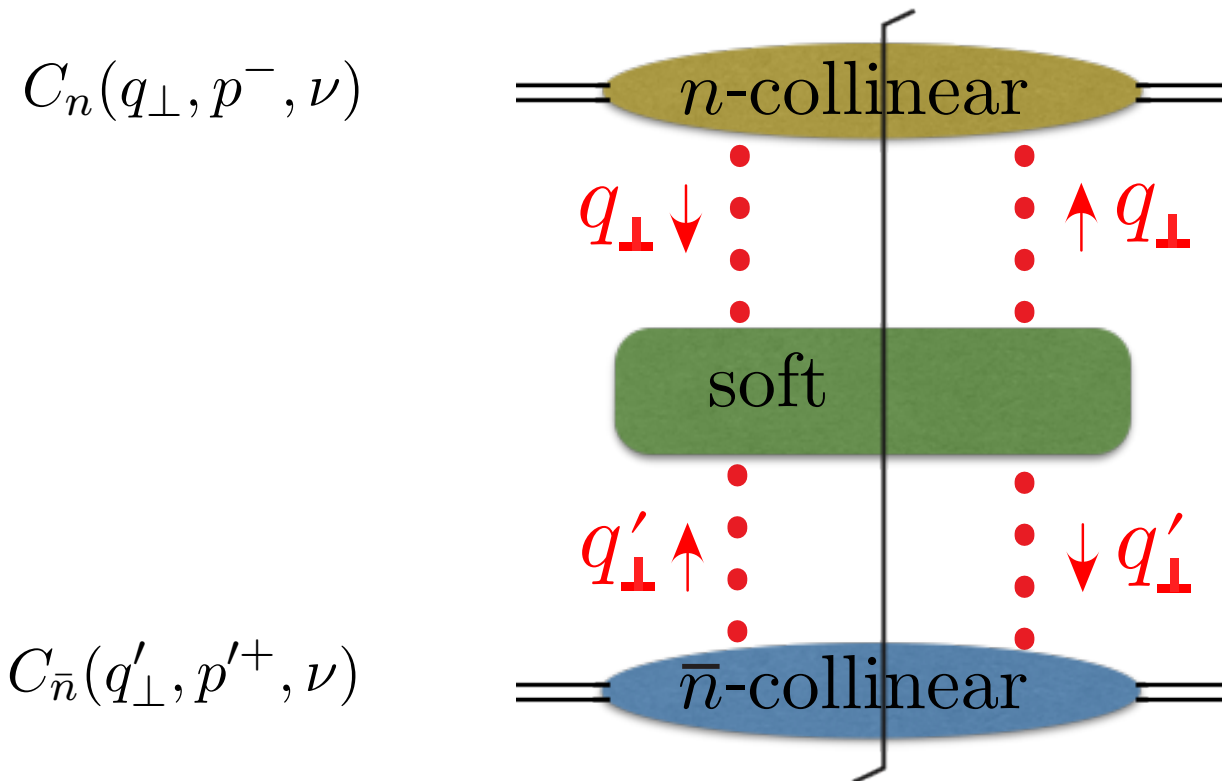
insert

$$O_{s(1,1)}^{AB}(q_\perp, q'_\perp) = O_s^{AB}(q_\perp, q'_\perp) + \int d^4x T [\mathcal{O}_s^{q_n A}(q_\perp) + \mathcal{O}_s^{g_n A}(q_\perp)] [\mathcal{O}_s^{q_{\bar{n}} A}(q'_\perp) + \mathcal{O}_s^{g_{\bar{n}} A}(q'_\perp)].$$

$$\begin{aligned} S_G(q_\perp, q'_\perp) &= \int d^2q''_\perp d^2q'''_\perp \delta^{AA'} \delta^{BB'} \delta^2(q_\perp - q''_\perp) \delta^2(q'_\perp - q'''_\perp) S_G^{AA'BB'}(q_\perp, q'_\perp, q''_\perp, q'''_\perp) \\ &= \frac{1}{V_2} \frac{\delta^{AA'} \delta^{BB'}}{(\vec{q}_\perp^2 \vec{q}'_\perp{}^2)^2} \sum_X \langle 0 | O_{s(1,1)}^{AB}(q_\perp, q'_\perp) | X \rangle \langle X | O_{s(1,1)}^{\dagger A'B'}(q_\perp, q'_\perp) | 0 \rangle. \end{aligned}$$

$$T_{(1,1)} = \int d^2q_\perp d^2q'_\perp C_n(q_\perp, p^-) S_G(q_\perp, q'_\perp) C_{\bar{n}}(q'_\perp, p'^+),$$

Collinear Matrix Elements



Lowest order

$$\langle 0 | O_{s(1,1)}^{AB}(q_\perp, q'_\perp) | 0 \rangle = -i 8\pi\alpha_s(\mu) \delta^{AB} \vec{q}_\perp^2 (2\pi)^2 \delta^2(\vec{q}_\perp + \vec{q}'_\perp).$$

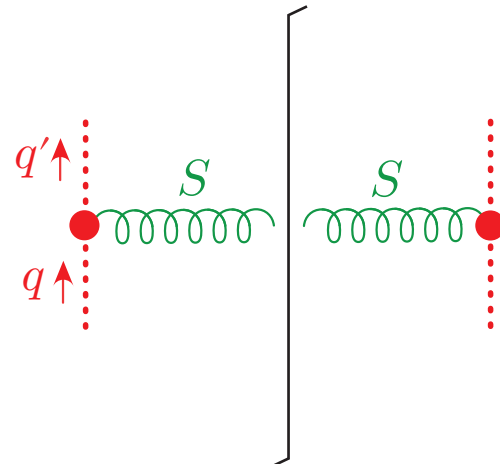


$$= S_G^{(0)}(q_\perp, q'_\perp) \equiv \frac{1}{V} \frac{1}{(\vec{q}_\perp^2 \vec{q}'_\perp^2)^2} \langle 0 | O_{s(1,1)}^{AB} | 0 \rangle \langle 0 | O_{s(1,1)}^{AB\dagger} | 0 \rangle$$

