

## Einstein, Kant, and the A Priori

Michael Friedman

Kant's original version of transcendental philosophy took both Euclidean geometry and the Newtonian laws of motion to be synthetic a priori constitutive principles—which, from Kant's point of view, function as necessary presuppositions for applying our fundamental concepts of space, time, matter, and motion to our sensible experience of the natural world. Although Kant had very good reasons to view the principles in question as having such a constitutively a priori role, we now know, in the wake of Einstein's work, that they are not in fact a priori in the stronger sense of being fixed necessary conditions for all human experience in general, eternally valid once and for all. And it is for precisely this reason that Kant's original version of transcendental philosophy must now be either rejected entirely or (at least) radically reconceived. Most philosophy of science since Einstein has taken the former route: the dominant view in logical empiricism, for example, was that the Kantian synthetic a priori had to be rejected once and for all in the light of the general theory of relativity.

Yet Hans Reichenbach took the latter route in his first published book: in *Relativitätstheorie und Erkenntnis Apriori* (1920) he proposed instead that Kantian constitutively a priori principles of geometry and mechanics should be *relativized* to a given time in a given theoretical context. Such principles still function, throughout the development from Newton to Einstein, as necessary presuppositions for applying our (changing) conceptions of space, time, and motion to our sensible experience, but they are no longer eternally valid once and for all. For example, while Euclidean geometry and the Newtonian laws of motion are indeed necessary conditions for giving empirical meaning to the Newtonian theory of universal gravitation, the situation in Einstein's general theory relativity is quite different. The crucial mediating role between abstract mathematical theory and concrete sensible experience is now played by the light principle and the principle of equivalence, which together insure that Einstein's revolutionary new description of gravitation by a four-dimensional geometry of variable curvature in fact says something about concrete empirical phenomena: namely, the behavior of light and gravitationally interacting bodies.

In my recent book, *Dynamics of Reason* (2001), I have taken up, and further developed, Reichenbach's idea. But my implementation of this idea of relativized constitutively a priori principles (of geometry and mechanics) essentially depends on an historical argument describing the developmental process by which the transition from

Newton to Einstein actually took place, as mediated, in my view, by the parallel developments in scientific philosophy involving, especially, Hermann von Helmholtz, Ernst Mach, and Henri Poincaré. However, since this argument depends on the concrete details of the actual historical process in question, it would therefore appear to be entirely contingent. How, then, can it possibly be comprehended within a properly *transcendental* philosophy? Indeed, once we have given up on Kant's original ambition to delineate in advance the a priori structure of all possible scientific theories, it might easily seem that a properly transcendental argument is impossible. We have no way of anticipating a priori the specific constitutive principles of future theories, and so all we can do, it appears, is wait for the historical process to show us what emerges a posteriori as a matter of fact. So how, more generally, can we develop a philosophical understanding of the evolution of modern science that is at once genuinely historical and properly transcendental?

Let us begin by asking how Kant's original transcendental method is supposed to explain the sense in which certain fundamental principles of geometry and mechanics are, in fact, both a priori and necessary. This method, of course, appeals to Kant's conception of the two rational faculties of sensibility and understanding. The answer to the question "how is pure mathematics possible?" appeals to the necessary structure of our pure sensibility, as articulated in the Transcendental Aesthetic of the *Critique of Pure Reason*; the answer to the question "how is pure natural science possible?" appeals to the necessary structure of our pure understanding, as articulated in the Transcendental Analytic. Yet there is an obvious objection to this procedure: how can such proposed transcendental explanations inherit the (assumed) a priori necessity of the sciences whose possibility they purport to explain unless we can also somehow establish that they are the *unique* such explanations? From our present point of view, for example, it does not appear that Kant's explanation of the possibility of pure mathematics is uniquely singled out in any way; on the contrary, our greatly expanded conception of purely logical or analytic truth suggests that an appeal to the faculty of pure sensibility may, after all, be explanatorily superfluous. Indeed, from the point of view of the anti-psychological approach to such questions that dominated much of twentieth-century analytic philosophy, it appears that all consideration of our subjective cognitive faculties is similarly explanatorily superfluous.

