

An Effective Field Theory for Factorization Violation (& Glauber Gluons)

in two parts:

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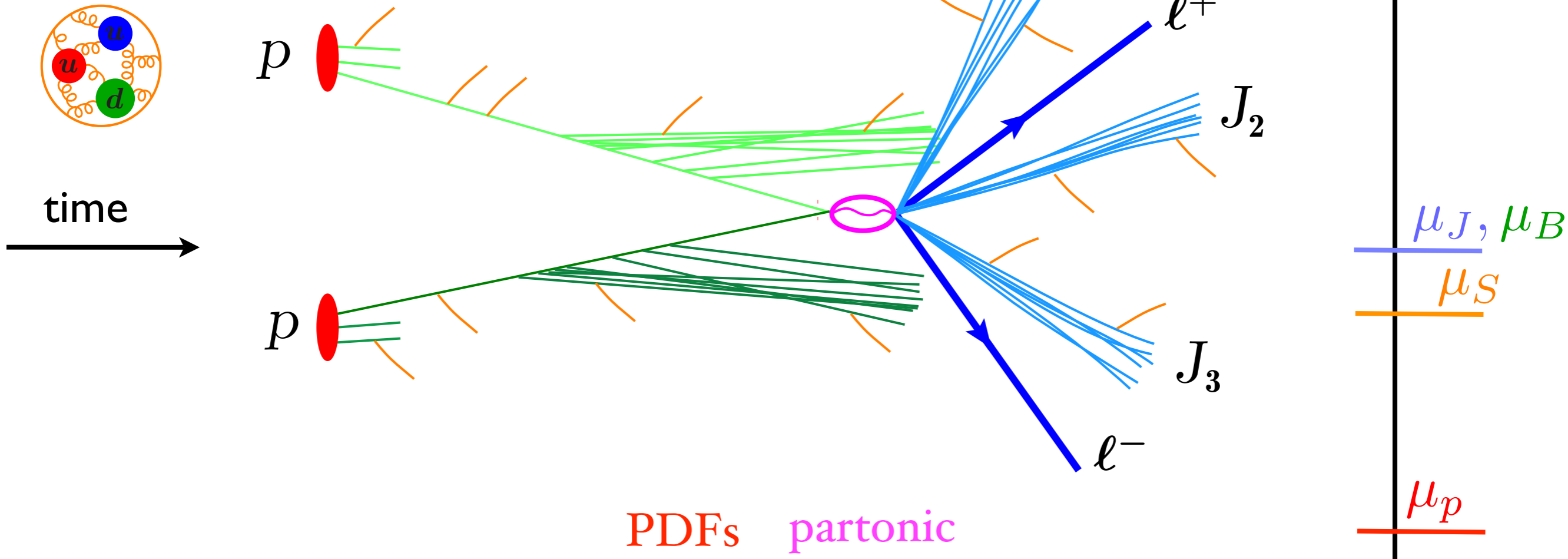
(paper to appear soon)

Workshop on QCD Factorization
University of Buffalo, November 2015

- Introduction:
 - Factorization Examples
 - Factorization Violation
 - why use an EFT?
- Modes & Operators in SCET
- SCET Glauber Operators & Forward Scattering
- Proof of a Power Counting Theorem (all orders, all powers)
- Matching Results for:
 - Wilson Lines
 - Mid-rapidity Soft Operator
- Complete Glauber Interaction Lagrangian
 - interactions between 2 or 3 rapidity sectors
- One-Loop Graphs
 - Rapidity Regulator, Subtractions, Matching

- Reggeization from Rapidity renormalization
- Forward Scattering and BFKL
 - Forward Scattering Collinear and Soft Functions
 - BFKL equation from RGE
- Glauber Exponentiation, eikonal phase
- Glauber's in Hard Scattering
 - The Cheshire Glauber
 - One and Two Loop results
- Glauber Effects with Spectators
- Summary

Hard Scattering Factorization:



Nonperturbative: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$

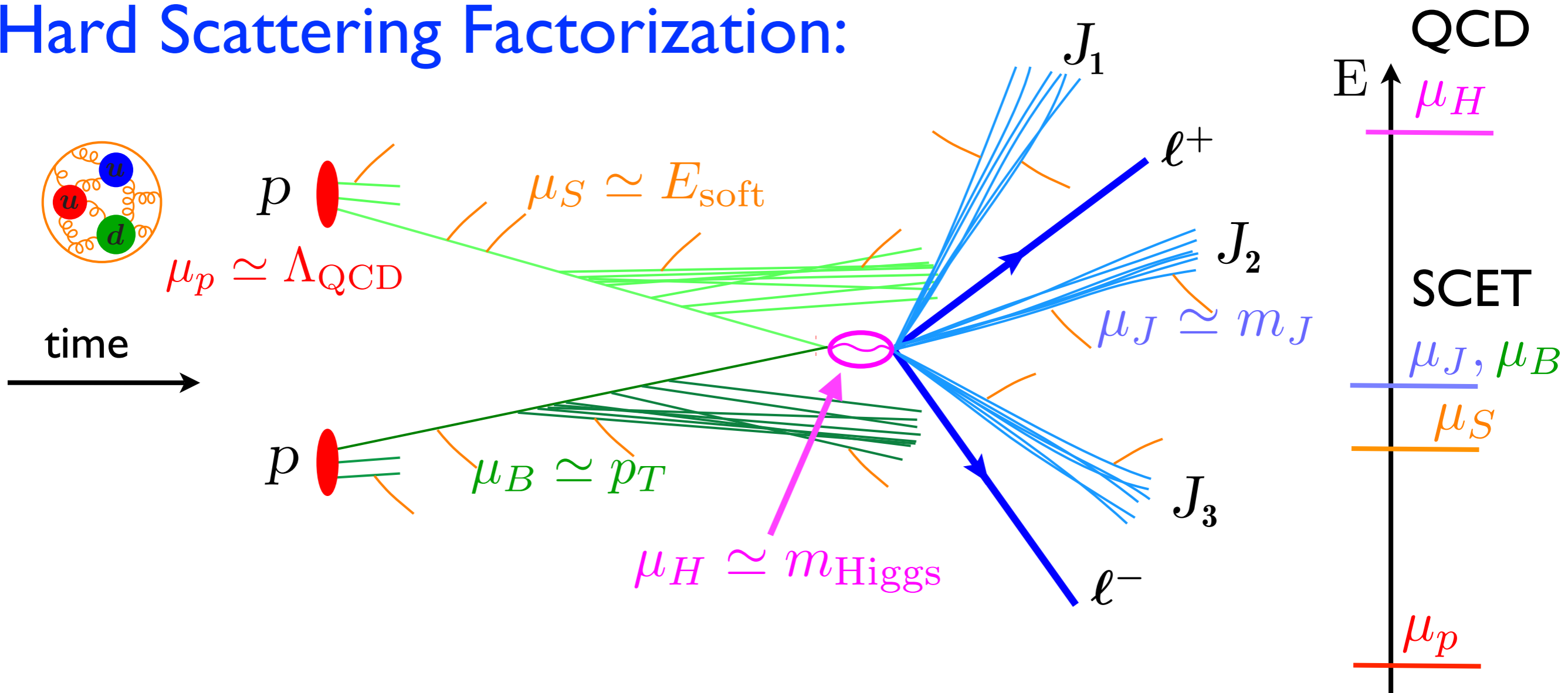
$\mu_p \simeq \Lambda_{\text{QCD}}$

hadronization
(In some cases by Operators, or is power suppressed)

eg. Perturbative: $\hat{\sigma}_{\text{fact}} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$

Used to Sum Logs

Hard Scattering Factorization:



Nonperturbative: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$

$\mu_p \simeq \Lambda_{\text{QCD}}$

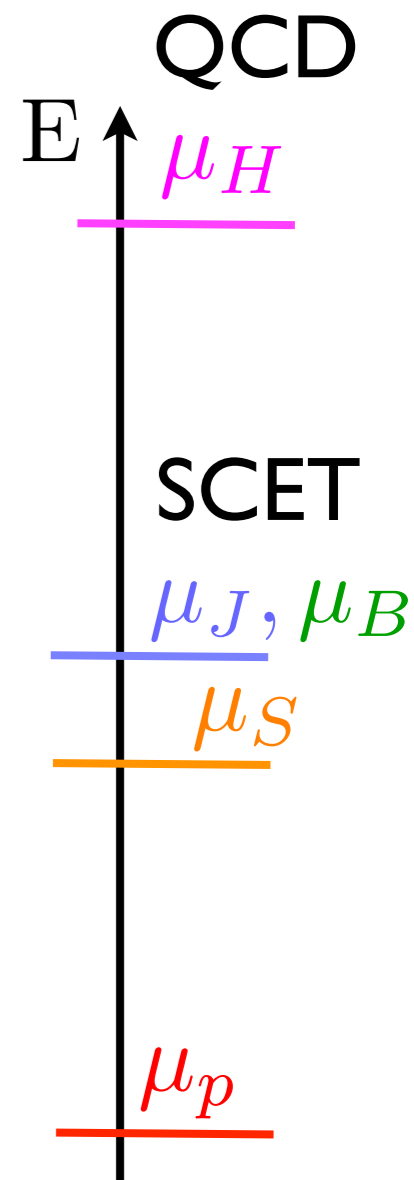
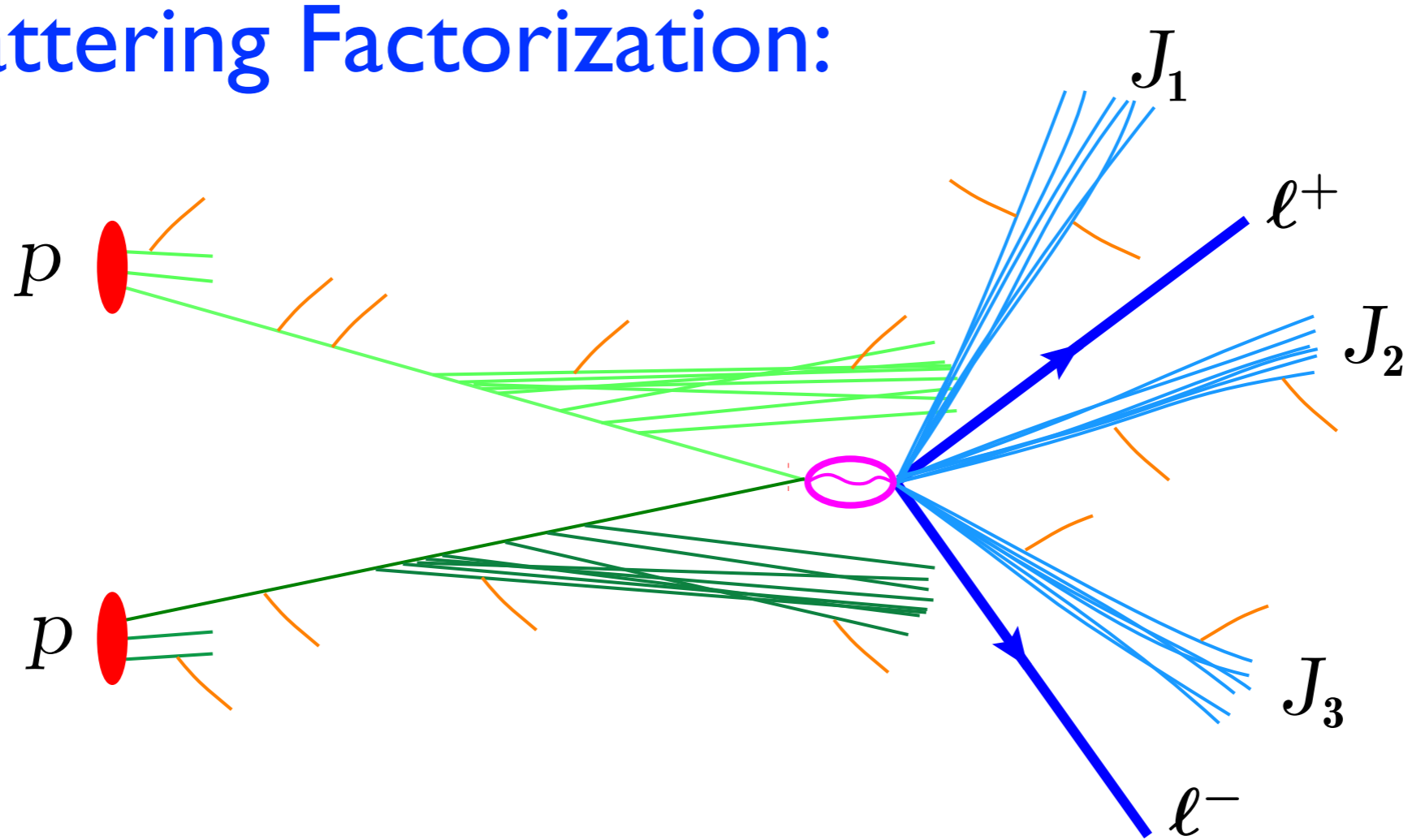
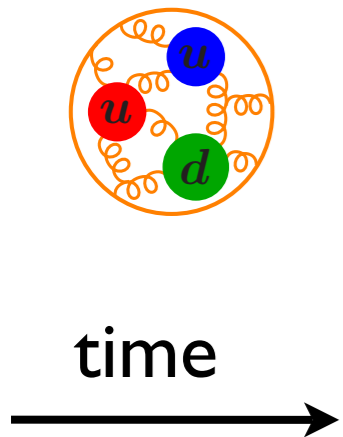
hadronization

eg. Perturbative: $\hat{\sigma}_{\text{fact}} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$ Used to Sum Logs

μ_B
 μ_H
 μ_J
 μ_S

Universal Functions: beam hard jet pert. soft

Hard Scattering Factorization:



In SCET:

factorized Lagrangian: $\mathcal{L}_{\text{SCET}_{\text{II},S,\{n_i\}}}^{(0)} = \mathcal{L}_S^{(0)}(\psi_S, A_S) + \sum_{n_i} \mathcal{L}_{n_i}^{(0)}(\xi_{n_i}, A_{n_i})$

factorized Hard Ops: $C \otimes (\mathcal{B}_{n_a \perp})(\mathcal{B}_{n_b \perp})(\mathcal{B}_{n_1 \perp})(\bar{\chi}_{n_2})(\chi_{n_3})(\mathcal{S}_{n_a} \mathcal{S}_{n_b} \mathcal{S}_{n_1} \mathcal{S}_{n_2} \mathcal{S}_{n_3})$
Soft Wilson Lines

quark & gluon operator building blocks:



factorized matrix elements defining **jet**, **soft**, ... functions

Examples of Factorization:

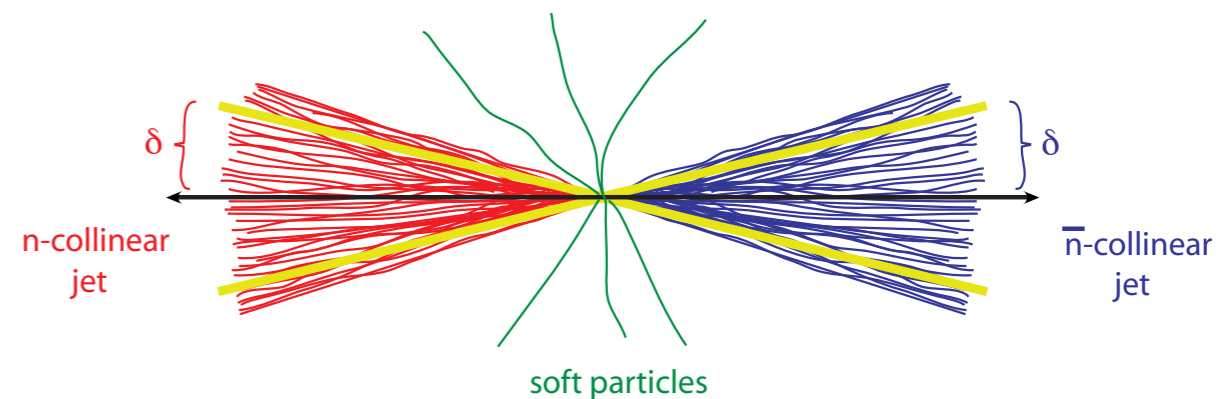
- Inclusive Higgs production $pp \rightarrow \text{Higgs} + \text{anything}$

$$d\sigma = \int dY \sum_{i,j} \int \frac{d\xi_a}{\xi_a} \frac{d\xi_b}{\xi_b} f_i(\xi_a, \mu) f_j(\xi_b, \mu) H_{ij}^{\text{incl}} \left(\frac{m_H e^Y}{E_{\text{cm}} \xi_a}, \frac{m_H e^{-Y}}{E_{\text{cm}} \xi_b}, m_H, \mu \right)$$

(PDFs, Glaubers cancel & Softs cancel)

- Dijet production $e^+ e^- \rightarrow 2 \text{ jets}$

thrust $\tau \ll 1$



$$\frac{d\sigma}{d\tau} = \sigma_0 \underbrace{H(Q, \mu)}_{\text{hard function}} Q \int d\ell d\ell' \underbrace{J_T(Q^2\tau - Q\ell, \mu)}_{\text{jet functions}} \underbrace{S_T(\ell - \ell', \mu)}_{\text{perturbative soft function}} \underbrace{F(\ell')}_{\text{non-perturbative soft function}}$$

(No PDFs, No Glaubers, Softs contribute)

- Higgs with a Jet Veto
(anti-kT jets, radius R)