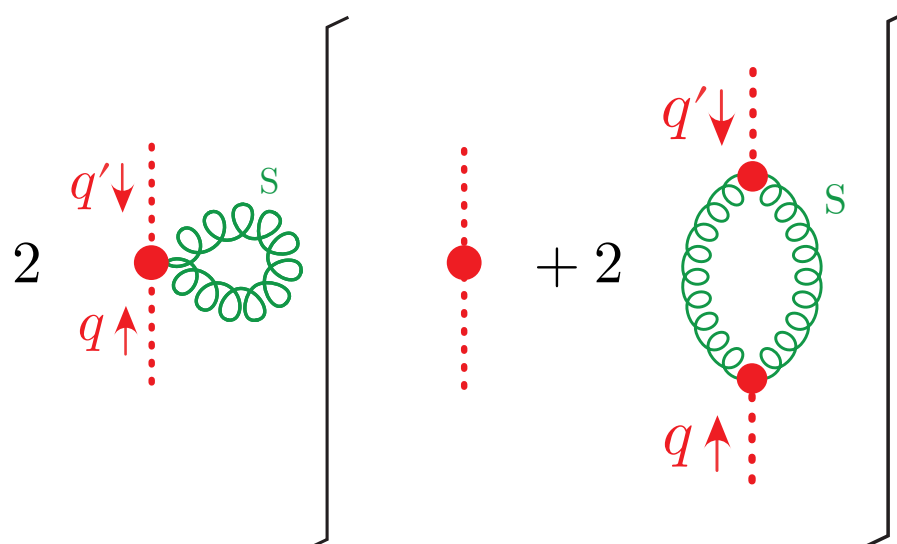
$$= \left( \frac{8\pi\alpha_s}{\vec{q}_\perp^2} \right)^2 \delta^{AA} (2\pi)^2 \delta^2(\vec{q}_\perp + \vec{q}'_\perp).$$

This form will allow to renormalize through a convolution





$$\begin{aligned}
&= (8\pi\alpha_s)^2 4\alpha_s C_A \delta^{AA} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2 \vec{q}_\perp^2 \vec{k}_\perp^2} (2\pi)^2 \delta^2(\vec{k}_\perp - \vec{q}_\perp - \vec{q}'_\perp) \\
&= (8\pi\alpha_s)^2 4\alpha_s C_A \delta^{AA} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp}{\vec{k}_\perp^2 \vec{q}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} (2\pi)^2 \delta^2(\vec{k}_\perp + \vec{q}'_\perp) \\
&= \frac{C_A \alpha_s}{\pi^2} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp \vec{k}_\perp^2}{(\vec{k}_\perp - \vec{q}_\perp)^2 \vec{q}_\perp^2} S_G^{(0)}(k_\perp, q'_\perp),
\end{aligned}$$



$$\begin{aligned}
&= -\frac{2(8\pi\alpha_s)^2 \alpha_s}{(\vec{q}_\perp^2)^3} C_A \delta^{AB} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp (\vec{q}_\perp^2)^2}{\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} (2\pi)^2 \delta(\vec{q}_\perp + \vec{q}'_\perp) \\
&= -\frac{C_A \alpha_s}{2\pi^2} w^2 \Gamma\left(\frac{\eta}{2}\right) \int d^2 k_\perp \frac{\vec{q}_\perp^2}{\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} S_G^{(0)}(q_\perp, q'_\perp),
\end{aligned}$$

$$\tilde{S}_G^{\text{bare}}(q_\perp, q'_\perp) \equiv \tilde{S}_G^{(0)}(q_\perp, q'_\perp) + \frac{\alpha_s C_A}{\pi^2} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2} \left[ \tilde{S}_G^{(0)}(k_\perp, q'_\perp) - \frac{\vec{q}_\perp^2}{2\vec{k}_\perp^2} \tilde{S}_G^{(0)}(q_\perp, q'_\perp) \right].$$

$$\tilde{S}_G(\vec{q}_\perp, \vec{q}'_\perp, \nu) = \int d^2 k_\perp Z(q_\perp, k_\perp) \tilde{S}_G^{\text{bare}}(k_\perp, q'_\perp).$$

$$Z(q_\perp, k_\perp) = \delta^2(\vec{q}_\perp - \vec{k}_\perp) - \frac{2C_A \alpha_s(\mu) w^2(\nu)}{\pi^2 \eta} \left[ \frac{1}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \delta^2(\vec{q}_\perp - \vec{k}_\perp) \int \frac{d^2 k_\perp \vec{q}_\perp^2}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right].$$

RRG

$$0 = \nu \frac{d}{d\nu} \tilde{S}_G^{\text{bare}}(q_\perp, q'_\perp) = \nu \frac{d}{d\nu} \int d^2 k_\perp Z^{-1}(q_\perp, k_\perp) \tilde{S}_G(k_\perp, q'_\perp, \nu).$$

extract anom.  
dim

$$\gamma_{S_G}(q_\perp, q'_\perp) = \frac{2C_A \alpha_s(\mu)}{\pi^2} \left[ \frac{1}{(\vec{q}_\perp - \vec{q}'_\perp)^2} - \delta^2(\vec{q}_\perp - \vec{q}'_\perp) \int d^2 k_\perp \frac{\vec{q}_\perp^2}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right].$$

BFKL

$$\nu \frac{d}{d\nu} \tilde{S}_G(q_\perp, q'_\perp, \nu) = \frac{2C_A \alpha_s(\mu)}{\pi^2} \int d^2 k_\perp \left[ \frac{\tilde{S}_G(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^2 \tilde{S}_G(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right].$$

We can also get the collinear BFKL equation by imposing nu independence on the rate

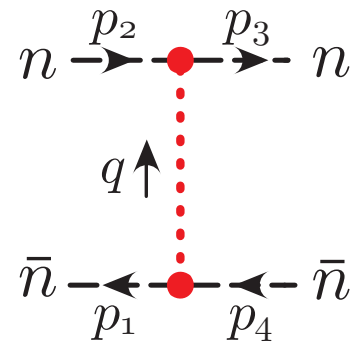
$$\gamma_C(q_\perp, q'_\perp) = -\frac{1}{2}\gamma_{SG}(q_\perp, q'_\perp).$$

$$\nu \frac{d}{d\nu} C_n(q_\perp, p^-, \nu) = -\frac{C_A \alpha_s}{\pi^2} \int d^2 k_\perp \left[ \frac{C_n(k_\perp, p^-, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^2 C_n(q_\perp, p^-, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right],$$

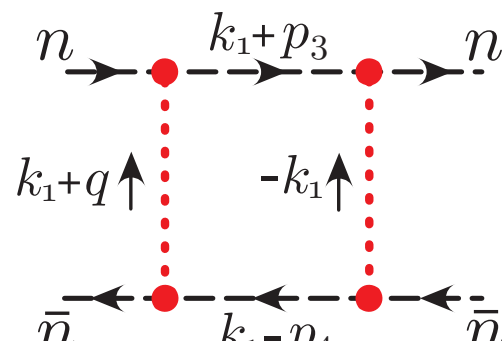
# Glauber Exponentiation and the Eikonal Phase

Expect iterations of Glauber exchanges to exponentiate

$$\begin{aligned}\phi(b_\perp) &= -\mathbf{T}_1^A \otimes \mathbf{T}_2^A g^2(\mu) \int \frac{d^{d-2}q_\perp (\iota^\epsilon \mu^{2\epsilon})}{\vec{q}_\perp^2} e^{i\vec{q}_\perp \cdot \vec{b}_\perp} \\ &= -\mathbf{T}_1^A \otimes \mathbf{T}_2^A g^2(\mu) \frac{\Gamma(-\epsilon)}{4\pi} \left( \frac{\mu |\vec{b}_\perp| e^{\gamma_E}}{2} \right)^{2\epsilon}.\end{aligned}$$



$$= \frac{-2ig^2}{\vec{q}_\perp^2} \left[ \bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[ \bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^A v_{\bar{n}} \right] \xrightarrow{\text{F.T.}_\perp} 2 \mathcal{S}^{n\bar{n}} i \phi(b_\perp),$$