In Kant's own intellectual context, however, explanations of scientific knowledge in terms of our cognitive faculties were the norm—for empiricists, rationalists, and (of course) Aristotelians. Everyone agreed, in addition, that the relevant faculties to consider were the senses and the intellect; what was then controversial was the precise nature and

relative importance of the two. Empiricist views, which denied the existence of the pure intellect or its importance for scientific knowledge, were, for Kant, simply out of the question, since they make a priori rational knowledge incomprehensible. Moreover, the conception of the pure intellect that was most salient for Kant was that of Leibniz, where the structure of this faculty is delineated, in effect, by the logical forms of traditional Aristotelian syllogistic. But this conception of the pure intellect, Kant rightly saw, is entirely inadequate for representing, say, the assumed infinite extendibility and divisibility of geometrical space, which had recently proven itself to be both indispensable and extremely fruitful in Newtonian mathematical physics. Nevertheless, Newton's own conception of space as the divine sensorium was also unacceptable on theological and metaphysical grounds, and so the only live alternative left to Kant was the one he actually came up with: space is a pure form of our sensibility (as opposed to the divine sensibility), wherein *both* (infinitely iterable) geometrical construction *and* the perception of spatial objects in nature (like the heavenly bodies) then become first possible.

It is of course entirely contingent that Kant operated against the background of precisely these intellectual resources, just as it is entirely contingent that Kant was born in 1724 and died in 1804. Given these resources, however, and given the problems with which Kant was faced, the solution he came up with is not contingent. On the contrary, the intellectual situation in which he found himself had a definite "inner logic"—mathematical, logical, metaphysical, and theological—which allowed him to triangulate, as it were, on a practically unique (and in this sense necessary) solution.

Beginning with this understanding of Kant's transcendental method and its associated rational necessity, we can then see a way forward for extending this method to post-Kantian developments in both the mathematical exact sciences and transcendental philosophy. We can trace out how the "inner logic" of the relevant intellectual situation evolves and changes after Kant in response to both new developments in the mathematical exact sciences themselves and the manifold and intricate ways in which post-Kantian scientific philosophers attempted to reconfigure Kant's original version of transcendental philosophy in light of these developments. That each of these successive new intellectual situations has its own "inner logic" implies that the enterprise does not collapse into total contingency; that, in addition, they successively evolve out of, and in light of, Kant's original system suggests that it may still count as transcendental philosophy.

Hermann von Helmholtz's neo-Kantian scientific epistemology, for example, had deep roots in Kant's original conception. In particular, Helmholtz developed a distinctive

conception of space as a “subjective” and “*necessary* form of our external intuition” in the sense of Kant; and, while this conception was certainly developed within Helmholtz’s *empirical* program in sensory psychology and psycho-physics, it nevertheless retained important “transcendental” elements. More specifically, space is “transcendental,” for Helmholtz, in so far as the principle of free mobility (which allows arbitrary continuous motions of rigid bodies) is a necessary condition for the possibility of spatial measurement—and, indeed, for the very existence of space and spatial objects. Moreover, the condition of free mobility represents a natural generalization of Kant’s original (Euclidean) conception of geometrical construction, in the sense that Euclidean constructions with straight-edge and compass, carried out within Kant’s form of spatial intuition, are generated by the group of specifically Euclidean rigid motions (translations and rotations). The essential point, however, is that free mobility also holds for the classical non-Euclidean geometries of constant curvature (hyperbolic and elliptic), and so it is no longer a “transcendental” and “necessary” condition of our spatial intuition, for Helmholtz, that the space constructed from our perception of bodily motion obeys the specific laws of Euclidean geometry. Nevertheless, Helmholtz’s generalization of the Kantian conception of spatial intuition is, in an important sense, the *minimal* (and in this sense unique) such generalization consistent with the nineteenth-century discovery of non-Euclidean geometries.

The great French mathematician Henri Poincaré then transformed Helmholtz’s conception in turn. In particular, Poincaré’s use of the principle of free mobility (which plays a central role in his philosophy of geometry) is explicitly framed by a hierarchical conception of the mathematical sciences, beginning with arithmetic and proceeding through analysis, geometry, mechanics, and empirical physics—where, in particular, each lower level of the hierarchy (after arithmetic) *presupposes* that all earlier levels are already in place.