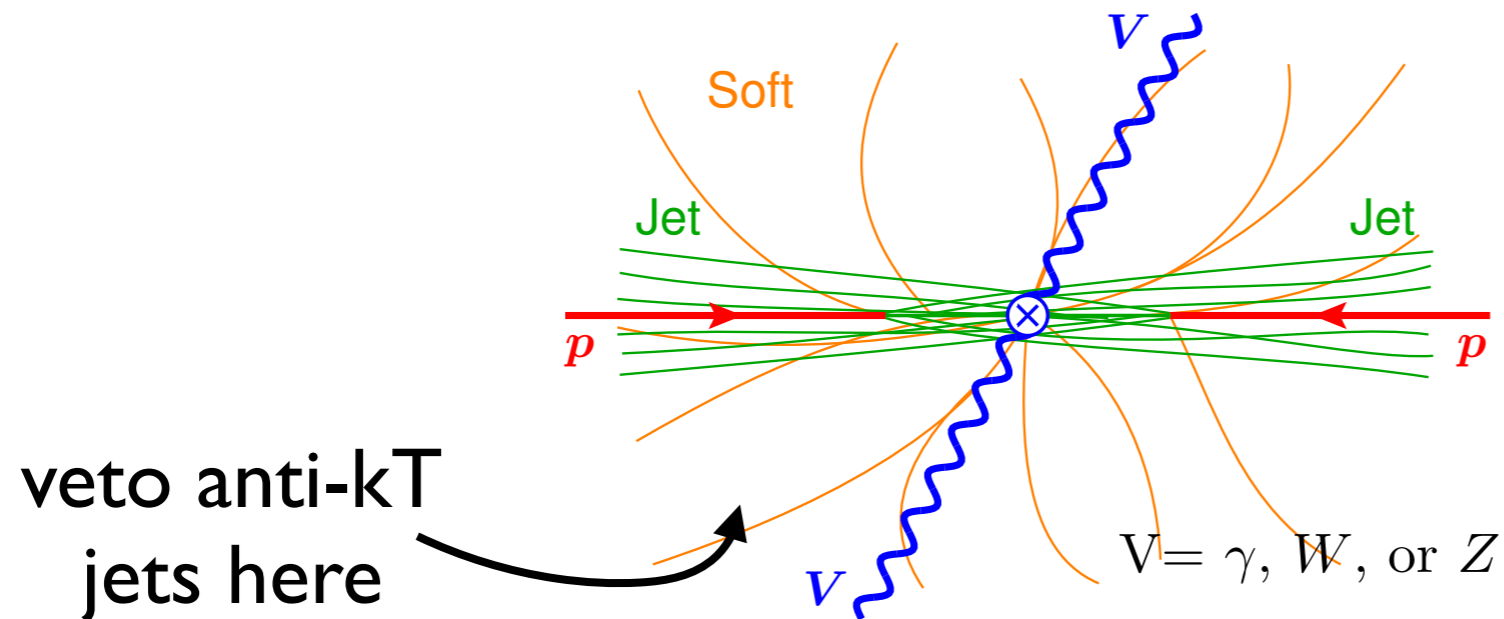
Berger, Marcantonini, IS
Tackmann, Waalewijn
Banfi, Salam, Zanderighi
Becher & Neubert
I.S., Tackmann, Walsh, Zuberi

$$\sigma_0(p_T^{\text{cut}}) = H_{gg}(m_H) \times [B_g(m_H, p_T^{\text{cut}}, R)]^2 \times S_{gg}(p_T^{\text{cut}}, R)$$

$$p_T^{\text{jet}} \leq p_T^{\text{cut}} \ll m_H$$

$$B_g = \mathcal{I}_{gj}(m_H, p_T^{\text{cut}}, R) \otimes f_j$$

$$\Lambda_{\text{QCD}} \ll p_T^{\text{cut}}$$



(PDFs, Softs contribute, Glaubers?)

- **Hard Amplitude Factorization (Quarks)**

$$\langle X_1 \cdots X_N; X_s | i \rangle \cong \mathcal{C}(P_i) \frac{\langle X_1 | \bar{\psi} W_1 | 0 \rangle}{\text{tr} \langle 0 | Y_1^\dagger W_1 | 0 \rangle} \cdots \frac{\langle X_N | W_N^\dagger \psi | 0 \rangle}{\text{tr} \langle 0 | W_N^\dagger Y_N | 0 \rangle} \langle X_s | Y_1^\dagger \cdots Y_N | 0 \rangle$$

Proof with well separated
Final State Collinear Particles (Feige, Schwartz)

$$\mathcal{O}_N^{(0)} = C_N^{(0)}(\{Q_i\}) \otimes \prod_{i=1}^N \left[\delta(\bar{n}_i \cdot Q_i - \bar{n} \cdot i\partial_n) X_{n_i}^{\kappa_i}(0) \right] \otimes \text{T} \left\{ \prod_{i=1}^N Y_{n_i}^{\kappa_i}(0) \right\}$$

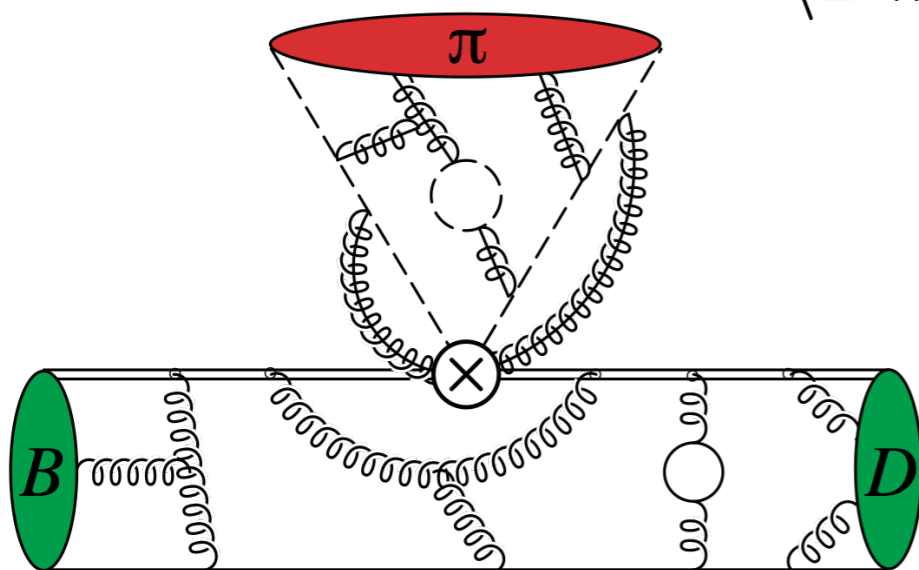
$$X_{n_i} = \{ \chi_{n_i}, \mathcal{B}_{n_i}^\mu \}$$

Simple in SCET
if we assume
Glauber's are absent

- **Exclusive Factorization**

$$\langle D\pi | (\bar{c}b)(\bar{u}d) | B \rangle = N \xi(v \cdot v') \int_0^1 dx T(x, \mu) \phi_\pi(x, \mu)$$

(Bauer, Pirjol, IS)



soft $\langle D^{(*)} | O_s | B \rangle = \xi(v \cdot v')$

n-collinear $\langle \pi | O_n(x) | 0 \rangle = f_\pi \phi_\pi(x)$

“Factorization Violation”

Phrase can mean different things.

In General: The anticipated form for a factorization formula is invalid.

Reasons Factorization could fail:

- **Measurement doesn't factor:** no simple factorization for the functions, or in SCET for the soft and collinear modes. (eg. Jade alg.)

- **Hard-Collinear Factorization fails.**

examples studied by Collins, Qiu, Aybat, Rogers, ...

$$H_2, H_3 \simeq \text{back-to-back} \\ H_1 + H_2 \rightarrow H_3 + H_4 + X$$

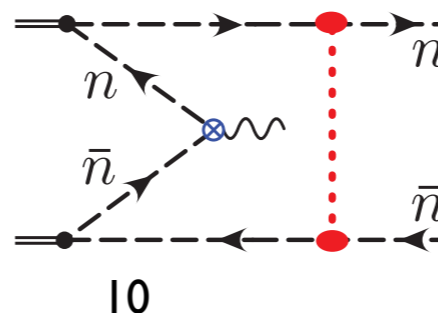
SCET: matching fails, cannot find a finite basis of Hard operators.

- **Divergent integrals**, not controlled by usual regulation procedures. (Requires more careful definition of functions.)

$$\int_0^1 \frac{dx}{x^2} \phi_\pi(x, \mu)$$

- Interactions that couple other modes and spoil factorization.

Glauber exchange



spectator-spectator
cancel in proof for Drell-Yan
(CSS = Collins, Soper, Sterman)

More Examples of Factorization Violation

- Violation of Cross Section factorization, for example PDFs entangled (CSS)

- Violation of Collinear Amplitude Factorization (Catani, de Florian, Rodrigo)
(Forshaw, Seymour, Siodmok)

$$|\mathcal{M}^{(1)}(p_1, p_2, \dots, p_n)\rangle \simeq \mathbf{Sp}^{(1)}(p_1, p_2; \tilde{P}; p_3, \dots, p_n) |\mathcal{M}^{(0)}(\tilde{P}, \dots, p_n)\rangle + \mathbf{Sp}^{(0)}(p_1, p_2; \tilde{P}) |\mathcal{M}^{(1)}(\tilde{P}, \dots, p_n)\rangle .$$

also superleading logs, talk by Forshaw

for space-like collinear limits

- Violation of Regge Amplitude Factorization (Del Duca, Falcioni, Magnea, Vernazza; Glover, Duhr)

$$\mathcal{M}_{\text{rs}}^{[8]} \left(\frac{s}{\mu^2}, \frac{t}{\mu^2}, \alpha_s \right) = 2\pi\alpha_s H_{\text{rs}}^{(0),[8]} C_r \left(\frac{t}{\mu^2}, \alpha_s \right) \left[A_+ \left(\frac{s}{t}, \alpha_s \right) + \kappa_{\text{rs}} A_- \left(\frac{s}{t}, \alpha_s \right) \right] C_s \left(\frac{t}{\mu^2}, \alpha_s \right)$$

$$A_{\pm} \left(\frac{s}{t}, \alpha_s \right) = \left(\frac{-s}{-t} \right)^{\alpha(t)} \pm \left(\frac{s}{-t} \right)^{\alpha(t)} \quad \text{violated at NNLL by } \frac{(i\pi)^2 \alpha_s^2}{\epsilon^2}$$




all of these violations are related to **Glauber exchange**

Plan: Add Glaubers to SCET

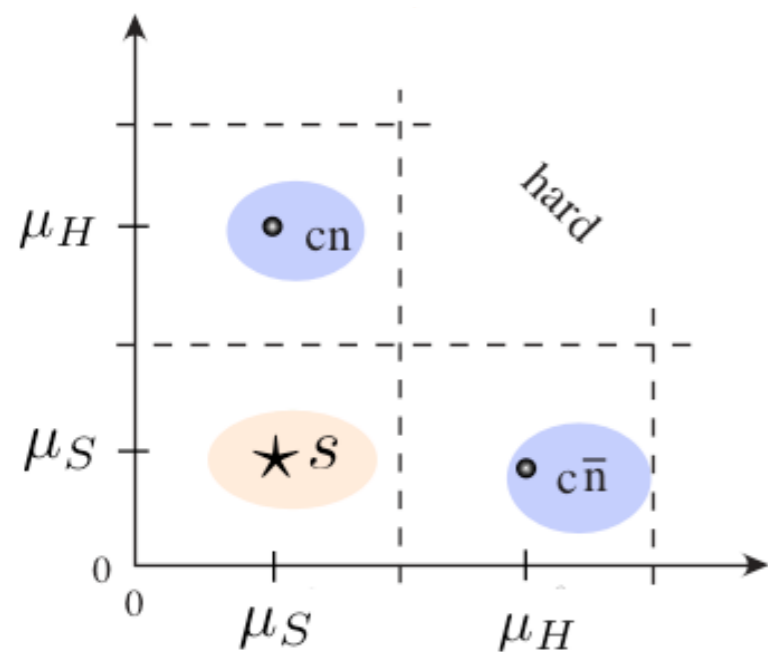
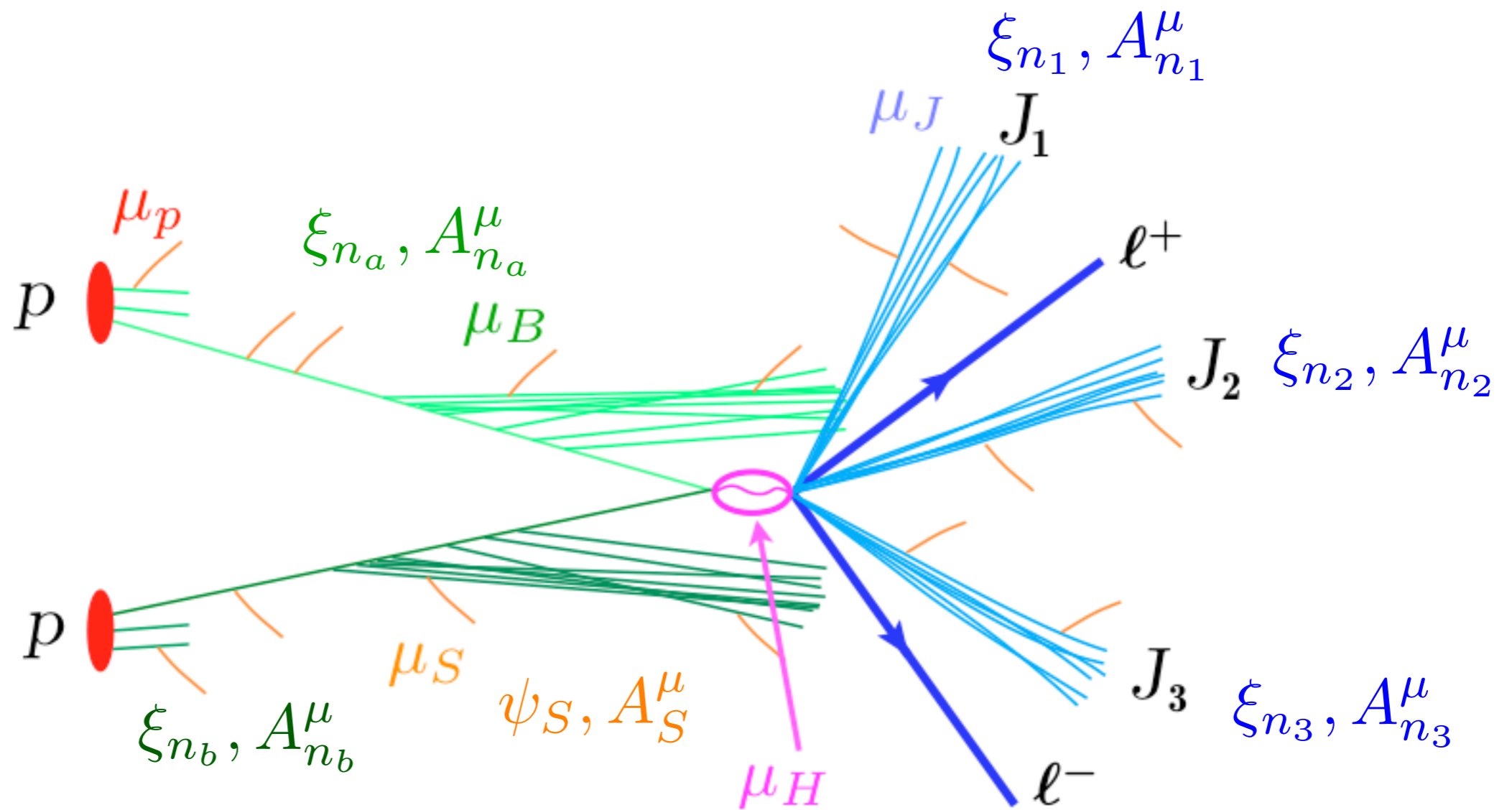
$$\mathcal{L}_{\text{SCET}_{\text{II}}}^{(0)} = \mathcal{L}_{\text{SCET}_{\text{II},\text{S},\{n_i\}}}^{(0)} + \mathcal{L}_G^{(0)}(\psi_S, A_S, \xi_{n_i}, A_{n_i})$$

Focus here
on SCET_{II}

Goals & Possible Advantages for EFT approach

- Hard Scattering and Forward Scattering in single framework
- Operator based: Can exploit symmetries, Gauge invariant
- $\overline{\text{MS}}$ style renormalization for rapidity divergences (counterterms, renormalization group equations, ...)
- Distinct Infrared Modes in Feyn. Graphs + Power Counting  derive when eikonal approximation is relevant
- Factorization violating interactions may also obey factorization theorems
- Valid to all orders in α_s & clear path using this formalism to study subleading power amplitudes (subleading ops & Lagrangians)
- Potential method to derive factorization results for less inclusive collider processes, predict things about UE, etc.