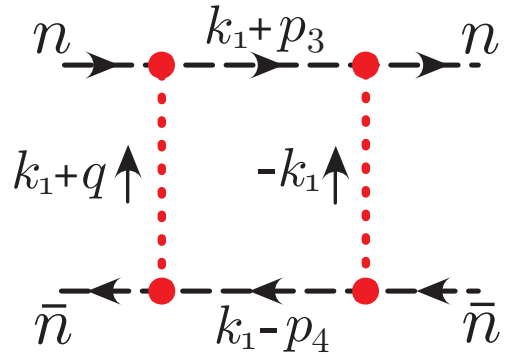


$$= -4ig^4 (T^A T^B \otimes \bar{T}^A \bar{T}^B) \mathcal{S}^{n\bar{n}} \int \frac{d^{d-2}k_\perp dk^z (|2k^z|^{-\eta} |2k^z|^{-\eta} \nu^{2\eta} \iota^{2\epsilon} \mu^{4\epsilon})}{\vec{k}_\perp^2 (\vec{k}_\perp + \vec{q}_\perp)^2 2(-k^z + \Delta + i0)}$$

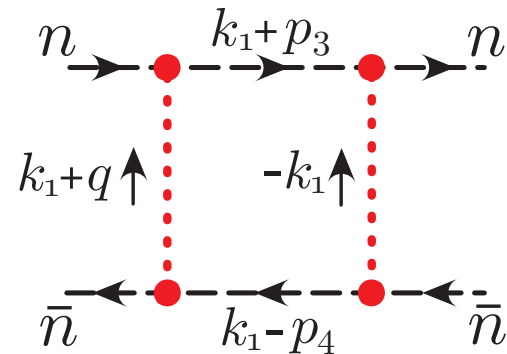
$$2\Delta = p_3^+ + p_4^- - (\vec{k}_\perp + \vec{p}_{3\perp})^2 / p_3^- - (\vec{k}_\perp - \vec{p}_{4\perp})^2 / p_4^+,$$

x space facilitates  
resummation

$$\int_{-\infty}^{+\infty} \bar{d}k^z e^{ixk^z} |2k^z|^{-\eta} = \kappa_\eta \frac{\eta}{2} |x|^{-1+\eta} ,$$



$$\begin{aligned}
 &= -4ig^4 (T^A T^B \otimes \bar{T}^A \bar{T}^B) \mathcal{S}^{n\bar{n}} \int \frac{\bar{d}^{d-2}k_\perp \bar{d}k^z (|2k^z|^{-\eta} |2k^z|^{-\eta} \nu^{2\eta} \iota^{2\epsilon} \mu^{4\epsilon})}{\vec{k}_\perp^2 (\vec{k}_\perp + \vec{q}_\perp)^2 2(-k^z + \Delta + i0)} \\
 &= -2g^4 \mathcal{S}_{(2)}^{n\bar{n}} I_\perp^{(1)}(q_\perp) \left( \kappa_\eta \frac{\eta}{2} \right)^2 \int_{-\infty}^{\infty} \bar{d}k_z dx dy d\alpha \theta(\alpha) |xy|^{-1+\eta} e^{i\alpha(k_z + \Delta) + ik_z(x-y)} \\
 &= -2g^4 \mathcal{S}_{(2)}^{n\bar{n}} I_\perp^{(1)}(q_\perp) \left( \kappa_\eta \frac{\eta}{2} \right)^2 \int_{-\infty}^{\infty} dx dy \theta(y-x) |xy|^{-1+\eta} e^{i\Delta(y-x)} \\
 &= 2\mathcal{S}_{(2)}^{n\bar{n}} i^2 g^4 I_\perp^{(1)}(q_\perp) \frac{1}{2!} [1 + \mathcal{O}(\eta)] , \quad I_\perp^{(1)}(q_\perp) = \int \frac{\bar{d}^{d-2}k_\perp (\iota^\epsilon \mu^{2\epsilon})^2}{\vec{k}_\perp^2 (\vec{k}_\perp + \vec{q}_\perp)^2} . \quad (244)
 \end{aligned}$$



$$\xrightarrow[\text{F.T.}_\perp]{} 2\mathcal{S}^{n\bar{n}} \frac{1}{2!} [i\phi(b_\perp)]^2 .$$

$$= -8ig^6 \mathcal{S}_{(3)}^{n\bar{n}} I^{(2)}(q_\perp) \int \frac{d^d k_1^z d^d k_2^z |2k_1^z|^{-\eta} |2k_1^z - 2k_2^z|^{-\eta} |2k_2^z|^{-\eta} \nu^{3\eta}}{4(-k_1^z + \Delta_1 + i0)(-k_2^z + \Delta_2 + i0)}$$

$$= 2ig^6 \mathcal{S}_{(3)}^{n\bar{n}} I_{\perp}^{(2)}(q_\perp) \int_{-\infty}^{\infty} d^d k_1^z d^d k_2^z dx dy dz d\alpha_1 d\alpha_2 \theta(\alpha_1)\theta(\alpha_2) \left(\kappa_\eta \frac{\eta}{2}\right)^3 |xyz|^{-1+\eta}$$

$$= -2\mathcal{S}_{(3)}^{n\bar{n}} i^3 g^6 I_{\perp}^{(2)}(q_\perp) \frac{1}{3!} [1 + \mathcal{O}(\eta)], \quad I_{\perp}^{(2)}(q_\perp) = \int \frac{d^{d-2} k_{1\perp} d^{d-2} k_{2\perp} (\iota^\epsilon \mu^{2\epsilon})^3}{(\vec{k}_{1\perp} + \vec{q}_\perp)^2 (\vec{k}_{2\perp} - \vec{k}_{1\perp})^2 \vec{k}_{2\perp}^2}.$$

$$\xrightarrow{\text{F.T.}_\perp} 2\mathcal{S}^{n\bar{n}} \frac{1}{3!} [i\phi(b_\perp)]^3,$$

$$\int d^{d-2} q_\perp e^{i\vec{q}_\perp \cdot \vec{b}_\perp} \sum_{N=0}^{\infty} \text{G.Box}_N^{2 \rightarrow 2}(q_\perp) = 2\mathcal{S}^{n\bar{n}} (\tilde{G}(b_\perp) - 1) \quad \tilde{G}(b_\perp) = e^{i\phi(b_\perp)},$$

$$\phi(b_\perp) = C_F g^2(\mu) \int \frac{d^2 q_\perp}{\vec{q}_\perp^2 + m^2} e^{i\vec{q}_\perp \cdot \vec{b}_\perp} = -2C_F \alpha_s(\mu) \ln \left( \frac{|\vec{b}_\perp| m e^{\gamma_E}}{2} \right).$$