This hierarchical conception of the mathematical sciences underlies Poincaré’s fundamental disagreement with Helmholtz. For Helmholtz, as we have seen, the principle of free mobility expresses the necessary structure of our form of external intuition, and, following Kant, Helmholtz views all empirical investigation as necessarily taking place within this already given form. Helmholtz’s conception is Kantian, that is, in so far as space has a “necessary form” expressed in the condition of free mobility, but it is also empiricist in so far as which of the three possible geometries of constant curvature obtains is then determined by experience. For Poincaré, by contrast, although the principle of free mobility is still fundamental, our actual perceptual experience of bodily “displacements” arising in accordance with this principle is far too imprecise to

yield the empirical determination of a specific mathematical geometry: our only option, at this point, is to *stipulate* Euclidean geometry by convention, as the simplest and most convenient idealization of our actual perceptual experience. In particular, experiments with putatively rigid bodies, for Poincaré, involve essentially physical processes at the level of mechanics and experimental physics, and these sciences, in turn, *presuppose* that the science of geometry is already firmly in place. In the context of Poincaré's hierarchy, therefore, the principle of free mobility expresses our necessary freedom to choose—by a “convention or definition in disguise”—which of the three classical geometries of constant curvature is the most suitable idealization of physical space.

One of the most important applications of Poincaré's hierarchical conception involves his characteristic perspective on the problem of absolute space and the relativity of motion explained in his discussion of the next lower level in the hierarchy: (classical) mechanics. Poincaré's key idea is that what he calls the (physical) “law of relativity” rests squarely on the “relativity and passivity of space” and therefore reflects the circumstance, essential to free mobility, that the space constructed from our experience of bodily displacements is both homogeneous and isotropic: all points in space, and all directions through any given point, are, necessarily, geometrically equivalent. Thus, Poincaré's conception of the relativity of motion depends on his philosophy of geometry, and this is especially significant, from our present point of view, because Poincaré's ideas on the relativity of motion were also inextricably entangled with the deep problems then afflicting the electrodynamics of moving bodies that were eventually solved (according to our current understanding) by Einstein's special theory of relativity.

I shall return to Einstein below, but I first want to emphasize that the connection Poincaré makes between his philosophy of geometry and the relativity of motion represents a continuation of a problematic originally prominent in Kant. Helmholtz, as we have seen, transformed Kant's philosophy of space and geometry, and Ernst Mach, among others, participated in a parallel transformation of Kant's approach to the relativity of motion—which finally eventuated in the modern concept of an inertial frame of reference. Neither Helmholtz nor Mach, however, established any kind of conceptual connection between the foundations of geometry and the relativity of motion—which, at the time, appeared to be entirely independent of one another. On Kant's original approach to transcendental philosophy, by contrast, the two were actually very closely connected. While Kant's answer to the question “how is pure mathematics possible?” essentially involved his distinctive perspective on Euclidean constructive operations, his answer to the question “how is pure natural science possible” involved an analogous constructive procedure by which Newton, from Kant's point of view, arrived at

successive approximations to “absolute space” via a definite sequence of rule-governed operations starting with our parochial perspective here on earth and then proceeding to the center of mass of the solar system, the center of mass of the Milky Way galaxy, the center of mass of a system of such galaxies, and so on *ad infinitum*. Indeed, the way in which Kant thereby established a connection between the problem of space and geometry and the problem of the relativity of motion was intimately connected, in turn, with both the overarching conception of the relationship between sensibility and understanding that frames his transcendental method and his characteristic perspective, more generally, on the relationship between constitutive and regulative transcendental principles. (“Absolute space,” in particular, is a forever unreachable regulative idea of reason.)