SCET Fields for various Modes



- dominant contributions from isolated regions of momentum space
- use subtractions rather than sharp boundaries to preserve symmetry

Relevant Modes

$\lambda \ll 1$ large Q

Infrared Structure of Amplitudes (Landau eqtns, CSS, ...)
Method of Regions (Beneke & Smirnov)

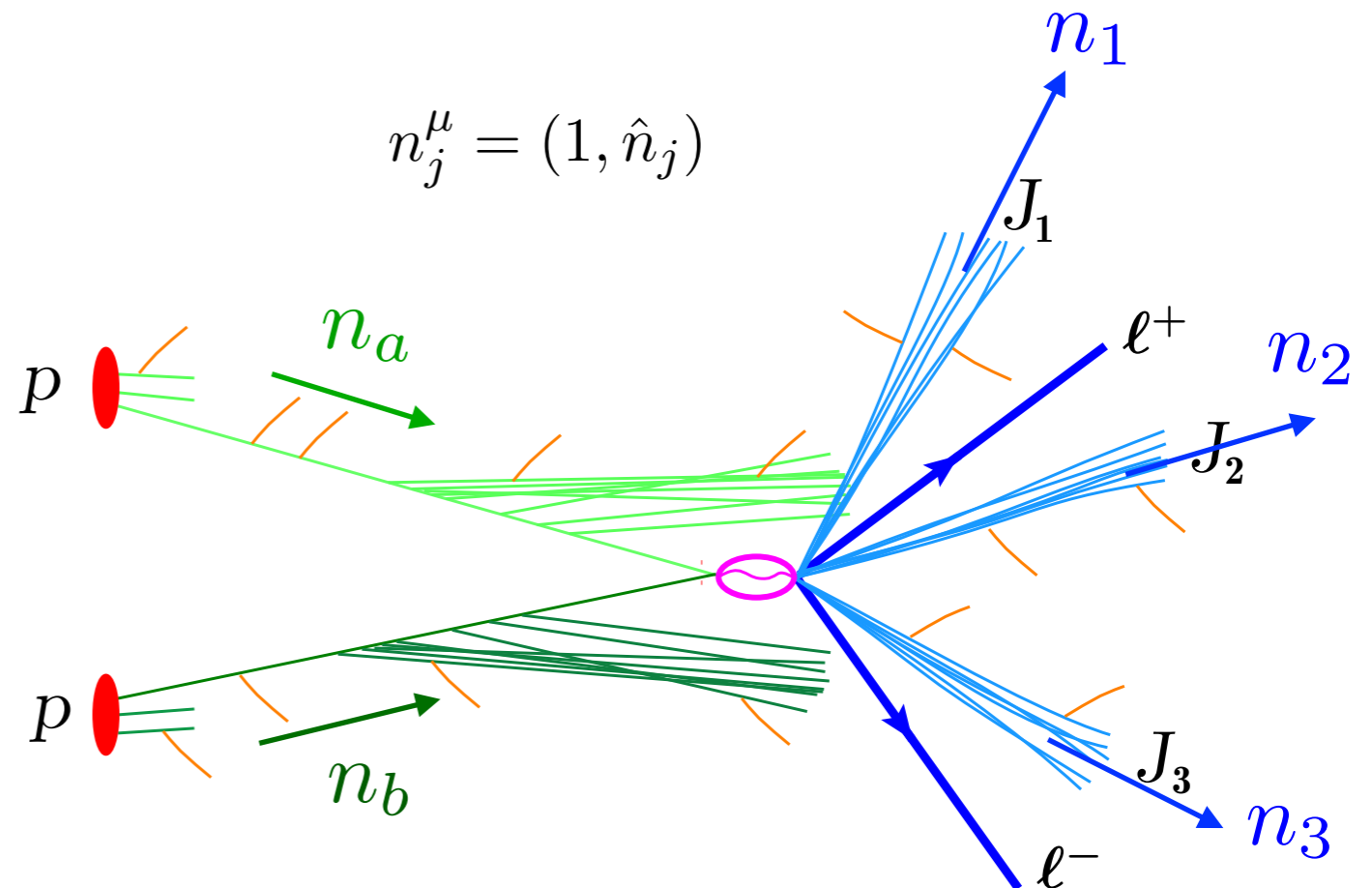
mode	fields	p^μ momentum scaling	physical objects
n_a -collinear	$\xi_{n_a}, A_{n_a}^\mu$	$(n_a \cdot p, \bar{n}_a \cdot p, p_{\perp a}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet a
n_b -collinear	$\xi_{n_b}, A_{n_b}^\mu$	$(n_b \cdot p, \bar{n}_b \cdot p, p_{\perp b}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet b
n_j -collinear	$\xi_{n_j}, A_{n_j}^\mu$	$(n_j \cdot p, \bar{n}_j \cdot p, p_{\perp j}) \sim Q(\lambda^2, 1, \lambda)$	collinear final state jet in \hat{n}_j
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation
ultrasoft	ψ_{us}, A_{us}^μ	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation
Glauber	—	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda), a + b > 2$	forward scattering potential
hard	—	$p^2 \gtrsim Q^2$	hard scattering

$$p^\mu = \bar{n}_i \cdot p \frac{n_i^\mu}{2} + n_i \cdot p \frac{\bar{n}_i^\mu}{2} + p_\perp^\mu$$

$$n_i^2 = 0$$

$$\bar{n}_i^2 = 0$$

$$n_i \cdot \bar{n}_i = 2$$



Principles used for Soft-Collinear Effective Theory

- **Matching** QCD & SCET must agree at long distances
short distance encoded by coefficients, giving H
- **Power Counting** for fields, states, amplitudes with loops
Rigorously track expansions
Power counting theorems
- **Symmetry** Gauge symmetry within sectors
Lorentz & Reparameterization symmetries

$$\mathcal{L}_{\text{SCET}_{\text{II}}}^{(0)} = \left[\mathcal{L}_{\text{S}}^{(0)}(\psi_{\text{S}}, A_{\text{S}}) + \sum_{n_i} \mathcal{L}_{n_i}^{(0)}(\xi_{n_i}, A_{n_i}) \right] \quad \text{Lagrangians factorize at leading power}$$

Relevant Modes

$\lambda \ll 1$ large Q

Infrared Structure of Amplitudes (Landau eqtns, CSS, ...)
Method of Regions (Beneke & Smirnov)

mode	fields	p^μ momentum scaling	physical objects	type
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n_b -collinear	$\xi_{n_b}, A_{n_b}^\mu$	$(n_b \cdot p, \bar{n}_b \cdot p, p_{\perp b}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet b	onshell
n_j -collinear	$\xi_{n_j}, A_{n_j}^\mu$	$(n_j \cdot p, \bar{n}_j \cdot p, p_{\perp j}) \sim Q(\lambda^2, 1, \lambda)$	collinear final state jet in \hat{n}_j	onshell
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	ψ_{us}, A_{us}^μ	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	—	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda), a + b > 2$	forward scattering potential	offshell
hard	—	$p^2 \gtrsim Q^2$	hard scattering	offshell

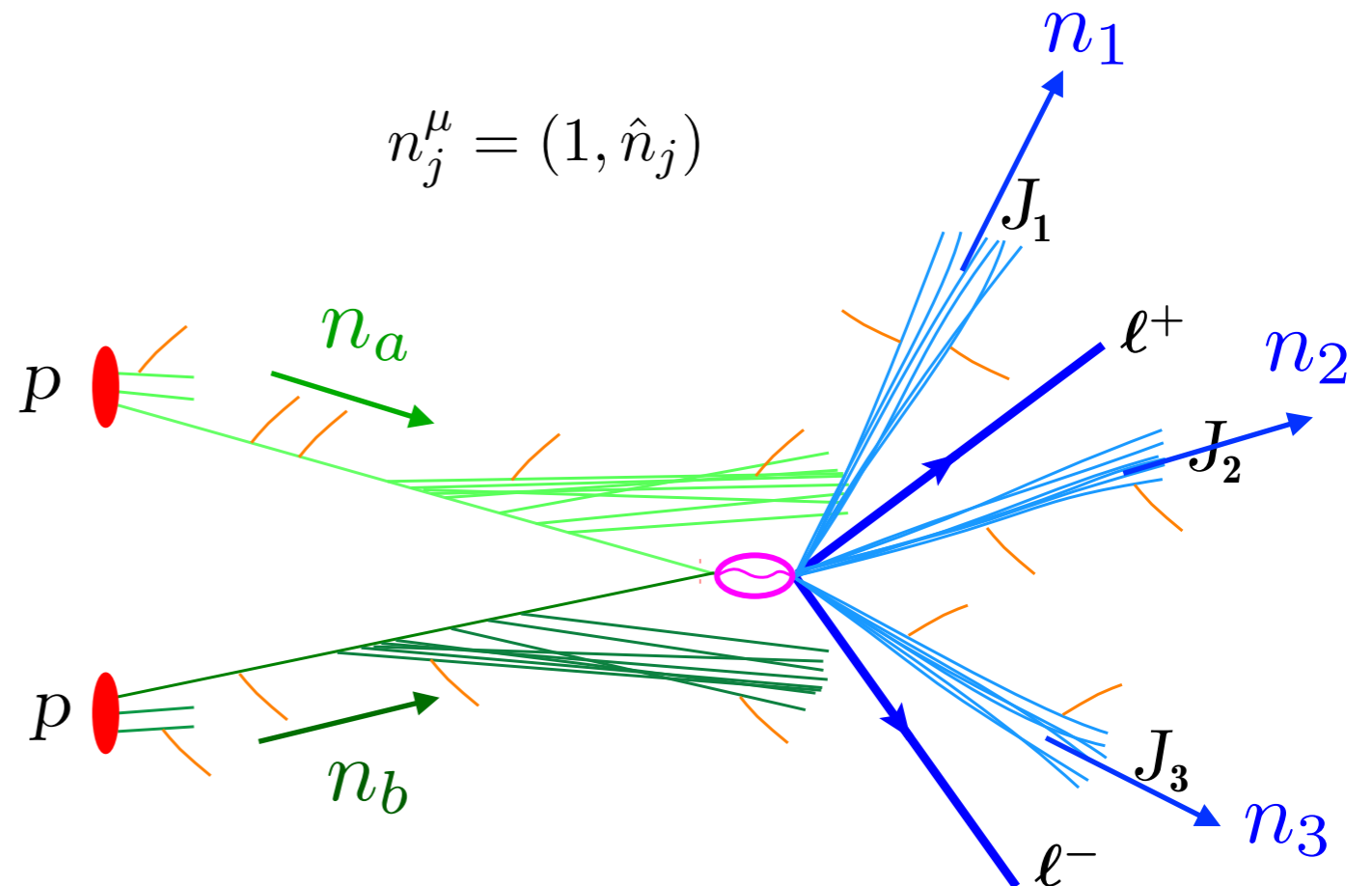
Integrate out these modes

$$p^\mu = \bar{n}_i \cdot p \frac{n_i^\mu}{2} + n_i \cdot p \frac{\bar{n}_i^\mu}{2} + p_\perp^\mu$$

$$n_i^2 = 0$$

$$\bar{n}_i^2 = 0$$

$$n_i \cdot \bar{n}_i = 2$$



SCET Operators & Hard Scattering:

$$\mathcal{L}_{\text{SCET II}}^{\text{hardscatter}} = \sum_K C_K^{\text{II}} \otimes O_K^{\text{II}}(\{\xi_{n_i}, A_{n_i}\}, \psi_S, A_S)$$

$$O_K^{\text{II}} = (\chi_{n_1, \omega_1^a} \chi_{n_1, \omega_1^b} \cdots) (\bar{\chi}_{n_1, \omega_1^c} \bar{\chi}_{n_1, \omega_1^d} \cdots) (\mathcal{B}_{n_1, \omega_1^e}^\perp \mathcal{B}_{n_1, \omega_1^f}^\perp \cdots) (\chi_{n_2, \omega_2^a} \cdots) (\bar{\chi}_{n_2, \omega_2^b} \cdots) (\mathcal{B}_{n_2, \omega_2^c}^\perp \cdots) \cdots \\ \times (\psi_{S, k_1^a}^{n_1} \psi_{S, k_2^b}^{n_1} \cdots) (\bar{\psi}_{S, k_1^c}^{n_1} \bar{\psi}_{S, k_1^d}^{n_1} \cdots) (\mathcal{B}_{S, k_1^e}^{\perp n_1} \mathcal{B}_{S, k_1^f}^{\perp n_1} \cdots) (\psi_{S, k_2^a}^{n_2} \cdots) (\bar{\psi}_{S, k_2^b}^{n_2} \cdots) (\mathcal{B}_{S, k_2^c}^{n_2 \perp} \cdots) \cdots$$

- Operator Building Blocks**

$$\chi_n = W_n^\dagger \xi_n \quad \text{“quark jet”}$$

$$\mathcal{B}_{n\perp}^\mu = \frac{1}{g} [W_n^\dagger i D_{n\perp}^\mu W_n] \quad \text{“gluon jet”}$$

Wilson lines generated by
integrating out offshell modes

$$W_n = P \exp \left(ig \int_{-\infty}^0 ds \bar{n} \cdot A_n(x + \bar{n}s) \right)$$

or

$$W_n = \bar{P} \exp \left(-ig \int_0^\infty ds \bar{n} \cdot A_n(x + \bar{n}s) \right)$$

analogous for soft fields

Direction dependence often cancelled by 0-bin subtractions.
(Can be fixed considering Glaubers.)

Determine Glauber Lagrangian

$$\mathcal{L}_{\text{SCET}_{\text{II}}}^{(0)} = \mathcal{L}_{\text{SCET}_{\text{II},\text{S},\{n_i\}}}^{(0)} + \mathcal{L}_G^{(0)}(\psi_S, A_S, \xi_{n_i}, A_{n_i})$$

Construction:

$\lambda \ll 1$

large Q

will do calculations with back-to-back collinear particles for simplicity

mode	fields	p^μ momentum scaling	physical objects	type
n -collinear	ξ_n, A_n^μ	$(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n -collinear “jet”	onshell
\bar{n} -collinear	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$	$(\bar{n} \cdot p, n \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	\bar{n} -collinear “jet”	onshell
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	ψ_{us}, A_{us}^μ	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	—	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda), a + b > 2$ (here $\{a, b\} = \{2, 2\}, \{2, 1\}, \{1, 2\}$)	forward scattering potential	offshell
hard	—	$p^2 \gtrsim Q^2$	hard scattering	offshell

Integrate out
these modes

Glaubers are offshell and must be integrated out (despite having $p^2 \sim \lambda^2$)

Otherwise one has problems with simultaneously having
gauge invariant operators and homogeneous power counting

Construction:

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large Q

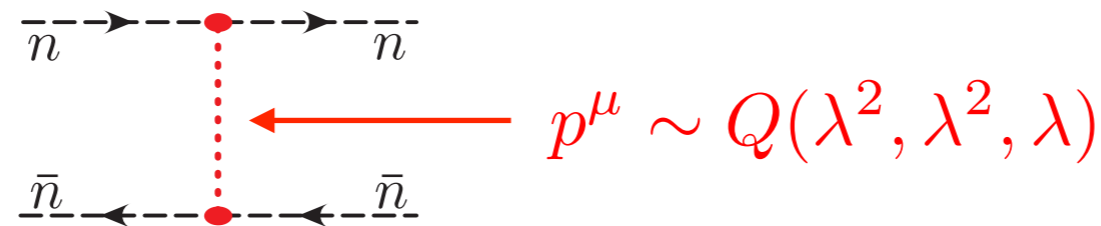
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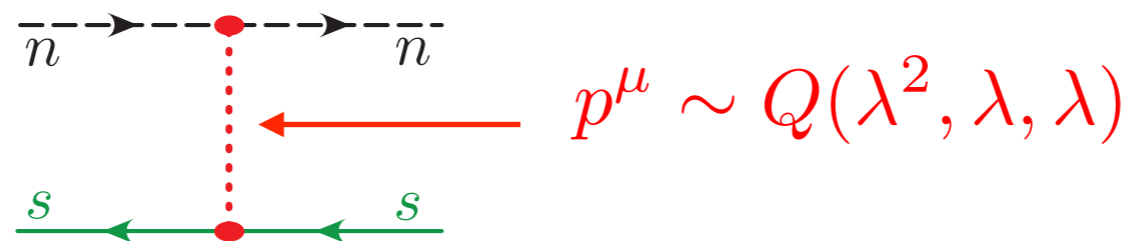
Integrate out

Need 3-types of Glauber momenta:

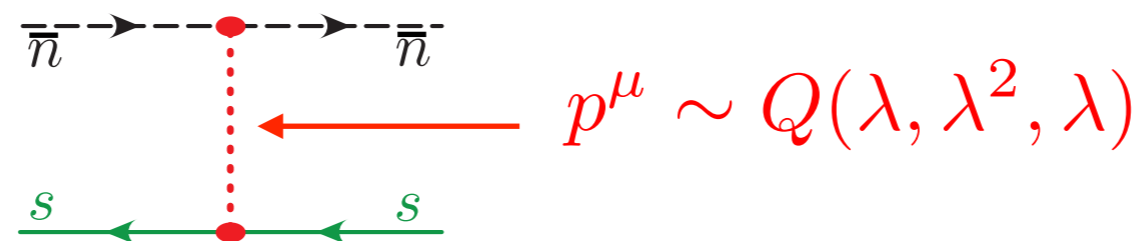
n - \bar{n}
fwd. scattering



n - S
fwd. scattering



\bar{n} - S
fwd. scattering



Construction:

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large Q

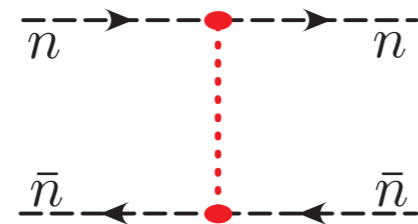
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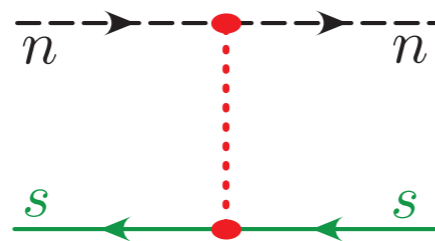
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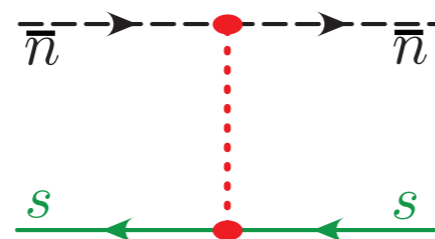
n - \bar{n}
fwd. scattering



n - S
fwd. scattering



\bar{n} - S
fwd. scattering



- $\frac{1}{k_\perp^2}$ potentials
- instantaneous in x^+, x^- (t and z)

Construction:

$\lambda \ll 1$

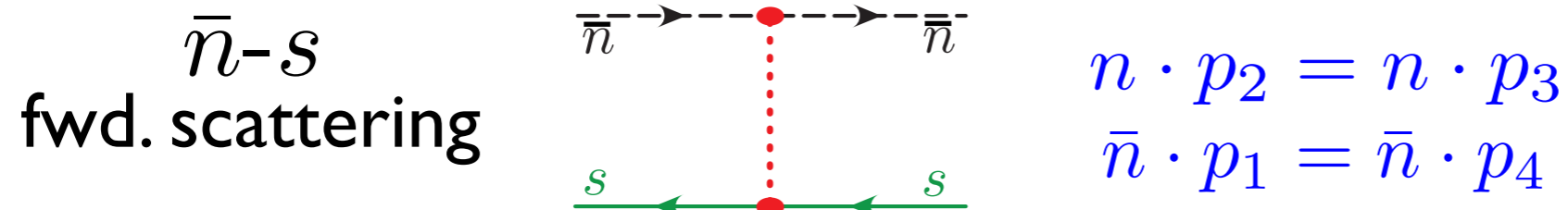
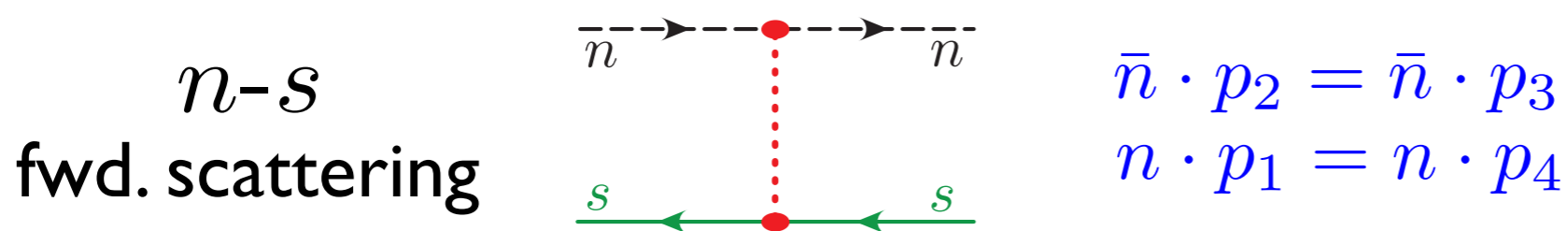
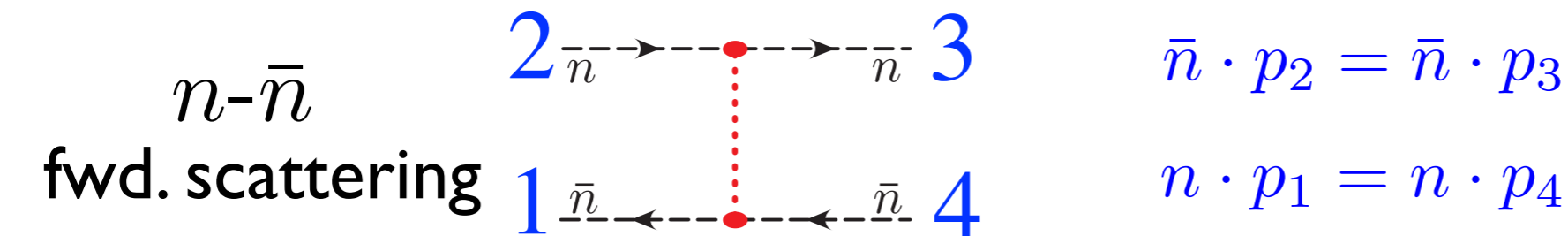
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n -collinear	ξ_n, A_n^μ	$(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n -collinear “jet”	onshell
\bar{n} -collinear	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$	$(\bar{n} \cdot p, n \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	\bar{n} -collinear “jet”	onshell
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	ψ_{us}, A_{us}^μ	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	—	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda), a + b > 2$ (here $\{a, b\} = \{2, 2\}, \{2, 1\}, \{1, 2\}$)	forward scattering potential	offshell
hard	—	$p^2 \gtrsim Q^2$	hard scattering	offshell

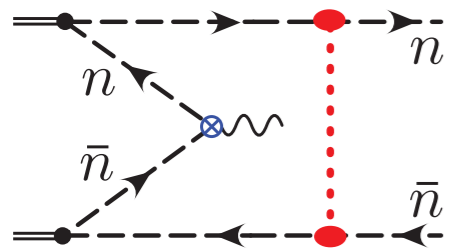
Need 3-types of Glauber momenta:

Integrate out



forward conditions

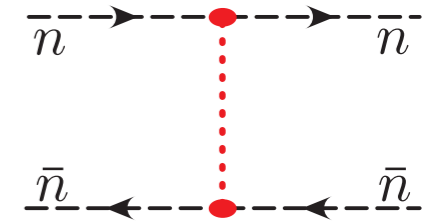
Glauber Exchange violates factorization:



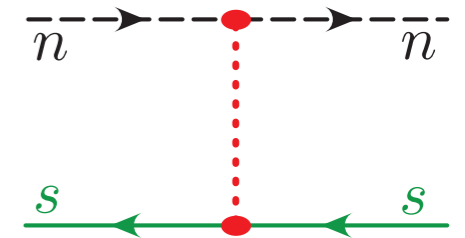
couples n -collinear,
 \bar{n} -collinear, and
soft modes

Glauber's dominate Forward Scattering:

$n-\bar{n}$
fwd. scattering



$n-S$
fwd. scattering



(small- x logs, reggeization, BFKL,
BK/BJMWLK, ...)

Construction:

$\lambda \ll 1$

large Q

mode	fields	p^μ momentum scaling	physical objects	type
n -collinear	ξ_n, A_n^μ	$(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n -collinear “jet”	onshell
\bar{n} -collinear	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$	$(\bar{n} \cdot p, n \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	\bar{n} -collinear “jet”	onshell
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	ψ_{us}, A_{us}^μ	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda), a + b > 2$ (here $\{a, b\} = \{2, 2\}, \{2, 1\}, \{1, 2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Power Counting formula for graph (any loop order, any power):

$$\sim \lambda^\delta \quad (\text{gauge invariant})$$

$$\delta = 6 - N^n - N^{\bar{n}} - N^{nS} - N^{\bar{n}S} + 2u,$$

$$+ \sum_k (k-8) V_k^{us} + \underbrace{(k-4)(V_k^n + V_k^{\bar{n}} + V_k^S)}_{\text{standard SCET}} + (k-3)(V_k^{nS} + V_k^{\bar{n}S}) + (k-2)V_k^{n\bar{n}}$$

$$\text{need } \sim \lambda^3 \quad \sim \lambda^2$$

standard SCET

Glauber

operators at leading power

Proof of Power Counting formula:

enumerate **vertices** from gauge invariant operators
of order $\sim \lambda^k$

V_k^n vertices with only n -collinear fields,

$V_k^{\bar{n}}$ vertices with only \bar{n} -collinear fields,

V_k^S vertices with only soft fields,

V_k^{nS} vertices that have both n -collinear and soft fields but do not have \bar{n} fields,

$V_k^{\bar{n}S}$ vertices with both \bar{n} -collinear and soft fields but not n fields,

$V_k^{n\bar{n}}$ vertices with both n and \bar{n} -collinear fields (with or without soft fields).

(includes scaling for external fields)

Proof of Power Counting formula:

count **# Loops** of various types, and **# Internal Propagators**

L^n : n -collinear loops with $k^\mu \sim Q(\lambda^2, 1, \lambda)$ loop momenta,

$L^{\bar{n}}$: \bar{n} -collinear loops with $k^\mu \sim Q(1, \lambda^2, \lambda)$ loop momenta,

L^S : soft loops with $k^\mu \sim Q(\lambda, \lambda, \lambda)$ loop momenta,

L^{nS} : soft-collinear Glauber loops with $k^\mu \sim Q(\lambda^2, \lambda, \lambda)$ loop momenta,

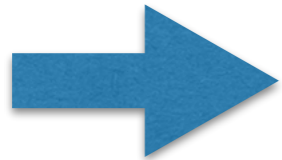
$L^{\bar{n}S}$: soft-collinear Glauber loops with $k^\mu \sim Q(\lambda, \lambda^2, \lambda)$ loop momenta,

$L^{n\bar{n}}$: n - \bar{n} Glauber loops with $k^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$ loop momenta,

I^n : internal n -collinear propagators,

$I^{\bar{n}}$: internal \bar{n} -collinear propagators,

I^S : internal soft propagators.



$$\delta = \sum_k k \left(V_k^n + V_k^{\bar{n}} + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}} \right)$$

$$+ 4L^n + 4L^{\bar{n}} + 4L^S + 5L^{nS} + 5L^{\bar{n}S} + 6L^{n\bar{n}} - 4I^n - 4I^{\bar{n}} - 4I^S$$

Proof of Power Counting formula:

Simplify with topological identities:

overall Euler:
$$1 = \sum_k (V_k^n + V_k^{\bar{n}} + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^n + L^{\bar{n}} + L^S + L^{nS} + L^{\bar{n}S} + L^{n\bar{n}} - I^n - I^{\bar{n}} - I^S$$

count disconnected components when we **erase** modes:

$$\bar{n} \quad N^{nS} = \sum_k (V_k^n + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^n + L^S + L^{nS} - I^n - I^S,$$

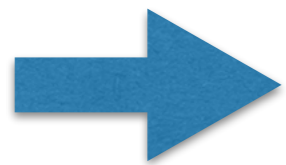
$$n \quad N^{\bar{n}S} = \sum_k (V_k^{\bar{n}} + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^{\bar{n}} + L^S + L^{\bar{n}S} - I^{\bar{n}} - I^S,$$

$$s \quad N^{n\bar{n}} = \sum_k (V_k^n + V_k^{\bar{n}} + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^n + L^{\bar{n}} + L^{n\bar{n}} - I^n - I^{\bar{n}}.$$

$$n, \bar{n} \quad N^S = \sum_k (V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^S - I^S,$$

$$s, \bar{n} \quad N^n = \sum_k (V_k^n + V_k^{nS} + V_k^{n\bar{n}}) + L^n - I^n,$$

$$s, n \quad N^{\bar{n}} = \sum_k (V_k^{\bar{n}} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^{\bar{n}} - I^{\bar{n}},$$



final result:

$$\delta = 6 - N^n - N^{\bar{n}} - N^{nS} - N^{\bar{n}S} + \sum_k (k-4)(V_k^n + V_k^{\bar{n}} + V_k^S) + (k-3)(V_k^{nS} + V_k^{\bar{n}S}) + (k-2)V_k^{n\bar{n}}$$

Proof of Power Counting formula:

Simplify with topological identities:

overall Euler:
$$1 = \sum_k (V_k^n + V_k^{\bar{n}} + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^n + L^{\bar{n}} + L^S + L^{nS} + L^{\bar{n}S} + L^{n\bar{n}} - I^n - I^{\bar{n}} - I^S$$

count disconnected components when we **erase** modes:

$$\bar{n} \quad N^{nS} = \sum_k (V_k^n + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^n + L^S + L^{nS} - I^n - I^S,$$

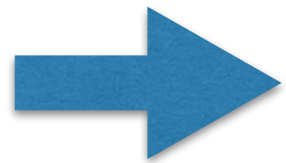
$$n \quad N^{\bar{n}S} = \sum_k (V_k^{\bar{n}} + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^{\bar{n}} + L^S + L^{\bar{n}S} - I^{\bar{n}} - I^S,$$

$$s \quad N^{n\bar{n}} = \sum_k (V_k^n + V_k^{\bar{n}} + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^n + L^{\bar{n}} + L^{n\bar{n}} - I^n - I^{\bar{n}}.$$

$$n, \bar{n} \quad N^S = \sum_k (V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^S - I^S,$$

$$s, \bar{n} \quad N^n = \sum_k (V_k^n + V_k^{nS} + V_k^{n\bar{n}}) + L^n - I^n,$$

$$s, n \quad N^{\bar{n}} = \sum_k (V_k^{\bar{n}} + V_k^{\bar{n}S} + V_k^{n\bar{n}}) + L^{\bar{n}} - I^{\bar{n}},$$



final result for SCET_I and SCET_{II}:

$$\delta = 6 - N^n - N^{\bar{n}} - N^{nS} - N^{\bar{n}S} + 2u$$

$$+ \sum_k (k-8)V_k^{us} + (k-4)(V_k^n + V_k^{\bar{n}} + V_k^S) + (k-3)(V_k^{nS} + V_k^{\bar{n}S}) + (k-2)V_k^{n\bar{n}}$$

Construction:

$\lambda \ll 1$

large Q

mode	fields	p^μ momentum scaling	physical objects	type
n -collinear	ξ_n, A_n^μ	$(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n -collinear “jet”	onshell
\bar{n} -collinear	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$	$(\bar{n} \cdot p, n \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	\bar{n} -collinear “jet”	onshell
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	ψ_{us}, A_{us}^μ	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda), a + b > 2$ (here $\{a, b\} = \{2, 2\}, \{2, 1\}, \{1, 2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Power Counting formula for graph (any loop order, any power):

$$\sim \lambda^\delta \quad (\text{gauge invariant})$$

$$\delta = 6 - N^n - N^{\bar{n}} - N^{nS} - N^{\bar{n}S} + 2u,$$

$$+ \sum_k (k-8) V_k^{us} + \underbrace{(k-4)(V_k^n + V_k^{\bar{n}} + V_k^S)}_{\text{standard SCET}} + (k-3)(V_k^{nS} + V_k^{\bar{n}S}) + (k-2)V_k^{n\bar{n}}$$

$$\text{need } \sim \lambda^3 \quad \sim \lambda^2$$

standard SCET

Glauber

operators at leading power

Construction:

$\lambda \ll 1$

large Q

mode	fields	p^μ momentum scaling	physical objects	type
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\bar{n} -collinear	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$	$(\bar{n} \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	\bar{n} -collinear "jet"	onshell
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
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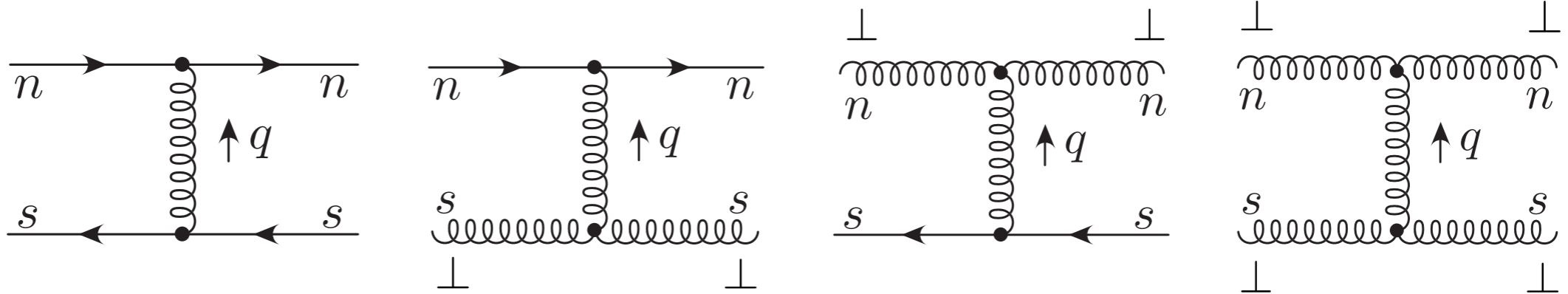
n - S fwd. scattering (2 rapidity sectors)

$s \gg t$

integrated out

calculate

$$\lambda^2 = \frac{t}{s} \ll 1$$



Construction:

$\lambda \ll 1$

large Q

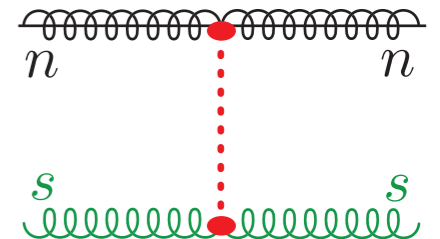
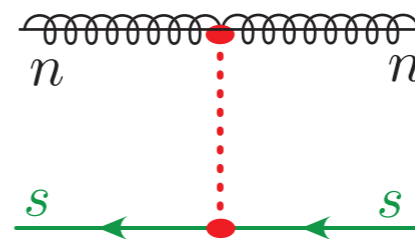
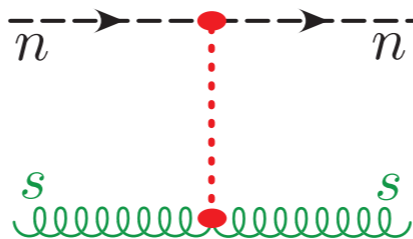
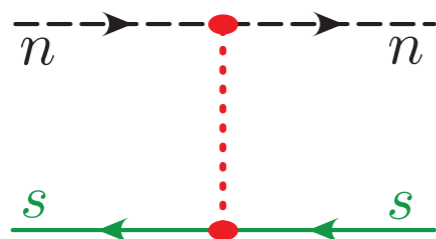
mode	fields	p^μ momentum scaling	physical objects	type
n -collinear	ξ_n, A_n^μ	$(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n -collinear "jet"	onshell
\bar{n} -collinear	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$	$(\bar{n} \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	\bar{n} -collinear "jet"	onshell
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
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n - S fwd. scattering

$s \gg t$

integrated out

$$\lambda^2 = \frac{t}{s} \ll 1$$



determine

$$\mathcal{O}(\lambda^3) : \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

$\lambda^2 \quad \lambda^{-2} \quad \lambda^3$

(2 rapidity sectors)

with bilinear octet operators

$$\psi_s^n = S_n^\dagger \psi_s$$

$$\mathcal{O}(\lambda^2) : \mathcal{O}_n^{qB} = \bar{\chi}_n T^B \frac{\vec{n}}{2} \chi_n,$$

$$\mathcal{O}(\lambda^3) : \mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}_S^n T^B \frac{\vec{n}}{2} \psi_S^n \right),$$

$$\mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\vec{n}}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{\vec{n}}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right)$$

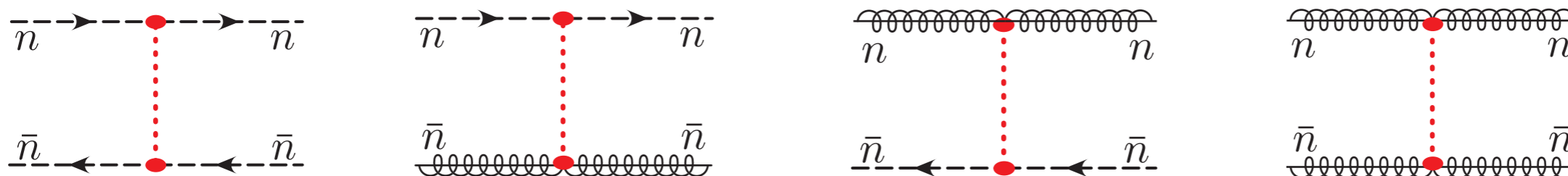
Construction:

$\lambda \ll 1$

large Q

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\bar{n} -collinear	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$	$(\bar{n} \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	\bar{n} -collinear “jet”	onshell
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n - \bar{n} fwd. scattering $s \gg t$



might
guess

$$\mathcal{O}(\lambda^2) : \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jB}$$

same \mathcal{O}_n^{iB}
analogous $\mathcal{O}_{\bar{n}}^{jC}$

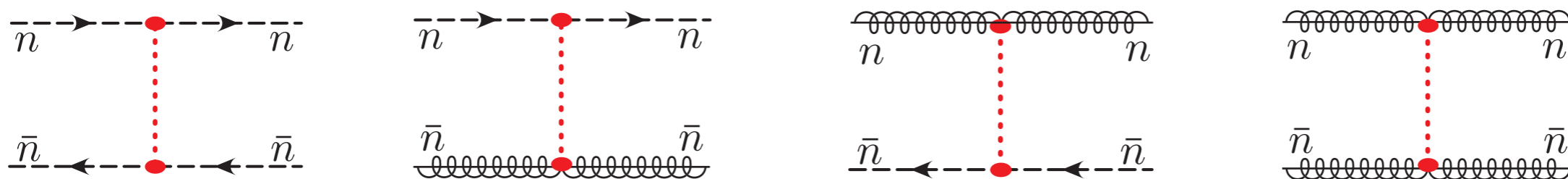
Construction:

$\lambda \ll 1$

large Q

mode	fields	p^μ momentum scaling	physical objects	type
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n - \bar{n} fwd. scattering $s \gg t$



actually $\mathcal{O}(\lambda^2)$:
(3 rapidity sectors)

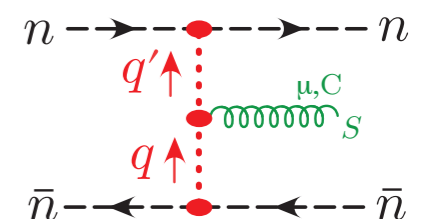
$$\sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC}$$

$\lambda^2 \quad \lambda^{-2} \quad \lambda^2 \quad \lambda^{-2} \quad \lambda^2$

same \mathcal{O}_n^{iB}
analogous $\mathcal{O}_{\bar{n}}^{jC}$

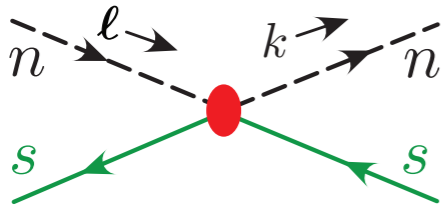
must allow for soft emission from **between** the rapidity sectors:

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \mathcal{P}_\perp^2 \delta^{BC} (+ \dots)$$

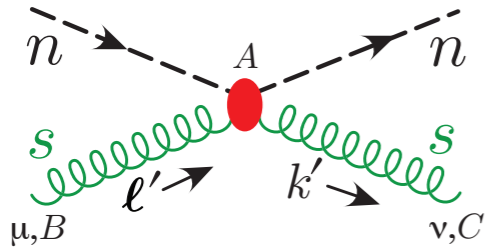


Feynman Rules (so far):

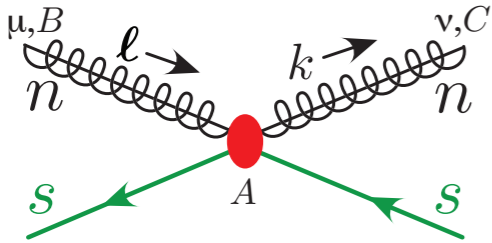
shorthand  = 



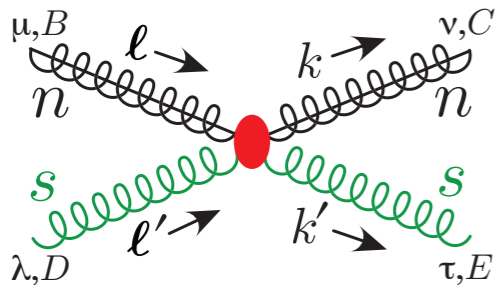
$$= \frac{-8\pi i \alpha_s}{(\vec{\ell}_\perp - \vec{k}_\perp)^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[\bar{v}_s \frac{\not{s}}{2} \bar{T}^A v_s \right]$$



$$= \frac{-8\pi \alpha_s f^{ABC}}{(\vec{\ell}'_\perp - \vec{k}'_\perp)^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[n \cdot k' g_\perp^{\mu\nu} - n^\mu \ell'_\perp{}^\nu - n^\nu k'_\perp{}^\mu + \frac{\ell'_\perp \cdot k'_\perp n^\mu n^\nu}{n \cdot k'} \right]$$



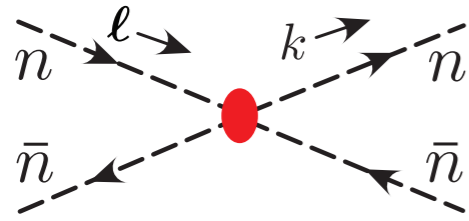
$$= \frac{-8\pi \alpha_s f^{ABC}}{(\vec{\ell}_\perp - \vec{k}_\perp)^2} \left[\bar{n} \cdot k g_\perp^{\mu\nu} - \bar{n}^\mu \ell_\perp{}^\nu - \bar{n}^\nu k_\perp{}^\mu + \frac{\ell_\perp \cdot k_\perp \bar{n}^\mu \bar{n}^\nu}{\bar{n} \cdot k} \right] \left[\bar{v}_s \frac{\not{s}}{2} \bar{T}^A v_s \right]$$



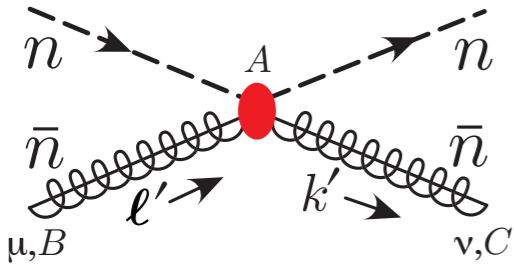
$$= \frac{8\pi i \alpha_s f^{ABC} f^{ADE}}{(\vec{\ell}_\perp - \vec{k}_\perp)^2} \left[\bar{n} \cdot k g_\perp^{\mu\nu} - \bar{n}^\mu \ell_\perp{}^\nu - \bar{n}^\nu k_\perp{}^\mu + \frac{\ell_\perp \cdot k_\perp \bar{n}^\mu \bar{n}^\nu}{\bar{n} \cdot k} \right] \\ \times \left[n \cdot k' g_\perp^{\lambda\tau} - n^\lambda \ell'_\perp{}^\tau - n^\tau k'_\perp{}^\lambda + \frac{\ell'_\perp \cdot k'_\perp n^\lambda n^\tau}{n \cdot k'} \right]$$

Feynman Rules (so far):

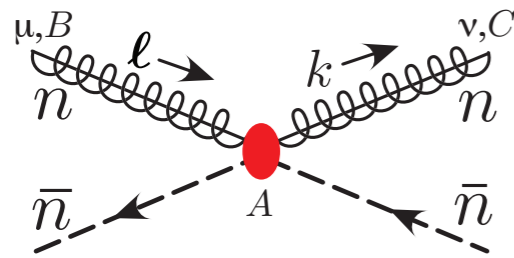
shorthand  = 



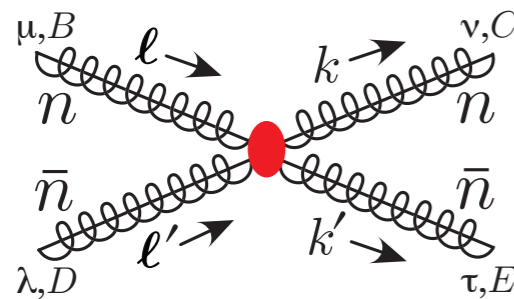
$$= \frac{-8\pi i \alpha_s}{(\vec{l}_\perp - \vec{k}_\perp)^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^A v_{\bar{n}} \right]$$



$$= \frac{-8\pi \alpha_s f^{ABC}}{(\vec{l}'_\perp - \vec{k}'_\perp)^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[n \cdot k' g_\perp^{\mu\nu} - n^\mu \ell'_\perp{}^\nu - n^\nu k'_\perp{}^\mu + \frac{\ell'_\perp \cdot k'_\perp n^\mu n^\nu}{n \cdot k'} \right]$$



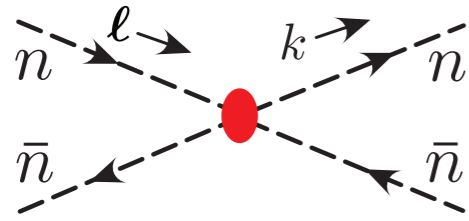
$$= \frac{-8\pi \alpha_s f^{ABC}}{(\vec{l}_\perp - \vec{k}_\perp)^2} \left[\bar{n} \cdot k g_\perp^{\mu\nu} - \bar{n}^\mu \ell_\perp{}^\nu - \bar{n}^\nu k_\perp{}^\mu + \frac{\ell_\perp \cdot k_\perp \bar{n}^\mu \bar{n}^\nu}{\bar{n} \cdot k} \right] \left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^A v_{\bar{n}} \right]$$



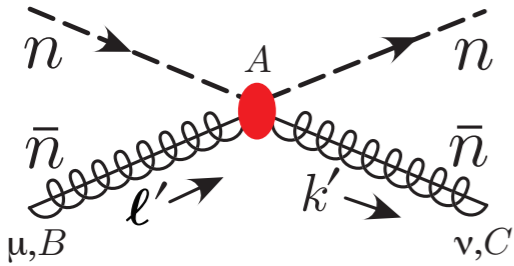
$$= \frac{8\pi i \alpha_s f^{ABC} f^{ADE}}{(\vec{l}_\perp - \vec{k}_\perp)^2} \left[\bar{n} \cdot k g_\perp^{\mu\nu} - \bar{n}^\mu \ell_\perp{}^\nu - \bar{n}^\nu k_\perp{}^\mu + \frac{\ell_\perp \cdot k_\perp \bar{n}^\mu \bar{n}^\nu}{\bar{n} \cdot k} \right] \\ \times \left[n \cdot k' g_\perp^{\lambda\tau} - n^\lambda \ell'_\perp{}^\tau - n^\tau k'_\perp{}^\lambda + \frac{\ell'_\perp \cdot k'_\perp n^\lambda n^\tau}{n \cdot k'} \right]$$

Feynman Rules (so far):

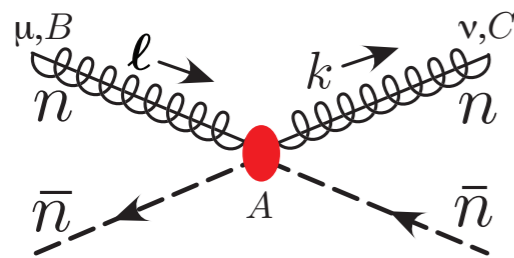
shorthand  = 



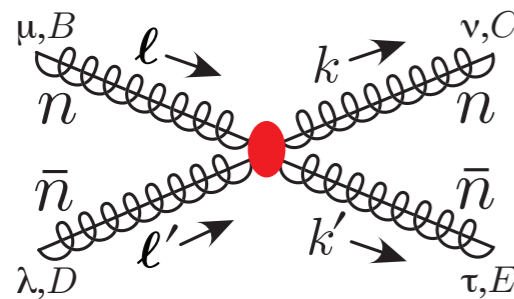
$$= \frac{-8\pi i \alpha_s}{(\vec{\ell}_\perp - \vec{k}_\perp)^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^A v_{\bar{n}} \right]$$



$$= \frac{-8\pi \alpha_s f^{ABC}}{(\vec{\ell}'_\perp - \vec{k}'_\perp)^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[n \cdot k' g_\perp^{\mu\nu} - n^\mu \ell'_\perp{}^\nu - n^\nu k'^\mu + \frac{\ell'_\perp \cdot k'_\perp n^\mu n^\nu}{n \cdot k'} \right]$$



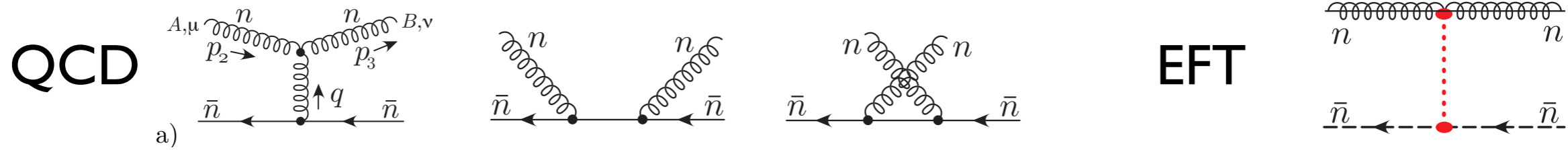
$$= \frac{-8\pi \alpha_s f^{ABC}}{(\vec{\ell}_\perp - \vec{k}_\perp)^2} \left[\bar{n} \cdot k g_\perp^{\mu\nu} - \bar{n}^\mu \ell_\perp{}^\nu - \bar{n}^\nu k^\mu + \frac{\ell_\perp \cdot k_\perp \bar{n}^\mu \bar{n}^\nu}{\bar{n} \cdot k} \right] \left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^A v_{\bar{n}} \right]$$



$$= \frac{8\pi i \alpha_s f^{ABC} f^{ADE}}{(\vec{\ell}_\perp - \vec{k}_\perp)^2} \left[\bar{n} \cdot k g_\perp^{\mu\nu} - \bar{n}^\mu \ell_\perp{}^\nu - \bar{n}^\nu k^\mu + \frac{\ell_\perp \cdot k_\perp \bar{n}^\mu \bar{n}^\nu}{\bar{n} \cdot k} \right] \\ \times \left[n \cdot k' g_\perp^{\lambda\tau} - n^\lambda \ell'^\tau - n^\tau k'^\lambda + \frac{\ell'_\perp \cdot k'_\perp n^\lambda n^\tau}{n \cdot k'} \right]$$

not purely \perp

Matching with arbitrary polarizations:



QCD calculation

$$0 = p^\mu A_\mu(p) = \frac{1}{2} \bar{n} \cdot p \, n \cdot A(p) + \frac{1}{2} n \cdot p \, \bar{n} \cdot A(p) + p_\perp \cdot A_\perp(p)$$

remove

used equations of motion

$$= \frac{g^2 f^{ABC}}{q^2} \left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^C v_{\bar{n}} \right] \left\{ 2 \bar{n} \cdot p_2 g_\perp^{\mu\nu} - 2 \bar{n}^\mu p_{2\perp}^\nu - 2 p_{3\perp}^\mu \bar{n}^\nu - n \cdot (p_2 + p_3) \bar{n}^\mu \bar{n}^\nu \right\}$$

$$= \frac{g^2 f^{ABC}}{q^2} \left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^C v_{\bar{n}} \right] \left\{ -\frac{q^2}{\bar{n} \cdot p_2} \right\} \bar{n}^\mu \bar{n}^\nu$$

sum & use equations of motion: $q^2 + \bar{n} \cdot p_2 \, n \cdot (p_2 + p_3) = -2 p_{2\perp} \cdot p_{3\perp}$

$$\frac{2g^2 f^{ABC}}{q^2} \left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^C v_{\bar{n}} \right] \left\{ \bar{n} \cdot p_2 g_\perp^{\mu\nu} - \bar{n}^\mu p_{2\perp}^\nu - p_{3\perp}^\mu \bar{n}^\nu + \frac{p_{2\perp} \cdot p_{3\perp}}{\bar{n} \cdot p_2} \bar{n}^\mu \bar{n}^\nu \right\}$$

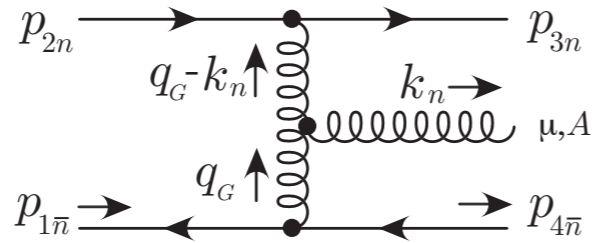
same as Glauber operator

Gluon Operators include Compton graphs in fwd.limit

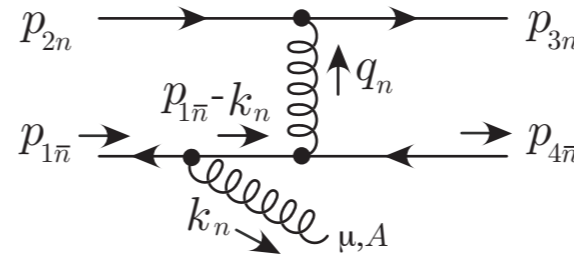
Wilson Lines in the operators are obtained from Matching:

eg. W_n^\dagger in χ_n

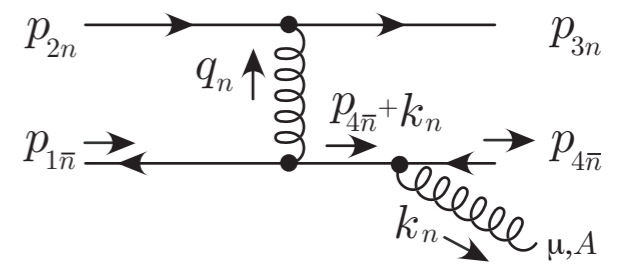
QCD



offshell glauber
onshell collinear

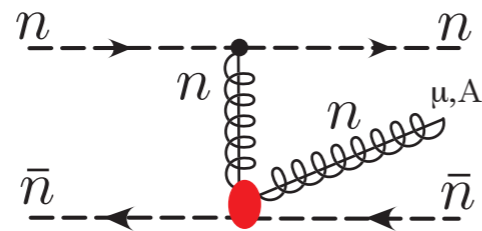


offshell hard
onshell collinear

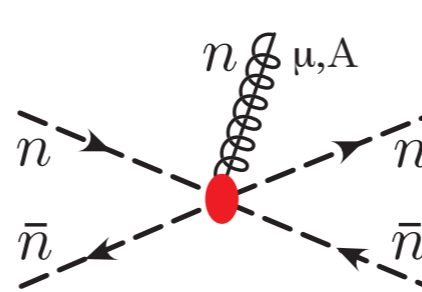


offshell hard
onshell collinear

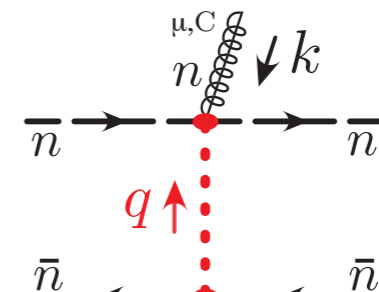
SCET



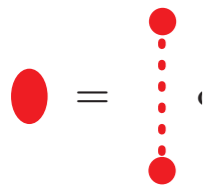
has onshell collinear



just offshell Glauber potential



from W
Wilson
lines



sums agree after using equations of motion

- This calculation differs from hard scattering Wilson lines due to onshell term on EFT side
- Similar for other operators