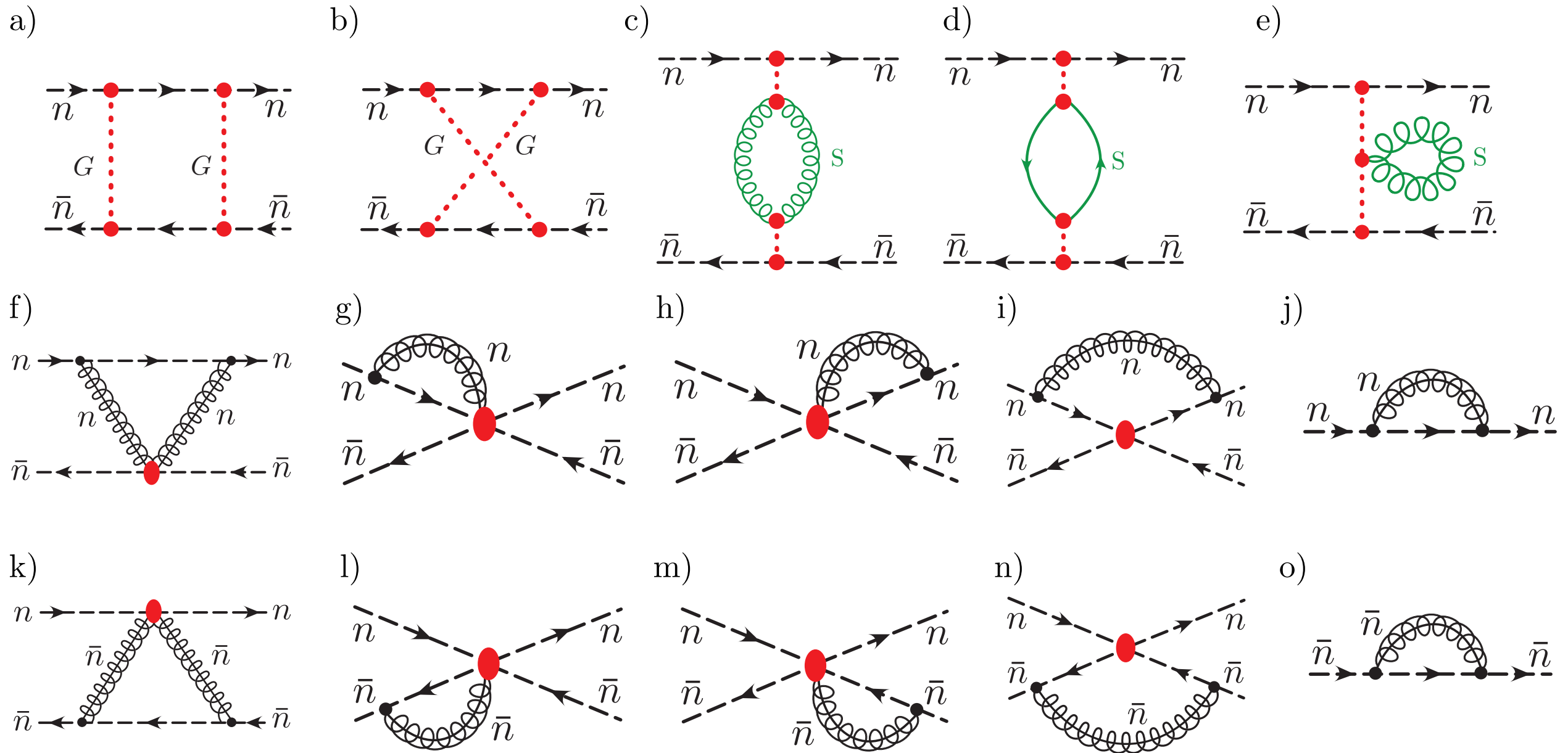
$$\begin{aligned}
G(q_\perp) &= (2\pi)^2 \delta^2(q_\perp) + \int d^2 b_\perp e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} (e^{i\phi(b_\perp)} - 1) \\
&= (2\pi)^2 \delta^2(q_\perp) + \frac{i4\pi C_F \alpha_s(\mu)}{(-t)} \frac{\Gamma(1 - iC_F \alpha_s(\mu))}{\Gamma(1 + iC_F \alpha_s(\mu))} \left( \frac{-t}{m^2 e^{2\gamma_E}} \right)^{iC_F \alpha_s(\mu)} \\
&= (2\pi)^2 \delta^2(q_\perp) + \frac{i4\pi C_F \alpha_s(\mu)}{(-t)} e^{i\delta(t, \alpha_s)},
\end{aligned}$$

$$\delta(t, \alpha_s) = C_F \alpha_s(\mu) \ln \left( \frac{-t}{m^2} \right) + 2 \sum_{k=1}^{\infty} \frac{(-1)^k \zeta_{2k+1}}{2k+1} (C_F \alpha_s(\mu))^{2k+1}.$$

This result holds even if we place insert in greens functions, as long as additional loop does not need rapidity regularization, will use this property later on.

Also note that source lines eikonalize, however this is not a general property of Glaubers (e.g. include interpolating fields for hadrons).

# What about NA-Corrections?



Soft graphs dress Glauber kernel lead to running of coupling  
 Therefore at least part of collinear must also exponentiate, but could be a remainder.

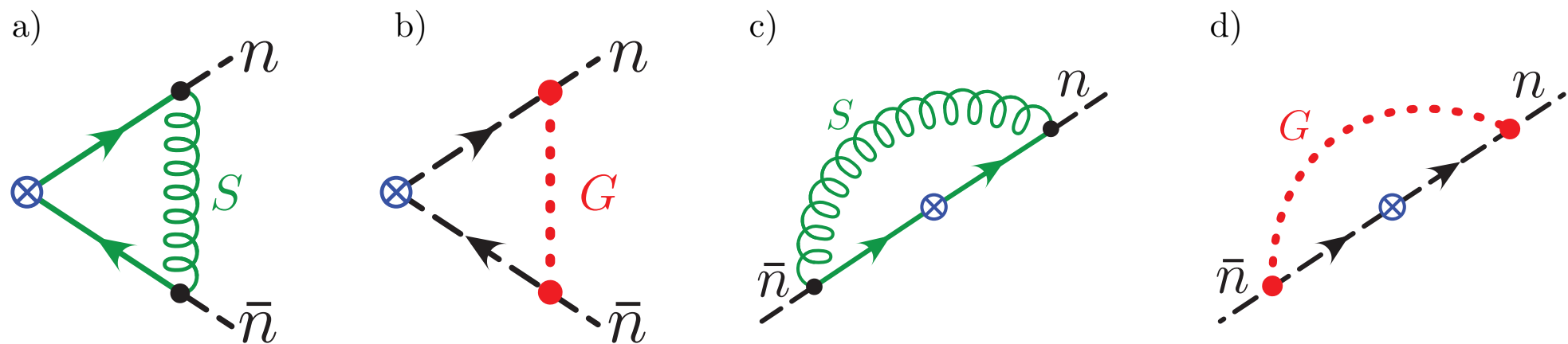


# Role of Glaubers in Hard Matching

Empirically Glaubers are not needed in hard matching (active only), but from the point of view of EFT (formally), they should be included as they are part of the theory.