Now it was Mach, as I have suggested, who first forged a connection between Kant’s original solution to the problem of “absolute space” and the late nineteenth-century solution based on the concept of an inertial frame of reference. (Kant, from a modern point of view, is constructing a sequence of better and better approximations to what we now call an inertial frame of reference.) And it is clear, moreover, that Poincaré was familiar with this late nineteenth-century solution as well. It is also clear, however, that Poincaré’s attempt to base his discussion of the relativity of motion on his philosophy of geometry runs into serious difficulties at precisely this point; for Poincaré is here forced to distinguish his “law of relativity” from what he calls the “principle of relative motion.” The latter applies only to inertial frames of reference, moving uniformly and rectilinearly with respect to one another, while the latter applies, as well, to non-inertial frames of reference in a state of uniform rotation: it follows from the “relativity and passivity” of space, for Poincaré, that uniform rotations of our coordinate axes should be just as irrelevant to the motions of a physical system as uniform translations. Therefore, the full “law of relativity,” as Poincaré says, “ought to impose itself upon us with the same force” as does the more restricted “principle of relative motion.” Poincaré must also admit, however, that the more extended “law of relativity” does not appear to be in accordance with our experiments (e.g., Newton’s famous rotating bucket experiment).

It is for this reason that Einstein’s appeal to what he calls the “principle of relativity” in his 1905 paper on special relativity is independent of Poincaré’s “law of relativity,” and it is also independent, accordingly, of Poincaré’s “conventionalist” philosophy of geometry. Einstein’s principle is limited, from the beginning, to inertial frames of reference (moving relative to one another with constant velocity and no rotation), and his concern is to apply this (limited) principle of relativity to both electromagnetic and mechanical phenomena. Thus, in particular, whereas Poincaré’s “law of

relativity” involves very strong a priori motivations deriving from his philosophy of geometry (based on the “relativity and passivity of space”), Einstein’s “principle of relativity” rests on the emerging experimental evidence suggesting that electro-magnetic and optical phenomena do not in fact distinguish one inertial frame from another. Einstein “conjectures” that this experimentally suggested law holds rigorously (and for all orders), and he proposes to “elevate” it to the status of a presupposition or postulate upon which a consistent electrodynamics of moving bodies may then be erected. Hence, Einstein’s understanding of the principle of relativity is also independent of Poincaré’s carefully constructed hierarchy of the mathematical sciences, and it is for precisely this reason, I suggest, that Poincaré himself could never accept Einstein’s theory.

Nevertheless, it appears overwhelmingly likely that, although Einstein did not embrace Poincaré’s “conventionalist” philosophy of geometry, Einstein’s use of the principle of relativity was explicitly inspired by Poincaré’s more general methodology described in *Science and Hypothesis*—according to which the fundamental principles of mechanics, in particular, are “conventions or definitions in disguise” arising from “experimental laws” that “have been elevated into principles to which our mind attributes an absolute value.” In Einstein’s case, the experimental law in question comprises the recent results in electrodynamics and optics, and Einstein now proposes to “elevate” both the principle of relativity and the light principle (which together imply that the velocity of light is invariant in all inertial frames) to the status of “presuppositions” or “postulates.” (To the best of my knowledge, this striking language in Einstein’s 1905 paper—together with its reappearance in 1921) has not been previously noted in the literature.) These two postulates together then allow us to “stipulate” a new “definition of simultaneity” (based on the assumed invariance of the velocity of light) implying a radical revision of the classical kinematics of space, time, and motion. In particular, whereas the fundamental kinematical structure of an inertial frame of reference, in classical mechanics, is defined by the Newtonian laws of motion, (a revised version of) this same structure, in Einstein’s theory, is rather defined by his two postulates.

A central contention of Kant’s original version of transcendental philosophy, as we know, is that the Newtonian laws of motion are not mere empirical laws but a priori constitutive principles on the basis of which alone the Newtonian concepts of space, time, and motion can then have empirical application and meaning. What we have just seen is that Einstein’s two fundamental “presuppositions” or “postulates” play a precisely parallel role in the context of special relativity. But we have also seen significantly more. For Poincaré’s conception of how a mere empirical law can be “elevated” to the status of a “convention or definition in disguise” is a continuation, in turn, of Kant’s original

conception of the constitutive a priori. Whereas Helmholtz's principle of free mobility generalized and extended Kant's original theory of geometrical construction within our "subjective" and "*necessary* form of external intuition," Poincaré's idea that specifically Euclidean geometry is then imposed on this form by a "convention or definition in disguise" represents an extension or continuation of Helmholtz's conception. In particular, specifically Euclidean geometry is applied to our experience by precisely such a process of "elevation," in which the merely empirical fact that this geometry governs, very roughly and approximately, our actual perceptual experience of bodily displacements gives rise to a precise mathematical framework within which alone our properly *physical* theories can subsequently be formulated.