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\text{II}(0)} = \sum_{n, \bar{n}} \sum_{i, j=q, g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_n \sum_{i, j=q, g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$



(3 rapidity sectors)

sum pairwise
on all collinears



(2 rapidity sectors)

sum on all
collinears

- Interactions with more sectors is given by T-products
- No Wilson coefficient ie. no new structures at loop level. [more later]

$$\mathcal{O}_n^{qB} = \bar{\chi}_n T^B \frac{\not{n}}{2} \chi_n$$

$$\mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\bar{n}}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}_{\bar{n}}^{qB} = \bar{\chi}_{\bar{n}} T^B \frac{\not{\bar{n}}}{2} \chi_{\bar{n}}$$

$$\mathcal{O}_{\bar{n}}^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{\bar{n}\perp\mu}^C \frac{n}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{\bar{n}\perp}^{D\mu}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \left\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \tilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \tilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g \tilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \tilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T i g \tilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \right\}^{BC}$$

$$\mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}_S^n T^B \frac{\not{n}}{2} \psi_S^n \right)$$

$$\mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{n}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right)$$

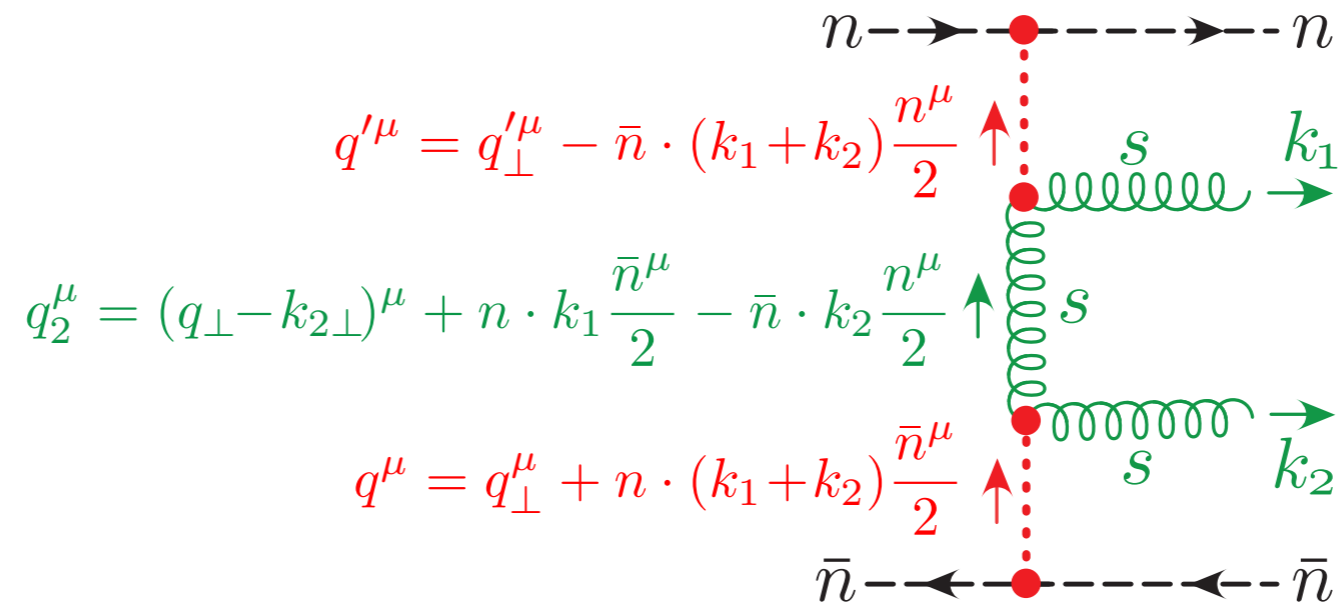
$$\mathcal{O}_s^{q_{\bar{n}} B} = 8\pi\alpha_s \left(\bar{\psi}_S^{\bar{n}} T^B \frac{\not{\bar{n}}}{2} \psi_S^{\bar{n}} \right)$$

$$\mathcal{O}_s^{g_{\bar{n}} B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{\bar{n}C} \frac{\bar{n}}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{\bar{n}D\mu} \right)$$

coming up
soon

Are there differences between $n-\bar{n}$ and $n-s$ forward scattering?

Yes. Consider:



Scattering is forward in one light-cone momentum at each soft vertex!

Allowed because:

$$n \cdot k_{1,2} \ll n \cdot p_{\bar{n}}$$

$$\bar{n} \cdot k_{1,2} \ll \bar{n} \cdot p_n$$

$\lambda \qquad \qquad \lambda^0$

there is an allowed routing of soft momentum through collinear lines

Soft \mathcal{O}_s^{BC} Operator

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \sum_i C_i O_i^{BC}$$

basis of $\mathcal{O}(\lambda^2)$ operators allowed by symmetries:

$$O_1 = \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu},$$

$$O_2 = \mathcal{P}_\perp^\mu \mathcal{S}_{\bar{n}}^T \mathcal{S}_n \mathcal{P}_{\perp\mu},$$

$$O_3 = \mathcal{P}_\perp \cdot (g\tilde{\mathcal{B}}_{S\perp}^n) (\mathcal{S}_n^T \mathcal{S}_{\bar{n}}) + (\mathcal{S}_n^T \mathcal{S}_{\bar{n}}) (g\tilde{\mathcal{B}}_{S\perp}^{\bar{n}}) \cdot \mathcal{P}_\perp,$$

$$O_4 = \mathcal{P}_\perp \cdot (g\tilde{\mathcal{B}}_{S\perp}^{\bar{n}}) (\mathcal{S}_{\bar{n}}^T \mathcal{S}_n) + (\mathcal{S}_{\bar{n}}^T \mathcal{S}_n) (g\tilde{\mathcal{B}}_{S\perp}^n) \cdot \mathcal{P}_\perp,$$

$$O_5 = \mathcal{P}_\mu^\perp (\mathcal{S}_n^T \mathcal{S}_{\bar{n}}) (g\tilde{\mathcal{B}}_{S\perp}^{n\mu}) + (g\tilde{\mathcal{B}}_{S\perp}^{n\mu}) (\mathcal{S}_n^T \mathcal{S}_{\bar{n}}) \mathcal{P}_\mu^\perp,$$

$$O_6 = \mathcal{P}_\mu^\perp (\mathcal{S}_{\bar{n}}^T \mathcal{S}_n) (g\tilde{\mathcal{B}}_{S\perp}^{n\mu}) + (g\tilde{\mathcal{B}}_{S\perp}^{n\mu}) (\mathcal{S}_{\bar{n}}^T \mathcal{S}_n) \mathcal{P}_\mu^\perp,$$

$$O_7 = (g\tilde{\mathcal{B}}_{S\perp}^{n\mu}) \mathcal{S}_n^T \mathcal{S}_{\bar{n}} (g\tilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}}),$$

$$O_8 = (g\tilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu}) \mathcal{S}_{\bar{n}}^T \mathcal{S}_n (g\tilde{\mathcal{B}}_{S\perp\mu}^n),$$

$$O_9 = \mathcal{S}_n^T n_\mu \bar{n}_\nu (ig\tilde{G}_s^{\mu\nu}) \mathcal{S}_{\bar{n}},$$

$$O_{10} = \mathcal{S}_{\bar{n}}^T n_\mu \bar{n}_\nu (ig\tilde{G}_s^{\mu\nu}) \mathcal{S}_n,$$

← octet Wilson line

← octet reps

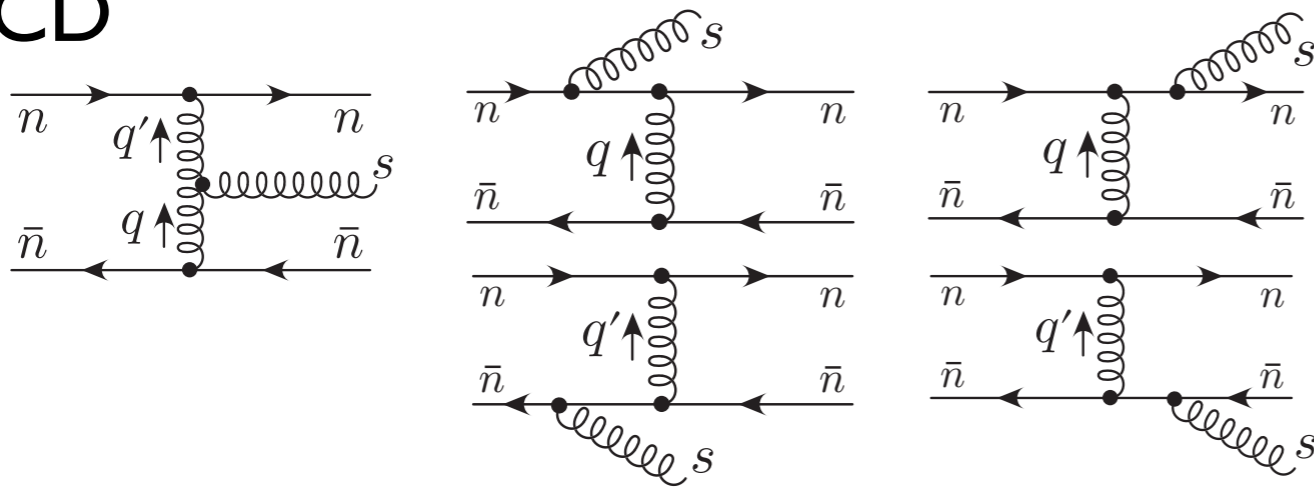
Restricted by: Hermiticity $O_i^\dagger|_{n\leftrightarrow\bar{n}} = O_i$, one \mathcal{S}_n , one $\mathcal{S}_{\bar{n}}$

operator identities: eg. $[\mathcal{P}_\perp^\mu (\mathcal{S}_n^T \mathcal{S}_{\bar{n}})] = -g\tilde{\mathcal{B}}_{S\perp}^{n\mu} (\mathcal{S}_n^T \mathcal{S}_{\bar{n}}) + (\mathcal{S}_n^T \mathcal{S}_{\bar{n}}) g\tilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu}$

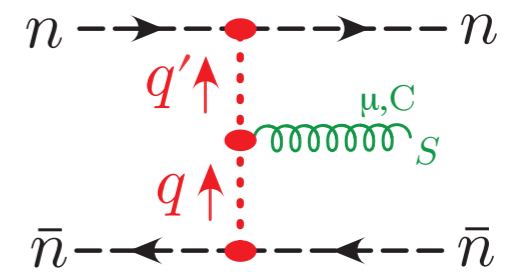
Matching with up to 2 soft gluons fixes all coefficients

One Soft Gluon:

QCD



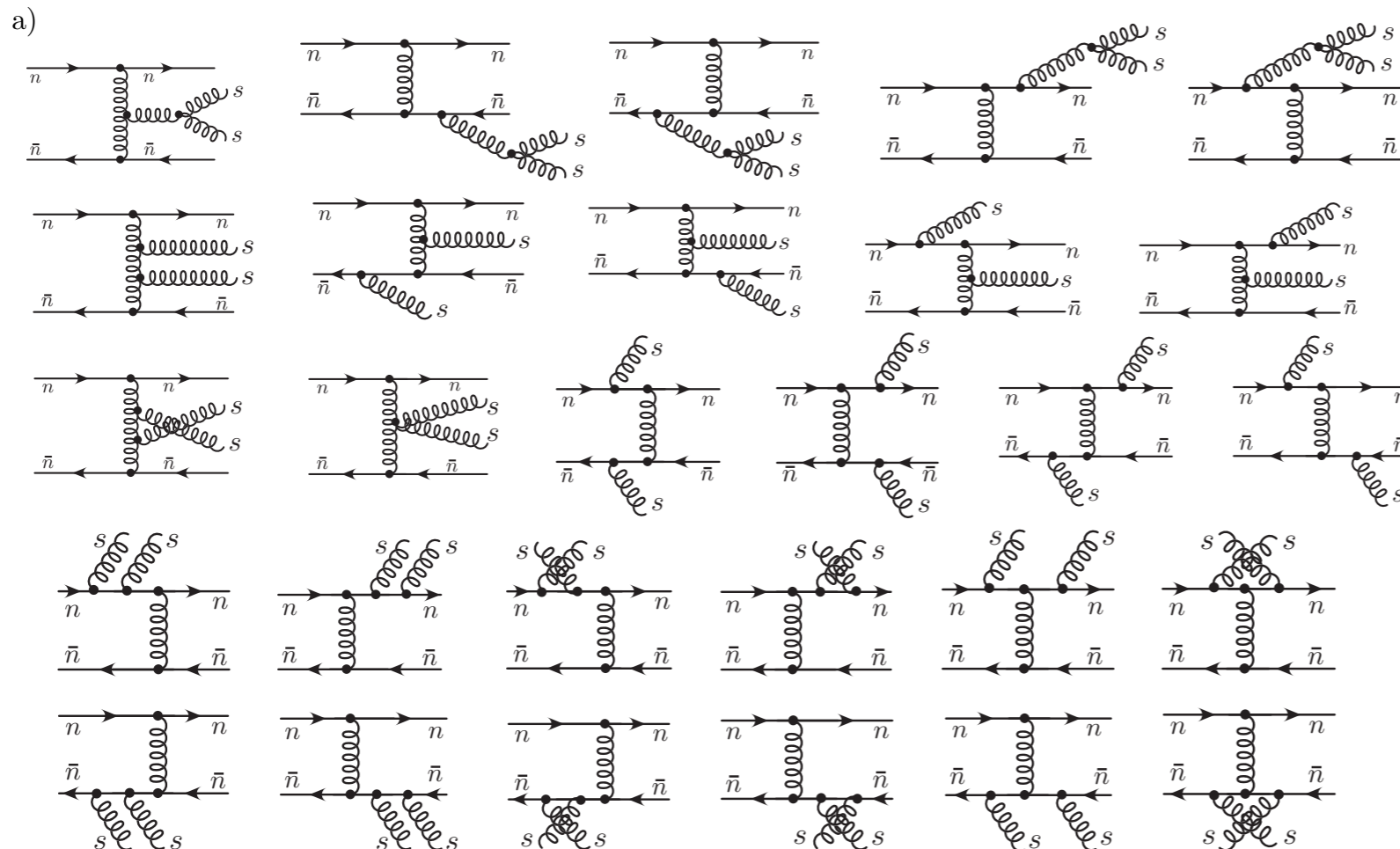
EFT



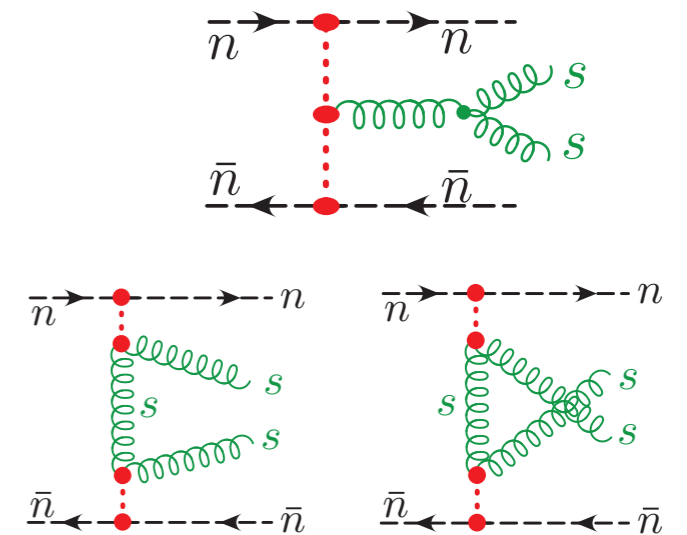
Lipatov vertex

Two Soft Gluons:

QCD



EFT



new vertex

Find:

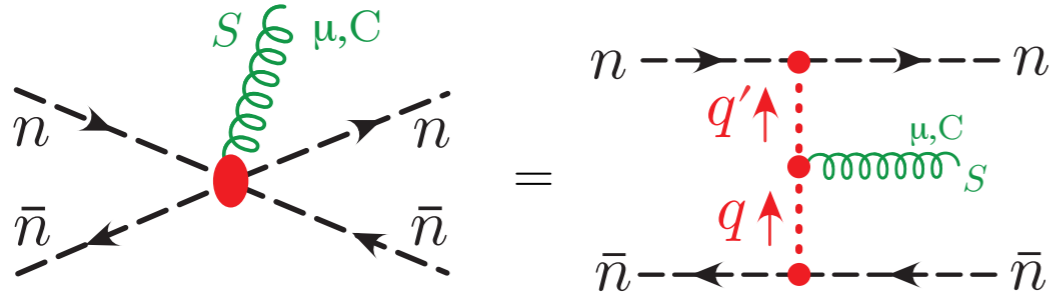
$$C_2 = C_4 = C_5 = C_6 = C_8 = C_{10} = 0,$$

$$C_1 = -C_3 = -C_7 = +1, \quad C_9 = -\frac{1}{2}$$

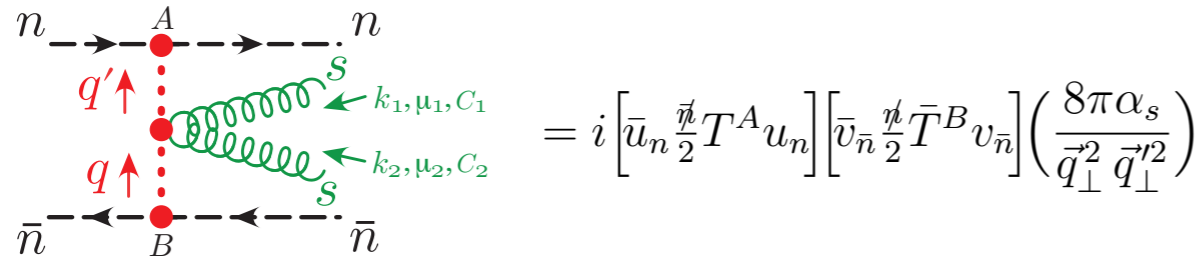
$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \left\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \tilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \tilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g \tilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \tilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}} \right. \\ \left. - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T i g \tilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \right\}^{BC}.$$