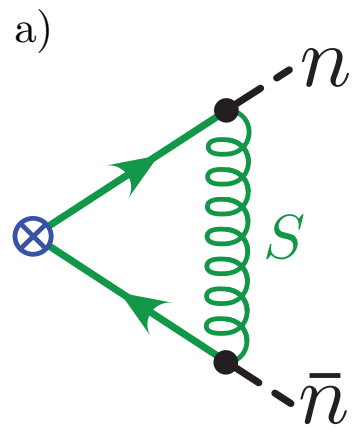
$$J_{\Gamma} = (\bar{\xi}_n W_n) S_n^{\dagger} \Gamma S_{\bar{n}} (W_{\bar{n}}^{\dagger} \xi_{\bar{n}})$$

113



BUT: If we include Glaubers we must make sure to subtract the Zero Bin from soft contribution

Once we include Glaubers, the direction of the soft Wilson line (in or out to infinity) becomes irrelevant. Glaubers measure whether FF is space like or timelike



$$\tilde{S} = \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \left[ \frac{-2h(\epsilon, \mu^2/m^2)}{\eta} + \ln \frac{\mu^2}{-\nu^2 - i0} \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right) + \frac{1}{\epsilon^2} - \frac{1}{2} \ln^2 \frac{\mu^2}{m^2} - \frac{\pi^2}{12} \right]$$

G-zero bin

$$\mathcal{S}^{(G)} = \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \left[ (i\pi) \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right) \right].$$

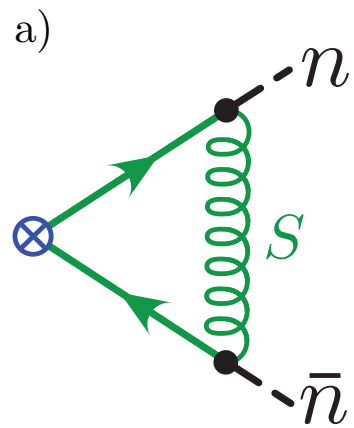
Wilson lines direction is inherited from  $\tilde{S}$

$$G = \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \left[ (i\pi) \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right) \right].$$

Glauber vanishes for space-like, direction of soft Wilson line flips

$$\tilde{S} - \mathcal{S}^{(G)} = Real$$

Alternatively, we can absorb this Glauber contribution into the soft Wilson line IF we choose the direction properly



$$= \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \left[ \frac{-2h(\epsilon, \mu^2/m^2)}{\eta} + \ln \frac{\mu^2}{-\nu^2 - i0} \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right) + \frac{1}{\epsilon^2} - \frac{1}{2} \ln^2 \frac{\mu^2}{m^2} - \frac{\pi^2}{12} \right]$$

G-zero bin

$$\mathcal{S}^{(G)} = \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \left[ (i\pi) \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right) \right].$$

$$G - \mathcal{S}^{(G)} = 0$$

$$G = \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \left[ (i\pi) \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right) \right].$$

Glauber vanishes for space-like,  
direction of soft Wilson line flips

# Persistence of Glauber Soft Zero Bin Correspondence

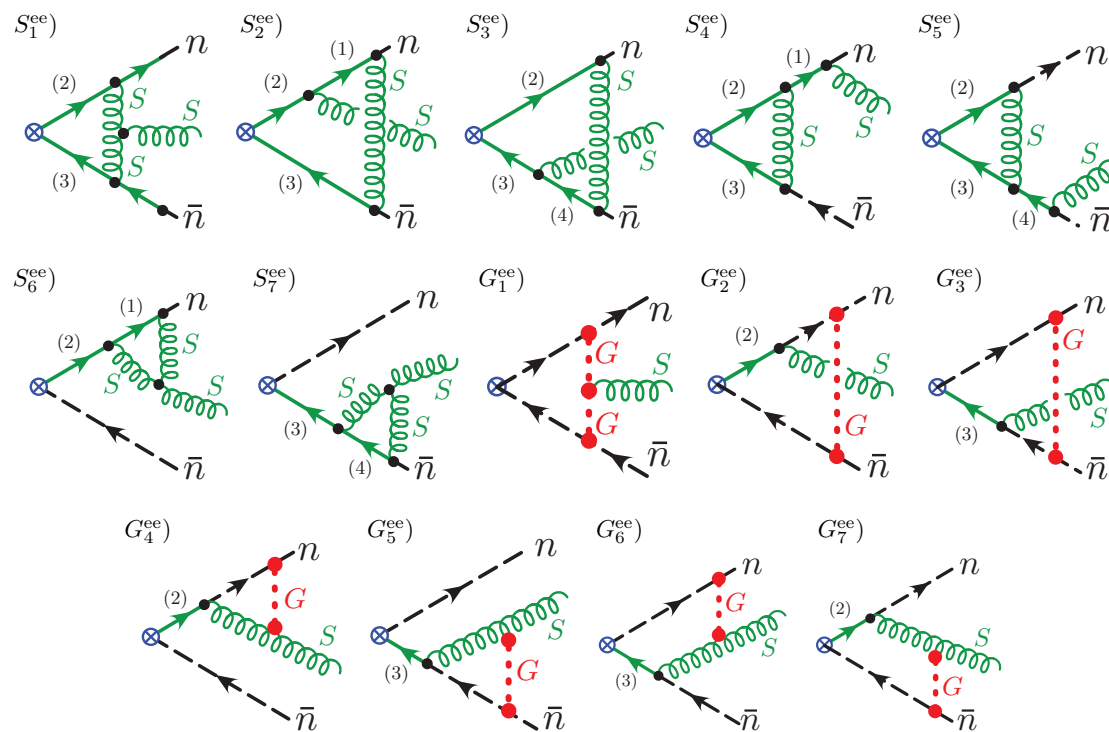
Allow for Soft radiation

General form of two loop soft current for three different kinematic situations (pp, ep, e+e-)

$$i\mathcal{A}_i^{\text{chan}} = (i\pi) \frac{g^3}{\pi} T^A \left( \frac{n^\mu}{n \cdot k} - \frac{\bar{n}^\nu}{\bar{n} \cdot k} \right) \left[ -\frac{1}{2} a_i^{\text{chan}} C_A n \cdot k \bar{n} \cdot k I_\perp^{(1)}(k_\perp) + b_i^{\text{chan}} C_F I_\perp^{(0)} - \frac{1}{2} c_i^{\text{chan}} C_A I_\perp^{(0)} \right].$$

$$I_\perp^{(0)} = \int \frac{d^{d-2} \ell_\perp (\iota^\epsilon \mu^{2\epsilon})}{\vec{\ell}_\perp^2}, \quad I_\perp^{(1)}(k_\perp) = \int \frac{d^{d-2} \ell_\perp (\iota^\epsilon \mu^{2\epsilon})^2}{\vec{\ell}_\perp^2 (\vec{\ell}_\perp + \vec{k}_\perp)^2},$$