This same process of "elevation," in Einstein's hands, then makes it clear how an extension or continuation of Kant's original conception can also accommodate new and surprising empirical facts—in this case, the very surprising empirical discovery (to one or another degree of approximation) that light has the same constant velocity in every inertial frame. It now turns out, in particular, that we can not only impose already familiar and accepted mathematical frameworks (Euclidean geometry) on our rough and approximate perceptual experience, but, in appropriate circumstances, we can also impose entirely unfamiliar ones (the kinematical framework of special relativity). Einstein's creation of special relativity, from this point of view, thus represents the very first instantiation of a relativized and dynamical conception of the a priori—which, in virtue of precisely its historical origins, has a legitimate claim to be considered as genuinely constitutive in the transcendental sense.

Yet Einstein's creation of the general theory of relativity in 1915 involved an even more striking engagement with Poincaré's "conventionalist" methodology, which, I contend, makes the transcendently constitutive role of this theory's fundamental postulates (the light principle and the principle of equivalence) even more evident.

The first point to make, in this connection, is that the principle of equivalence (together with the light principle) plays the same role in the context of the general theory that Einstein's two fundamental "presuppositions" or "postulates" played in the context of the special theory: namely, they define a new inertial-kinematical structure for describing space, time, and motion. Because Newtonian gravitation theory involves an instantaneous action at a distance (and therefore absolute simultaneity), it was necessary after special relativity to develop a new theory of gravitation where the interactions in question propagate with the velocity of light. And Einstein solved this problem, via the principle of equivalence, by defining a new inertial-kinematical structure wherein the freely falling trajectories in a gravitational field replace the inertial trajectories described

by free particles affected by no forces at all. The principle of equivalence, in this sense, replaces the classical law of inertia holding in both Newtonian mechanics and special relativity. But the principle of equivalence itself rests on a well-known empirical fact: that gravitational and inertial mass are equal, so that all bodies, regardless of their mass, fall with exactly the same acceleration in a gravitational field. In using the principle of equivalence to define a new inertial-kinematical structure, therefore, Einstein has “elevated” this merely empirical fact (recently verified to a quite high degree of approximation by Lorand von Eötvös) to the status of a “convention or definition in disguise”—just as he had earlier undertaken a parallel “elevation” in the case of the new concept of simultaneity introduced by the special theory.

Nevertheless, Einstein did not reach this understanding of the principle of equivalence all at once. He first operated, instead, within an essentially three-dimensional understanding of special relativity, and he proceeded (in the years 1907–1912) to develop relativistically acceptable models of the gravitational field by considering the inertial forces (like centrifugal and Coriolis forces) arising in non-inertial frames of reference within this framework. It was in precisely this context, in particular, that Einstein finally (in 1912) came upon the example of the uniformly rotating frame (the rotating disk), and it is at this point (and only at this point) that he then arrived at the conclusion that the gravitational field may be represented by a non-Euclidean geometry.

It was in precisely the context of this line of thought, finally, that Einstein found that he now had explicitly to oppose Poincaré’s “conventionalist” philosophy of geometry. Yet Einstein’s argument—as described in *Geometrie und Erfahrung* (1921)—was far from a simple rejection of Poincaré’s methodology in favor of a straightforward “empiricism.” For Einstein also famously says, in the same work, that “*sub specie aeterni*” Poincaré is actually correct—so that, in particular, Einstein’s reliance on a Helmholtzian conception of “practically rigid bodies” is here merely provisional. I have suggested, therefore, that we can best understand Einstein’s procedure as one of delicately situating himself *between* Helmholtz and Poincaré. Whereas Einstein had earlier followed Poincaré’s general “conventionalist” methodology in “elevating” the principle of relativity (together with the light principle) to the status of a “presupposition” or “postulate,” he here follows Helmholtz’s “empiricism” in rejecting Poincaré’s more specific philosophy of geometry in favor of “practically rigid bodies.” It does not follow, however, that Einstein is also rejecting his earlier embrace of Poincaré’s general “conventionalist” (or perhaps we should say “elevationist”) methodology. Indeed, Einstein had already side-stepped Poincaré’s specific philosophy of geometry in the case of special relativity, and for essentially the same reason he explicitly opposes it here:

Poincaré's rigid hierarchy of the sciences, in both cases, stands in the way of the radical new innovations Einstein himself proposes to introduce.

But why was it necessary, after all, for Einstein to engage in this delicate dance between Helmholtz and Poincaré? The crucial point is that Einstein thereby arrived at a radically new conception of the relationship between the foundations of (physical) geometry and the relativity of space and motion. These two problems, as we have seen, were closely connected in Kant, but they then split apart and were pursued independently in Helmholtz and Mach. In Poincaré, as we have also seen, the two were perceptively reconnected once again, in so far as Poincaré's hierarchical conception of the mathematical sciences incorporated both a modification of Helmholtz's philosophy of geometry and a serious engagement with the late nineteenth-century concept of inertial frame. Indeed, it is for precisely this reason, as we now see, that Poincaré's scientific epistemology was so important to Einstein. Einstein could not simply rest content with Helmholtz's "empiricist" conception of geometry, because the most important problem with which he was now faced was to connect the foundations of geometry with the relativity of motion. But Einstein could not rest content with Poincaré's conception either, because his new models of gravitation had suggested that geometry has genuine physical content.

Einstein's radically new way of reconfiguring the relationship between the foundations of geometry and the relativity of motion therefore represents a natural (but also entirely unexpected) extension or continuation of the same conception of dynamical and relativized constitutive a priori principles he had first instantiated in the creation of special relativity. Just as he had earlier shown how an extension or continuation of Kant's original conception could accommodate new and surprising empirical facts (the discovery of the invariance of the velocity of light), Einstein here shows how a further extension of this same tradition can do something very similar in facilitating, for the first time, the application of a non-Euclidean geometry to nature. In this case, however, it is not the relevant empirical fact (the well-known equality of gravitational and inertial mass) that is surprising, but the entirely unforeseen connection between this fact and the new geometry. And what makes this connection itself possible, for Einstein, is precisely the principle of equivalence—which thereby constitutively frames the resulting physical space-time geometry of general relativity in just the same sense that Einstein's two fundamental "presuppositions" or "postulates" had earlier constitutively framed his mathematical description of the electrodynamics of moving bodies in special relativity. Whereas the particular geometry in a given general relativistic space-time is now determined empirically (by the distribution of mass and energy in accordance with

Einstein's field equation), the principle of equivalence itself is not empirical in this sense. This principle is instead *presupposed*—as a transcendently constitutive condition—for any such geometrical description of space-time to have genuine empirical meaning in the first place.

The historicized version of transcendental philosophy I am attempting to exemplify therefore sheds striking new light, I believe, on the truly remarkable depth and fruitfulness of Kant's original version. Kant's particular way of establishing a connection between the foundations of geometry and the relativity of motion—which, as we have seen, lies at the heart of his transcendental method—has not only led, through the intervening philosophical and scientific work of Helmholtz, Mach, and Poincaré, to a new conception of the relativized a priori first instantiated in Einstein's theories, it has also led, through this same tradition, to a radically new reconfiguration of the connection between geometry and physics in the general theory of relativity itself. There can be no question, of course, of Kant having "anticipated" this theory in any way. The point, rather, is that Kant's own conception of the relationship between geometry and physics (which was limited, of necessity, to Euclidean geometry and Newtonian physics) then set in motion a remarkable series of successive reconceptualizations of this relationship (in light of profound discoveries in both pure mathematics and the empirical basis of mathematical physics) that finally eventuated in Einstein's theory.