Form is unique to all loops since there are no hard α_s corrections to this matching (more later)

More Feynman Rules:



$$= i \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[\frac{8\pi\alpha_s}{\vec{q}_\perp^2 \vec{q}'_\perp^2} i g f^{ABC} \left(q_\perp^\mu + q'_\perp{}^\mu - n \cdot q \frac{\bar{n}^\mu}{2} - \bar{n} \cdot q' \frac{n^\mu}{2} - \frac{n^\mu \vec{q}'_\perp^2}{n \cdot q} - \frac{\bar{n}^\mu \vec{q}_\perp^2}{\bar{n} \cdot q'} \right) \right] \left[\bar{v}_{\bar{n}} \frac{\not{\bar{n}}}{2} \bar{T}^B v_{\bar{n}} \right]$$



$$= i \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[\bar{v}_{\bar{n}} \frac{\not{\bar{n}}}{2} \bar{T}^B v_{\bar{n}} \right] \left(\frac{8\pi\alpha_s}{\vec{q}_\perp^2 \vec{q}'_\perp^2} \right)$$

$$\times \left\{ g^2 f_{C_1 A E} f_{C_2 B E} \left[-g_\perp^{\mu_1 \mu_2} - \frac{n^{\mu_1} (2q_\perp^{\mu_2} + k_{2\perp}^{\mu_2})}{n \cdot k_1} + \frac{(2q'_\perp{}^{\mu_1} - k_{1\perp}^{\mu_1}) \bar{n}^{\mu_2}}{\bar{n} \cdot k_2} + \frac{\bar{n}^{\mu_1} n^{\mu_2} - n^{\mu_1} \bar{n}^{\mu_2}}{2} \right. \right. \\ \left. \left. + \frac{n^{\mu_1} \bar{n}^{\mu_2}}{n \cdot k_1 \bar{n} \cdot k_2} \left(\vec{q}_\perp \cdot \vec{q}'_\perp + \vec{k}_{1\perp} \cdot \vec{k}_{2\perp} + \vec{k}_{1\perp} \cdot \vec{q}'_\perp - \vec{k}_{2\perp} \cdot \vec{q}_\perp - \frac{1}{2} n \cdot k_2 \bar{n} \cdot k_2 - \frac{1}{2} n \cdot k_1 \bar{n} \cdot k_1 \right) \right. \right. \\ \left. \left. + n^{\mu_1} n^{\mu_2} \left(\frac{\vec{q}'_\perp^2}{n \cdot q} + \frac{\bar{n} \cdot k_2}{2n \cdot k_1} \right) + \bar{n}^{\mu_1} \bar{n}^{\mu_2} \left(\frac{-\vec{q}_\perp^2}{\bar{n} \cdot k_1 \bar{n} \cdot q'} + \frac{n \cdot k_1}{2\bar{n} \cdot k_2} \right) \right] \right. \\ \left. + g^2 f_{C_2 A E} f_{C_1 B E} \left[-g_\perp^{\mu_1 \mu_2} + \frac{\bar{n}^{\mu_1} (2q'_\perp{}^{\mu_2} - k_{2\perp}^{\mu_2})}{\bar{n} \cdot k_1} - \frac{(2q_\perp^{\mu_1} + k_{1\perp}^{\mu_1}) n^{\mu_2}}{n \cdot k_2} + \frac{n^{\mu_1} \bar{n}^{\mu_2} - \bar{n}^{\mu_1} n^{\mu_2}}{2} \right. \right. \\ \left. \left. + \frac{\bar{n}^{\mu_1} n^{\mu_2}}{n \cdot k_2 \bar{n} \cdot k_1} \left(\vec{q}_\perp \cdot \vec{q}'_\perp + \vec{k}_{1\perp} \cdot \vec{k}_{2\perp} + \vec{k}_{2\perp} \cdot \vec{q}'_\perp - \vec{k}_{1\perp} \cdot \vec{q}_\perp - \frac{1}{2} n \cdot k_2 \bar{n} \cdot k_2 - \frac{1}{2} n \cdot k_1 \bar{n} \cdot k_1 \right) \right. \right. \\ \left. \left. + n^{\mu_1} n^{\mu_2} \left(\frac{\vec{q}'_\perp^2}{n \cdot q} + \frac{\bar{n} \cdot k_1}{2n \cdot k_2} \right) + \bar{n}^{\mu_1} \bar{n}^{\mu_2} \left(\frac{-\vec{q}_\perp^2}{\bar{n} \cdot k_2 \bar{n} \cdot q'} + \frac{n \cdot k_2}{2\bar{n} \cdot k_1} \right) \right] \right\}$$

plus analogs
replacing collinear
quarks by gluons

One Loop EFT graphs

- QCD topologies will appear more than once (soft, collinear, ...)
- Each dominated by one invariant mass scale & one rapidity
- Require invariant mass regulator (dim.reg.)

Requires rapidity regulator for Glauber potential $|2k^z|^{-\eta} \nu^\eta$
and for Wilson lines

Use **Chiu, Jain, Neill, Rothstein** works like $\overline{\text{MS}}$:

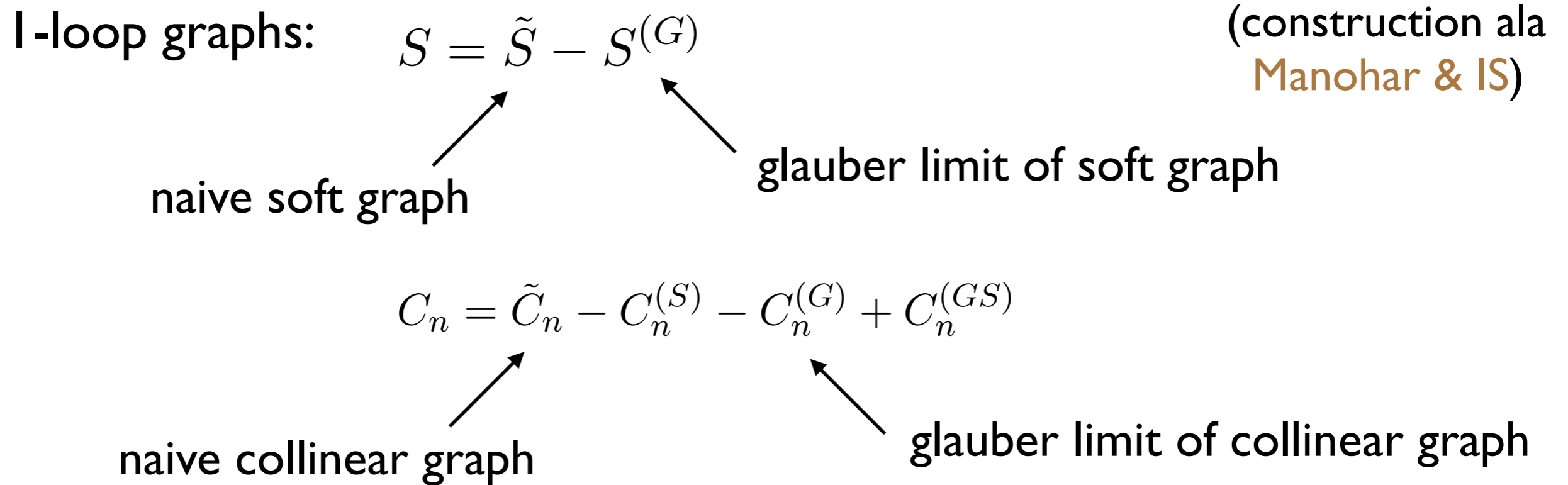
$$S_n = \sum_{\text{perms}} \exp \left\{ \frac{-g}{n \cdot \mathcal{P}} \left[\frac{w |2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} n \cdot A_s \right] \right\} \quad \frac{1}{\eta} \quad \ln(\nu)$$

$$W_n = \sum_{\text{perms}} \exp \left\{ \frac{-g}{\bar{n} \cdot \mathcal{P}} \left[\frac{w^2 |\bar{n} \cdot \mathcal{P}|^{-\eta}}{\nu^{-\eta}} \bar{n} \cdot A_n \right] \right\}$$

\updownarrow

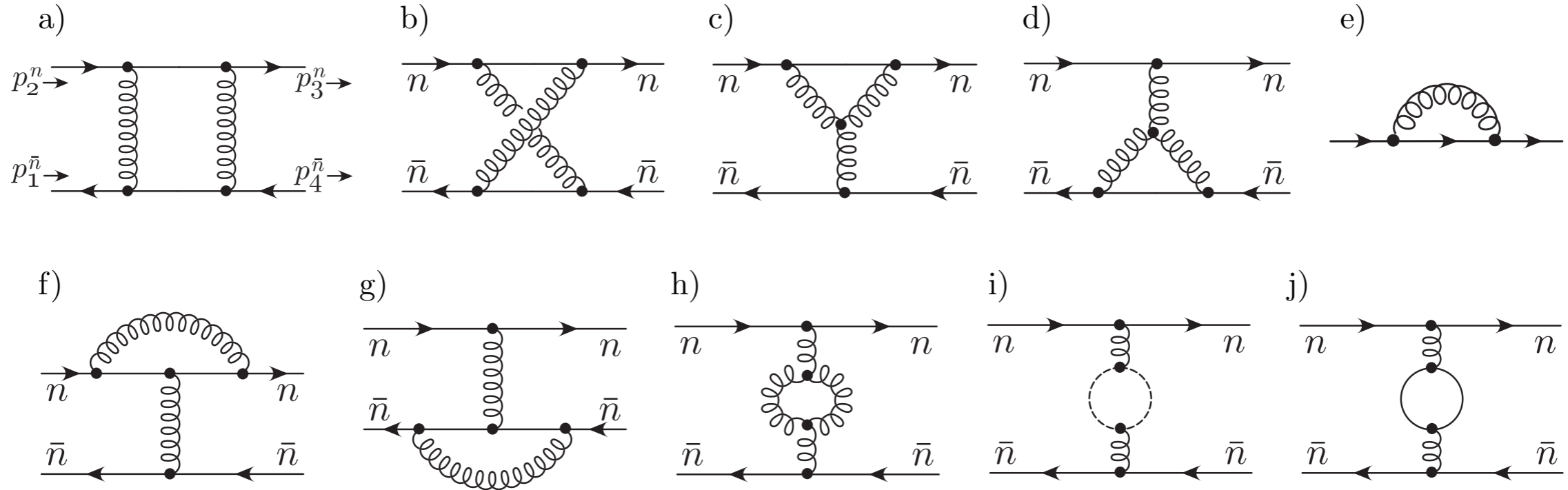
$$\frac{1}{\epsilon} \quad \ln(\mu)$$

- Zero-bin subtractions, avoid double counting IR regions



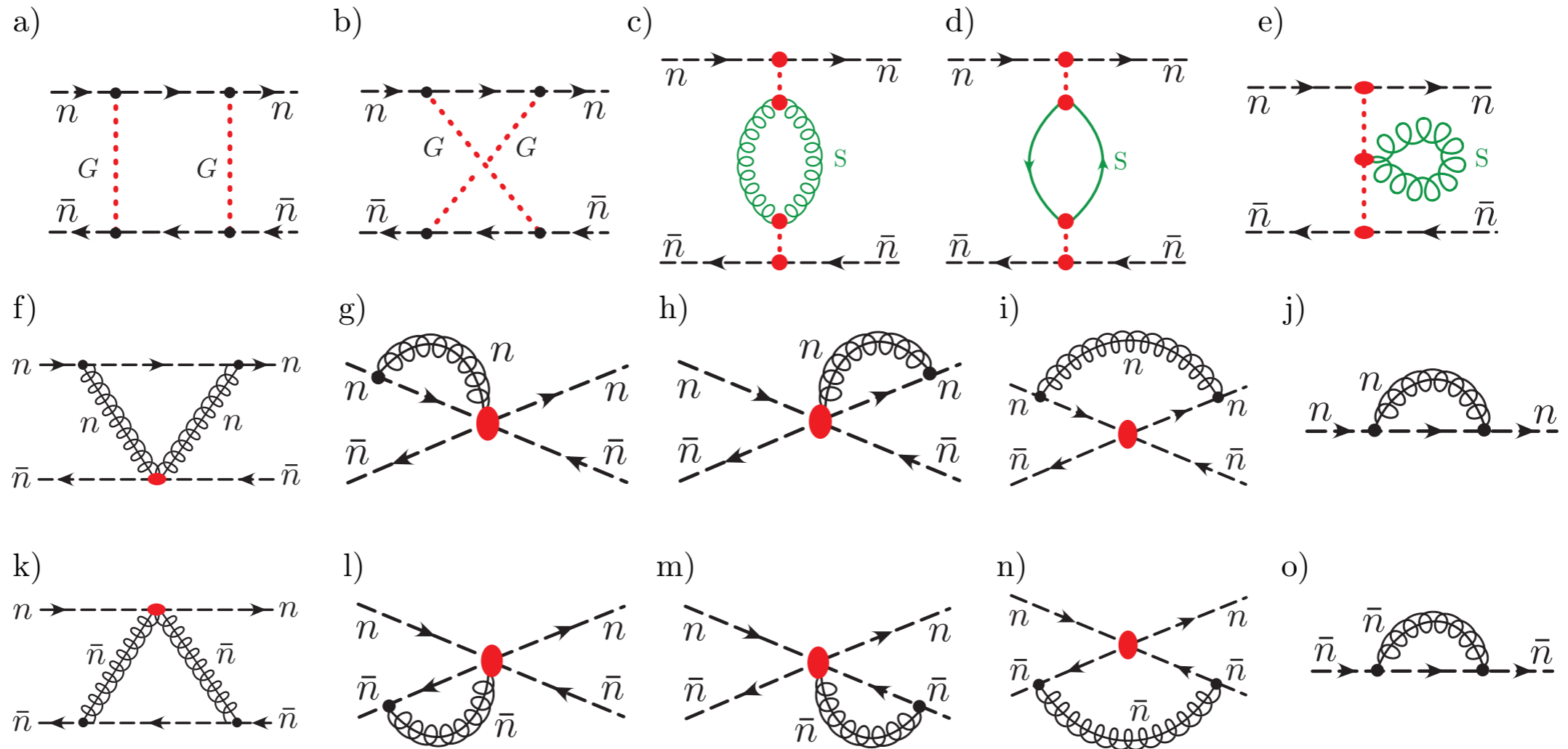
eq. One Loop $q\bar{q}$ scattering

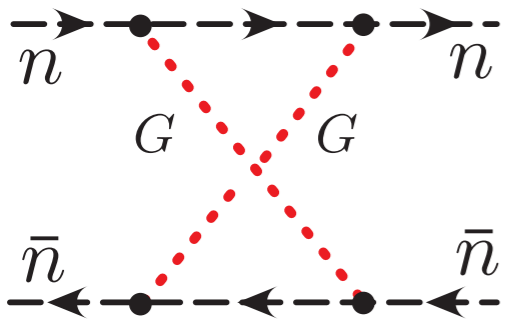
QCD graphs with leading power contributions, $s \gg t$



eq. One Loop $q\bar{q}$ scattering

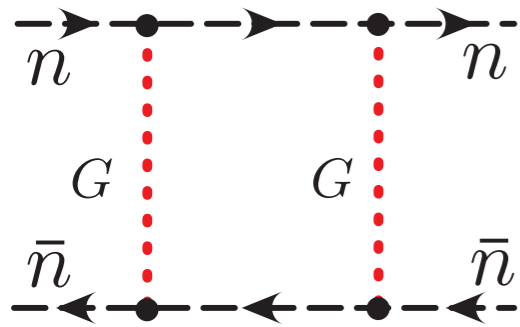
Leading Power EFT graphs (Glauber, Soft, & Collinear Loops)



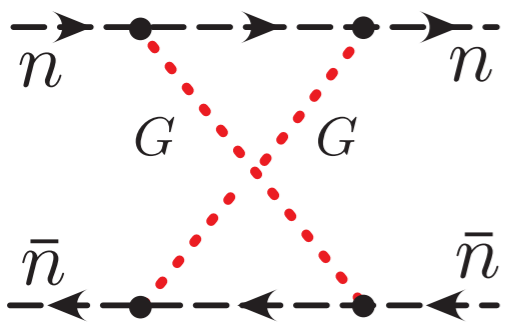


$$I_{\text{Gcbox}} = \int \frac{d^{d-2}k_{\perp} d k^+ d k^-}{2(\vec{k}_{\perp}^2)(\vec{k}_{\perp} + \vec{q}_{\perp})^2 \left(k^+ + p_3^+ - (\vec{k}_{\perp} + \vec{q}_{\perp}/2)^2/p_2^- + i0 \right) \left(+k^- + p_1^- - (\vec{k}_{\perp} + \vec{q}_{\perp}/2)^2/p_1^+ + i0 \right)}$$

Must regulate light-cone singularities.