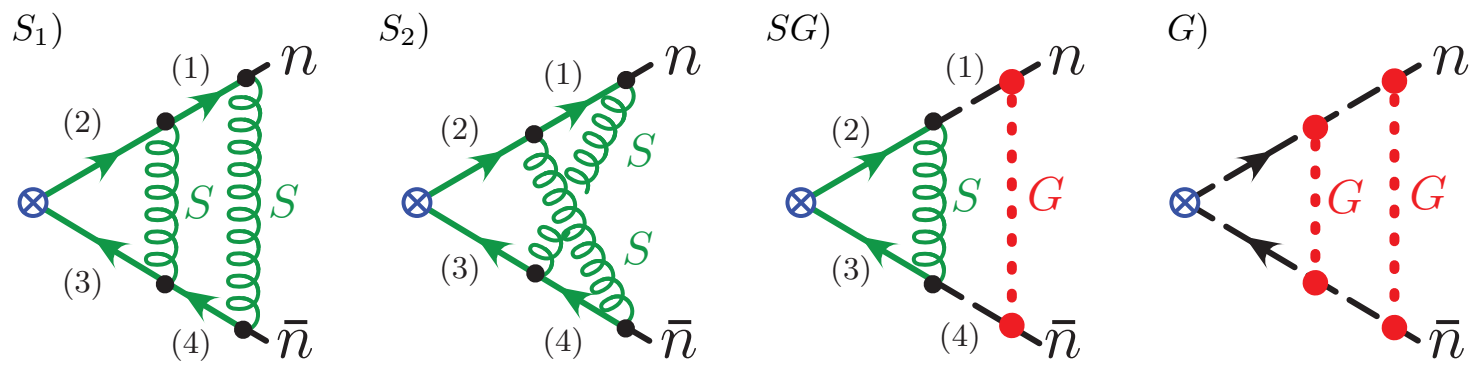
e.g. e+e-



Considering all possible subtractions of soft lines leads to the same conclusion

	$G_1$	$G_{2+3}$	$G_{4+5}$	$G_6$	$G_7$	$S_1^{(2,3)}$	$S_1^{(23)}$	$S_{2+3}^{(13,24)}$	$S_{2+3}^{(23)}$	$S_{2+3}^{(1,\dots,4)}$	$S_{4+5}^{(23)}$	$S_{4+5}^{(2,3)}$	$S_{6+7}^{(1,4)}$
$C_{AI_\perp}^{(1)}:$													
$a_i^{ee}$	$-\frac{1}{4}$	0	0	$+\frac{1}{4}$	$+\frac{1}{4}$	$+\frac{1}{2}$	$-\frac{1}{4}$	0	0	0	0	0	0
$a_i^{ep}$	0	0	0	$+\frac{1}{4}$	0	$+\frac{1}{4}$	0	0	0	0	0	0	0
$a_i^{pp}$	$-\frac{1}{4}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0
$C_{FI_\perp}^{(0)}:$													
$b_i^{ee}$	0	$-\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0
$b_i^{ep}$	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}^*$	0	$+\frac{1}{2}^*$	0
$b_i^{pp}$	0	$-\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0	0
$C_{AI_\perp}^{(0)}:$													
$c_i^{ee}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$+\frac{1}{4}$	0	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	$+\frac{1}{4}$
$c_i^{ep}$	0	0	0	0	0	$+\frac{1}{4}^*$	0	0	0	$-\frac{1}{2}^*$	0	0	$+\frac{1}{4}^*$
$c_i^{pp}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0

We may also consider two loop virtuals which now will include effects of Liptov vertex



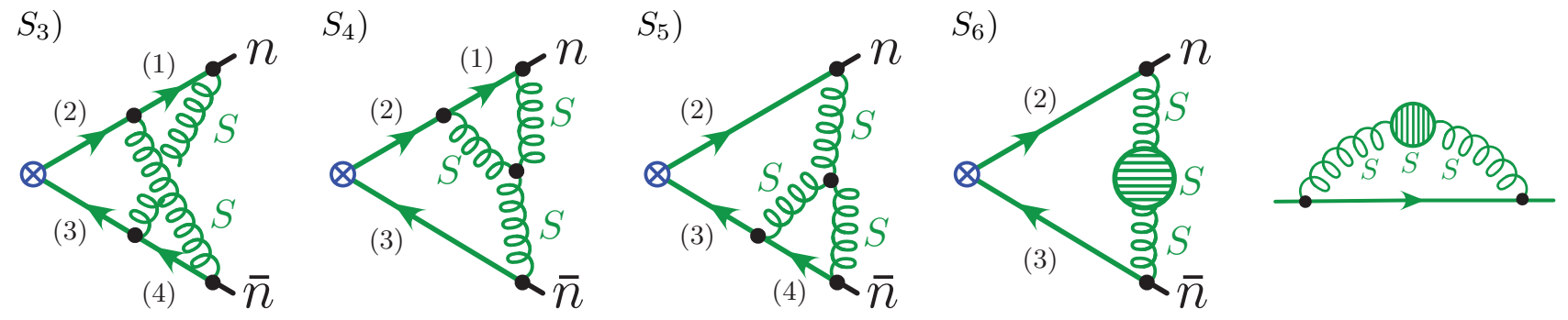
Abelian

naive soft graph

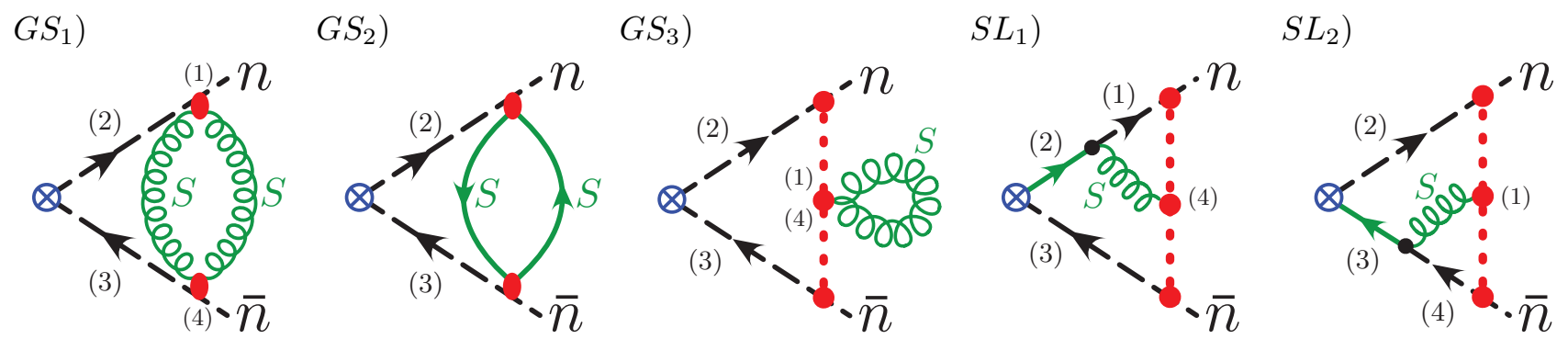
$$S_1 + S_2 + SG + G = \tilde{S}_1 + \tilde{S}_2,$$

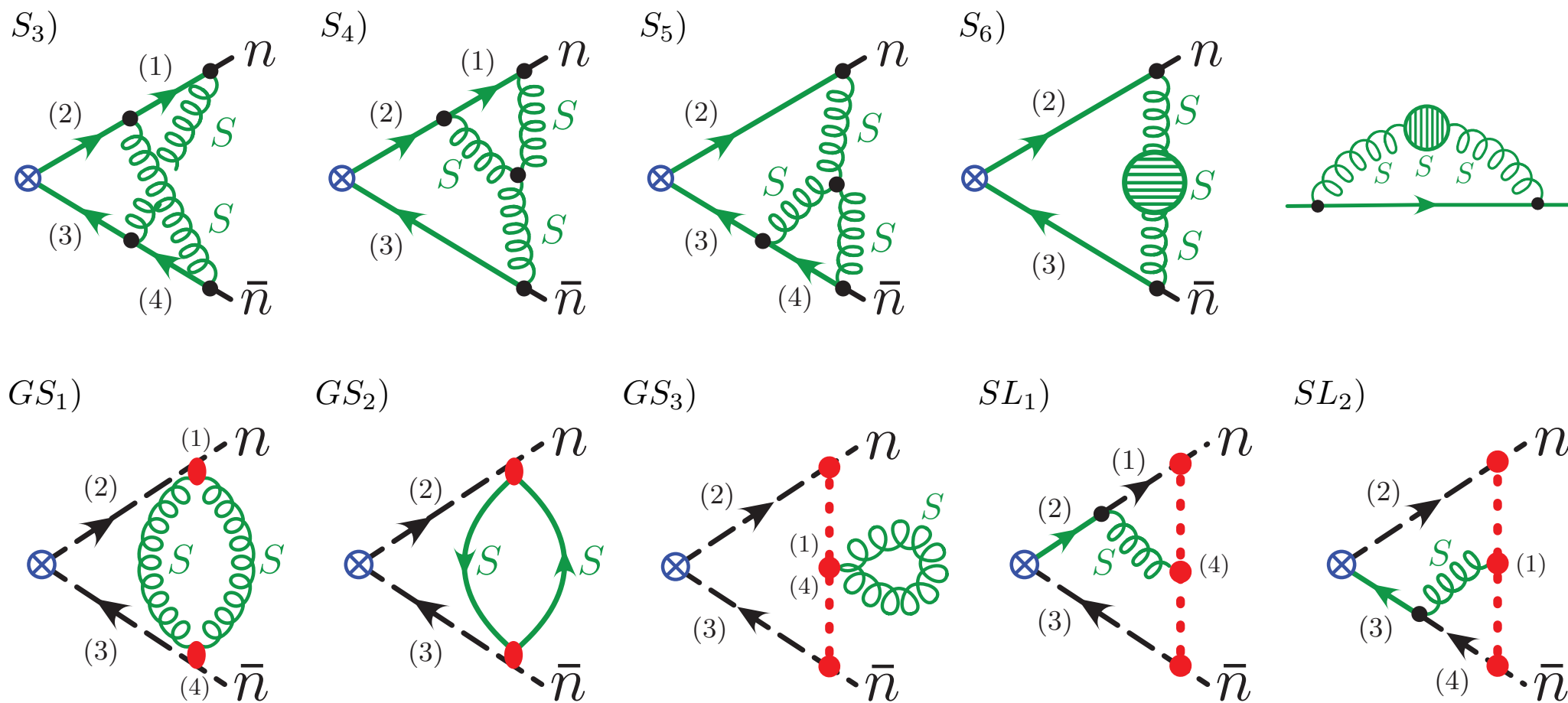
subtracted

i.e. soft subtractions = Glaubers



non-Abelian

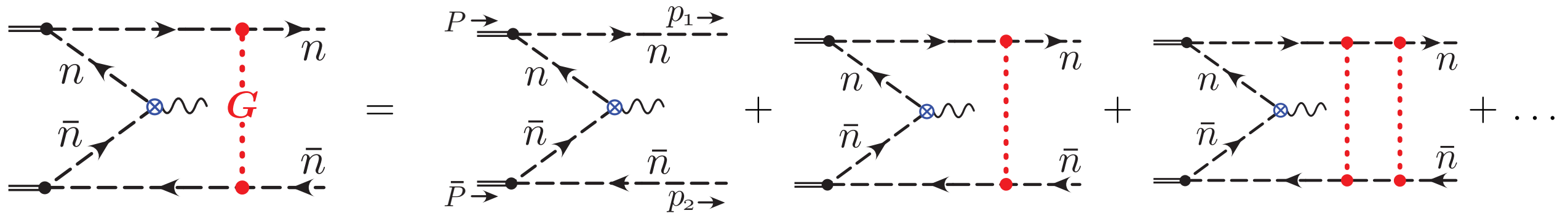




As before we find that the  
Glaubers sit inside the softs

$$S_3 + S_4 + S_5 + S_6 + GS_1 + GS_2 + GS_3 + LS_1 + LS_2 = \tilde{S}_3 + \tilde{S}_4 + \tilde{S}_5 + \tilde{S}_6 .$$

# Spectator-Spectator



Note: no corresponding soft diagrams. Also fermions don't eikonalize

Sum over all boxes

$$\mathcal{S}^\gamma i^4 \int d^d k \frac{-2G(k_\perp) (-1)^2}{[k^+ - \Delta'_1 + i0][k^+ - \Delta_1 + i0][k^- - \Delta_2 + i0][k^- - \Delta'_2 + i0]}$$

$$= \mathcal{S}^\gamma \int d^{d-2} k_\perp G(k_\perp) E(p_{1\perp} + k_\perp, p_{2\perp} - k_\perp).$$

$$= \mathcal{S}^\gamma \int d^{d-2} b_\perp e^{-i\Delta \vec{p}_\perp \cdot \vec{b}_\perp} \tilde{E}'(b_\perp, q_\perp) e^{i\phi(b_\perp)}.$$

$$q_\perp = (-p_{2\perp} - p_{1\perp}) \quad \Delta p = (p_{2\perp} - p_{1\perp})/2$$