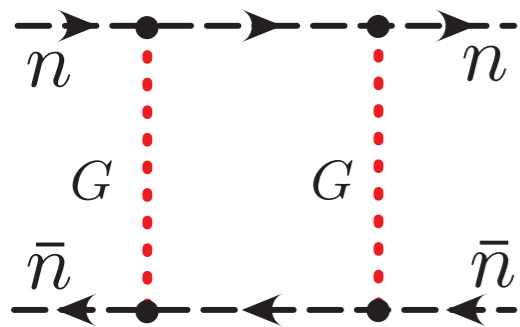
$$I_{\text{Gbox}} = \int \frac{d^{d-2}k_{\perp} d k^+ d k^-}{2(\vec{k}_{\perp}^2)(\vec{k}_{\perp} + \vec{q}_{\perp})^2 \left(k^+ + p_3^+ - (\vec{k}_{\perp} + \vec{q}_{\perp}/2)^2/p_2^- + i0 \right) \left(-k^- + p_4^- - (\vec{k}_{\perp} + \vec{q}_{\perp}/2)^2/p_1^+ + i0 \right)}$$



$$I_{\text{Gcbox}} = \int \frac{d^{d-2}k_{\perp} d k^0 d k^z |k^z|^{-2\eta} (\nu/2)^{2\eta}}{(\vec{k}_{\perp}^2)(\vec{k}_{\perp} + \vec{q}_{\perp})^2 (k^+ - \Delta_1(k_{\perp}) + i0) (k^- - \Delta_2(k_{\perp}) + i0)}$$

$$= 0$$

k^0 poles on same side



$$I_{\text{Gbox}} = \int \frac{d^{d-2}k_{\perp} d k^0 d k^z |k^z|^{-2\eta} (\nu/2)^{2\eta}}{(\vec{k}_{\perp}^2)(\vec{k}_{\perp} + \vec{q}_{\perp})^2 (k^+ - \Delta_1(k_{\perp}) + i0) (-k^- - \Delta_2(k_{\perp}) + i0)}$$

$$= -i \int \frac{d^{d-2}k_{\perp} d k^z |k^z|^{-2\eta} (\nu/2)^{2\eta}}{(\vec{k}_{\perp}^2)(\vec{k}_{\perp} + \vec{q}_{\perp})^2 (-2k^z - \Delta + i0)}$$

$$= \frac{-i}{4\pi} \int \frac{d^{d-2}k_{\perp}}{(\vec{k}_{\perp}^2)(\vec{k}_{\perp} + \vec{q}_{\perp})^2} \left[(\nu/2)^{2\eta} (-2i\pi) \csc(2\pi\eta) \sin(\pi\eta) (i\Delta)^{-2\eta} \right]$$

$$= \left(\frac{-i}{4\pi} \right) \int \frac{d^{d-2}k_{\perp}}{(\vec{k}_{\perp}^2)(\vec{k}_{\perp} + \vec{q}_{\perp})^2} \left[-i\pi + \mathcal{O}(\eta) \right]$$

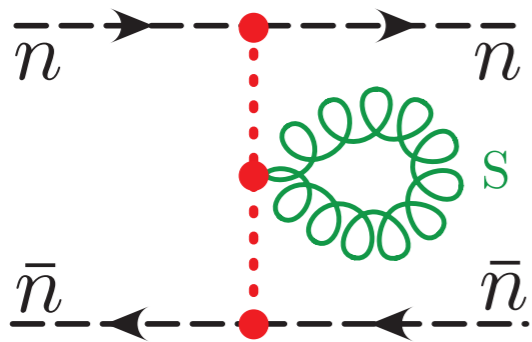
generic,
Glauber phase

independent of Δ
hence collinears behave
like eikonal propagators
here (but not always!)

Same for other Glauber
boxes (softs, gluons, ...).
Just replace color.

Rapidity divergences from Pure Wilson line terms

e)

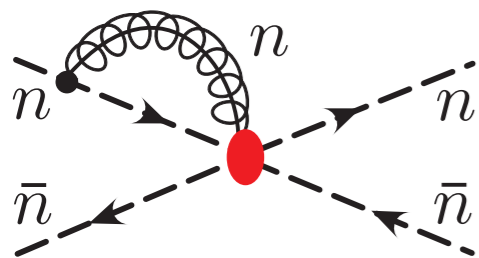


$$= \frac{4g^4}{t} \mathcal{S}_3^{n\bar{n}} w^2 \iota^\epsilon \mu^{2\epsilon} \int \bar{d}^d k \frac{|k_z|^{-\eta} \nu^\eta}{[k^2 - m^2](n \cdot k)(\bar{n} \cdot k)}$$

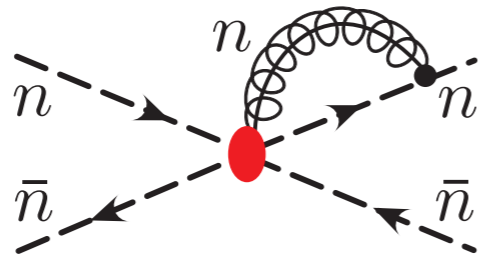
$$S = \tilde{S} - S^{(G)}$$

Subtraction in Glauber limit makes $\pm i0$ irrelevant.

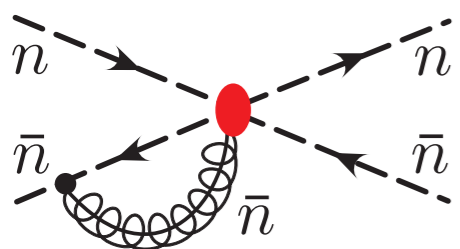
g)



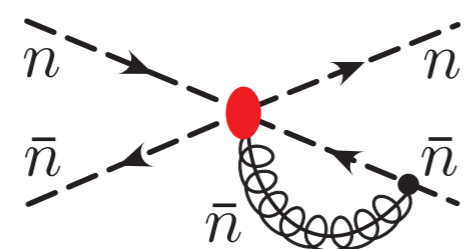
h)



l)



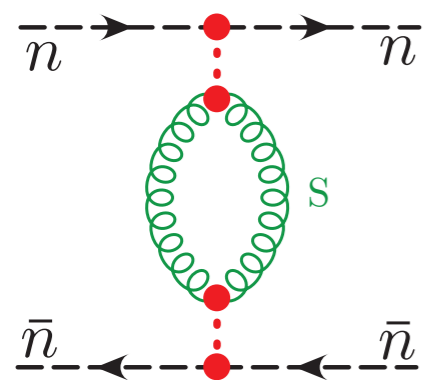
m)



$$= \mathcal{S}_3^{n\bar{n}} \frac{2w^2 g^4}{t} \left[\int \bar{d}^d k \frac{(\iota^\epsilon \mu^{2\epsilon} |\bar{n} \cdot k|^{-\eta} \nu^\eta) \bar{n} \cdot (k + p_3)}{[k^2 - m^2](k + p_3)^2(\bar{n} \cdot k)} + \int \bar{d}^d k \frac{(\iota^\epsilon \mu^{2\epsilon} |n \cdot k|^{-\eta} \nu^\eta) n \cdot (k + p_4)}{[k^2 - m^2](k + p_4)^2(n \cdot k)} \right]$$

More Rapidity divergences from Wilson lines

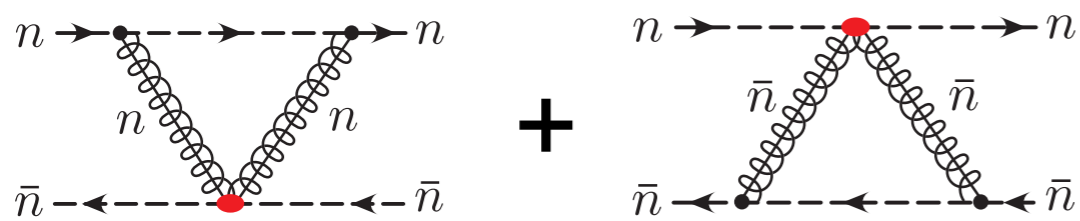
$$\mu^2 \sim \nu^2 \sim -t$$



$$= -\frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left\{ \frac{8}{\eta} g(\epsilon, \mu^2/t) + \frac{4}{\epsilon^2} + \frac{4}{\epsilon} \ln\left(\frac{\mu^2}{\nu^2}\right) + \boxed{4 \ln\left(\frac{\mu^2}{\nu^2}\right) \ln\left(\frac{\mu^2}{-t}\right)} - 2 \ln^2\left(\frac{\mu^2}{-t}\right) + \frac{\pi^2}{3} \right. \\ \left. + 2 \left(-\frac{11}{3\epsilon} - \frac{11}{3} \ln\frac{\mu^2}{-t} - \frac{67}{9} \right) \right\}$$

rapidity divergent

β -function without ghosts



$$= \frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left[\left\{ \frac{4}{\eta} g(\epsilon, \mu^2/t) - \frac{4}{\epsilon} \ln\left(\frac{\nu}{\bar{n} \cdot p_3}\right) - 4 \ln\left(\frac{\nu}{\bar{n} \cdot p_3}\right) \ln\left(\frac{\mu^2}{-t}\right) - \frac{3}{\epsilon} - 3 \ln\left(\frac{\mu^2}{-t}\right) - 6 + \frac{4\pi^2}{3} \right\} \right. \\ \left. + \left\{ \frac{4}{\eta} g(\epsilon, \mu^2/t) - \frac{4}{\epsilon} \ln\left(\frac{\nu}{n \cdot p_4}\right) - 4 \ln\left(\frac{\nu}{n \cdot p_4}\right) \ln\left(\frac{\mu^2}{-t}\right) - \frac{3}{\epsilon} - 3 \ln\left(\frac{\mu^2}{-t}\right) - 6 + \frac{4\pi^2}{3} \right\} \right] \\ = \frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left\{ \frac{8}{\eta} g(\epsilon, \mu^2/t) - \frac{4}{\epsilon} \ln\left(\frac{\nu^2}{s}\right) - \boxed{4 \ln\left(\frac{\nu^2}{s}\right) \ln\left(\frac{\mu^2}{-t}\right)} - \frac{6}{\epsilon} - 6 \ln\left(\frac{\mu^2}{-t}\right) - 12 + \frac{8\pi^2}{3} \right\}.$$

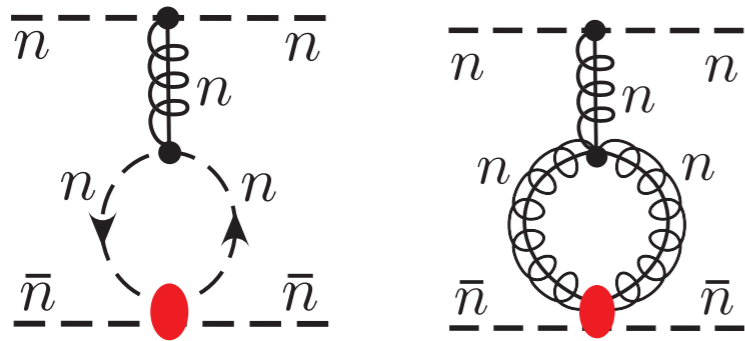
opposite sign for this
rapidity divergence

$$\mu^2 \sim -t$$

$$\nu^2 \sim s$$

Non-trivial 0-bin example:

b)



a collinear propagator here is
double counting Glauber region

graphs already accounted for
by soft loops

$$C = \tilde{C} - C^{(S)} - C^{(G)} + C^{(SG)}$$

find $\tilde{C} - C^{(S)} = 0$

$$C^{(G)} - C^{(SG)} = 0$$

$C = 0$ no double counting

One Loop Results & Matching

$$\mathcal{S}_1^{n\bar{n}} = -\left[\bar{u}_n T^A T^B \frac{\not{\eta}}{2} u_n\right] \left[\bar{v}_{\bar{n}} \bar{T}^A \bar{T}^B \frac{\not{\eta}}{2} v_{\bar{n}}\right], \quad \mathcal{S}_2^{n\bar{n}} = C_F \left[\bar{u}_n T^A \frac{\not{\eta}}{2} u_n\right] \left[\bar{v}_{\bar{n}} \bar{T}^A \frac{\not{\eta}}{2} v_{\bar{n}}\right],$$

$$\mathcal{S}_3^{n\bar{n}} = C_A \left[\bar{u}_n T^A \frac{\not{\eta}}{2} u_n\right] \left[\bar{v}_{\bar{n}} \bar{T}^A \frac{\not{\eta}}{2} v_{\bar{n}}\right], \quad \mathcal{S}_4^{n\bar{n}} = T_F n_f \left[\bar{u}_n T^A \frac{\not{\eta}}{2} u_n\right] \left[\bar{v}_{\bar{n}} \bar{T}^A \frac{\not{\eta}}{2} v_{\bar{n}}\right].$$

$m =$ gluon mass IR regulator

$$\text{Glauber Loops} = \frac{i\alpha_s^2}{t} \mathcal{S}_1^{n\bar{n}} \left[8i\pi \ln\left(\frac{-t}{m^2}\right) \right]$$

$$\text{Soft Loops} = \frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left\{ -\frac{8}{\eta} h(\epsilon, \mu^2/m^2) - \frac{8}{\eta} g(\epsilon, \mu^2/t) - 4 \ln\left(\frac{\mu^2}{\nu^2}\right) \ln\left(\frac{m^2}{-t}\right) \right. \\ \left. - 2 \ln^2\left(\frac{\mu^2}{m^2}\right) + 2 \ln^2\left(\frac{\mu^2}{-t}\right) - \frac{2\pi^2}{3} + \frac{22}{3} \ln\frac{\mu^2}{-t} + \frac{134}{9} \right\} \leftarrow \text{no } 1/\epsilon \text{ poles}$$

(after coupling renormalization)

$$+ \frac{i\alpha_s^2}{t} \mathcal{S}_4^{n\bar{n}} \left[-\frac{8}{3} \ln\left(\frac{\mu^2}{-t}\right) - \frac{40}{9} \right].$$

$$\text{Collinear Loops} = \frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left\{ \frac{8}{\eta} h\left(\epsilon, \frac{\mu^2}{m^2}\right) + \frac{8}{\eta} g\left(\epsilon, \frac{\mu^2}{-t}\right) + 4 \ln\left(\frac{\nu^2}{s}\right) \ln\left(\frac{-t}{m^2}\right) + 2 \ln^2\left(\frac{m^2}{-t}\right) + 4 + \frac{4\pi^2}{3} \right\}$$

$$+ \frac{i\alpha_s^2}{t} \mathcal{S}_2^{n\bar{n}} \left[-4 \ln^2\left(\frac{m^2}{-t}\right) - 12 \ln\left(\frac{m^2}{-t}\right) - 14 \right]$$

One Loop Results & Matching

$$\begin{aligned} \mathcal{S}_1^{n\bar{n}} &= -\left[\bar{u}_n T^A T^B \frac{\not{\eta}}{2} u_n\right] \left[\bar{v}_{\bar{n}} \bar{T}^A \bar{T}^B \frac{\not{\eta}}{2} v_{\bar{n}}\right], & \mathcal{S}_2^{n\bar{n}} &= C_F \left[\bar{u}_n T^A \frac{\not{\eta}}{2} u_n\right] \left[\bar{v}_{\bar{n}} \bar{T}^A \frac{\not{\eta}}{2} v_{\bar{n}}\right], \\ \mathcal{S}_3^{n\bar{n}} &= C_A \left[\bar{u}_n T^A \frac{\not{\eta}}{2} u_n\right] \left[\bar{v}_{\bar{n}} \bar{T}^A \frac{\not{\eta}}{2} v_{\bar{n}}\right], & \mathcal{S}_4^{n\bar{n}} &= T_F n_f \left[\bar{u}_n T^A \frac{\not{\eta}}{2} u_n\right] \left[\bar{v}_{\bar{n}} \bar{T}^A \frac{\not{\eta}}{2} v_{\bar{n}}\right]. \end{aligned}$$

$m = \text{gluon mass IR regulator}$

$$\text{Glauber Loops} = \frac{i\alpha_s^2}{t} \mathcal{S}_1^{n\bar{n}} \left[8i\pi \ln\left(\frac{-t}{m^2}\right) \right]$$

$$\begin{aligned} \text{Soft Loops} &= \frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left\{ -\frac{8}{\eta} h(\epsilon, \mu^2/m^2) - \frac{8}{\eta} g(\epsilon, \mu^2/t) - 4 \ln\left(\frac{\mu^2}{\nu^2}\right) \ln\left(\frac{m^2}{-t}\right) \right. \\ &\quad \left. - 2 \ln^2\left(\frac{\mu^2}{m^2}\right) + 2 \ln^2\left(\frac{\mu^2}{-t}\right) - \frac{2\pi^2}{3} + \frac{22}{3} \ln\frac{\mu^2}{-t} + \frac{134}{9} \right\} \\ &\quad + \frac{i\alpha_s^2}{t} \mathcal{S}_4^{n\bar{n}} \left[-\frac{8}{3} \ln\left(\frac{\mu^2}{-t}\right) - \frac{40}{9} \right]. \end{aligned} \quad = 4K$$

$$\begin{aligned} \text{Collinear Loops} &= \frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left\{ \frac{8}{\eta} h\left(\epsilon, \frac{\mu^2}{m^2}\right) + \frac{8}{\eta} g\left(\epsilon, \frac{\mu^2}{-t}\right) + 4 \ln\left(\frac{\nu^2}{s}\right) \ln\left(\frac{-t}{m^2}\right) + 2 \ln^2\left(\frac{m^2}{-t}\right) + 4 + \frac{4\pi^2}{3} \right\} \\ &\quad + \frac{i\alpha_s^2}{t} \mathcal{S}_2^{n\bar{n}} \left[-4 \ln^2\left(\frac{m^2}{-t}\right) - 12 \ln\left(\frac{m^2}{-t}\right) - 14 \right] \end{aligned}$$

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{5}{9} n_f$$

Note: full 2-loop cusp anomalous dimension appears in finite part of one-loop soft.

$i\pi$ purely from Glauber loop

$$\begin{aligned} \text{Total SCET} &= \frac{i\alpha_s^2}{t} \mathcal{S}_1^{n\bar{n}} \left[8i\pi \ln \left(\frac{-t}{m^2} \right) \right] + \frac{i\alpha_s^2}{t} \mathcal{S}_2^{n\bar{n}} \left[-4 \ln^2 \left(\frac{m^2}{-t} \right) - 12 \ln \left(\frac{m^2}{-t} \right) - 14 \right] \\ &+ \frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left\{ -4 \ln \left(\frac{s}{-t} \right) \ln \left(\frac{-t}{m^2} \right) + \frac{22}{3} \ln \frac{\mu^2}{-t} + \frac{170}{9} + \frac{2\pi^2}{3} \right\} \\ &+ \frac{i\alpha_s^2}{t} \mathcal{S}_4^{n\bar{n}} \left[-\frac{8}{3} \ln \left(\frac{\mu^2}{-t} \right) - \frac{40}{9} \right] \end{aligned}$$

rapidity divergences cancel
leave behind large log

Total SCET = Total QCD ($s \gg t$)

- IR divergences are all reproduced
- no hard matching (no loops with momenta $\sim s$)

offshell lines in loop graphs are always sequestered in tree level subcomponents



No loop corrections to $\mathcal{L}_G^{\text{II}(0)}$

Conclusion (so far)

- Constructed an EFT for $s \gg t$, Fwd. Scattering & Fact. Violation
- Universal Operators that can be used for many processes & problems

More in Ira's talk

Part II