Exponentiates in impact parameter space



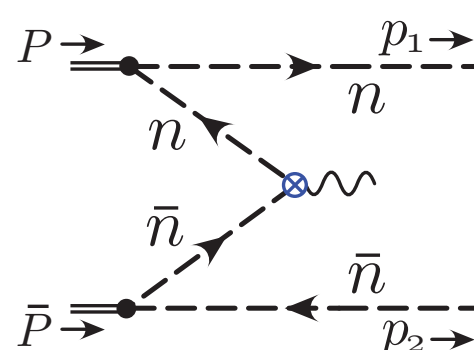
When does this phase cancel?

$$\begin{aligned} & \int d^{d-2} \Delta p_{\perp} |\mathcal{A}_{SS}(\Delta p_{\perp}, q_{\perp})|^2 & (337) \\ &= |\mathcal{S}^{\gamma}|^2 \int d^{d-2} \Delta p_{\perp} \int d^{d-2} b_{\perp} d^{d-2} b'_{\perp} e^{i\Delta \vec{p}_{\perp} \cdot (\vec{b}'_{\perp} - \vec{b}_{\perp})} \tilde{E}'(b_{\perp}, q_{\perp}) \tilde{E}'^{\dagger}(b'_{\perp}, q_{\perp}) e^{i\phi(b_{\perp}) - i\phi(b'_{\perp})} \\ &= |\mathcal{S}^{\gamma}|^2 \int d^{d-2} b_{\perp} |\tilde{E}'(b_{\perp}, q_{\perp})|^2 \end{aligned}$$

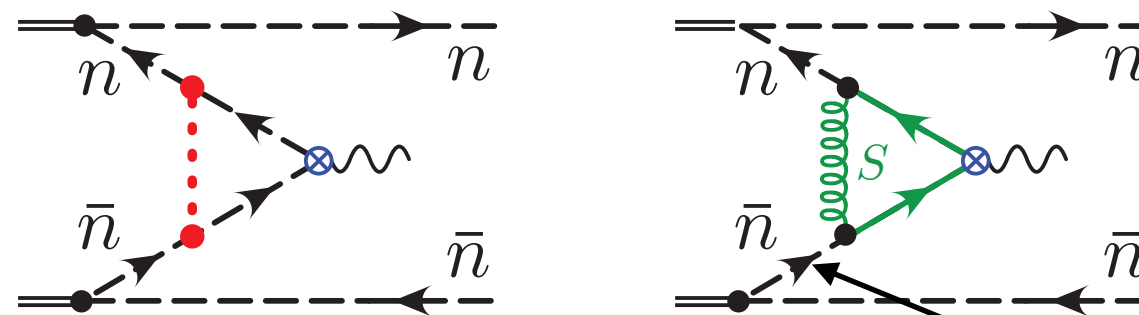
As long as we integrate over  $\Delta p_{\perp}$

Much easier to prove violation than non-violation

So it seems that Glaubers are absorbable into Wilson line when we consider partonic scattering, what about hadronic?



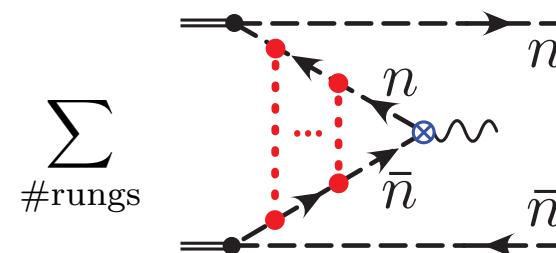
$$\begin{aligned}
 &= S^\gamma \frac{i \bar{n} \cdot (p_1 - P)}{(P - p_1)^2} \frac{i n \cdot (\bar{P} - p_2)}{(\bar{P} - p_2)^2} \\
 &= S^\gamma \left[ \frac{1}{\vec{p}_{1\perp}^2} \frac{1}{\vec{p}_{2\perp}^2} \right] \left[ \frac{\bar{n} \cdot p_1 \bar{n} \cdot (P - p_1)}{\bar{n} \cdot P} \frac{n \cdot p_2 n \cdot (\bar{P} - p_2)}{n \cdot \bar{P}} \right] \\
 &\equiv S^\gamma E(p_{1\perp}, p_{2\perp}),
 \end{aligned}$$



only difference from <sup>a)</sup> previous calculation is that line is not longer on shell. Just shifts  $\Delta$  which we know from Glauber loop computation does not effect result.

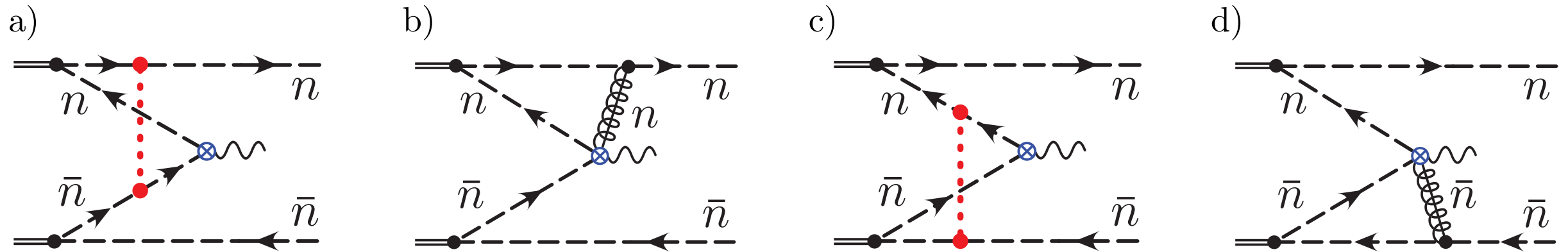
So again Glauber absorbable into S Wilson line

Moreover, the entire series can be summed to generate a phase



$$\sum_{\#rungs} = S^\gamma E(p_{1\perp}, p_{2\perp}) e^{i\phi(0)/2}.$$

# What About Active Collinear?



$$C_n^{(G)}(\text{Fig.35b}) = 2S^\gamma \frac{n \cdot (\bar{P} - p_2)}{(\bar{P} - p_2)^2} \int d^d k \frac{G^0(k_\perp) |\bar{n} \cdot k|^{-\eta} \nu^\eta}{[k^- + i0][ -k^+ - \Delta_1 + i0][k^+ - \Delta'_1 + i0]} .$$

$$\text{Fig.35a} = 2S^\gamma \frac{n \cdot (\bar{P} - p_2)}{(\bar{P} - p_2)^2} \int d^d k \frac{G^0(k_\perp) |2k^z|^{-\eta} \nu^\eta}{[k^- - \Delta_2 + i0][ -k^+ - \Delta_1 + i0][k^+ - \Delta'_1 + i0]} ,$$

Rapidity regulator makes differences irrelevant

Glauber Effect absorbed into Collinear Wilson line.  
 These Glaubers are also benign

# Conclusions

Set up systematic EFT to address the question of Glauber Gluons (Completes SCET)

Reproduced BFKL and Reggeization from EFT

Showed when and how Glaubers may be absorbed into Wilson line

Glauber exponentiation and (non)eikonalization

EFT opens door for systematic treatment of power corrections.